The Inverse Relationship between Farm Size and Productivity: Refocusing the Debate[ǂ]

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Abstract

The relationship between farm size and productivity is a recurrent topic in development economics, almost as old as the discipline itself. This paper emphasizes the importance of choice of productivity measures in the inverse relationship literature. First, we seek to clarify the common measures, their relationships, and advantages and limitations in empirical work. Second, we argue that total factor productivity (TFP), not land productivity, is the appropriate indicator for most policy questions. Lastly, using a pseudo-panel of Brazilian farms that are aggregated at the farm size and municipality levels over the period 1985-2006, we provide new evidence on the inverse relationship between farm size and both land productivity and TFP. The inverse relationship between size and land productivity is alive and well. The relationship between TFP and size, in contrast, has evolved with modernization during this period, becoming increasingly U-shaped or even positive. Policy implications are discussed.

Keywords: Inverse relationship, agriculture, farm size, total factor productivity (TFP), Brazil.

JEL Codes: O13, Q12

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1. Introduction

The relationship between farm size and productivity is a recurrent topic in development economics, almost as old as the discipline itself. John Stuart Mill observed an inverse relationship as early as 1848, later positing that this had changed due to increasing capital intensity of farming (Lipton, 2009). The issue appeared in the works of Marx, resurfaced with Lenin and Chayanov in the early 20th century, and has captivated modern agricultural and development economists for over fifty years. Debate around the nature and causes of this relationship continues despite a mountain of empirical analysis, posing a puzzling question for 21st century researchers (Binswanger et al., 1995; Eastwood et al., 2010). Conventional economic wisdom expects resources to be allocated such that returns to land are equalized across farms; however, the empirical research on developing countries contradicts this and frequently identifies an inverse relationship. Policy-makers in developing countries have engaged the debate, as an inverse relationship between farm size and productivity indicates a role for small farms in development strategies and the potential for land reform to simultaneously generate improvements in equity and efficiency.

Harnessing such a relationship to inform policy requires accurate interpretation of the empirical evidence as well as an understanding of its causes, the channels through which it operates, and the factors that condition its strength. Theoretical explanations for this phenomenon often result from household heterogeneity and/or (multiple) market failures, for example Sen's (1966) dual labor market hypothesis, Eswaran and Kotwal's (1986) model of household endowments with credit constraints, and Feder's (1985) model of moral hazard and costly monitoring of hired labor. Risk aversion (Barrett, 1996) and plot-level behavioral and agronomic issues (Bevis and Barrett, 2020) provide alternative explanations. Measurement error (Lamb, 2003; Carletto et al., 2013; Carletto et al., 2015; Desiere and Jolliffe, 2018; Dillon et al., 2019; Gourlay et al., 2019; Abay et al., 2019a; Abay et al., 2019b) and omitted variables, such as soil quality (Bhalla and Roy, 1988; Benjamin 1995; Assunção and Braido, 2007; Barrett et al. 2010), are two empirical issues that could lead to a spuriously observed inverse relationship. Attempts to sort out the relative importance of these mechanisms have been mixed.

Adding to the confusion is the variety of productivity measures and empirical approaches that have been used. As with Sen (1962), Deolalikar (1981), Assunção and Braido (2007), Barrett et al. (2010), Deininger et al. (2018), Dillon et al. (2019), and Abay et al. (2019b), much of the early literature used land productivity—output per unit of land—as a measure of performance.[1] Conditioning land productivity on input use by estimating a production function is a second commonly used approach that generates an alternative measure of performance (Bardhan, 1973; Carter, 1984; Barrett et al., 2010; Ali and Deininger, 2015; Muyanga and Jayne, 2016). Controlling for a partial set of inputs (Bhalla and Roy, 1988; Desiere and Jolliffe, 2018) is distinct from estimating a full production function. Still others employ value added per unit of land (Heltberg, 1998; Carletto et al., 2013; Henderson, 2015), profit per unit of land (Heltberg, 1998; Foster and Rosenzweig, 2017), profit (Benjamin, 1995; Lamb, 2003; Ali and Deininger, 2015), or technical efficiency (Helfand and Levine, 2004; Kagin et al., 2016). Despite the recognition that partial measures such as land productivity are problematic (Berry and Cline, 1979; Binswanger et al., 1995; Muyanga and Jayne, 2016), they continue to be used, often alongside alternative

[1] The literature often uses the terms yield and land productivity interchangeably. We only use yield when talking about a physical measure of productivity for a single product (tons/hectare). Land productivity is more appropriate in a multiple-output context, requiring a method for aggregation.

productivity measures, and are frequently discussed synonymously with a more general notion of productivity. Where multiple productivity measures are used, the distinctions between the relationships being estimated are seldom addressed directly. As Barrett (1996) notes, this literature "habitually, perhaps cavalierly," uses physical yields and productivity synonymously. Conceptual clarity is needed on how these measures relate to each other and to farm size, and which is most relevant from a policy perspective.

We do not attempt to explain the IR, as do many of the contributions in this field. Rather, we seek to clarify the relationships between the various productivity measures used in this literature and explore the implications of the choice of measure. We show that an inverse relationship between farm size and a partial productivity measure, such as land productivity, is neither necessary nor sufficient for an inverse relationship between farm size and a comprehensive measure of productivity, such as total factor productivity. As such, these measures are not generally comparable. An inverse relationship may be observed when using land productivity, but not necessarily when using a comprehensive measure of productivity. Where comprehensive measures of productivity are more relevant and of interest, a focus on land productivity effectively introduces omitted variable bias by not controlling for the intensity with which other inputs are used. In fact, Bardhan (1973), Berry and Cline (1979), Carter (1984), and Heltberg (1998) are all examples where, in the presence of an inverse relationship between farm size and land productivity, the use of more comprehensive productivity measures leads to an attenuated, if not direct, farm size – productivity relationship. This highlights the importance of how productivity is measured when assessing its relationship with farm size, and for drawing policy conclusions and recommendations from these relationships.

The lack of an explicit focus on total factor productivity is a curious feature of the inverse relationship literature, especially given the early and widespread acknowledgement of its superiority over partial measures. From a policy perspective, total factor productivity is likely the most relevant measure where poverty alleviation, equity and the productive use of all resources are pressing concerns. Policy discussions of the future of small farms, for example, emphasize the role of small farms in agricultural development in part because of their superior efficiency (Hazell, 2005; Hazell et al. 2010). This argument leans heavily on the inverse farm size – productivity relationship, but requires that small farms be more efficient with their use of all resources and not just land. Whereas a farm size – land productivity relationship does not provide clarity on this issue, a farm size – total factor productivity relationship does.

In this light, we argue that the inverse relationship literature needs to shift its focus from land productivity to total factor productivity. In fact, empirical studies assessing the productivity – farm size relationship in the developed world, such as Garcia et al. (1982), Alvarez and Arias (2004), and Rasmussen (2010), almost exclusively use measures of technical efficiency or total factor productivity.[2] Similarly, the literature estimating national level agricultural productivity is clear in its use of total factor productivity as a preferred measure (Fuglie, 2008; and Headey et al., 2010).

We illustrate the importance of productivity measures with new empirical evidence on the farm size – productivity relationship across regions of Brazil from 1985 to 2006 . Our evidence is only suggestive because we are unable to correct for potential issues of measurement error in

[2] We have chosen not to focus on the literature estimating a stochastic production frontier to explore technical efficiency, as it is still an infrequent, albeit important, approach taken in the existing literature on developing countries.

farm size, output, and inputs that have been identified in recent literature. However, this period in Brazil provides an excellent case study because it includes regions with relatively advanced agricultural sectors, those characterized by more traditional agricultural production, and others experiencing rapid agricultural transformation, allowing us to assess the farm size – productivity relationship and its dynamics at different stages of agricultural development. Using a pseudo-panel of farms aggregated at the municipality by farm size level, we show that estimating the farm size – productivity relationship using land productivity is potentially misleading. While we always identify an inverse relationship using land productivity, we find disparate results when using total factor productivity. In the modern agricultural regions of Brazil, we find a direct relationship between farm size and total factor productivity, and in the rapidly transforming region of the Center-West we identify dynamics that suggest the inverse relationship is disappearing over time. The analysis highlights that the relationship between total factor productivity and farm size has evolved with modernization, shedding some light on the issues raised by Mill over 150 years ago.

The remainder of this paper is organized as follows. In Section 2 we seek to clarify the common measures, their relationships, and their advantages and limitations in empirical work. Section 3 presents the empirical exercise, generating new evidence on the relationship between size and productivity in several macro regions of Brazil. In Section 4 we summarize and conclude with policy implications.

2. Measures of Agricultural Productivity

Farm size may be related to a broad range of economic outcomes, such as employment, poverty, inequality, food security, efficiency and growth. While these are important issues connected to the role of farm size in development, here, as with most of the literature on the inverse relationship (IR), we focus specifically on the concept of productivity. The following discussion seeks to clarify the relationships between the various productivity measures most commonly used in the literature, allowing us to draw conclusions on the impact that choice of measure may have on finding an IR and the potential implications for policy.

2.1 The Unconditional Relationship between Land Productivity and Farm Size

Historically, land productivity is the most commonly used measure in the literature on the inverse relationship. Where alternative productivity measures are used, the relationship between land productivity and farm size is often a starting point, serving as a benchmark for the expansive existing literature. Land productivity, q, is a partial measure of productivity:

$$Land\ Productivity = \frac{Q}{A} = q = \psi_u(A) \tag{1}$$

where A is the area of the farm, Q is an index of agricultural output, q is agricultural output per unit of land, and $\psi_u(A)$ connotes that land productivity may be a function of farm size. In a world where farm size and land productivity are unrelated we have $\frac{\partial \psi_u(A)}{\partial A} = 0$. However, the regularity with which empirical work finds $\frac{\partial \psi_u(A)}{\partial A} < 0$ has led to the stylized fact that they are inversely related, generating an abundance of interest in the relationship and its potential explanations. Figure 1 displays this relationship using data from Brazil for the years 1985, 1996, and 2006. While the relationship is potentially non-linear and may not be monotonic, for now we focus on the first order approximation.

The relationship captured by $\psi_u(A)$ is unconditional (u) in the sense that it is the simple bivariate relationship between land productivity and farm size. Factors that may be causing or influencing this relationship have not been controlled for. Using land productivity as a measure is inherently limited—as would be any partial measure of productivity—whenever there is more than one factor of production. If use of other factors vary systematically with farm size, the IR between land productivity and farm size may simply reflect more input intensive practices of small farms. Higher land productivity may reflect overuse of fertilizer, for example, which would not necessarily reflect any underlying productivity advantage of small farms. In such situations, estimates of the farm size – land productivity relationship introduces omitted variable bias into estimates of the underlying farm size – productivity relationship. From this perspective, a focus on the relationship between land productivity and the size of farms may be misplaced.

Similarly, analysis using different partial productivity measures may result in conflicting policy recommendations. Indeed, Sen's (1962) seminal contribution revealed precisely this type of systematic relationship between the intensity of labor use and farm size, leading to his formal exposition of the dual labor market hypothesis (Sen 1966). Figure 2 illustrates the problem in the case of Brazil. While there is an inverse relationship between land productivity and farm size, there is a direct relationship between labor productivity and size. Analysis of the farm size and productivity relationship using labor productivity suggests that larger farms are more productive than are their smaller counterparts. Policy recommendations from the two partial measures of productivity would differ, underscoring the need for a comprehensive measure of productivity when identifying any relationship with farm size.

2.2 The Conditional Relationship between Land Productivity and Farm Size

In spite of its limitations, the unconditional relationship between land productivity and farm size continues to be used, even if in conjunction with comprehensive measures of productivity. A more appropriate approach is to use a conditional relationship, where the relationship is conditioned on a vector of controls, $X(A)$, that are potentially correlated with both land productivity and farm size:

$$q = \psi_u(A) = g\big(X(A), \psi_c(A)\big) \tag{2}$$

The conditional (c) relationship, $\psi_c(A)$, should differ from the unconditional relationship to the extent that the controls explain the unconditional IR. For example, the impact of varying input intensities can be controlled for by including those inputs as controls, household heterogeneity can be controlled for with household fixed effects, market failures controlled for with regional fixed effects, and other omitted variables such as soil quality can be introduced. This is a useful approach for exploring the theoretical channels that explain the IR and is a strategy commonly used by researchers in recent empirical studies of the farm size – productivity relationship (Assunção and Braido, 2007; Barrett et al., 2010; Desiere and Jolliffe, 2018; Gourlay et al., 2019).

As discussed above, partial measures such as land productivity are potentially misleading when there are other factors of production. At the very least, understanding any relationship between productivity and farm size requires empirical analysis that controls for the intensity with which other factors of production are used. For exposition, assume that land, labor (L), and capital (K) are the only factors of production and that their intensities, labor per unit of land and capital per unit of land, are given by l and k, respectively. Then (2) becomes:

$$q = \psi_u(A) = g\big(k(A), l(A), \psi_c(A)\big) \tag{3}$$

showing that the IR as identified by the unconditional relationship between land productivity and farm size, $\frac{\partial \psi_u(A)}{\partial A}$, is composed of the relationship between capital intensity and farm size, labor intensity and farm size, and any conditional relationship between farm size and land productivity,

$\frac{\partial \psi_c(A)}{\partial A}$. When differences in the use of other factors of production are controlled for, the conditional relationship between farm size and productivity is revealed, providing a more comprehensive measure of productivity. If the two measures diverge, the unconditional relationship suffers from omitted variable bias.

Exploring (3) highlights how omitted variables can lead to ambiguity in how the land productivity and farm size relationship, as captured by $\psi_u(A)$, is related to the more general productivity and farm size relationship captured by $\psi_c(A)$. Differentiating (3) with respect to farm size shows:

$$\left(\frac{\partial \psi_u}{\partial A}\right) = \left(\frac{\partial g}{\partial k}\right)\left(\frac{\partial k}{\partial A}\right) + \left(\frac{\partial g}{\partial l}\right)\left(\frac{\partial l}{\partial A}\right) + \left(\frac{\partial g}{\partial \psi_c}\right)\left(\frac{\partial \psi_c}{\partial A}\right) \tag{4}$$

Assuming, quite reasonably, that output per unit of land is increasing in both capital and labor per unit of land, it is plausible for the conditional relationship to be positive $\left(\frac{\partial \psi_c}{\partial A} > 0\right)$ even if the unconditional relationship is negative $\left(\frac{\partial \psi_u}{\partial A} < 0\right)$ if, as is often the case, $\frac{\partial k}{\partial A}$, $\frac{\partial l}{\partial A}$, or both are negative.[3] In short, an unconditional IR is neither a necessary nor a sufficient condition for an inverse relationship between the broader measure of productivity and farm size captured by $\psi_c(A)$.

When, as in (3), the conditional relationship includes all factors of production as controls, the approach is equivalent to estimating a production function and the conditional relationship can be interpreted as total factor productivity (TFP). Across a host of policy objectives – for example, improving the efficiency of resource use in the rural economy or alleviating poverty

[3] Abay et al. (2019a) show clear evidence of input intensities declining with farm size in four African countries. The same is true in all regions using our Brazil data.

among agricultural households – policymakers are best informed by comprehensive measures such as TFP that take into account the productivity with which all resources are utilized.

2.3 Profitability and Farm Size

Empirical studies have often looked to the farm size – profitability relationship as an alternative to measuring the farm size – productivity relationship. While assessing profitability raises its own practical challenges, the use of a profit rate to measure farm performance faces the same conceptual issues as does the use of productivity. Partial measures of profitability – such as profit (or value added) per unit of land – are potentially misleading, and for most policy considerations a comprehensive profit rate is most relevant.

Profit is expressed as:

$$\Pi = Q - p_L L - p_K K - p_A A \tag{5}$$

where p_L is the price of labor, p_K the price of capital, and p_A the price of land. In expression (5) if the output quantity index, Q, is constructed using prices in the aggregation process, it can be interpreted as the value of output. The level of profit can be expressed as the product of output and profit per unit of output: $Q\frac{\Pi}{Q}$. Regardless of whether the profit per unit of output rises or falls with size, we would expect the level of output to dominate in the determination of the level of profit. A large farm that produces a value of output of 1,000, for example, should generate more profit than a small farm that produces 10. The level of profit, then, is not a particularly good measure for comparing the productivity of farms of different sizes.

It is not profit per se that matters but rather profitability, requiring the transformation of the profit level into a profit rate. Profit per unit of land, as used by Carletto et al. (2013) among others, is one approach:

$$Profit\ per\ unit\ of\ land = \pi_A = \frac{\Pi}{A} = q - p_L l - p_K k - p_A = \phi(A) \quad (6)$$

Profit per unit of land is a measure of farm performance that controls for the levels of other inputs additively, providing an improvement over land productivity. Notions of value added are similar, however they fall short of profit measures as they control only for intermediate inputs and not the complete set of factors of production. Despite being an improvement over land productivity and value-added, profit per unit of land is itself problematic because it is fundamentally a partial measure. The finding of a systematic inverse relationship with farm size, $\frac{\partial \phi(A)}{\partial A} < 0$, provides limited information to policy makers because, as with productivity measures, it is the profitability of overall resource use that matters.

To highlight this, note that partial profitability measures potentially provide conflicting perspectives on the relative profitability of farms:

$$\pi_A = \frac{\Pi}{A} = \frac{\Pi}{K}\frac{K}{A} = \pi_K * k \quad (7)$$

Here we see that the profit per unit of land is the product of profit per unit of capital, π_K, and capital intensity. An observed inverse relationship between profit per unit of land and farm size could be associated with declining capital intensity as farm size increases, even if profit per unit of capital is increasing. If true, then the use of one partial measure or the other would lead to conflicting policy recommendations. Overcoming these limitations requires the use of a comprehensive measure of profitability. Indeed, Binswanger et al. (1995) advocate normalizing profit by “capital invested” or “assets,” an approach that is appropriate as long as the assets included are restricted to those used in agricultural production and do not include all of household wealth. In practice, this approach to measuring profits has rarely been used, in part due to inexistent or imprecise information about the value of assets used in production. Where partial profit rates have been employed, bias can arise from incomplete information on input prices as well as unobservable inter-farm variation in prices that is potentially correlated with farm size.

2.4 TFP as a Comprehensive Measure

Comprehensive measures of either productivity or profitability are the appropriate means to measure the efficiency of resource use and, in most cases, will provide the information necessary for effective policy design.[4] Total factor productivity can be defined as the ratio of output to all inputs used, where output and input quantity indices are typically required to aggregate physical quantities. TFP can be written as:

$$TFP = \frac{Q}{Inputs} = \varphi(A) \tag{8}$$

TFP effectively captures the productivity with which all inputs are used in the production process, and in this sense is a comprehensive measure of productivity. If this measure is a function of farm size, i.e. $\frac{\partial\varphi(A)}{\partial A} \neq 0$, then there is an unambiguous difference in how productively farms of different sizes utilize resources in agricultural production. An understanding of the determinants of $\varphi(A)$ would support effective policy design, whether the objective is poverty reduction or economic growth, because these are concerned with the use of all resources available to farms.

[4] Under certain conditions, TFP can be shown to be a monotonic transformation of profitability.

Although this is widely acknowledged, an explicit focus on TFP is seldom the approach of empirical analyses of the IR in developing economies.

While the difference is rarely noted, empirical analyses of the IR that estimate a production function effectively pivot from estimating the farm size – land productivity relationship towards estimating the farm size – TFP relationship. To illustrate this point, assume a standard Cobb-Douglas production function homogenous of degree t, where T is the unobserved measure of total factor productivity and production is a function of labor, capital, and land:

$$f(L,K,A) = TL^{\alpha}K^{\beta}A^{\gamma} \tag{9}$$

If, as has often been confirmed, CRS holds, then farm size disappears from the right hand side of (9) after dividing through by farm size.[5] If not, then the natural log of the production function takes the form:

$$lnq = (t-1)lnA + lnT + \alpha lnl + \beta lnk \tag{10}$$

and if, as in (8), there exists a relationship between total factor productivity and size, $\varphi(A)$, we have:

$$lnq = (t-1)lnA + ln\varphi(A) \ + \alpha lnl + \beta lnk \tag{11}$$

From (11) it is clear that the conditional relationship identified in (3), $\psi_c(A)$, is composed of the relationship between TFP and farm size ($\varphi(A)$) as well as any deviations from CRS in the production function (as captured by $(t-1)lnA$).

[5] To see this, if we assume the production function is homogenous of degree t then $f(\lambda L, \lambda K, \lambda A) = \lambda^t f(L,K,A)$, with constant returns to scale (CRS) holds if $t = 1$. Setting $\lambda = \frac{1}{A}$ implies that $f(l,k,1) = A^{-t}\, f(L,K,A)$, implying that when the production function is expressed in intensities we have $q = \frac{f(L,K,A)}{A} = A^{t-1}f(l,k,1)$.

Equation (11) highlights two useful features of the production function approach. First, if CRS holds then the conditional relationship, $\psi_c(A)$, captures the relationship between TFP and farm size, $\varphi(A)$. Second, if CRS does not hold then it will be difficult to empirically differentiate whether a conditional relationship is driven by non-CRS, a relationship between TFP and farm size, or a combination of the two. One cannot have confidence in tests of returns to scale if there is also a relationship between farm size and TFP. This highlights the importance of interpreting any observed conditional relationship as including both a relationship between farm size and TFP and any potential deviations from CRS.[6]

Thus, when empirical researchers estimate a production function to explore the relationship between farm size and productivity they are, in effect, estimating the relationship between farm size and TFP and not farm size and land productivity. All too often, empirical work that takes this approach estimates the unconditional relationship first (non-parametrically, and parametrically with some controls), followed by the estimation of a production function and then interpretation of the two approaches as if they were exploring the same relationship. However, the conditional and unconditional relationships are by no means the same, can plausibly take different signs, and will almost certainly have different magnitudes.

2.5 TFP and Land Productivity Redux

The relationship between TFP and output per unit of land can be explored further. TFP is a unit-less measure, but multiplying and dividing by $\frac{1}{A}$ allows the measure to be rewritten as:

[6] Future research should seek to develop an approach to disentangle the relationship between farm size and these two sources of productivity. Aragon et al. (2019) have taken a step in this direction, proposing a sequential approach which estimates RTS first and then uses this to correct their estimates of the farm size - TFP relationship. While this recognizes the need to account for deviations from CRS, the first stage estimates of returns to scale likely suffer from omitted variables bias.

$$TFP = \frac{q}{\tau} = \varphi(A) \qquad (12)$$

where TFP is expressed as land productivity normalized by inputs per unit of land, τ.

Taking a derivative with respect to farm size:

$$\frac{\partial TFP}{\partial A} = \left(\frac{\partial q}{\partial A}\right)\left(\frac{1}{\tau}\right) - \frac{\left(\frac{\partial \tau}{\partial A}\right)q}{\tau^2} \qquad (13)$$

Employing a little bit of algebra (see Appendix A):

$$\frac{\partial TFP}{\partial A} = \left(\frac{\partial q}{\partial A}\right)\left(\frac{1}{\tau}\right)\left[\frac{\varepsilon_{q,A} - \varepsilon_{\tau,A}}{\varepsilon_{q,A}}\right] \qquad (14)$$

where $\varepsilon_{q,A}$ is the elasticity of land productivity with respect to farm size, and $\varepsilon_{\tau,A}$ is the elasticity of input use per unit of area with respect to farm size. If there is an empirically observed inverse relationship between the partial measure and farm size such that $\frac{\partial q}{\partial A} < 0$, then we know $\varepsilon_{q,A}$ is negative. This implies that one of two possibilities must hold:

(i) $\frac{\partial TFP}{\partial A} < 0$ *and* $\varepsilon_{q,A} < \varepsilon_{\tau,A}$

(ii) $\frac{\partial TFP}{\partial A} > 0$ *and* $\varepsilon_{q,A} > \varepsilon_{\tau,A}$

If (i) is true then an IR between a partial measure and farm size reflects an IR between productivity and farm size as measured by TFP. When this is the case either input use per unit of land is increasing in farm size or it is decreasing, but slower than the rate at which output per unit of land is decreasing. If (ii) is true then use of a partial measure is generating an incorrect indication about the productivity and farm size relationship, and TFP is actually directly related to farm size. However, this requires that $0 > \varepsilon_{q,A} > \varepsilon_{\tau,A}$. In such a case, input use per unit of land is negatively related to farm size and is relatively elastic compared to output per unit of land. Use of a partial measure implies policy recommendations inconsistent with those that would result if a comprehensive measure were used. This discussion highlights the conclusion that an IR between a partial measure of productivity and farm size is neither necessary nor sufficient for the existence of an IR between farm size and a comprehensive measure of productivity such as TFP.

The conditions set out in (i) and (ii) provide a framework for considering how a modernizing agricultural sector can lead to a changing farm size – productivity relationship. Depending upon the stage of development and the institutional structure, partial measures of productivity may fail to capture the dynamics of the farm size – productivity relationship. Land productivity may provide an adequate proxy for TFP at an early stage of development, even if the magnitudes of the two relationships differ. At an intermediate stage of development characterized by mechanization and technical improvements, capital and the ability to adopt modern technologies become increasingly important. Substitution away from labor may move large farms towards a more efficient mix of factors of production. In such a context condition (ii) might hold, with a direct relationship between TFP and farm size emerging even as an IR continues to exist for land productivity. Further agricultural development could realign the relationships between TFP, land productivity, and farm size as institutions improve and distortions in land, labor, and capital markets begin to disappear. In such an environment, the inverse relationship between land productivity and farm size could disappear, implying $\varepsilon_{q,A} \geq 0$ and both land productivity and TFP could conceivably exhibit a direct relationship with farm size.[7] We return to this discussion following the empirical exercise on Brazil.

[7] There is evidence of a direct relationship between land productivity and farm size among grain farmers in the U.S. (Key, 2019), and for specific crops in Brazil (Filho and Vian, 2016).

3. Empirical Analysis

We now provide an example using data on Brazilian agriculture. The intention here is not to explain the relationship between farm size and productivity by controlling for its potential determinants. Rather, we seek to use a regional analysis within Brazil to highlight how the choice of measure influences the observed relationship and how these patterns can change across stages of agricultural development. Our evidence is only suggestive because we are unable to correct for the measurement issues in farm size, outputs, and inputs that recent literature has focused on. We discuss this further below. The results provide an important counterpoint to much of the literature that has focused on countries in Africa and Asia where the overwhelming majority of farms have less than 2 hectares (Eastwood et al., 2010). Mean and median farm size in Brazil, in contrast, were around 65 and 10 hectares in 2006.

3.1 Data and Variables

The data come from the 1985, 1995/1996, and 2006 rounds of the Brazilian agricultural census. For confidentiality reasons, we constructed a pseudo-panel in which all farms in the census are aggregated into five farm size classes within each municipality of Brazil.[8] Aggregation requires that we assume homogeneity within each observation (for example, farms with 0–5 ha in the municipality of Cachoeira). We call these "representative-farms," as they reflect the average behavior of a given farm size in a given municipality. The pseudo-panel approach has

[8] The size classes in hectares (ha) are 0-5 ha, 5-20 ha, 20-100 ha, 100-500 ha, and 500+ ha. To protect the confidentiality of the farms, the Brazilian Institute of Geography and Statistics (IBGE) requires that each aggregate observation have at least 3 farms. As the aggregation was conducted on site prior to analysis, we are not able to expand the number of farm size bins. However, previous work using the underlying Brazilian census data found little difference in qualitative results across alternative bin specifications (Helfand et al., 2014; Moreira et al., 2007; Helfand and Levine, 2004).

been used recently to study agricultural productivity growth by Key (2019) and Rada et al. (2019). Antmann and McKenzie (2007) demonstrate that, in the context of mobility studies, pseudo-panels can be used to consistently estimate parameters of interest. The averaging within cells (representative farms) in each period reduces the influence of individual-level measurement error, and the fact that it is not a true panel of farms makes it less vulnerable to non-random attrition. They show the approach is also robust to some forms of non-classical measurement error.

We begin with 47,365 representative farms for all of Brazil across the three survey years. Due to concern about the comparability of a small number (84) of extremely large observations, we remove all representative farms in the Northeast and South over 4,000 ha and all of those over 5,000 ha in the North, Southeast, and Center-West. We then identify land productivity outliers taking into account the IR shown in Figure 1 and potential non-linearities. Thus, rather than trim the tails of the unconditional land productivity distribution, we use a quadratic specification to regress land productivity on farm size with municipal fixed effects and survey year dummy variables.[9] From this regression we identify and remove outliers, defined as all representative farms with residuals greater than four standard deviations from their size specific predicted values. Together, the data cleaning exercises remove 1.8% of the initial sample.[10]

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[9] The changing composition of Brazil's municipalities—rising from around 4100 to over 5500 in the period of study—requires the construction of geographic units that are spatially consistent over time. We create 3,861 consistent geographic units—called minimum comparable areas—and continue to refer to them as "municipalities" for simplicity.

[10] See Appendix Table A1 for the results of data cleaning from each stage of the process. Sensitivity analyses using alternative approaches to trimming the data had no qualitative impact on our results, and only a negligible impact on the magnitude of the estimated coefficients. The alternatives included 1) the same as in the core approach but using a linear rather than a quadratic specification, 2) only trimming the 84 extremely large representative farms, 3) trimming the 84 and the top and bottom 1% of the unconditional land productivity distribution, 4) trimming the 84, the top and bottom 1%, and using the quadratic specification as in the core approach, and 5) the same as the core approach but trimming residuals greater than three rather than four standard deviations.

The Census data were gathered by the Brazilian Institute of Geography and Statistics (IBGE) through end of season in-person farmer interviews based on recall. Output is measured as the real value of total agricultural production, deflated to 2006 with a price index developed from the data in Gasques et al. (2010). Farm size is measured in hectares (ha), and unlike in many African and Asian countries the overwhelming majority of farms operate a single plot. Additional factors of production used in the production function are family labor, purchased inputs including hired labor, and an index of capital. The number of male, female, and child family members working on each farm are used to develop a family labor index measured in adult male equivalents. The index assigns weights of 1.0 to men, 0.75 to women and 0.5 to children under 14.[11] In 2006 around two thirds of family labor was provided by men, and over 90% of working family members were 14 years or older. The real value (R$2006) of purchased inputs, including expenditure on fertilizer, seeds, hired labor, fuel, energy, soil amendments, and other items, are deflated with the same price index used for output. A proxy for the total capital stock is calculated as a quantity index comprised of machine, animal, and tree capital stock sub-indices following Moreira et al. (2007) and Butzer et al. (2012). The machine capital stock index values tractors of five horsepower classes, trucks, harvesters and other agricultural equipment using a constant set of sale prices drawn from the Instituto de Economia Agrícola in São Paulo. The stock of animal capital is measured in cattle equivalents of the nine most important animal stocks and aggregated with a set of time-invariant relative prices (following the approach in Hayami and Ruttan, 1985). The stock of tree capital is measured as the present discounted value of expected future profits

[11] The weights are drawn from Moreira et al. (2007), and reflect average hours worked on-farm according to data in the national households survey (PNAD).

for thirteen different tree crops, using region-specific estimates of expected profits. The sub-indices are aggregated using region-specific weights estimated by regressing output on the three capital stock sub-indices in the base year 1985.[12]

Additionally, we control for unexpected shocks in rainfall and temperature to each municipality in each survey year utilizing data described in Wilmott and Matsuura (2001). These quarterly shocks are measured as standardized deviations from 25-year moving averages ending in the year prior to each Census. The data are transformed into categorical variables capturing extremely low, below average, average, above average, and extremely high values relative to the historical municipal average. Weather shocks between -1 and 1 standard deviations are treated as normal weather years and are the reference category, with extremely high and extremely low values occurring at more than ± 1.645 standard deviations.[13]

3.2 Measurement error

The data used are drawn from a nation-wide decennial census and are potentially subject to multiple sources of measurement error. The literature on measurement error and its implications for the IR has grown rapidly in recent years. Of greatest concern are non-classical types of measurement error that are correlated with farm size. Carletto et al. (2013), Carletto et al. (2015), Abay et al. (2019b) and Dillon et al. (2019) examine measurement error in self-reported farm size relative to more accurate approaches to measuring land (GPS or compass-

[12] While there are many assumptions that go into the construction of the capital stock index, capturing the capital invested in perennial crops and animals is an improvement over most of the literature on Brazil that often uses tractors as the sole proxy for capital. The Census data used to construct the indices here relies on the number of machines, trees, and animals present on farm at the end of the season. Because these are stocks and the recall period is shorter, bias should be less of a concern for these variables than for inputs that are used irregularly or are more marginal to the production process.

[13] Further discussion of the entire dataset can be found in Rada et al. (2019).

and-rope). They demonstrate clearly that farmers report area with error, that this error varies systematically with farm size, and that whereas small farms tend to overestimate farm size, large farms tend to underestimate their size. The implications for the IR literature are mixed, as Carletto et al. (2013) and Abay et al. (2019b) find that the IR becomes stronger when measurement error in farm size is the sole correction made, but Carletto et al. (2015) and Dillon et al. (2019) both find that correcting for such measurement error partially mitigates the IR in some of their data but has no statistically significant impact elsewhere.

Similarly, several recent papers have explored the implications of non-classical measurement error in output. Desiere and Jolliffe (2018), Gourlay et al. (2019), and Lobell et al. (2020) show non-classical measurement error in self-reported output when compared to “crop cuts” as the gold standard measure. Importantly, small farms over-report output more so than larger farms in their data. Conditional on GPS land measurement, the IR disappears in these papers when they utilize the more objective measure of output. Abay et al. (2019b) explore measurement error in both farm size and output, and concur that in their data the IR disappears when land is measured objectively and then crop cuts are used to correct for measurement error in production. However, they caution that the IR strengthens when land is self-reported and measurement error in output alone is corrected.

Lastly, measurement error in the use of inputs such as labor is potentially an issue. Relative to weekly surveys conducted in-person or by phone, end of season surveys of labor usage can contain substantial errors (Arthi et al., 2018; Gaddis et al., 2019). The implications for the IR, however, are ambiguous because the degree and direction of recall bias depends on a number of factors that can offset each other. Overestimation is likely to be greatest when surveys ask about hours worked per person per plot, but can be substantially smaller or even underestimated when focusing on total household hours per farm. At this level, the authors conclude that labor productivity might be underestimated in Tanzania (Arthi et al., 2018) and overestimated in Ghana (Gaddis et al., 2019).

How can this brief review of the recent literature on measurement error guide our empirical analysis of Brazil? First, we recognize that these are serious concerns. Because we do not have more objective measures that could be used to correct the data, our results should only be considered suggestive. Second, the pseudo-panel approach based on cohort averages should reduce the influence of classical measurement error, and may even help diminish some of the non-classical measurement error. In the case of land measurement, for example, the evidence from Africa suggests that the largest errors happen for the smallest of farms (under 0.5 acres), with the sign of the bias flipping from positive to negative somewhere between 0.75 ha (Abay et al., 2019b) and 2.0 ha (Carletto et al., 2015). Since our smallest farm size class is 0-5 ha, over- and under-estimation in this group may partially cancel. Third, while the literature suggests that the largest errors occur on the smallest of farms, it has little to say about measurement error for the larger farms included in our study. In the case of measurement error in output, for example, ninety five percent of parcels in the Ethiopian sample used by Desiere and Jolliffe (2018) were smaller than 1 ha, and mean plot size in the Ugandan sample used by Gourlay et al. (2019) is under 0.18 ha. What happens to measurement error in area and output as we move from farms of 10 to 100 to 1000 ha is an open question, and the IR in land productivity continues out this far in our data. Fourth, any sources of measurement error that are correlated with size, but constant over time, would not explain how the farm size – productivity gradient changes over time. This is an important aspect of our empirical analysis. Finally, Abay et al. (2019b) provide a unifying framework for thinking about data with multiple sources of non-classical measurement error. In this case, they show that the "signs and magnitude of resulting biases in estimates of a key parameter are analytically ambiguous." Thus, any attempt to correct for some, but not all, sources of measurement error could "prove inferior to a `second best' approach that uses multiple variables measured with error" (p. 183). In light of this discussion, we remain agnostic on measurement error and make no attempt to correct for it, reiterating that our results are only suggestive.

3.3 Empirical Methodology

We estimate an average production function assuming a Cobb-Douglas technology. Output and inputs for a representative farm in municipality m of size s in year t are normalized by area A_{mst}. Estimating the model using intensities imposes constant returns to scale on the technology coefficients and forces any deviation from CRS into the estimated relationship between farm size and productivity. Because of the difficulties discussed in Section 2 of distinguishing deviations from CRS from other causes of an IR, and because our focus here is not on explaining the IR, this approach simplifies the interpretation of the results. Survey year specific dummy variables for five farm size classes, δ_{st}, are used to flexibly capture the relationship between farm size and TFP. The farm size class 0-5 ha in 1985 is excluded and used as a reference. While this structure allows the farm size and productivity relationship to change over time, the technology coefficients are assumed to be time invariant. This assumption forces technical change into our measure of TFP. The estimated equation takes the form:

$$ln y_{mst} = \beta_0 + \boldsymbol{\beta} \boldsymbol{x}_{mst} + \boldsymbol{\alpha} \boldsymbol{w}_{mt} + \delta_{st} + \lambda_m + \varepsilon_{mst} \tag{15}$$

where y_{mst} is aggregate output per unit of land, $\boldsymbol{x}_{mst}$ is a vector of logged factors of production per unit of land (capital, family labor, and purchased inputs including hired labor), $\boldsymbol{w}_{mt}$ is a vector of municipality-specific rainfall and temperature shocks in each period, and λ_m are municipal fixed effects. The relationship between farm size and TFP in each year is identified from within-municipality variation. The parameters are estimated using ordinary least squares, with standard errors clustered at the municipal level. Because the number of farms represented by each representative farm varies, each observation is weighted by the number of farms that it represents.

With the above approach the systematic portion of TFP is a function of $\beta_0, \delta_{st}, \text{and } \lambda_m$, but it is only the component that varies by farm size and over time for each region that is of interest here. This size-specific component in each period can be calculated as:

$$TFP_{st} = e^{\delta_{st}} \tag{16}$$

A TFP index is then calculated for each size class in each period using the size class 0-5 ha in 1985 as a base level set to 100.

While the use of municipality fixed effects controls for time-invariant differences across municipalities, such as soil quality, omitted variables that vary across farm size within municipalities remains a concern. Similarly, endogeneity of inputs could lead to bias in our estimated coefficients. This is a limitation of the production function approach in the IR literature, and remains a concern here.[14]

[14] Estimation of profit or cost functions is a potential solution, but as discussed in Section 2 the necessary input price data frequently does not exist in developing countries to make this strategy feasible.

3.4 Empirical Results

By focusing on a regional analysis we are able to examine the relationship between farm size and productivity in light of each region's characteristics and stage of development. The five macro-regions of Brazil differ in both the type of predominant agricultural activities and the degree of modernization. They include the Amazon rainforest in the North, a large semi-arid region in the Northeast, a highly mechanized and commercial agriculture in the Southeast, a predominance of family farms in the South, and the Cerrado (savannahs) of the Center-West where grains have rapidly expanded and agriculture has modernized in recent decades. We restrict attention to the North, Center-West, and Southeast, three macro-regions that capture sufficient regional variation in Brazilian agriculture to illustrate our argument.[15] Descriptive statistics for output and input intensities for these regions in 2006 are shown in Table A2 of the appendix. Differences in input intensities reflect the heterogeneity in agricultural production across regions. The more traditional agricultural region in the North relies more heavily on family labor, whereas the mechanized Southeast and Center-West use capital and purchased inputs more intensively. We also observe that the intensities of capital and labor decline with farm size, whereas the intensity of purchased inputs declines through the first three or four size classes, and then inverts. This was not the case in 1985. The use of purchased inputs on farms in the 500-ha class has grown more rapidly than in all the other size classes during this period.

Figure 3 shows the unconditional relationship between land productivity and farm size class for the three regions under study. Despite considerable regional heterogeneity in their

[15] The results for the Northeast are similar to those in the North, and the results in the South are most similar to those in the Southeast.

agricultural activities and agrarian structures, each region mirrors the country as a whole in displaying a strong inverse relationship between land productivity and farm size. There is no evidence of it disappearing during this period.

The estimated coefficients from region-specific estimates of equation (15) are shown in Table A3 of the appendix, which generate the TFP estimates presented in Figures 4 through 6. Recall from Section 2 that these relationships potentially include the influence of deviations from constant returns to scale.[16] In the North (Figure 4), we estimate an inverse relationship between farm size and TFP. It is not, however, a linear relationship, but rather an emerging U-shaped inverse relationship with farms over 500 ha becoming more productive than medium-sized farms. The significance tests in Table 1 confirm this, showing that while the productivity of farms between 20 ha and 500 ha is statistically less than the smallest farms in all periods, the largest farms are not statistically different from the smallest farms after the first period. Thus, while a strong negative relationship would be found in this region when using land productivity, a U-shaped relationship begins to emerge when TFP is used and linearity is not imposed.

The Center-West (Figure 5) demonstrates a more dynamic pattern. Table 1 shows that the farm size – TFP relationship in the Center-West in 1985 looked very similar to the inverse relationship in the North. However, by 2006 the inverse relationship had disappeared in the Center-West, with the TFP of all farm sizes being statistically indistinguishable from that of the smallest farms. The point estimates show that the largest farms in the Center-West were 46%

[16] Note that the variables are measured per unit of land, and thus the sum of the coefficients in Table A3 does not indicate the returns to scale (RTS). We do not investigate RTS because we are unable to identify deviations from CRS separately from other causes of a size – productivity relationship. And because of potential non-linearities in this relationship, RTS do not have to be constant over all farm sizes, as they are with a Cobb-Douglas. Thus, where CRS does not hold this would be captured in our size dummies δ_{st}.

less productive than the smallest farms in 1985, yet by 2006 they were 8% more productive, albeit statistically insignificantly so. Once again, a U-shape begins to emerge, driven by rapid growth of the productivity of larger farms. Increased use of purchased inputs played an important role in this transformation, as they grew roughly three to four times as fast on farms over 500 ha than on farms in the middle three size classes. This is the clearest case of a strong inverse relationship becoming reversed over the 21 year period. Using land productivity to measure the farm size – productivity relationship in a rapidly modernizing agricultural region such as the Center-West would completely miss this transformation.

The Southeast, in contrast, shows a positive non-linear relationship between farm size and TFP. The relationship was statistically flat in 1985, although the point estimates show that even in 1985 the largest farms were 25% more productive than the smallest. Rapidly rising TFP at the upper end of the farm size distribution makes the relationship more positive over time, and by 2006 the largest farms were 48% more productive than the smallest, and statistically so. Once again, the relationship appears non-linear. This contrasts sharply with the persistent IR found in the Southeast when using land productivity as a measure.[17]

3.5 Discussion

In comparison to much of the development literature surrounding the IR, the Brazilian data used here represent a very heterogeneous group of farms and span a much greater range of farm sizes. A more accurate comparison group to the international literature might be farms

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[17] The empirical results obtained here are comparable to those reported at the national level by Rada et al. (2019) using a similar dataset. One difference is that they find somewhat faster TFP growth for the smallest farm size class, resulting in a more pronounced U-shape in 2006. The principal differences in empirical methodology are that they estimate TFP growth separately for each farm size class, and do not explore regional heterogeneity.

less than 100 ha, which indeed make up approximately 90% of all Brazilian farms. Even when restricting our analysis to this subset of farms, the use of land productivity would still show a marked inverse relationship while the use of TFP would reveal a negative relationship that has disappeared in the more modernizing regions. Perhaps more importantly, inclusion of the largest farm size class reveals that these farms have notably higher productivity in the more modern regions, and it is only when TFP is used that this becomes apparent. These are commercial farms that are unlikely to be included in most household surveys in developing countries, but they are present in the Agricultural Census data used here.

The regional analysis of Brazil provides insight into how the farm size – productivity relationship that was discussed in Section 2 can evolve with the modernization of agriculture. In the least developed regions of the country, the North and Northeast, the inverse relationship persists through the 100-500 ha size class regardless of the productivity measure used, and it is only with TFP that an emerging U-shape begins to appear. In the Center-West, where farms over 500 ha operated 80% of the land and accounted for around 75% of output in 2006, modernization of agriculture in this period converted an initially strong negative TFP relationship into one that was statistically flat by the end of the period. And in the Southeast, the most modern region of the country, the use of TFP reveals that the largest farms had higher productivity than all other size classes as early as 1985, but that this only became statistically significant in 2006. While it is beyond the scope of this paper to explain the causes of these changes, we note that conditions (i) and (ii) from Section 2 provide insight. They suggest that modernization has led output per ha to fall more slowly than inputs per ha as farm size rises. The use of modern inputs and technology appears to have successfully inverted the size – TFP relationship. Future research should seek to address whether these changes are due to increasing returns to scale above a certain size, diminishing importance of market failures, measurement error or other factors.

4. Conclusions and Policy Implications

We have sought to address an important weakness of the development economics literature on the inverse relationship between farm size and productivity. We argued that a variety of productivity measures are used when estimating this relationship, that the choice of measure matters for its identification and interpretation, and that total factor productivity is, in most cases, the preferred and most informative measure for policy. Furthermore, we argued that a commonly used measure – land productivity – is problematic and potentially misleading when used in modernizing agricultural contexts or when assessing a full range of farm sizes. Where comprehensive measures of productivity are more relevant and of interest, a focus on land productivity introduces omitted variable bias by not controlling for the intensity with which other inputs are used. Our conceptual discussion provides a framework for assessing the implications of the choice of productivity measure. Theoretically, it is clear that an inverse relationship between land productivity and farm size is neither necessary nor sufficient for an inverse relationship to exist between farm size and TFP.

How much does this critique matter? We conduct an empirical analysis at the regional level in Brazil using a pseudo-panel from 1985 to 2006 to contrast the land productivity – farm size relationship with the TFP – farm size relationship. While the analysis is only suggestive due to potential bias stemming from non-classical measurement error or endogeneity of inputs, the results indicate that the choice of productivity measure matters greatly. As in many developing country contexts, there exists an inverse relationship between land productivity and farm size for

Brazil, and within each of its macro-regions in every period. In contrast, the TFP and farm size relationship varies across time and space. The regional analysis of the TFP and farm size relationship shows 1) land productivity is not always an appropriate proxy for TFP; 2) the relationship is dynamic, changing with agricultural modernization; 3) the relationship is non-linear, often characterized by a U-shape; and 4) the very largest farms, such as those with more than 500 ha, are important to consider when assessing any relationship between farm size and productivity.

From a policy perspective, our findings have important implications for the debate about the future of small farms in developing countries. When using TFP, we see that superior productivity of small farms in traditional agricultural contexts is fully consistent with emergent productivity advantages for larger commercial farms in modernizing agricultural sectors. As economies develop, superior productivity may not continue to provide a valid argument for the importance and future of small farms, as we expect larger farms to play a more important role in driving national-level agricultural productivity growth. As such, it is increasingly unlikely that redistributive land reform could positively impact both equity and efficiency. However, this does not imply that small farms will, nor should, disappear. We expect them to remain important for generating livelihoods for rural households, providing food security, and contributing to the development of rural economies. Total factor productivity gains among small farmers will also continue to be essential for poverty alleviation. Importantly, rather than resting on an inverse farm size – productivity relationship, policy that seeks to impact both equity and efficiency should focus on ensuring that smallholders have access to the productivity gains experienced by their larger counterparts. Thus, policies that help build human capital, facilitate adoption of new technologies, and enhance access to markets via a reduction in transactions costs will continue to be indispensable for reducing rural poverty in developing countries.

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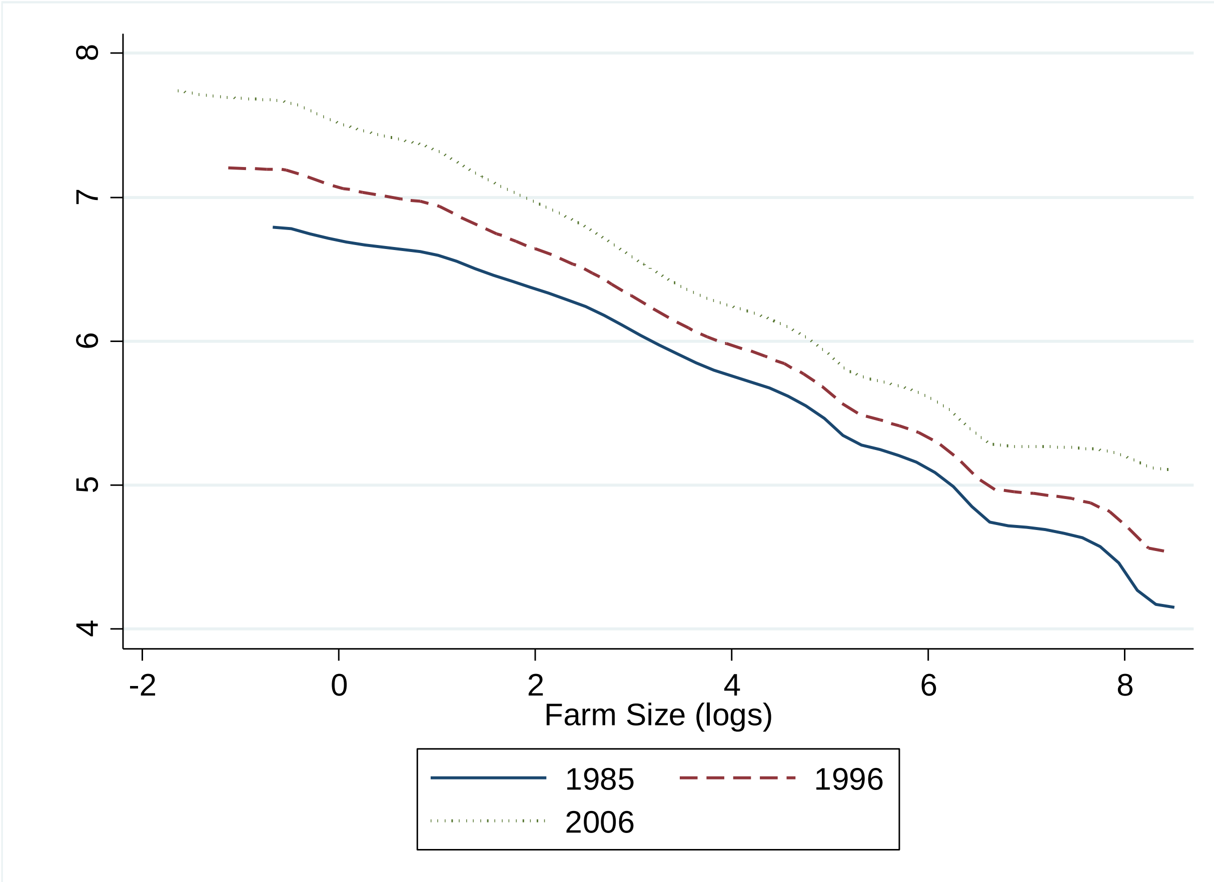
Figure 1: Farm Size and Land Productivity, Brazil (2006 R$/ha)



Note: Smoothed as a local polynomial regression with bandwidth of 1.25 and Epanechnikov kernel.

Figure 2: Land and Labor Productivity, Brazil 2006

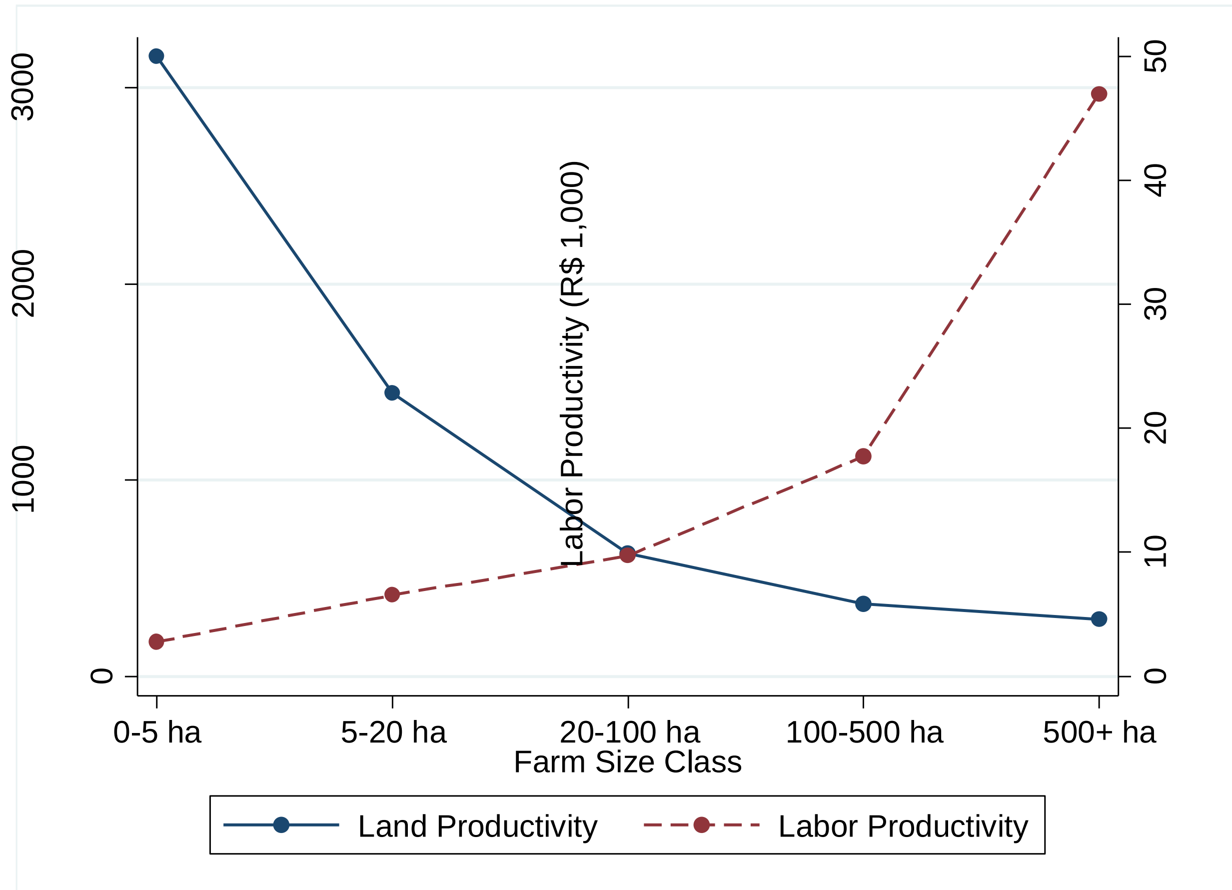


Figure 3: Land Productivity in Brazil by Region

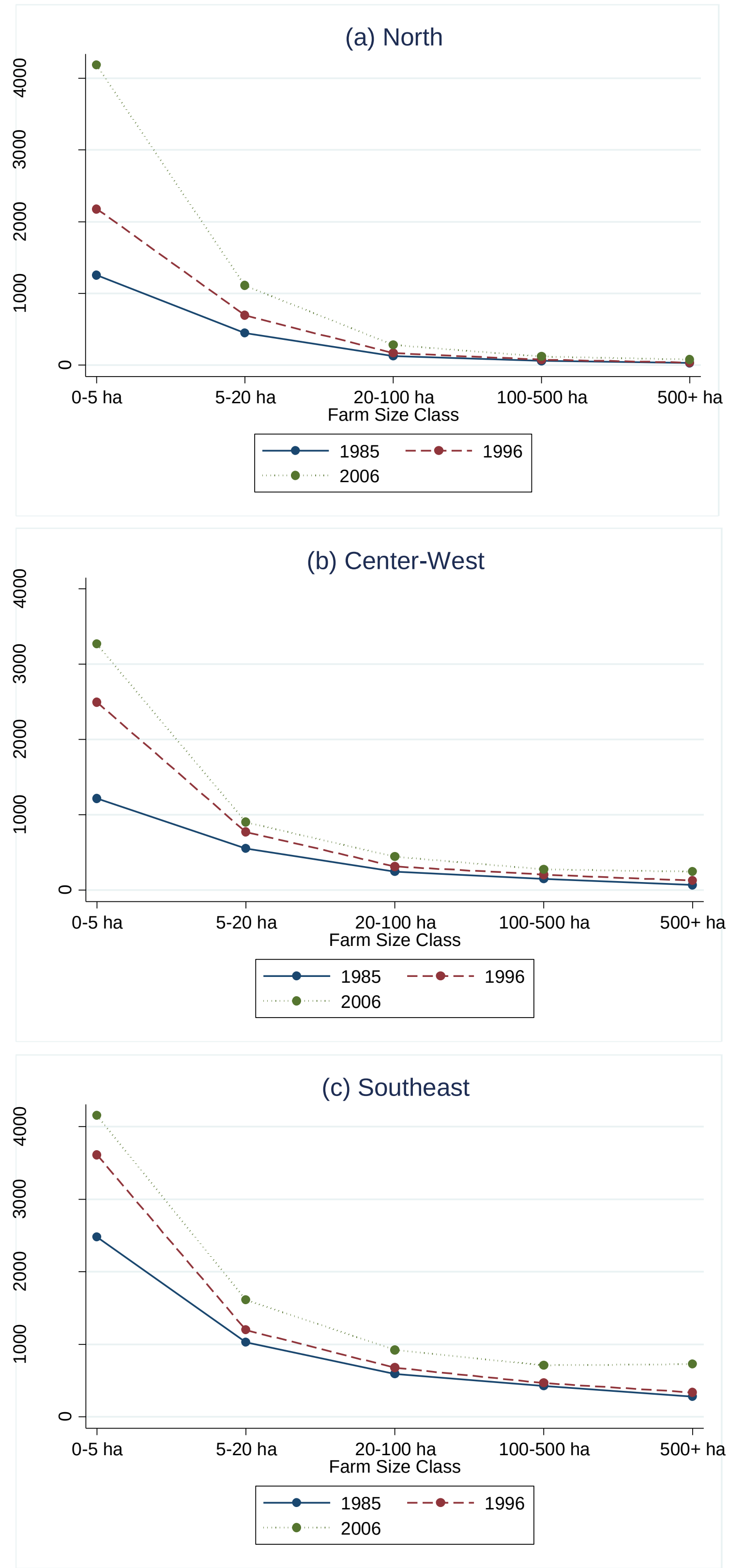


Figure 4: Total Factor Productivity in Brazil's North

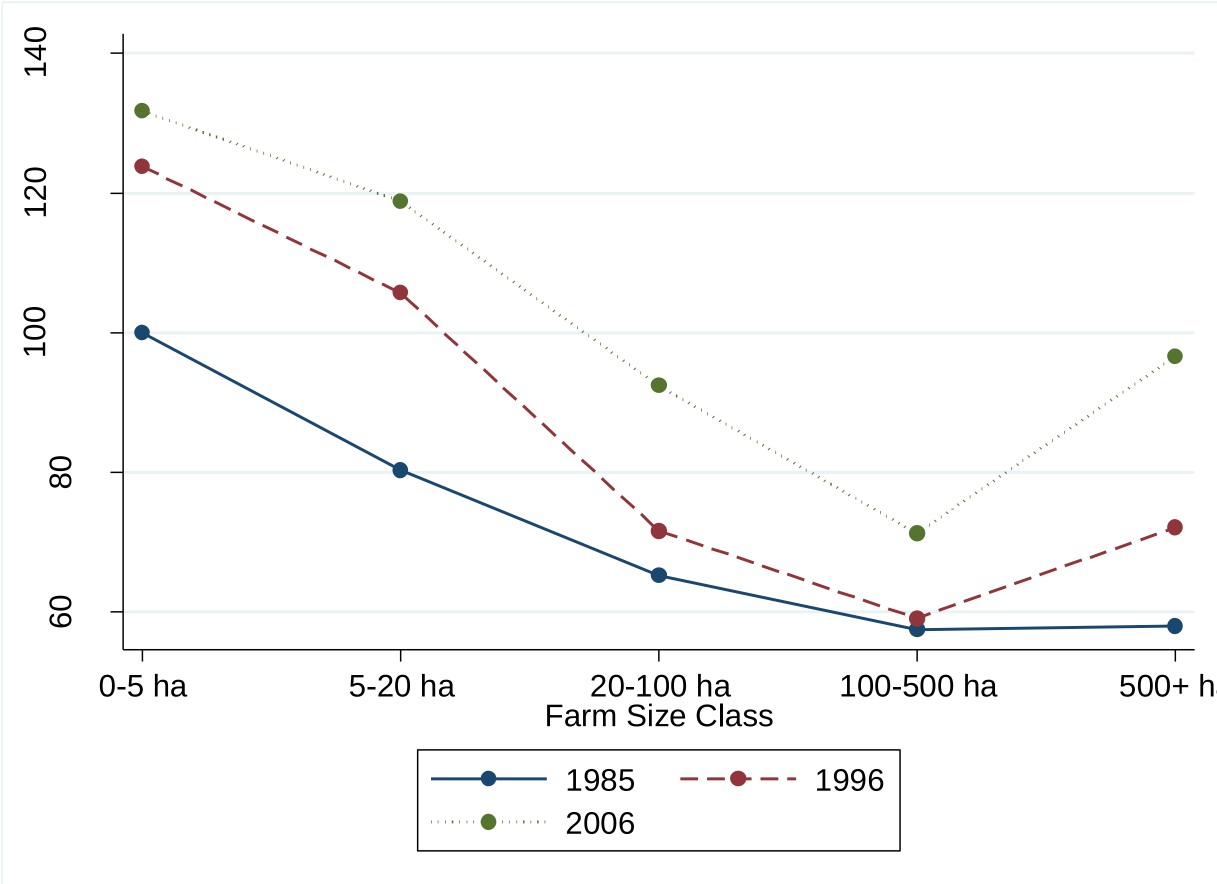


Figure 5: Total Factor Productivity in Brazil's Center-West

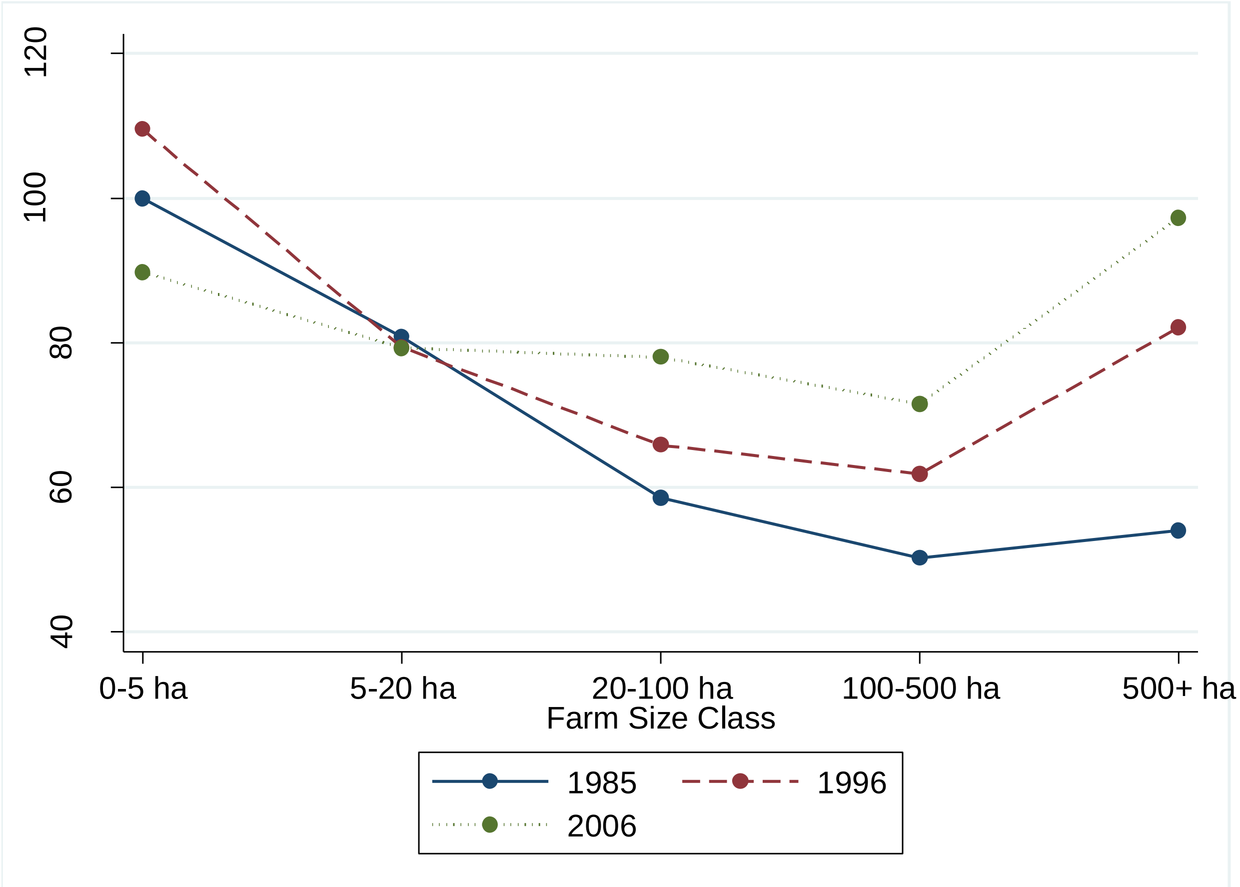


Figure 6: Total Factor Productivity in Brazil's Southeast

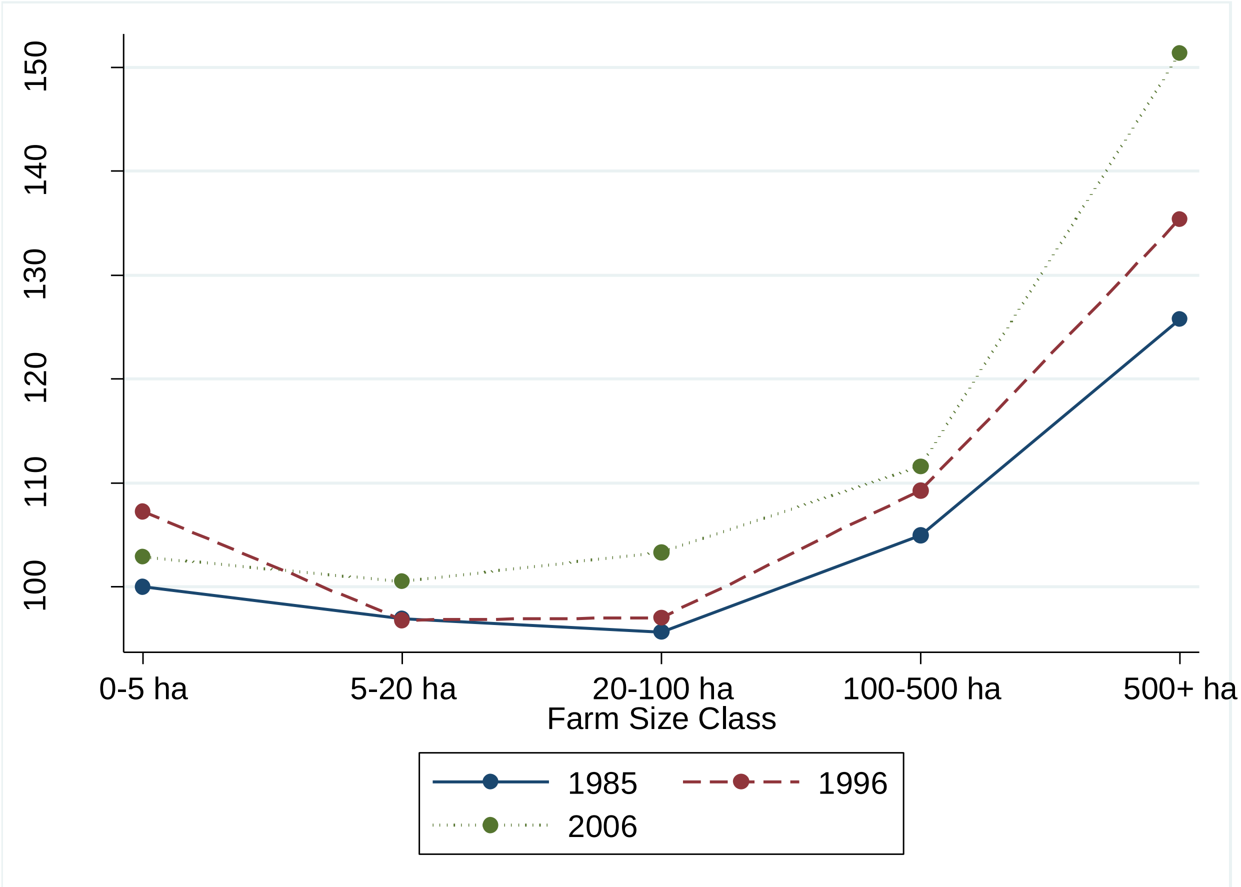


Table 1. Percentage Difference in TFP Relative to 0-5 ha Farms

	North			Center-West			Southeast		
	1985	1996	2006	1985	1996	2006	1985	1996	2006
5-20 ha	-19.71**	-14.63*	-9.79	-19.22***	-27.49***	-11.65	-3.07	-9.22**	-2.25
	(0.018)	(0.068)	(0.481)	(0.001)	(0.002)	(0.334)	(0.369)	(0.011)	(0.578)
20-100 ha	-34.69***	-42.11***	-29.85*	-41.43***	-39.88***	-13.10	-4.42	-8.94	1.11
	(0.005)	(0.001)	(0.070)	(0.000)	(0.003)	(0.283)	(0.510)	(0.190)	(0.870)
100-500 ha	-42. 53***	-52.25***	-45.89**	-49.83***	-43.53***	-20.30	5.05	2.59	8.61
	(0.009)	(0.001)	(0.027)	(0.000)	(0.013)	(0.286)	(0.637)	(0.816)	(0.441)
500 + ha	-41.97*	-41.72	-26.68	-46.01**	-24.99	8.37	24.98	27.82	47.66**
	(0.100)	(0.146)	(0.453)	(0.036)	(0.361)	(0.795)	(0.144)	(0.117)	(0.013)
N	1,888	1,888	1,888	3,038	3,038	3,038	17,742	17,742	17,742

Base farm size bin, 0-5 ha. P-values from significance tests are in parentheses: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Appendix A

Proof of expression (14).

$$\frac{\partial TFP}{\partial A} = \left(\frac{\partial q}{\partial A}\right)\left(\frac{1}{\tau(A)}\right) - \frac{\left(\frac{\partial \tau(A)}{\partial A}\right) q}{\tau(A)^2}$$

$$\frac{\partial TFP}{\partial A} = \left(\frac{\partial q}{\partial A}\right)\left(\frac{1}{\tau(A)}\right)\left[1 - \frac{\left(\frac{\partial \tau(A)}{\partial A}\right) q}{\tau(A)\left(\frac{\partial q}{\partial A}\right)}\right]$$

$$\frac{\partial TFP}{\partial A} = \left(\frac{\partial q}{\partial A}\right)\left(\frac{1}{\tau(A)}\right)\left[1 - \frac{\left(\frac{\partial \tau(A)}{\partial A}\right)\left(\frac{1}{\tau(A)}\right)}{\left(\frac{\partial q}{\partial A}\right)\left(\frac{1}{q}\right)}\right] = \left(\frac{\partial q}{\partial A}\right)\left(\frac{1}{\tau(A)}\right)\left[1 - \frac{\left(\frac{\partial \tau(A)}{\partial A}\right)\left(\frac{A}{\tau(A)}\right)}{\left(\frac{\partial q}{\partial A}\right)\left(\frac{A}{q}\right)}\right]$$

$$\frac{\partial TFP}{\partial A} = \left(\frac{\partial q}{\partial A}\right)\left(\frac{1}{\tau(A)}\right)\left[\frac{\varepsilon_{q,A}}{\varepsilon_{q,A}} - \frac{\varepsilon_{\tau(A),A}}{\varepsilon_{q,A}}\right]$$

$$\frac{\partial q}{\partial A} = \left(\frac{\partial TFP}{\partial A}\right)\tau(A)\left[\frac{\varepsilon_{q,A}}{\varepsilon_{q,A} - \varepsilon_{\tau(A),A}}\right]$$

Appendix B

Table A1: Data Cleaning and Sample Size, by Region and Farm Size Class

	Farm Size Class (ha)	Observations (N)	Less Farm Size Outliers	Less Land Productivity Outliers	Percent Dropped from Cleaning
North	0-5	315	315	310	1.6
	5-20	420	420	403	4.0
	20-100	459	459	437	4.8
	100-500	443	443	433	2.3
	500 +	323	315	305	5.6
Center-West	0-5	537	537	520	3.2
	5-20	619	619	605	2.3
	20-100	681	681	673	1.2
	100-500	672	672	659	1.9
	500 +	613	595	581	5.2
Southeast	0-5	3,850	3,850	3,805	1.2
	5-20	3,991	3,991	3,927	1.6
	20-100	4,024	4,024	3,944	2.0
	100-500	3,896	3,896	3,871	0.6
	500 +	2,235	2,215	2,195	1.8
Brazil[1]		*47,365*	*47,281*	*46,515*	*1.8*

[1]The sample size in Brazil includes data from the Northeastern and Southern regions.

Table A2: Descriptive Statistics, 2006

	Farm Size Class (ha)	Output (R$ 2006/ha)	Capital (Index/ha)	Family Labor (Adult Male Equivalents/ha)	Purchased Inputs (R$ 2006 per ha)	Share of Farms (%)	Share of Area (%)	Share of Output (%)
	0-5	4,185.02	965.03	1.75	862.4	19.4%	0.3	6.9
	5-20	1,110.46	333.92	0.24	214.4	17.4	1.6	12.0
North	20-100	282.24	129.06	0.05	73.5	43.0	17.4	32.2
	100-500	120.65	88.47	0.01	58.2	16.9	26.4	20.9
	500 +	78.98	54.90	0.01	73.0	3.3	54.3	28.0
	0-5	3,265.49	2,628.76	0.72	1,971.5	8.5	0.1	0.8
	5-20	902.03	851.41	0.16	507.0	20.2	0.7	2.4
Center-West	20-100	444.53	378.38	0.04	231.9	37.8	5.0	8.3
	100-500	276.48	279.76	0.01	210.4	21.2	13.8	14.3
	500 +	247.08	127.28	0.01	246.5	12.3	80.4	74.2
	0-5	4,152.85	3,903.90	0.85	1,892.5	28.8	1.1	5.5
	5-20	1,611.97	1,797.21	0.18	1,061.6	32.1	6.2	11.7
Southeast	20-100	923.89	906.85	0.04	554.7	28.5	21.8	23.5
	100-500	711.38	555.44	0.01	556.7	9.1	32.9	27.2
	500 +	726.50	276.13	0.01	715.4	1.5	37.9	32.1

Table A3: Estimated Technology Coefficients

	North	Center-West	Southeast
Capital per ha	0.201***	0.171***	0.121***
	(0.033)	(0.035)	(0.014)
Family Labor per ha	0.267***	0.142***	0.191***
	(0.056)	(0.039)	(0.026)
Purchased Inputs per ha	0.315***	0.430***	0.510***
	(0.035)	(0.042)	(0.019)
Constant	3.862***	3.667***	3.309***
	(0.225)	(0.290)	(0.019)
Weather Shocks	Yes	Yes	Yes
AMC FE	Yes	Yes	Yes
Time-varying Size Dummies	Yes	Yes	Yes
R-squared	0.96	0.90	0.91
N	1,888	3,038	17,742

Standard errors in parentheses: * p<0.10, ** p<0.05, *** p<0.01.
Dependent variable is logged output; all independent variables are logged; all variables normalized by farm size.