Real-Time Indicator of Weekly Inflation with A Mixed-Frequency Unobserved Component Model with Stochastic Volatility

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Abstract

This paper builds short-run and long-run coincident indicators of inflation at the weekly frequency. We propose a mixed-frequency unobserved component model in which the common permanent and transitory inflation components have time-varying stochastic volatilities (MF-UCSV model). The key aspects of the model are its flexibility to describe the changing inflation over time, and its ability to represent distinct time series properties across price indices at mixed frequencies. The model is estimated using Bayesian Gibbs Sampler and data on weekly commodity inflation, monthly consumer inflation, expenditures inflation, and quarterly GDP deflator inflation. The empirical results show that the constructed weekly inflation indicator closely matches monthly consumer and expenditure inflation. Additionally, the paper proposes and estimates a measure of high frequency trend inflation, which are in line with survey forecast and core inflation, and provides alternative to existing trend measures. We also study the changing persistence of inflation, and find that although it has reduced since the 1990s, it was due to different components over time. Interestingly, we also find that inflation volatility increased during the Great Recession, but this did not change the mean-reversion property of inflation. Overall, the model provides a strategy for real-time multivariate tracking and nowcasting of inflation at the weekly frequency, as new data are released.

1 Introduction

Inflation is one of the most watched economic series by policymakers and the public in general. Monetary policymakers continuously monitor inflation releases to update their expectation about future economic conditions and to control price stability. Market practitioners also rely on inflation reports in forming expectations when negotiating long-term nominal commitments. There is a growing recent literature on rich data environment (large datasets) and mixed frequency framework that has yielded major advances in assessing real economic activity, nowcasting, and forecasting output. However, this method has not been extensively applied to study inflation dynamics. Building an inflation indicator based on a set of variables is challenging given the important time-varying properties of inflation.[1] This is especially the case across price indices at different frequencies. Aruoba and Diebold (2010) construct a real-time monthly inflation indicator with the same framework used in Aruoba, Diebold and Scotti's (ADS, 2009) business condition indicator. However, the model does not provide a characterization of changing inflation dynamics, in particular, the evolving local mean and time-varying volatility.

This paper proposes a framework with underlying trend and cycle components representing long- and short-run inflation dynamics, which are used to construct high frequency coincident indicators of inflation. The proposed model encompasses price measures sampled at different frequencies, including weekly, monthly and quarterly price indices. The output is estimated weekly inflation indicators, which depict historical inflation trend and cycle, and that can be used to assess current inflation in real-time. The possibility of a high frequency inflation indicator providing more timely measurement than the official publication is very appealing. Official inflation measures can only be observed at the monthly frequency and with publication lags. For example, U.S. CPI is announced at the middle of the month for measures of inflation for the month prior.

The increasing availability of data at higher frequency has sparked interest in mixed frequency models. On one hand, mixed-frequency factor model and MIDAS (mixed-data sampling) model have become important tools for nowcasting and forecasting, using daily or weekly time series. Monteforte and Moretti (2013), Modugno (2013), Breitung and Roling (2014) examine the predictability of commodity prices and asset prices with this framework. On the other hand, researchers from other fields such as computer science and statistics have advanced methods to study high frequency data. With the recent large data set collected by electronic commerce system, such as Amazon, Walmart in the U.S. or Alibaba in China, among many others, daily price information is extracted and aggregated in few minutes us-

[1] see e.g. Cogley and Sargent (2005), Stock and Watson (2007), etc.

ing web crawler technology.[2] These measures are gradually accepted by private agents to complement the official inflation publications. However, the existing methods (e.g. machine learning) are designed to predict but not to obtain inferences regarding time series dynamics. In particular, the data collecting and filtering approaches that are used by high frequency price indices (e.g. online price index and commodity price index) are distinct from those that are used by official statistical agencies, and a formal statistical treatment of inflation dynamics at high frequency is still lacking.

Our approach involves formal modeling of inflation dynamics characterizing its trend, cycle, and volatility, while allowing for mixed-frequency. In particular, this paper proposes a mixed frequency small-scale unobserved component factor model with stochastic volatility - the MF-UCSV model. In the proposed model, underlying inflation process is approximated as a sum of unobserved common permanent and transitory factors. The permanent component captures long-run trend inflation, while the transitory component captures short-run deviations of inflation from its trend value. Additionally, the variances of the permanent and transitory disturbances are allowed to evolve over time according to a stochastic volatility process. Thus, the persistence of the inflation process is summarized by the relative importance of the variability of permanent and transitory components. The key aspects of the model are its flexibility to describe the changing inflation over time, and its ability to represent distinct time series properties across price indices at mixed frequencies. Price measures differ in terms of data collecting process, categories, utilization, but are highly correlated and may be driven by a set of common latent factors. The proposed flexible model allows for the potential distinct dynamics of underlying inflation, and also extracts common trend and common cyclical movements across the series. The underlying inflation indicators are extracted from a set of weekly, monthly, and quarterly price indices. The results indicate that the weekly inflation index tracks historical inflation dynamics well in the sense that it successfully identifies important inflation cycle and trend phases, including their severity and duration. Additionally, the estimated inflation indicator closely matches monthly consumer and expenditures inflation at the weekly frequency in real-time. Overall, the model provides a strategy for real-time multivariate tracking and nowcasting of inflation as new data are released. In particular, the real-time trend inflation estimates are in line with survey forecast and core inflation, which provide alternative to existing trend measures.

Our paper has several contributions to the literature. To our knowledge, this is the first

[2] For e.g. The Billion Prices Project (BPP) operated by MIT Sloan and Harvard Business School use big data to estimate dynamics in prices and implications for economic theory. This project uses prices collected from hundreds of online retailers on a daily basis to build inflation index and already has been applied to measure Argentina's inflation. In China, the companies Alibaba and Tsinghua University are collaborating to publish a daily internet-based consumer price index (icpi). This project not only provides the aggregate price index, but also price indices in sub-categories.

paper that builds high frequency short-run and long-run U.S. inflation coincident indicators that can be updated in real-time. Previous works do not use high frequency data or do not use them to build coincident indicators. Aruoba and Diebold (2010) build a U.S. inflation indicator but based on monthly and quarterly series. Similar to Aruoba and Diebold (2010), Modugno (2013) uses a dynamic factor model with three factors corresponding to weekly, monthly and quarterly series to forecast U.S. Consumer Price Index and Harmonised Index of Consumer Price for the Euro Area. Monteforte and Moretti (2013) use MIDAS regression framework with daily data to forecast inflation. However, these papers do not construct a short-run and long-run coincident indicators of inflation as proposed here.

Second, this paper takes into account potential nonstationarity in inflation dynamics. Many researchers suggest models that take account of slow-varying local mean for inflation perform reasonably well in forecasting inflation. Atkeson and Ohanian (2001) show that forecasts from simple random walk model cannot be statistically beaten by alternatives. Following their work, Stock and Watson (2007, 2016) proposed characterization of quarterly rate of inflation as an unobserved component model with stochastic volatility. Faust and Wright (2013) compare various inflation forecasts and find that models based on stationary specifications for inflation do consistently worse than non-stationary models. However, the related literature that focuses on extracting inflation indicator (Diebold and Aruoba, 2010) or nowcasting (Giannone, et al. 2006) with mixed frequency framework are all based on stationary specifications for inflation. This raises the question on whether the consideration of nonstationarity in these frameworks could also improve characterization of the dynamic properties of latent inflation. For example, the temporal aggregation of latent autoregressive factor is also autoregressive, while the GDP deflator inflation is better approximated as integrated moving average. In our framework, the factor loading along with changing volatilities can solve this problem by providing appropriate approximation for each series. Our mixed frequency MF-UCSV model takes into account nonstationarity and yields an estimated trend inflation at high frequency. Trend inflation is an important tool for monetary policy as it conveys information on long run inflation expectations.

Finally, filtering out the noise in multiple inflation measures has not been done in the mixed frequency literature. Generally, there are two approaches in the literature to approximate trend inflation. Clark (2011), Faust and Wright (2013), among others, use measures of long-run inflation expectations from surveys forecasts (either Survey of Professional Forecasters or Blue Chip) to capture trend inflation. Survey-based trend leads to an improvement in the accuracy of model-based forecasts (see, e.g., Ang, et al. 2007). However, surveys of inflation expectation can not be replicated as it is a result of a combination of many objective (models) and subjective information. Alternatively, a range of studies has modeled trend inflation as an unobserved component (e.g. Stock and Watson 2007, Cogley and Sbordone 2008, or Mertens 2011). In this paper, we follow this approach, using time series smoothing methods to extract trend inflation, which is additionally obtained from a multivariate framework. The use of factor model mitigates the problem of estimating weights separately and downweighting sectors that may have large variations over time.

This article is organized as follows. The model is described in section 2, along with the estimation method, which uses the Gibbs Sampler used for simulating the posterior distribution of the parameters. The third section presents and interprets the empirical results. The conclusion are discussed in the fourth section.

2 The model

2.1 The Underlying Inflation Process

Following Stock and Watson (2007), inflation is characterized by an unobserved component model with stochastic volatility. We assume that the underlying inflation process evolves daily. This assumption can be adjusted to other frequencies, like weekly or monthly.

Let π_t denote the underlying inflation at day t, which evolves following a stochastic process:

$$\pi_t = \tau_t + \eta_t \tag{1}$$

where τ_t represents the permanent component of underlying inflation and η_t represents the transitory component. Permanent component takes the form of a simple random walk by equation (2):

$$\tau_t = \tau_{t-1} + \sigma_{\tau,t}\varepsilon_{\tau,t} \tag{2}$$

and transitory component has a finite order AR(p) representation:

$$\Phi(L)\eta_t = \sigma_{\eta,t}\varepsilon_{\eta,t} \tag{3}$$

where function $\Phi(L)$ is a finite lag polynomial with order p, and has all the roots outside the unit cycle, $\varepsilon_{\tau,t}$ and $\varepsilon_{\eta,t}$ are mutually independent $i.i.d.$ $N(0,1)$ stochastic processes. $\sigma_{\tau,t}$ and $\sigma_{\eta,t}$ represent the variability of innovations to permanent component and transitory component. They together determine the relative importance of random walk disturbance. To model the changing volatility of inflation components, it is assumed that their log-volatility follows a random walk with no drift,

$$ln(\sigma^2_{\tau,t}) = ln(\sigma^2_{\tau,t-1}) + \nu_{\tau,t} \tag{4}$$

$$ln(\sigma^2_{\eta,t}) = ln(\sigma^2_{\eta,t-1}) + \nu_{\eta,t} \tag{5}$$

where $\nu_{\tau,t} \sim N(0,\sigma^2_{\nu\tau})$ and $\nu_{\tau,t} \sim N(0,\sigma^2_{\nu\eta})$. The magnitudes of time variation in $\sigma_{\tau,t}$ and $\sigma_{\eta,t}$ depend on the variances of $\nu_{\tau,t}$ and $\nu_{\eta,t}$. In particular, a large $\sigma^2_{\nu\tau}$ means the variability of trend components can undergo large period changes, which affect the inflation persistence indirectly.

2.2 Relate Inflation Factors with Observed Inflation Indices

A vector of price measures and other variables displaying comovement is modeled to depend on the latent permanent and transitory inflation factors. The daily economic variable is a linear combination of daily common permanent and transitory components. Let y^i_t denote the ith daily price measures at day t and we have below the relationship:

$$y^i_t = \beta_i \tau_t + \gamma_i \eta_t + u^i_t \tag{6}$$

where $u^i_t \sim N(0,\sigma^2_{u_i})$ are contemporaneously and serially uncorrelated innovations that capture idiosyncratic shocks to the specific price measures. β_i and γ_i are the factor loadings on the common permanent and transitory components.

In the mixed frequency framework, the relationship between the observed data and daily variables need to be specified. Most of the economic variables are observed at lower frequency, for example, CPI inflation and GDP inflation are monthly and quarterly measures respectively. Inflation measures growth rate of price level, then relationship between observed inflation series and underlying daily variables depends on the temporal aggregation of price index. Here we approximate the price index observed at low frequency as the systematic sampling of the daily variables, i.e. end of period value. Thus, the inflation measures can be processed as flow variable.[3] Our approximation method is different from the commonly used method for approximating GDP growth rate in Mariano and Murasawa (2003). Their method is doable but will complicate our model in high frequency, by making the state variable extremely large and computation unattainable. In contrast, our approximation can cast the model into a linear form. Let $\tilde{y}^i_t$ denote the ith observed flow variable in low frequency. Then $\tilde{y}^i_t$ is the intra-period sums of the corresponding daily values,

[3] A comprehensive description of temporal aggregation of flow and stock variables can be seen in Aruoba, et al (2008).

$$\begin{aligned}\tilde{y}_t^i &= \begin{cases}\sum_{j=0}^{D_i-1} y_{t-j}^i & if\ y_t^i\ can\,be\,observed\\ NA & otherwise\end{cases} \\ &= \begin{cases}\beta_i \sum_{j=0}^{D_i-1} \tau_{t-j} + \gamma_i \sum_{j=0}^{D_i-1} \eta_{t-j} + u_t^{*i} & if\ y_t\ can\,be\,observed\\ NA & otherwise\end{cases}\end{aligned} \tag{7}$$

where D_i is the number of days per observational period. For example, D_i for monthly CPI of January equals 31. u_t^{*i} adds up the daily white noise disturbances and thus follows $MA(D_i - 1)$ process. Here we can appropriately treat u_t^{*i} as white noise following Aruoba et al. (2009).

Following Harvey(1990), we apply the accumulator variables to handle temporal aggregation. This could greatly reduce the state of the system. Let $C_{\tau,t}$ and $C_{\eta,t}$ denote the permanent and transitory component accumulator:

$$\begin{aligned}C_{\tau,t} &= \theta_t C_{\tau,t-1} + \tau_t \\ &= \theta_t C_{\tau,t-1} + \tau_{t-1} + \sigma_{\tau,t}\varepsilon_{\tau,t}\end{aligned} \tag{8}$$

and

$$\begin{aligned}C_{\eta,t} &= \theta_t C_{\eta,t-1} + \eta_t \\ &= \theta_t C_{\eta,t-1} + \Phi(L)\eta_{t-1} + \sigma_{\eta,t}\varepsilon_{\eta,t}\end{aligned} \tag{9}$$

where θ_t is an indicator variable which is defined as:

$$\theta_t = \begin{cases}0 & If\ t\ is\,the\,first\,day\,of\,the\,period\\ 1 & otherwise\end{cases}$$

Then equation (7) can be written as:

$$\tilde{y}_t^i = \begin{cases}\beta_i C_{\tau,t}^i + \gamma_i C_{\eta,t}^i + u_t^{*i} & if\ y_t\ can\,be\,observed\\ NA & otherwise\end{cases}$$

2.3 State-Space Form

A more compact state-space representation of the MF-UCSV model is the following:

$$Y_t = C_t^{'} \alpha_t + w_t \tag{10}$$

$$\alpha_{t+1} = A_t \alpha_t + R_t v_t \tag{11}$$

$$\Lambda_{t+1} = \Lambda_t + \zeta_t \tag{12}$$

$$w_t \sim N(0, H), \quad v_t \sim N(0, Q_t) \tag{13}$$

$$\zeta_t \sim N(0, W) \tag{14}$$

where Y_t is an $N \times 1$ vector of observed variables with missing values. State vector α_t includes 8 state variables, w_t and v_t are Gaussian and orthogonal measurement and transitory shocks. The time-varying variance matrix Q_t is a diagonal matrix with elements of $\sigma^2_{\tau,t}$ and $\sigma^2_{\tau,t}$. Λ_t is the vector of unobserved log-volatilities, and W is a diagonal matrix containing the variance of log-volatility disturbances.

There are two special cases nested in our model. First, the changing volatility crucially depend on the covariance matrix W. When we set $W = 0$, Λ_t is constant, then we return to the normal mixed-frequency dynamic factor model. In this case, Kalman filter and smoother can be used to extract the state variables and the corresponding state disturbances. The algorithm is classical Kalman filter in the textbook. Second, instead of shutting off the stochastic volatility, we may assume $\sigma^2_{\tau,t} = \sigma^2_{\eta,t}$ and reduce the dimension of W to unity. This is the case of common stochastic volatility. Koopman (2004) propose a method using importance sampling and Kalman filter to estimate the model. These two models can be estimated by maximizing the log likelihood function. For our model with great flexibility in setting the conditional variance, MLE is not feasible.

2.4 Estimation

We use Bayesian MCMC method with Metropolis-within-Gibbs sampler to estimate our model. The estimation procedure, model identification, and priors will be described briefly, and more details can be obtained in the appendix.

Sampling of the parameters, including latent factors and volatilities, can be proceeded in several steps. First, since the model can be cast into state-space form, the unobserved state variables τ_t and η_t can be easily drawn using Kalman smoother (Koopman and Durbin, 2003). Second, conditioning on α_t and Y_t, elements in C_t' and H can be drawn row by row in equation (10). Taking the ith measurement equation $\tilde{y}_t^i = \beta_i C^i_{\tau,t} + \gamma_i C^i_{\eta,t} + u_t^{*i}$, we can draw the β_i, γ_i and variance of u_t^{*i} following the conventional method for linear model. Third, equation (11) can be broken down to equation (3), which is AR(p) model with heteroscedastic disturbance. Dividing by $\sigma_{\eta,t}$, one can obtain a standard linear regression model and draw the AR coefficients from the conjugate normal distribution. Forth, we use Jacquier, Polson, and Rossi (1994)'s algorithm and Kim, Shephard and Chib (1998)'s Metropolis rejection method to draw the stochastic volatilities, which are the unobserved components in equation (12). Fifth, conditional on the log-volatilites, $\sigma^2_{\nu\tau}$ and $\sigma^2_{\nu\eta}$ in covariance matrix W can be drawn from conjugate inverse gamma distribution.

3 Empirical Application: Real Time Inflation Index and Trend Inflation

3.1 Data

The empirical application uses weekly GSCI commodity price index, monthly CPI-all items, monthly personal consumption expenditure deflator, and quarterly GDP deflator. The inflation measures are observations on 100 times first difference of the logarithm of each price indices. The sample ranges from 1970/02/01 through 2016/12/31. The extracted inflation indicator can be updated weekly, by including the high frequency commodity price index GSCI inflation (Goldman Sachs Commodity Index). GSCI index is a weighted future prices that almost covering all the sectors of commodities. It is published by Standard and Poor's and recognized as a leading measure of general price movements in the global economy. In this paper, the daily GSCI is averaged to build our weekly GSCI index. Similar high frequency indices include daily CRB index (Commodity Research Bureau Index) which is calculated by Commodity Research Bureau, World Market Price of Raw Materials (RMP) produced by OCED and other energy prices. The commodity price index is obtained from Global Financial Data, and all other price measures are from FRED Economic Data.

We choose the data set for the following reasons. First, a small-scale factor model is sufficient to achieve our goal and illustrate the implementation of our model. Second, we only use data up to weekly frequency since daily observations are far too noisy. Third, the indicators are all price measures that assess the change of inflation from different aspects. The choice of the variable set can also be extended beyond, for example, asset prices, monetary base and survey data. These variables have some predicative power for future rate of inflation, thus sometimes are used in the literature. However, correlations between those variables and inflation are weak, which may disturb our signal extraction. So we exclude them in our estimation.

Examination of our data indicates our model is an appropriate approximation to different inflation measures. First, we use the Augmented Dickey–Fuller (ADF) test to examine the stationarity of the series. The test is done for three sample ranges: 1970 to 1983, 1985 to 2016 and the full sample period 1970 to 2016. The first sample period corresponds to the Great Inflation, while the second sub-sample includes Inflation Stabilization period when both the level of inflation and the volatility declined dramatically. The ADF test in Table 1 suggests a unit root in pre-1984 period and the full sample period for low frequency inflation measures (monthly and quarterly). However, the null hypothesis of unit root in the post-1984 period is rejected. This may suggest that innovations of transitory component tend to play a greater role in the inflation process.

Second, equation (1) to (3) imply that the first-order autocorrelation is negative for the first difference of inflation. Table 2 presents estimated autocorrelation for the change in inflation over three sample periods. The first-order autocorrelation is negative for each of the measures in all sample periods. For GDP inflation, $\Delta\pi_t$ is negatively correlated, with the first autocorrelation much larger in absolute magnitude in the second period than the first.

3.2 Model Implementation

We assume that the transitory component follows AR(1) process. Modeling the persistence with AR(1) process would be inadequate, high-order dynamics nevertheless is not statistically better, as the transitory shock would decay too quickly when we assume the latent factors evolve daily.

The equations applied to the data are

$$\begin{bmatrix} \tilde{y}_t^{GSCI} \\ \tilde{y}_t^{CPI} \\ \tilde{y}_t^{PCE} \\ \tilde{y}_t^{GDPD} \end{bmatrix} = \begin{bmatrix} 0 & 0 & \beta_1 & \gamma_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta_2 & \gamma_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta_3 & \gamma_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \beta_4 & \gamma_4 \end{bmatrix} \begin{bmatrix} \tau_t \\ \eta_t \\ C_{\tau,t}^W \\ C_{\eta,t}^W \\ C_{\tau,t}^M \\ C_{\eta,t}^M \\ C_{\tau,t}^Q \\ C_{\eta,t}^Q \end{bmatrix} + \begin{bmatrix} u_t^{GSCI} \\ u_t^{CPI} \\ u_t^{PCE} \\ u_t^{GDPD} \end{bmatrix}, \tag{15}$$

$$
\begin{bmatrix} \tau_t \\ \eta_t \\ C^W_{\tau,t} \\ C^W_{\eta,t} \\ C^M_{\tau,t} \\ C^M_{\eta,t} \\ C^Q_{\tau,t} \\ C^Q_{\eta,t} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \phi & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & \theta^W_t & 0 & 0 & 0 & 0 & 0 \\ 0 & \phi & 0 & \theta^W_t & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & \theta^M_t & 0 & 0 & 0 \\ 0 & \phi & 0 & 0 & 0 & \theta^M_t & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & \theta^Q_t & \\ 0 & \phi & 0 & 0 & 0 & 0 & & \theta^Q_t \end{bmatrix} \begin{bmatrix} \tau_{t-1} \\ \eta_{t-1} \\ C^W_{\tau,t-1} \\ C^W_{\eta,t-1} \\ C^M_{\tau,t-1} \\ C^M_{\eta,t-1} \\ C^Q_{\tau,t-1} \\ C^Q_{\eta,t-1} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_{\tau,t} \\ v_{\eta,t} \end{bmatrix}, \quad (16)
$$

$$
\begin{bmatrix} u^{GSCI}_t \\ u^{CPI}_t \\ u^{PCE}_t \\ u^{GDPD}_t \end{bmatrix} \sim N(0_{4\times 1}, H), \quad \begin{bmatrix} v_{\tau,t} \\ v_{\eta,t} \end{bmatrix} \sim N(0,\ Q_t),
$$

$$
H = \begin{bmatrix} \sigma_1^2 & 0 & 0 & 0 \\ 0 & \sigma_2^2 & 0 & 0 \\ 0 & 0 & \sigma_3^2 & 0 \\ 0 & 0 & 0 & \sigma_4^2 \end{bmatrix}, \quad (17)
$$

$$
Q_t = \begin{bmatrix} \sigma^2_{\tau,t} & 0 \\ 0 & \sigma^2_{\eta,t} \end{bmatrix}, \quad (18)
$$

$$
\begin{bmatrix} ln(\sigma^2_{\tau,t}) & 0 \\ 0 & ln(\sigma^2_{\eta,t}) \end{bmatrix} = \begin{bmatrix} ln(\sigma^2_{\tau,t-1}) & 0 \\ 0 & ln(\sigma^2_{\eta,t-1}) \end{bmatrix} + \begin{bmatrix} \nu_{\tau,t} \\ \nu_{\eta,t} \end{bmatrix}, \quad (19)
$$

$$
\begin{bmatrix} \nu_{\tau,t} \\ \nu_{\eta,t} \end{bmatrix} \sim N(0,\ W),
$$

$$
W = \begin{bmatrix} \sigma^2_{\nu\tau} & 0 \\ 0 & \sigma^2_{\nu\eta} \end{bmatrix}. \quad (20)
$$

We identify the model and set the prior hyperparameters in the following ways: First, we restrict the factor loadings β_4 and γ_4 to be 1 to identify the scale of factor loadings and of the unobserved components (See equation (15)). Then, we obtain the initial guess value of β_i , γ_i , ϕ and σ_i as estimates of the state-space model using MLE with time-invariant variability of state disturbances. Along with factor loadings, the initial guess of the latent state variables in α_t can also be estimated. Second, for the initial guess of time-varying volatilities $\sigma^2_{\tau,t}$ and $\sigma^2_{\eta,t}$, we estimate a GARCH(1,1) model to obtain the conditional variance. Third, the prior distributions of β_i , γ_i and ϕ are conjugate independent diffuse normal with mean fixed to initial guess value and their variance set to 10^3. Forth, we impose independent inverse gamma with degrees of freedom to 1 for σ_i in H, $\sigma^2_{\nu\tau}$ and $\sigma^2_{\nu\eta}$ in W. Finally, following Del Negro and Otrok (2008), we fix the initial condition of stochastic volatility to zero.

3.3 Results

3.3.1 Inflation Indicator and Factor Loading

We build the coincident inflation indicator as the sum of latent permanent component and transitory component. The extracted weekly inflation indicator is plotted in Figure 1.[4] Several observations and desirable properties are noteworthy: First, our estimated inflation indicators are available at high frequency, whereas the monthly CPI and PCE inflation are released only monthly and with weeks of lags. Therefore, our inflation indicator can be applied to nowcast CPI and PCE inflation.

Second, our inflation indicator broadly coheres with the dynamics of inflation in the past 50 years. The Great Inflation in 1970s is apparent, along with the inflation stabilization staring from 1982. The average annual inflation indicator is 6.3648 during 1970s, compared with an average value of 2.3418 after 1982 in our estimation. For the Great Inflation, we find the first peak occurred on October 6, 1974 with a weekly indicator value of 0.199632, and the second peak was on November 30, 1980, with an indicator value of 0.188673. The recent 2007 recession experienced unprecedented price decline. However, this deflation was quite brief and lasted two months from 10/26/2018 to 01/04/2019. In addition, the estimated inflation indicator also indicates varying volatility of inflation, which is consistent with the observations in the literature (Stock and Waston, 2007). We will examine this property in the later sections with estimated conditional volatility.

Third, our inflation indicator coheres with consumer inflation and expenditure inflation but plays no leading role in identifying the turning points. Figure 2 graphs the weighted inflation indicator along with monthly CPI inflation and PCE inflation. The fact that the weekly inflation indicator has no leading performance can be explained in two ways. On one hand, commodity price is made up of commodity future contracts, thus may convey limited leading information in the consumer and personal expenditure inflation. On the other hand, monthly indicators account for a large part of the extracted inflation factor as indicated by the values of the factor loading. It is not surprising that the weekly inflation indicator tracks monthly consumer and personal expenditure inflation well. Adding leading variables such as term premium and M2 may improve the leading performance of our indicator.

Estimated factor loadings measure the sensitivity of input variables to latent permanent and transitory components. The full sample posterior mean estimates of the factor loadings are reported in Table 3. The relative importance of our chosen indicators is given by the full sample posterior mean estimate of the factor loadings. For the common trend component,

[4] The estimated inflation factors follow daily evolution. But information of price comes on Friday of each week as assumed in our model, so we aggregate the daily inflation factors to get the weekly inflation index as plotted. By doing so, we can mimic the real time updating of inflation index.

the monthly inflation indices have the highest posterior means, and followed by the quarterly GDP inflation. The weakest contribution comes from weekly commodity inflation (0.26). This suggests that commodity inflation is less persistent and thus few of the variation itself are from the variation of common persistent component.

Among inflation measures at different sampling frequencies, monthly CPI and PCE inflation capture both the persistent and transitory components well, comparing with quarterly inflation index in extracting low frequency movements and with weekly commodity inflation in modeling high frequency variations. This may suggest that transitory shocks in general price level vanish within a quarter. Thus, using quarterly average of inflation may overrate the model implied persistence of inflation either in univariate time series model or multivariate VAR model and New Keynesian model.

3.3.2 Trend Inflation and Volatilities

The model provides a measure of trend inflation. Figure 3 plots the full sample posterior means of τ_t , $\sigma^2_{\tau,t}$ and $\sigma^2_{\eta,t}$. The estimated trend inflation is quite smooth and shows substantial variation over time. Regarding the recent 2007-2009 recession, trend inflation did not plunge deeper and go under zero line, but recovered steadily to the Fed inflation target. Compared with inflation indicator which indicates a short period deflation, trend inflation only suggests a pressure of disinflation. Therefore, the decline of price in 2008 was more likely due to a one-time large shock which decayed very quickly.

There are important similarities between $\sigma^2_{\tau,t}$ and $\sigma^2_{\eta,t}$, most notably the larger variation in 1970s coincided with high trend inflation, and persistently low volatility in 1990s followed by a remarkable increase in early 2000s. Stock and Watson (2007) suggest the recent rise of volatility as the potential reason for the decreased forecastability of inflation in recent decades. However, there are also differences between the changes in these two series. In 1990s, volatility of permanent component decreased strikingly compared to 1980s and 1970s, whereas there was only slight decrease in the volatility of transitory component. During 2000s, the volatility of both permanent and transitory components increased, but transitory component increased much more than permanent component. The persistence of inflation which depends on the relative importance of the variances of the permanent and transitory innovations is also examined. The change in inflation indicator has a negative first-order autocorrelation which summarizes the persistence of inflation process (Cecchetti, et al. 2007). The analytical expression can be calculated as:

$$\rho_{\Delta\pi} = \frac{Cov(\Delta\pi_t, \Delta\pi_{t-1})}{Var(\Delta\pi_t^2)} = \frac{-\frac{1-\phi}{1+\phi}\sigma^2_{\eta,t}}{\sigma^2_{\tau,t} + \frac{2}{1+\phi}\sigma^2_{\eta,t}} \tag{21}$$

Note that $\rho_{\Delta\pi}$ has a range that depends on the AR coefficient ϕ. With the estimated value, the closer it is to -0.845, the less persistent the inflation process is. Additionally, the higher $\sigma^2_{\tau,t}$ is relative to $\sigma^2_{\eta,t}$, the closer inflation is to a pure random walk, and the closer the first-order autocorrelation of $\Delta\pi_t$ is to zero. By contrast, when $\sigma^2_{\eta,t}$ is dominant, inflation is close to a stationary AR process. From our estimation of the weekly inflation indicator, $\rho_{\Delta\pi}$ decreased by 74.12% from 1970s to the current decade. Alternatively, when inflation does change unexpectedly, how much of the surprise should we assume to be part of the new trend? We calculate the share of inflation surprise that the model currently attributes to the new trend. In our estimation, 54% of the unexpected inflation change assumed to be part of a new trend in 1970s, and this share decreased to 22% in 1990s and 19% in current decade. Overall, inflation persistence has reduced since the 1990s, but due to different components over time. In this period with a lower persistence, inflation tended to revert to a stable trend during the 2000s, whereas in the 70s and 80s the trend moved to track inflation.

Comparing with univariate model using quarterly data, our model tends to overrate the role of high frequency variations. This is shown in the higher contribution of transitory innovations to the variability of inflation process. It is not surprising that quarterly data filter out the high frequency innovations due to temporal aggregation. In contrast, weekly data highlights the volatile movements in commodity inflation, thus assign higher weights to them in signal extracting process (See Table 3 the factor loadings). Modeling transitory components as autoregressive process rather than white noise also weight more on the variability of transitory shocks.

Our model hinders the smoothness to stochastic volatility. The two spikes in 1974 and 2008 are more likely to be occasional large jumps in inflation. The 1974 spike was due to the oil crisis, and the2008 spike was due to the recent financial crisis. Hence it is possible that 2007 recession can be viewed as a temporary period with a high level of volatility in a longer period when moderate volatility is the norm.

Figure 4 compares model implied trend inflation with core inflation (CPI and PCE). Our trend estimates are broadly in line with the alternative measures of trend inflation. They together reflect the common low frequency variability in inflation series. However, there are important differences between trend inflation and distinct core inflation measures. Core CPI inflation is much persistently higher than trend inflation and core PCE deflator during late 1970s and early 1980s, which indicates that sectors in CPI categories besides food and energy also contribute to large short-term variations.

Figure 5 plots our model implied trend inflation along with the median 10-year ahead forecast that has been reported in the Survey of Professional Forecasters since 1991. Trend inflation lines up with the survey forecasts but lies below trend inflation during 1990s and the current decade. The reason is that survey forecasts are always upward biased. Especially, long-term forecasts of PCE inflation from the SPF have often been a bit higher than long-term projections from the FOMC. After a slightly decline together in 1990s, survey forecasts kept stable henceforth while trend inflation became volatile. Although many concerns disinflation due to large output gaps and unemployment in 2004 and 2008 (Williams, 2009), the substantial increase in expectations anchoring mute these pressures and revert the trend to local mean.

A subtle feature in Figure 5 is that the model implied trend inflation leads long-run survey forecast movements. This is especially obvious for the drop around 1997 and the decline after 2012. This raises the question of how inflation expectation reacts to the changes in trend inflation. A simple linear regression between one-year ahead inflation expectation and trend inflation indicates that there exists statistically significant evidence that trend inflation help forecasting trend inflation expectation. This suggests a rise of inflation that is not accompanied by a rise of inflation expectations is less likely to persist.

4 Concluding Remarks

This article introduces a mixed-frequency unobserved component model with stochastic volatility and estimate the model using Bayesian Gibbs Sampler. MF-UCSV model provides a flexible mixed-frequency framework for extracting high frequency inflation indicator, estimating trend inflation, and describing persistence of inflation. Inflation indicator and trend inflation could be flagged in the real-time as new data are released. The framework allows ragged-edge data, publication lags and non-synchronization in real time monitoring and nowcasting.

Our paper supports the desirability of using models that account for a slowly-varying trend. The changing time series properties of inflation imparts the forecasting performance of most univariate and activity-based inflation forecast (Stock and Watson, 2007). Apart from accounting for local mean and varying volatility in this paper, one could also apply methods that take account of parameter instability, such as time-varying coefficients VAR by Cogley and Sargent (2005). It is noteworthy that researches which impose a structural break also do well in some specific models (Goren, et al., 2013). However, our initial try of a regime switching model fails to detect a structural break endogenously. One possibility is that high frequency data contains too many noises which can largely disturb the inference of Markov Switching model.

Compared with the conventional way of modeling low-frequency movement from quarterly data and high frequency variations from daily or weekly data separately, we estimate the variability of both components jointly. However, our model underrate the transitory innovation due to the quickly decay in transitory dynamics. This suggests a mixed frequency model with monthly and quarterly observations should be a future research direction to examine the relative importance of transitory components.

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Appendix

A. Details of the Gibbs Sampler

We describe in more detail the sampling steps and related posterior distributions that compose our Gibbs sampler procedure. The model in state-space form is

$$y_t = C_t^{'} \alpha_t + w_t \tag{22}$$

$$\alpha_{t+1} = A_t \alpha_t + R_t v_t \tag{23}$$

$$\Lambda_{t+1} = \Lambda_t + \zeta_t \tag{24}$$

$$w_t \sim N(0, H), \quad v_t \sim N(0, Q_t) \tag{25}$$

$$\zeta_t \sim N(0, W) \tag{26}$$

The parameters to be estimated are $[\{\tau_t, \eta_t\}, \{\beta_i, \gamma_i\}, \{\phi\}, \{\sigma_{\tau,t}, \sigma_{\eta,t}\}, \{\sigma_{\nu\tau}, \sigma_{\nu\eta}\}, \{\sigma_{u_i}\}]$. We partition them into 5 blocks:

$$\theta_1 = \{\tau_t, \eta_t\}$$

$$\theta_2 = \{\beta_i, \gamma_i, \sigma_{u_i}\}$$

$$\theta_3 = \{\phi_i\}$$

$$\theta_4 = \{\sigma_{\tau,t}, \sigma_{\eta,t}\}$$

$$\theta_5 = \{\sigma_{\nu\tau}, \sigma_{\nu\eta}\}$$

and let $y_t = [\tilde{y}_t^1, \tilde{y}_t^2, \cdots, \tilde{y}_t^n]$ denote the observed variables.

A.1 Step 1: drawing the unobserved state variables $\theta_1 = \{\tau_t, \eta_t\}$ from $f(\theta_1 | Y_t, \theta_{\neq 1})$

In the first step of the Gibbs sampler, we draw the state variables in α_t which contains the unobserved permanent component τ_t and transitory component η_t. Since the model is in state-space form, the posterior distribution of the state vector can be obtained via the Kalman smoother proposed by Koopman and Durbin (2003). The posterior distribution of the state vector in the linear Gaussian state-space model is also Gaussian with conditional mean $\hat{\alpha}_t = E(\alpha_t | Y_T)$ and conditional covariance $V_t = Cov(\alpha_t | Y_T)$. The derivation of the conditional mean and covariance matrix follows classical forward recursion of Kalman filter and backward recursion of Kalman smoother. The classical Kalman filter recursion is

$$E(\alpha_{t+1}|Y_t) = a_{t+1} = A_t a_t + K_t \upsilon_t$$

$$Cov(\alpha_{t+1}|Y_t) = P_{t+1} = A_t P_t L_t^{'} + R_t Q_t R_t^{'}$$

where

$$K_t = A_t P_t C_t F_t^{-1}$$

$$\upsilon_t = y_t - C_t^{'} a_t$$

$$L_t = A_t - K_t C_t^{'}$$

$$F_t = C_t^{'} P_t C_t + H.$$

The smoothing backward recursion is

$$r_{t-1} = C_t F_t^{-1} \upsilon_t + L_t^{'} r_t$$

$$\hat{\alpha}_t = a_t + P_t r_{t-1}$$

$$N_{t-1} = C_t F_t^{-1} C_t^{'} + L_t^{'} N_t L_t$$

$$V_t = P_t - P_t N_{t-1} P_t$$

for $t = T, T-1, \ldots 1$, with $r_n = 0$, and $N_n = 0$.

Since the state variables are not all stationary, the unconditional mean and variance is not appropriate to initialize the Kalman filter. We adopt the exact recursions for calculating the mean and mean square error matrix of the state vector in the case where the initial state vector is diffuse. The initial state vector is specified as

$$\alpha_1 = a + T\delta + R_0 \varepsilon_0$$

where $\varepsilon_0 \sim N(0, Q_0)$, δ is a $q \times 1$ vector of unknown quantities. The $m \times q$ matrix T and the $m \times (m-q)$ matrix R_0 are selection matrices, and satisfy$T^{'} R_0 = 0$ and $\delta = T^{'} \alpha_1$. The vector δ is random and we assume that

$$\delta \sim N(0, \kappa I_q)$$

where $\kappa \to \infty$. Therefore the initial conditions for the state vector become

$$E(\alpha_1) = a, \; and \; Var(\alpha_1) = P$$

where $P = \kappa P_\infty + P_*$, $P_\infty = TT^{'}$, $P_* = R_0 Q_0 R_0^{`}$.

The mean squared error P_t in the classical filtering is decomposed into

$$P_t = \kappa P_{\infty,t} + P_{*,t} + O(\kappa^{-1})$$

where the term $P_{\infty,t}$ will disappear after a limited number of updates d in the exact Kalman filter. Therefore, the state filtering equations apply without change for $t > d$.

For the initial d time periods, the exact filtering equations are

$$a_{t+1} = A_t a_t + K_{\infty,t} \upsilon_t$$

$$P_{\infty,t+1} = A_t P_{\infty,t} L_{\infty,t}^{'}$$

$$P_{*,t+1} = A_t P_{*,t} L_{\infty,t}^{'} - K_{\infty,t} F_{\infty,t} K_{*,t}^{'} + R_t Q_t R_t^{'}$$

where

$$K_{\infty,t} = A_t P_{\infty,t} C_t F_{\infty,t}^{-1}$$

$$\upsilon_t = y_t - C_t^{'} a_t$$

$$L_{\infty,t} = A_t - K_{\infty,t} C_t^{'}$$

$$F_{\infty,t} = C_t^{'} P_{\infty,t} C_t$$

$$K_{*,t} = (A_t P_{*,t} C_t + K_{\infty,t} F_{*,t}) F_{\infty,t}^{-1}$$

$$F_{*,t} = C_t^{'} P_{*,t} C_t + H$$

with the initialization $a_1 = a$, $P_{*,t} = P_*$ and $P_{\infty,t} = P_\infty$.

The the initial d time periods state smoothing recursion is given by

$$\hat{\alpha}_t = a_t + P_{*,t} r_{t-1}^{(0)} + P_{\infty,t} r_{t-1}^{(1)}$$

$$\begin{aligned} V_t &= P_{*,t} - P_{*,t} N_{t-1}^{(0)} P_{*,t} - P_{\infty,t} N_{t-1}^{(1)} P_{\infty,t} \\ &\quad - (P_{\infty,t} N_{t-1}^{(1)} P_{\infty,t})^{'} - P_{\infty,t} N_{t-1}^{(2)} P_{\infty,t} \end{aligned}$$

where

$$r_{t-1}^{(0)} = L_{\infty,t}^{'} r_{t-1}^{(0)}$$

$$r_{t-1}^{(1)} = C_t (F_{\infty,t}^{-1} \upsilon_t - K_{*,t}^{'} r_t^{(0)}) + L_{\infty,t}^{'} r_t^{(1)}$$

$$N_{t-1}^{(0)} = L_{\infty,t}^{'} N_t^{(0)} L_{\infty,t}$$

$$N_{t-1}^{(1)} = C_t F_{\infty,t}^{-1} C_t' + L_{\infty,t}' N_t^{(1)} L_{\infty,t} - L_{\infty,t} N_t^{(0)} K_{*,t} C_t' - (L_{\infty,t} N_t^{(0)} K_{*,t} C_t')'$$

$$N_{t-1}^{(2)} = C_t (K_{*,t}' N_t^{(0)} K_{*,t} - F_{\infty,t}^{-1} K_{*,t} F_{\infty,t}^{-1}) C_t' + L_{\infty,t}' N_t^{(2)} L_{\infty,t} - L_{\infty,t} N_t^{(2)} K_{*,t} C_t' - (L_{\infty,t} N_t^{(1)} K_{*,t} C_t')'$$

for $t = d, d-1, \ldots 1$, with $r_d^{(0)} = r_d$, $r_d^{(1)} = 0$ and $N_d^{(0)} = N_d$, $N_d^{(1)} = N_d^{(2)} = 0$. With the above results, the conditional mean and covariance matrix are obtained and used to sample the state vector from $N(\hat{\alpha}_t, V_t)$.

A.2 Step 2: drawing the factor loadings $\theta_2 = \{\beta_i, \gamma_i, \sigma_{u_i}\}$

Conditioning on α_t and Y_t, factor loading in C_t' and variances in H can be drawn row by row in equation (22). Taking the ith measurement equation:

$$\tilde{y}_t^i = \beta_i C_{\tau,t}^i + \gamma_i C_{\eta,t}^i + u_t^{*i},$$

where $C_{\tau,t}^i$ and $C_{\tau,t}^i$ are obtained in the first step. Conditioning on all the variables, β_i, γ_i and variance of u_t^{*i} can be drawn following the conventional method for linear model. We state the priors in terms of their precision ($\bar{H}$). Then the prior of β_i, for example, is $\beta_i \sim N(\bar{\beta}_i, \bar{H}^\dagger)$, where $\bar{H}^\dagger$ is the inverse of $\bar{H}$. The data evidence is summarized as

$$\beta_i \sim N(\hat{\beta}_i, h^{-1}(X'X)^{-1})$$

where $h^{-1} = \sigma_{u^{*i}}^2$. Then we can draw β_i from the posterior distribution:

$$\beta_i \sim N((\bar{H} + \hat{H})^{-1}(\hat{H}\hat{\beta}_i + \bar{H}\bar{\beta}_i), (\bar{H} + \hat{H})^{-1}).$$

where $\bar{\beta}$ is the prior mean. Finally, γ_i can be drawn is a similar way.

The natural prior for the reciprocal of the variance of u_t^{*i} is

$$\iota s^2 h \sim \chi_\iota^2.$$

The data evidence is summarized as

$$((\tilde{y}^i - \beta_i C_\tau^i + \gamma_i C_\eta^i)'(\tilde{y}^i - \beta_i C_\tau^i + \gamma_i C_\eta^i))h \sim \chi_T^2.$$

We sample σ_{u_i} from the posterior distribution:

$$((\tilde{y}^i - \beta_i C_\tau^i + \gamma_i C_\eta^i)'(\tilde{y}^i - \beta_i C_\tau^i + \gamma_i C_\eta^i) + \iota s^2)h \sim \chi_{T+\iota}^2.$$

A.3 Step 3: drawing $\theta_3 = \{\phi_i\}$ from $f(\theta_3|\eta_t, \sigma^2_{\eta,t})$

Equation (23) can be broken down to $\phi(L)\eta_t = \sigma_{\eta,t}\varepsilon_{\eta,t}$, which is AR(p) model with heteroscedastic disturbance. Dividing by $\sigma_{\eta,t}$, one can obtain a standard linear regression model

$$\phi(L)\eta^*_t = \varepsilon_{\eta,t}$$

Then the autoregressive coefficients can be draw in a similar way as drawing β in step 2.

A.4 Step 4: drawing $\theta_4 = \{\sigma_{\tau,t}, \sigma_{\eta,t}\}$ from $f(\theta_4|Y_T, \theta_{\neq 4})$

We use Jacquier, Polson, and Rossi (1994)'s algorithm and Kim, Shephard and Chib(1998)'s Metropolis rejection method to draw the stochastic volatility, that is the unobserved components in equation (24). To sample the stochastic volatilities, notice that conditional on all the parameters and on the states vectors, the orthogonal innovations $x_{\tau,t} = \sigma_{\tau,t}\varepsilon_{\tau,t}$ and $x_{\eta,t} = \sigma_{\eta,t}\varepsilon_{\eta,t}$ are observable. We can proceed on a univariate basis because the stochastic volatilities are mutually independent. Jacquier, et. al. adopted a date-by-date blocking scheme and developed the conditional kernel. We take the $\sigma_{\tau,t}$ for example and $\sigma_{\eta,t}$ can be obtained in the same way.

Let $h_t = log(\sigma^2_{\tau,t})$, the conditional distribution of h_t is

$$p(h_t|h_{-t}, x_{\tau,t}, \sigma_{\nu\tau}) = p(h_t|h_{t-1}, h_{t+1}, \varepsilon_\tau, \sigma_{\nu\tau})$$

from Markov properties of stochastic volatility. By Bayes's theorem, the conditional kernel can be expressed as

$$\begin{aligned} p(h_t|h_{t-1}, h_{t+1}, x_{\tau,t}, \sigma_{\nu\tau}) &\propto p(x_{\tau,t}|h_t)p(h_t|h_{t-1})p(h_t|h_{t+1}) \\ &\propto h_t^{-1.5} exp(-\frac{x^2_{\tau,t}}{2h_t}) exp(-\frac{(lnh_t - \frac{1}{2}(lnh_{t-1} + lnh_{t+1}))^2}{\sigma^2_{\nu\tau}}). \end{aligned}$$

Since the normalization constant in the kernel is costly to compute, we use a Metropolis step to generate a sequence of random draws from the approximate distribution. The Metropolis sampler involves calculating a ratio to decide accepting or rejecting the draw from the approximation distribution. Here we use the approximation distribution

$$q(h_t) \propto N(\mu_t, \sigma^2_{\nu\tau}/2)$$

where

$$\mu_t = \frac{h_{t-1} + h_{t+1}}{2} + \frac{\sigma^2_{\nu\tau}}{4}(x^2_{\tau,t} exp(-\frac{h_{t-1} + h_{t+1}}{2}) - 1).$$

The acceptance probability is specified as

$$r_t = \frac{f^*}{g^*}$$

where

$$logf^* = -\frac{1}{2}h_t - \frac{x_{\tau,t}^2}{2}exp(-h_t),$$

and

$$log(g^*) = -\frac{1}{2}h_t - \frac{x_{\tau,t}^2}{2}\{exp(-h_t^*)(1+h_t^*) - h_t exp(-h_t^*)\},$$

$h_t^* = \frac{h_{t-1}+h_{t+1}}{2}$. Then the accept-reject procedure to sample h_t is first to propose a value of h_t from $q(h_t)$ and second to accept this value with probability r_t. If the value is rejected, we set $h_t^m = h_t^{m-1}$, where m denote the mth iteration.

A.5 Step 5: drawing $\theta_5 = \{\sigma_{\nu\tau}, \sigma_{\nu\eta}\}$ from $f(\theta_5|\theta_4)$

Conditioning on the log-volatilites, $\sigma_{\nu\tau}^2$ and $\sigma_{\nu\eta}^2$ in covariance matrix W can be drawn from conjugate inverse gamma distribution as in step 2. For example, the dynamics of log-volatilites is random walk with only $\sigma_{\nu\tau}$ unknown. Assume the prior for $\sigma_{\nu\tau}$ is

$$p(\sigma_{\nu\tau}) = IG(\frac{\upsilon_0}{2}, \frac{\delta_0}{2})$$

Then the posterior inverse gamma is

$$p(\sigma_{\nu\tau}|h_T) = IG(\frac{\upsilon_1}{2}, \frac{\delta_1}{2})$$

where $\upsilon_1 = \upsilon_0 + T$, and $\delta_1 = \delta_0 + \sum_{t=1}^{T}(\Delta lnh_t^2)$.

Table 1: Augmented Dicky-Fuller Unit Root Test

	1970 to 1983	1984 to 2016	1970 to 2016
GSCI inflation	-24.13	-42.021	-48.727
CPI inflation	-1.861	-4.823	-1.914
PCE infaltion	-1.578	-3.06	-1.722
GDP Deflator inflation	-2.57	-3.816	-2.23

Note: The ADF test includes a constant. The number of lags is chosen based on SIC criteria.

Table 2: Autocorrelations of the First Difference of Inflation

	1970 to 1983			1984 to 2016			1970 to 2016		
lags	1	2	3	1	2	3	1	2	3
GSCI inflation	-0.513	0.008	0.032	-0.476	-0.006	-0.011	-0.523	0.013	0.042
CPI inflation	-0.461	0.12	-0.111	-0.158	-0.294	-0.080	-0.267	-0.153	-0.085
PCE inflation	-0.336	-0.048	-0.099	-0.233	-0.226	-0.059	-0.264	-0.175	-0.068
GDP inflation	-0.219	-0.103	-0.031	-0.404	-0.066	-0.009	-0.303	-0.074	-0.029

Table 3: Factor loading Estimates

Parameters	Posterior Mean
Common Persistent component	
β_1	0.262034
β_2	1.151936
β_3	1.103199
β_4	1.0
Transitory component	
γ_1	-8.687555
γ_2	13.347247
γ_3	31.176756
γ_4	1.0

Note: β_1 and γ_1 are the factor loadings on GSCI commodity inflation; β_2 and γ_2 are the factor loadings on CPI inflation; β_3 and γ_3 are the factor loadings on PCE inflation; β_4 and γ_4 are the factor loadings on GDP deflator inflation. The estimated AR coefficient in equation (3) is -0.69.

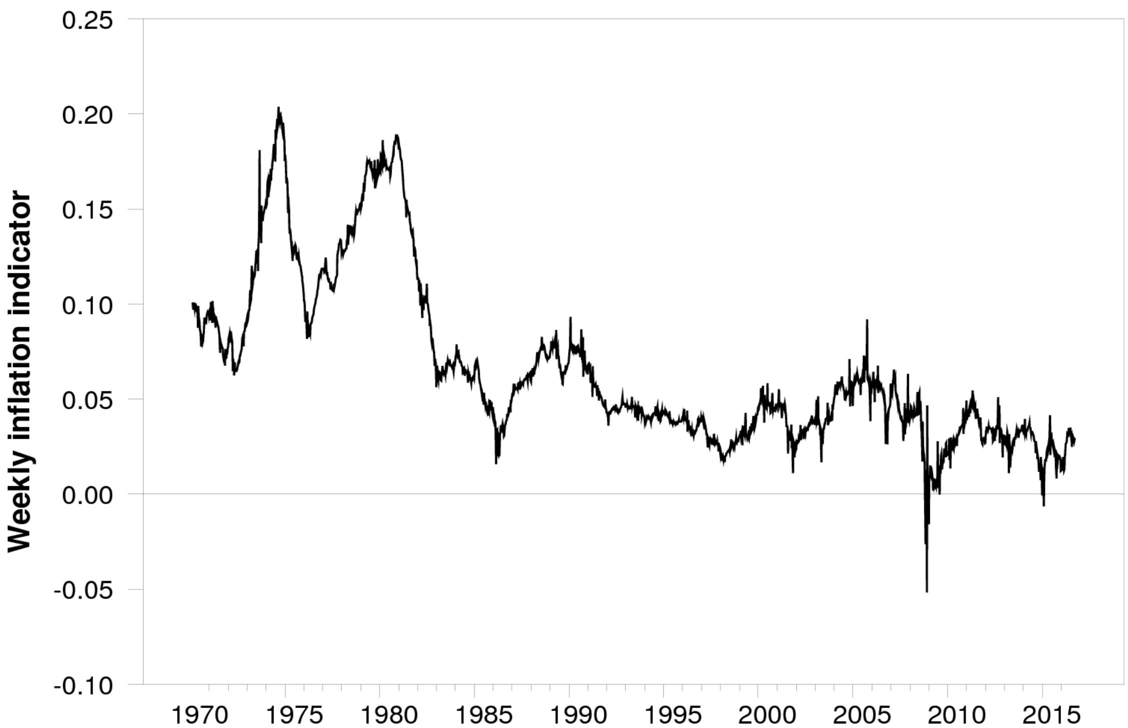

Figure 1: Extracted Weekly Inflation Index

Note: The weekly inflation indicator is the weekly sum of the daily inflation factors.

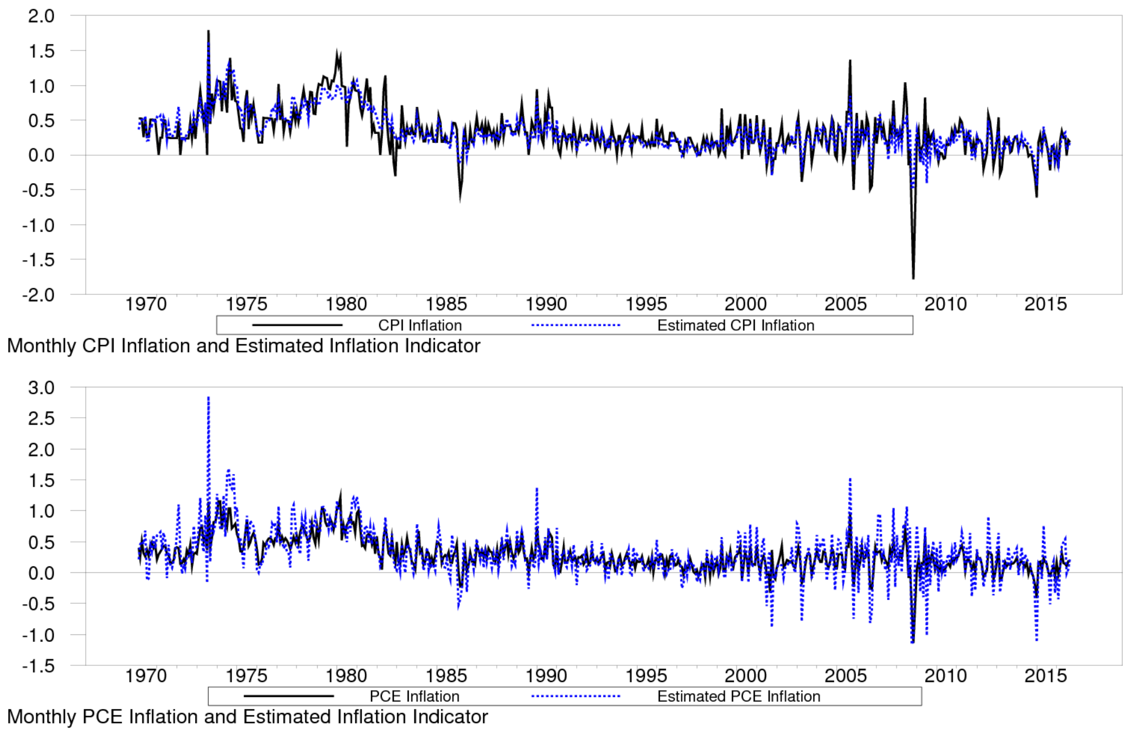

Figure 2: Estimated Inflation Indicator, CPI and PCE Deflator

Note: The upper graph shows estimated monthly trend inflation along with CPI inflation; the lower one shows estimated monthly trend inflation along with PCE inflation.

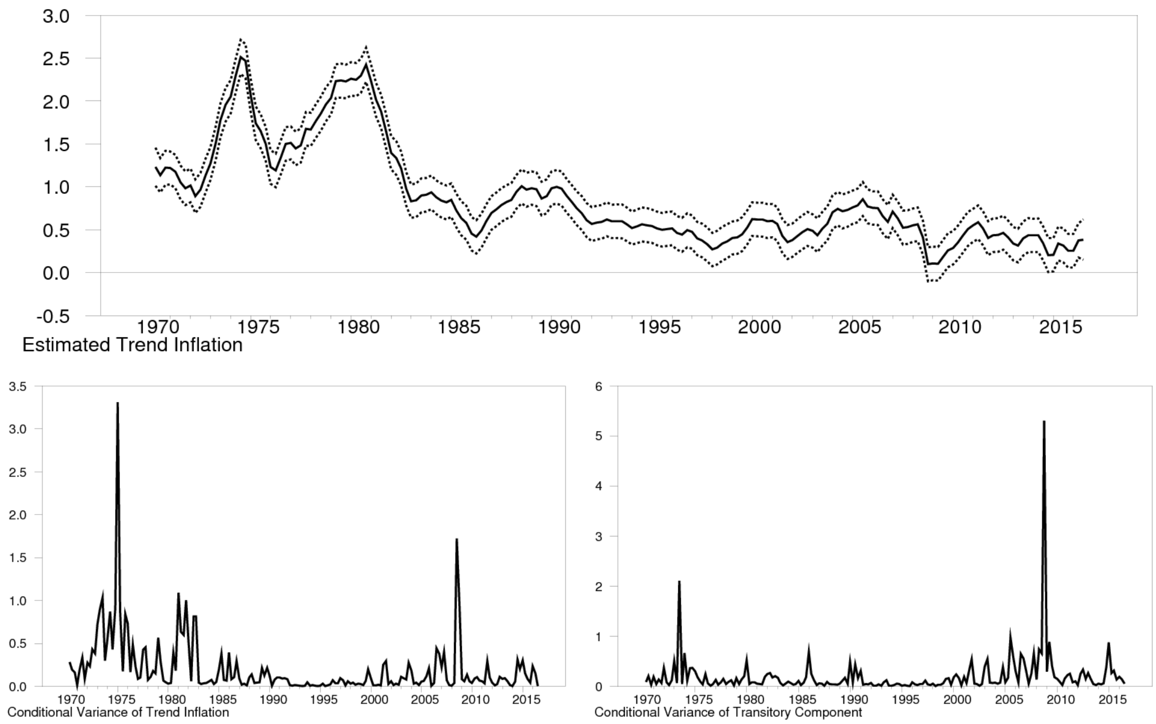

Figure 3: Estimated Trend Inflation and Stochastic Volatility

Note: The upper graph shows the weekly trend inflation; the lower left graph depicts the stochastic volatilies of persistent component; the lower right graph depicts the stochastic volatilities of transitory component of inflation.

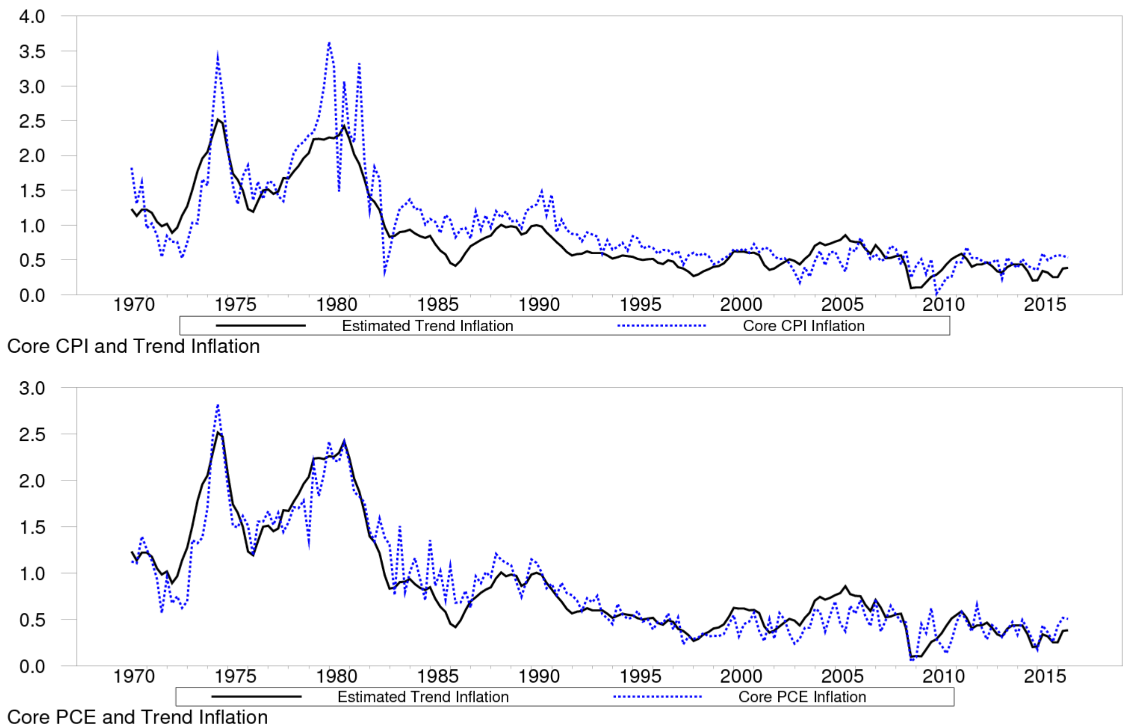

Figure 4: Trend Inflation and Core Inflation

Note: Trend inflation are monthly trend inflation component .

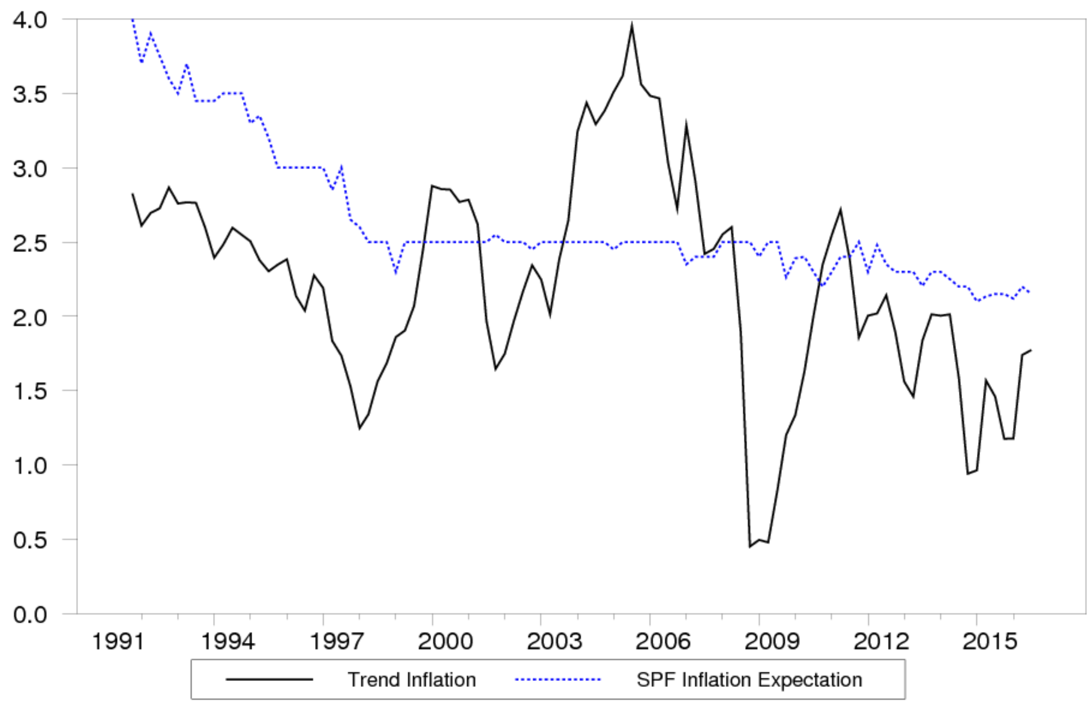

Figure 5: Estimated Trend Inflation and Inflation Expectation

Note: Long-term inflation expectation is measured with Survey of Professional Forecasters 10-years inflation expectation.