FRICTIONAL GOODS MARKETS: THEORY AND APPLICATIONS∗

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Abstract
We analyze economies where informed buyers direct their search to sellers posting favorable terms and uninformed buyers search randomly, nesting existing theories that have one or the other. One application concerns the cost of inflation, which was previously measured to be high in models using random search and low in models using directed search. With the fractions of informed and uninformed buyers disciplined by data, we find the cost is quite low. This is in part because, in addition to the usual costs, we uncover a novel benefit from inflation: it improves market composition by impinging more heavily on inefficient high-price sellers. We also discuss the relationship between inflation, markups and price dispersion. Other applications analyze theoretically and numerically the impact of changes in credit conditions and information frictions.

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[Some consumers] receive no information on the price in this market. (It is natural to think of them as tourists, having no local information.) A second type of consumer (resident) receives some information... It would be interesting to develop models with both types of consumers and, I suspect, would result in a different structure of equilibrium. Diamond (1971)

1 Introduction

A genuinely classic issue in economics concerns the impact of inflation on welfare, as well as on observables like price dispersion, markups, etc. To study such phenomena, perhaps especially welfare, we believe that it is important to use a microfounded theory where money ameliorates explicit trading frictions since, after all, inflation is a tax on monetary exchange. This project develops a framework where money is a medium of exchange, and uses it to study several issues, both theoretically and quantitatively. A central feature is the role of information: consumers can either be informed or uninformed about sellers’ terms of trade, and the former direct their search while the latter search randomly. In addition, we allow agents to use credit (at a cost, so it does not completely drive out currency) to incorporate a primary effect of inflation: it provides an incentive to substitute out of cash and into alternative payment instruments.

In terms of motivation, first note that at some point there emerged a consensus that the impact of inflation was small, from studies using Walrasian theory augmented with reduced-form devices like money-in-utility or cash-in-advance assumptions, e.g., Cooley and Hansen (1989) or Lucas (2000). See Rocheteau and Nosal (2017) and Diercks (2017) for more discussion and references, but as a rough average over many studies, eliminating a 10% annual inflation was found to be worth around 0.5% of consumption. That was challenged by economists working on models based on search and bargaining, e.g., Lagos and Wright (2005), where the same policy is worth closer to 5.0% of consumption – an order of magnitude higher. However, in similar environments using competitive search, which means directed search and price posting rather than random search and bargaining, e.g., Rocheteau and Wright (2005, 2009), the result goes back down to around 1%.
To understand this, we nest models with directed search and posting and those with random search and bargaining. In one version, following Lester (2011), a fraction $\lambda$ of buyers are informed about the terms offered by all sellers, while the rest are not and search at random. In another version, using noisy search as in Burdett and Judd (1983), a fraction $\lambda$ observe the terms offered by $h > 1$ sellers but not all of them, while others see $h = 1$ (this is related to Acemoglu and Shimer (2000) in a much simpler setup). In both versions, ex-ante homogeneous sellers behave differently in equilibrium: some post a low price $p$ and high quantity or quality $q$ to make informed buyers want to visit them; others cater only to the less-well informed at less-attractive terms. However, the first version has only two values of $(p, q)$ posted, while the second has a density of $(p, q)$ on an interval plus a mass of firms posting the most attractive terms.

Different from Burdett-Judd or Lester, we give buyers an option to bargain that they may or may not exercise in equilibrium.\(^1\) Also, those authors use basically static, partial equilibrium, nonmonetary models, while we require dynamic, general equilibrium, monetary theory. We use a New Monetarist model, and in particular build on Lagos and Wright (2005), although in principle we could use Shi (1997), Molico (2006) or Menzio et al. (2013), which are similar in spirit if technically different.\(^2\) As discussed in the survey by Lagos et al. (2017), this approach is natural for studying money and/or credit because it tractably captures an asynchronization of expenditures and receipts, and moreover, it easily accommodates random, directed or noisy search, as well as bargaining or posting, and this project is all about understanding how these details of market microstructure matter.

In these models, limited commitment and imperfect monitoring or record keeping hinder credit, and thereby make monetary exchange useful. However, we want money

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1 Papers that are related, in the sense of analyzing whether agents bargain or post, and in the latter case whether buyers or sellers post, as well as different specifications for the matching process, include Acemoglu and Shimer (2000), Craig and Rocheteau (2008), Gill and Thanassoulis (2009), Delacroix and Shi (2013), Stacey (2017), Moen et al. (2017) and Shi and Delacroix (2018).

2 An advantage of Molico (2006) and Menzio et al. (2013) is that they deliver endogenous distributions of liquidity that can affect price dispersion in interesting ways, but that also renders them more complicated. Still, as suggested by a referee, we discuss a version with an endogenous distribution, but relegate details to a Supplemental Appendix.
and credit. Why? First, as explained in the discussion surrounding Lemma 1, pure-currency economies with posting display a nuisance indeterminacy of steady state that is eliminated with costly credit. Second, it adds discipline by generating statistics, like the ratio of credit to money purchases, that we can compare to data. Third, from a public finance perspective, a serious analysis of inflation should give consumers the opportunity to substitute out of cash, and not just into autarky, which is the only option in pure-currency economies. Fourth, we derive a few results on credit conditions that one would miss in nonmonetary theories. For all these reasons, plus realism, we want both, although obviously pure-credit or pure-currency economies are special cases.

To highlight an analytic result, note that in very many models the optimal monetary policy is the Friedman rule, i.e., the inflation rate corresponding to a nominal interest rate of $i = 0$. But here $i > 0$ can be desirable due to market-composition effects that have not been analyzed elsewhere. Namely, in equilibrium there are low- and high-price sellers, and the latter are inefficient, surviving only by exploiting uninformed buyers. Inflation is effectively a tax impinging more heavily on these inefficient sellers, partially offsetting its negative effects, and on net $i > 0$ can be optimal in the baseline model. Moreover, in a version where agents choose to be informed at a cost, and in the version with noisy search, $i > 0$ is always optimal. Below we explain in terms of information externalities and second-best theory how this affects the welfare results, as well as some empirical observations deemed interesting in the literature.

In summary, the paper has these components: (1) A workhorse model in monetary

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3See Gu et al. (2016) and references therein for more on money and credit. As in that paper, credit here is a payment instrument, like cash, not a way to smooth over the life cycle, like mortgages or student loans. According to The Economist (Oct 15, 2016), US merchants paid over $40 billion to process charge card transactions in 2015, and despite policy reforms aimed at reducing this, the cost keeps going up. So payment systems are not costless. In terms of modeling, using transaction costs does not constitute a “deep” theory of money and credit, but it has much precedent (Rocheteau and Nosal (2017) give many citations). As to what the cost represents, we are agnostic. A narrow interpretation pushed by Gomis-Porqueras et al. (2014) is that credit makes it harder to avoid taxation. Wallace (2013) emphasizes monitoring. Also relevant are costs of record keeping, screening and enforcement.

4We can match the finding in Benabou (1992b), e.g., of a negative relation between markups and inflation through a channel different from Benabou (1992a) or Head and Kumar (2005). We can also match in a novel way the positive relation between price dispersion and inflation in Parsley (1996) or Debelle and Lamont (1997), although note that other papers get different results, including Reinsdorf (1994) and Caglayan et al. (2008).
economics is extended by having a fraction $\lambda$ of agents more informed than others. (2) Ultimately $\lambda$ is determined from data, and we use this to quantitatively reassess the cost of inflation. (3) Using information theory provides a gateway to other applications – e.g., we show the price level and price dispersion can actually increase with $\lambda$. (4) Letting agents use money and credit is technically convenient, empirically relevant, and provides another gateway to applications – e.g., we show agents use too much credit and too little cash. (5) Incorporating endogenous information and noisy search yields new insights and affect quantitative results in interesting ways. (6) Even with all these ingredients the setup is tractable, delivering clean results on existence, uniqueness, efficiency and comparative statics. (7) The whole is greater than the sum of its parts – e.g., with $\lambda = 0$ or $\lambda = 1$, $i = 0$ is always optimal; with $\lambda \in (0, 1)$ it is not.

As regards quantitative findings: in a benchmark model eliminating 10% inflation is worth 1.1% of consumption; with endogenous information it is worth 0.7%; and with noisy search it is worth 0.3%. These numbers are low because, while inflation has costs, it also has benefits due to the above-mentioned composition effects. To see how credit matters, eliminating currency has a welfare cost of about 4% of consumption; this is sizable but far less than in the model without credit, where the number is closer to 40%. We conclude from this that one can go far wrong by studying inflation in models without credit, or, symmetrically, studying credit without money. In terms of information, going from the calibrated $\lambda$ to having all buyers informed is worth around 1% or 2% of consumption; going from $\lambda = 0$ to $\lambda = 1$ is worth much more. We conclude that in principle information frictions can be quantitatively very important.

Sections 2 and 3 present a baseline model and results. Section 4 focuses on information. Section 5 discusses the calibration and quantitative findings. Section 6 concludes.\(^5\)

\(^5\)An alternative (our original) motivation is understanding retail markets better. In addition to having money and credit, plus directed and random search, the framework is built to capture these salient features of retail: price dispersion; quality dispersion; high and variable markups; and mainly posted prices but also some bargaining. Some of these features are self evident; others require documentation. On price and quality dispersion, see Ellison and Ellison (2005, 2014), and Jaimovich et al. (2015). Data on markups and the use of money and credit are discussed below. For related work on retail see, e.g., Faig and Jerez (2005), Gourio and Rudanko (2014), Paciello et al. (2014) or Liu et al. (2015).
2 The Benchmark Model

Each period in discrete time has two subperiods: first there is a decentralized market, or DM, with frictions detailed below; then there is a frictionless centralized market, or CM. As mentioned above this is a natural environment for studying money and credit because agents face an asynchronization of expenditures and receipts: sometimes they want something in the DM, but as their income accrues in the CM, they either have to work to acquire cash before consuming, or use debt to work after consuming. A measure of infinite-lived agents are called buyers, and a measure $N$ are called sellers. In the DM buyers want to acquire a good $q$ produced by sellers. In the CM all agents trade a different good $x$ and labor $\ell$, plus pay taxes, settle debts and adjust cash balances. Period utility is $U(x) + u(q) - \ell$ for buyers and $U(x) - q - \ell$ for sellers, with the usual properties. All agents discount at $\beta \in (0, 1)$ between the CM and DM.

Buyers and sellers meet bilaterally in the DM and trade $(p, q)$, where $p$ is the payment denominated in the next CM’s numeraire. If $q$ represents quantity then $P = p/q$ is the unit price, but if $q$ represents unobserved quality then we would say $p$ is the price: considering two wines at $10 and $100 a bottle, someone not knowing the latter is superior would say it has a higher price, but it might actually be a better deal. In any case we call $P = p/q$ the markup. The DM partitions sellers into submarkets with the same $(p, q)$, and in each submarket agents match randomly with arrival rates depending on tightness, or the buyer-seller ratio $n$, in that submarket. Thus a submarket is identified by $\Gamma = (p, q, n)$. This is all fairly standard. What is less standard is this: each period, with probability $\lambda$ a buyer is informed, meaning (for now) he sees $\Gamma$ in every submarket, and with probability $1 - \lambda$ he uninformed, meaning he only knows the distribution across submarkets. Here $\lambda$ is exogenous; in Section 4.2 it is endogenous. In either case, uninformed buyers are called tourists and informed buyers locals.\footnote{We use these labels in deference to Diamond (1971), but one need not take them literally. Suppose Mr. A knows which shops have good deals on apples but not bananas, and vice versa for Ms. B. On days when Mr. A and Ms. B both need apples, he is a local and she is a tourist, and vice versa when they need bananas. This is equivalent to having different locations, and agents randomly transiting between them, where they know more about prices in some locations than others.}
As a benchmark, buyers in the CM do not know if they will be informed in the next DM; later we let them know. Anticipating some results, as in Lester (2011), equilibrium in the baseline model features two submarkets, one with local shops catering to the informed, and one with tourist shops serving only the uninformed. Local shops offer favorable terms to attract informed customers. Since their terms are attractive, buyers at local shops do not bargain, but accept the posted \((p, q)\). Note that this is true for locals and for tourists that end up at local shops by pure luck. Tourist shops only get tourists, and when they do, we can interpret them as bargaining. However, as usual in strategic bargaining theory, agents are never observed bargaining on the equilibrium path: a buyer always accepts the terms a seller offers, but the threat of bargaining disciplines the offer. The main impact of this assumption is that it precludes sellers from extracting all the rents from uniformed consumers.

Fig. 1 depicts the DM structure, where \(N_L\) and \(N_T\) are the measures of local and tourist shops. If \(\omega_j\) denotes the ex ante probability a buyer goes to submarket \(j\), before knowing if he will be informed, then

\[
\omega_T = \frac{(1 - \lambda)N_T}{N_L + N_T} \quad \text{and} \quad \omega_L = \frac{N_L + \lambda N_T}{N_L + N_T}.
\]

Thus, \(\omega_T\) is the probability of being uninformed times the probability of finding a tourist shop. As there is a measure 1 of buyers, \(\omega_j\) is also the measure of buyers in submarket \(j\). Market tightness in each submarket is

\[
n_T = \frac{1 - \lambda}{N_T + N_L} \quad \text{and} \quad n_L = n_T + \frac{\lambda}{N_L}.
\]
Within a submarket agents meet according to a CRS matching technology: for a seller, the probability of meeting a buyer is $\alpha(n)$; for a buyer the probability of meeting a seller is $\alpha(n)/n$. As is standard, $\alpha(n)$ is strictly increasing if $\alpha(n) < 1$, $\alpha(n)/n$ is strictly decreasing if $\alpha(n) < n$, $\alpha(0) = 0$, and $\alpha(\hat{n}) = 1$ for some $\hat{n} \in (0, \infty]$. We assume $\alpha''(n) < 0$ for $n < \hat{n}$ which ensures the first-order conditions are necessary and sufficient. Notice $n_L \geq n_T$, with $n_L > n_T$ if $\lambda > 0$; hence local submarkets are tighter.

Consider a buyer in the CM (a seller is similar). His state is his net worth, $A = \phi m + \tau - d - \gamma(d)$, where $m$ is money brought from the previous DM, $\phi$ is the price of $m$ in terms of numeraire, $\tau$ is a lump sum transfer used to inject currency, $d$ is debt from the previous DM, and $\gamma(d)$ is the transaction cost from using debt, where $\gamma'(d), \gamma''(d) > 0 \quad \forall d > 0$. We assume $\gamma(0) = \gamma'(0) = 0$ to guarantee $d > 0$, since as usual interior solutions simplify the presentation. Also, while the transaction cost here is borne by buyers, the results are similar if it is instead borne by sellers, with one caveat: if the cost is borne by sellers they may in principle want to charge more for using credit, a practice that is sometimes but not always banned by platform rules or state regulations. In any case, we abstract from that complication in this paper.

A buyer’s CM problem is therefore

$$W(A) = \max_{x,\ell,\hat{m}} \left\{ U(x) - \ell + \beta V(\phi \hat{m}) \right\} \quad \text{st} \quad x = w\ell - \phi \hat{m} + A,$$

where $w$ is the wage, $\hat{m}$ is money taken out of the CM, and $V$ is the DM value function, depending on real balances at tomorrow’s prices, $z \equiv \phi \hat{m}$. We focus on stationary equilibrium, where $W$ and $V$ are independent of time, and to ease notation adopt a CM technology $x = \ell$ since that means $w = 1$. Then, after eliminating $\ell$, we get

$$W(A) = A + \max_x \left\{ U(x) - x \right\} + \max_z \left\{ -(1+\pi)z + \beta V(z) \right\},$$

where $1 + \pi = \phi/\phi_{+1}$ is inflation, equal to the growth of the money supply in stationary equilibrium. For buyers the FOC for $z > 0$ is $1 + \pi = \beta V'(z)$, and for sellers $z = 0$ because they do not need liquidity in the DM. For both, $z$ is independent of $A$ and $W'(A) = 1$, as usual in models following Lagos and Wright (2005).
This means buyers’ DM trading surplus is \( S = u(q) - p - \gamma(d) \). Similarly, sellers’ DM surplus is net revenue \( R = p - q \), and expected profit is \( \Pi = \alpha(n)R - k \), where \( k \) is a fixed cost for sellers entering the DM. Let \( q^* \) be the efficient quantity defined by \( u'(q^*) = 1 \). Then, to guarantee some sellers enter, impose

\[
k < (1 - \eta)[u(q^*) - q^*],
\]

where \( \eta = \min_n \{\eta(n)\} \) and \( \eta(n) = n\alpha'(n)/\alpha(n) \) the elasticity of matching. As mentioned earlier, in equilibrium there are at most 2 types of sellers: local shops that set terms to attract the informed buyers; and tourist shops that cater exclusively to the uninformed. Given there are at most 2 types of submarkets, by CRS we can assume there is 1 representative submarket of each type. Also, while (3) implies \( N_L > 0 \), it is possible to have \( N_T > 0 \) or \( N_T = 0 \), as discussed below.

Consider first submarket \( L \). As standard, the outcome can be found by maximizing local buyer’s expected surplus subject to free entry by sellers:

\[
\max_{p,q,n} \left\{ \frac{\alpha(n)}{n} [u(q) - p - \gamma(p - z)] \right\} \text{ st } \alpha(n)(p - q) = k.
\]

Then \( \Gamma_L = (p_L, q_L, n_L) \) solves \( k = \alpha(n_L)(p_L - q_L) \) plus the FOC’s wrt \( q \) and \( n \),

\[
\begin{align*}
  u'(q_L) &= 1 + \gamma' (p_L - z) \\
  p_L - q_L &= \frac{(1 - \eta_L)[u(q_L) - q_L - \gamma(p_L - z)]}{\eta_Lu'(q_L) + 1 - \eta_L}.
\end{align*}
\]

By (6), the seller gets a fraction \( (1 - \eta_L)/[\eta_Lu'(q) + 1 - \eta_L] \) of the DM trade surplus.

Consider next submarket \( T \), where sellers bargain, or post the bargaining outcome. We use Kalai bargaining, which has advantages over the Nash solution in models with liquidity considerations (Aruoba et al. (2007)). Kalai’s solution is found by maximizing the buyer’s surplus subject to him getting a share \( \theta \) of the total surplus:

\[
\max_{p,q} \{u(q) - p - \gamma(p - z)\} \text{ st } p - q = (1 - \theta)[u(q) - q - \gamma(p - z)].
\]

Now \( \Gamma_T = (p_T, q_T, n_T) \) solves \( \alpha(n_T)(p_T - q_T) = k \) plus the FOC’s

\[
\begin{align*}
  u'(q_T) &= 1 + \gamma' (p_T - z) \\
  p_T - q_T &= (1 - \theta)[u(q_T) - q_T - \gamma(p_T - z)].
\end{align*}
\]
Conveniently, the conditions for $\Gamma_T$ are the same as those for $\Gamma_L$, except $1 - \theta$ replaces $(1 - \eta_L)/[\eta_L u'(q) + 1 - \eta_L]$. We assume $\theta \leq \eta(n)$ for all $n$ so that buyers can bargain at local shops, but prefer to accept $(p_L, q_L)$.

Buyers’ DM payoff is the CM continuation value plus the expected surplus,

$$ V(z) = W(z + \tau) + \omega_L \frac{\alpha(n_L)}{n_L} [u(q_L) - p_L - \gamma(p_L - z)] $$

$$ + \omega_T \frac{\alpha(n_T)}{n_T} [u(q_T) - p_T - \gamma(p_T - z)] . 

(10)

Trade depends on $z$ at tourist shops, where customers bargain, but not local shops. Therefore,

$$ V'(z) = 1 + \omega_L \frac{\alpha(n_L)}{n_L} \gamma'(d_L) + \omega_T \frac{\alpha(n_T)}{n_T} [u'(q_T)q_T' - p_T' - \gamma'(d_T)(p_T' - 1)] , $$

(11)

where $q_T'$ and $p_T'$ are derivatives wrt to $z$. Inserting (11) into $(1 + \pi) = \beta V'(z)$, we get

$$ i = \omega_L \frac{\alpha(n_L)}{n_L} \gamma'(d_L) + \omega_T \frac{\alpha(n_T)}{n_T} [u'(q_T)q_T' - p_T' - \gamma'(d_T)(p_T' - 1)] , $$

(12)

where $i$ is the nominal interest rate defined by the Fisher equation $1 + i = (1 + \pi) / \beta$.

(As usual, $i$ is the return agents require in the next CM to give up a dollar in this CM, and we can price such trades whether or not they occur in equilibrium.)

The Fisher equation makes it equivalent to use $i$ or $\pi$ as our policy variable. In any case, we adopt the usual restriction $i > 0$, or $\pi > \beta - 1$, but consider the limit $i \to 0$, or $\pi \to \beta - 1$, called the Friedman rule. Given all this we have the following:

**Definition 1** A stationary equilibrium is a nonnegative list $\langle \Gamma_L, \Gamma_T, z \rangle$ such that $\Gamma_j$ solves the relevant conditions in each submarket given $z$, and $z$ solves the money demand problem given $(\Gamma_L, \Gamma_T)$. It is a monetary equilibrium if $z > 0$.

### 3 Baseline Analytic Results

While we are mainly interested in monetary equilibrium, let us mention these results for pure-credit economies (see the Supplemental Appendix for a proof):

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7 After finding equilibrium we check $(p_L, q_L)$ is such that buyers do not want to bargain on the equilibrium path or off – i.e., they do not want to bring a different $z$ and bargain.
**Proposition 1** Equilibrium with $z = 0$ exists and is unique. If $N_L, N_T > 0$ then $n_L > n_T$, $p_L < p_T$, $P_L < P_T$, $R_L < R_T$, and $q_L > q_T$.

Much of this carries over to monetary economies – e.g., local shops have lower revenue per unit, $R_L < R_T$, and since free entry implies $\Pi_L = \Pi_T$ they must make it up on the volume, which means $n_L > n_T$. Of course this is only relevant if $N_T > 0$, which naturally requires $\lambda$ not too big.

Moving to monetary equilibrium, we have the following convenient results (proofs of all results below are in the Appendix):

**Lemma 1** In monetary equilibrium (i) $z < \hat{p} = \max \{p_L, p_T\}$; (ii) $V''(z) < 0 \forall z < \hat{p}$.

From (i), buyers must cash out in some shops. Below we show $p_T > p_L$, so they cash out in tourist shops, while they may or may not in local shops. Also, (ii) means (12) has unique solution, due to costly credit, which is one reason to include it.\(^8\)

For monetary equilibrium we seek $z > 0$ such that $T(z) = i$, where $T(z)$ is given by the RHS of (12). For $N_T(z) > 0$, this can be written as

$$T(z) = \frac{\omega_L \alpha(n_L)}{n_L} \gamma'(p_L - z) + \frac{\omega_T \alpha(n_T)}{n_T} [u'(q_T) q_T' - p_T' - \gamma'(p_T - z)(p_T' - 1)]; \quad (13)$$

for $N_T(z) = 0$ it can be written as

$$T(z) = \frac{\alpha(n_L)}{n_L} \gamma'(p_L - z). \quad (14)$$

**Proposition 2** Monetary equilibrium exists and is unique if $i$ is not too big.

**Proposition 3** In monetary equilibrium the comparison between local and tourist shops in Proposition 1 holds.

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\(^8\)In particular, $V(z)$ is twice differentiable with $V''(z) < 0$ due to a smooth cost of credit $\gamma(d)$, which avoids an indeterminacy of monetary steady states analyzed in a series of papers following Green and Zhou (1998). See Jean et al. (2010) for details, but to understand the idea, heuristically, consider indivisible DM goods. If sellers think all buyers bring $m = X$ to market, they all post $p = X$ as long $X$ is not too small; and if they all post $p = X$, buyers all bring $m = X$ as long as $X$ is not too big. So $p = m = X$ is an equilibrium for any $X$ in some range. A similar indeterminacy arises with divisible goods. These problems are eliminated, however, with costly credit. Intuitively, for the case with indivisible goods, even if all sellers charge $p = X$, a buyer can bring $m < X$ and put the difference on his credit card. The demand for money is then driven by the desire to reduce the costly use of credit.
We now give the effects of monetary policy, which may be surprising in that they are all unambiguous except $\partial P_j/\partial i$ (and examples show that can go either way). In competitive search theory, generally, it is not clear the solution is monotone or even continuous in parameters, but we make progress using the methods of monotone comparative statics (see Choi (2015) for a discussion in the context of liquidity models).

**Proposition 4** In monetary equilibrium $z$ and $\Gamma_j$ are almost-everywhere differentiable wrt $i$, with $\partial z/\partial i < 0$, $\partial p_j/\partial i < 0$, $\partial q_j/\partial i < 0$, $\partial n_j/\partial i > 0$ and $\partial d_j/\partial i > 0$, while $\partial P_j/\partial i$ is ambiguous.

Since buyers are constrained in at least some trades, $N_T = 0$ implies $q_L < q^*$, and $N_T > 0$ implies $q_T < q^*$ but we can have $q_L < q^*$ or $q_L = q^*$. Fig. 2 shows the different outcomes in $(\lambda, i)$ space. Area $A_1$ has $N_L, N_T > 0$, with credit used in tourist but not local shops, as $i$ is sufficiently low that buyers carry enough cash to pay $p_L$. Area $A_2$ has $N_L, N_T > 0$, with credit used in all shops, as higher $i$ makes them carry less cash. Area $A_3$ has $N_T = 0$, as $\lambda$ is high enough to eliminate tourist shops. Also note that for a given $\lambda$ there is a bound for $i$ above which monetary equilibrium breaks down.9

Now consider efficiency. Since CM payoffs are constant with respect to the interventions considered here, welfare is well measured by the sum of DM payoffs net of

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9The calibration below puts us in $A_2$, but even before that an empirical point can be made about the finding in Benabou (1992b) of a negative relation between markups and inflation. While the impact of $i$ on $P_j$ is ambiguous, even if $\partial P_T/\partial i > 0$ and $\partial P_L/\partial i > 0$, average $P$ in the DM can fall with $i$ since $N_T$ goes down and $N_L$ up. Also, average $P$ across the DM and CM can fall since the former shrinks. We show later how this pans out numerically, but the point is that it matters which markup we consider.
entry, production and transaction costs:

$$\Omega = \sum_j \omega_j \left\{ \frac{\alpha(n_j)}{n_j} [u(q_j) - q_j - \gamma (p_j - z)] - \frac{k}{n_j} \right\}.$$  \hspace{1cm} (15)

For two reasons $N_T > 0$ is undesirable. First $q_T < q_L$, which means tourist shops are inefficient compared to local shops on the intensive margin. Second $n_T < n_L$, so with $\alpha(n)$ concave the number of DM meetings is not maximized given the measures of buyers and sellers, which is inefficient on the extensive margin. Hence, it would be good to regulate or tax submarket $T$ away, but we do not take this advice too seriously, because it may be difficult to identify those shops or dictate their terms of trade.\(^{10}\)

Absent such draconian measures, consider monetary policy. It is easy to check $q_L, q_T \to q^*$ as $i \to 0$, but that need not give full efficiency due to composition effects (the mix of $N_L$ and $N_T$). In fact, we claim inflation is always bad when $\lambda = 1$ or $\lambda = 0$, but can be good when $\lambda \in (0, 1)$, which as mentioned above shows the value of an integrated model. To verify this consider area $A_1$ in Fig. 2, where higher $i$ has two opposing effects: (1) $z$ and hence the surplus in submarket $T$ falls; and (2) $N_T$ falls, making buyers less likely to end up in tourist shops. There is an area $A_1^* \neq \emptyset$ in the lower right of $A_1$ where the net effect is positive, since $N_T$ is small, and hence the loss from reducing the surplus is dominated by the gain from downsizing submarket $T$.\(^{11}\)

**Proposition 5** In monetary equilibrium, $i = 0$ is optimal for $\lambda$ near 0 or 1, while $i > 0$ is optimal for $(\lambda, i) \in A_1^*$.

Similar second-best logic applies to the cost of credit. From an individual buyers’ perspective money and credit are both costly, but from a social perspective the former is not, because the revenue from monetary expansion can be rebated to tax payers while the resources used to implement credit constitute a deadweight loss. Thus, individuals

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\(^{10}\)Still, we worked it out. In the baseline calibration presented below, eliminating tourist shops requires a 19% tax on their profit. Alternatively, we can subsidize information acquisition in the version with $\lambda$ endogenous, but the required subsidy is almost 100% – i.e., it makes information nearly free.

\(^{11}\)This result is less trite than saying, e.g., that people smoke too much, and since cigarettes tend to be cash goods, inflation is beneficial for health reasons. That is trite without saying why cigarettes cannot be taxed directly or purchased on credit. In our environment it may be difficult to identify and hence tax tourist shops directly, and it is an equilibrium outcome that they use more cash, not an assumption.
tend to use too much credit and too little cash, and raising the higher cost of credit offsets that. In fact, raising the cost of credit can be good even if $\gamma(d)$ is rebated to agents.\footnote{This would make sense if the private cost of using credit is that DM transactions can be taxed, while using cash avoids this, as in higher cost of credit; then we can rebate the tax proceeds to agents. It would also make sense if the transaction cost were paid to agents — say, bankers — to monitor or enforce credit arrangements, as opposed to a deadweight loss. Note that other than in this short discussion we do not distinguish between these alternative interpretations, because in the quantitative analysis total transaction costs as a fraction of output are fairly small (see Section 5).}

Proposition 6 says that DM output can increase with the cost of credit, intuitively, because in the DM the cost of holding cash is sunk while the cost of credit is not. When the cost of credit increases, buyers carry more money, and that can increase $q$.

To formalize this, define a ranking by saying $\gamma_2(d)$ is more costly than $\gamma_1(d)$ when $\gamma_2^{-1}(\xi)$ is weakly flatter than $\gamma_1^{-1}(\xi)$ $\forall \xi > 0$. Then $\lambda \in \{0, 1\}$ implies welfare rises with the cost of credit, which would be counterintuitive without the discussion in the previous paragraph. For $\lambda \in (0, 1)$, however, in $A^*_1$, the same region identified in Proposition 5, welfare falls as credit becomes more costly.

**Proposition 6** In monetary equilibrium: (a) for $\lambda$ near 0 or 1, $n$ and $d$ fall while $p$, $q$, $z$ and $\Omega$ rise as $\gamma$ becomes more costly; (b) for $(\lambda, i) \in A^*_1$, $\Omega$ falls as $\gamma$ becomes more costly.

### 4 More on Information

Here we expand on a few novel implications related to information theory.

#### 4.1 Exogenous Information

Several well-known papers predict that prices and/or price dispersion fall with exogenous increases in buyer information (Salop and Stiglitz (1977); Varian (1980); Burdett and Judd (1983); Stahl (1989)). Yet as Ellison and Ellison (2005) say “evidence from the Internet... challenged the existing search models, because we did not see the tremendous decrease in prices and price dispersion that many had predicted.” Similarly, Baye et al. (2006) say “Reductions in information costs over the past century have neither
reduced nor eliminated the levels of price dispersion observed.” While the framework is
not designed to apply specifically to Internet shopping, it speaks to these issues.13

It is not hard to check that higher $\lambda$ implies prices fall, consistent with conventional
wisdom, if $z = 0$. In monetary equilibrium, however, this is less clear because money
demand changes with $\lambda$. To see how it works, first note from (4)-(9) that $\Gamma_j$ depends on
$z$ but not directly on $\lambda$ or other endogenous variables, and since $\frac{\partial p_j}{\partial z} > 0$, the effect
of $\lambda$ on $p_j$ is the same as the effect of $\lambda$ on $z$. We formalize this as follows:

**Proposition 7** In monetary equilibrium, $\frac{\partial z}{\partial \lambda} \geq 0$ iff $\frac{\partial p_j}{\partial \lambda} \geq 0$. While $z$ is mono-
tone in $\lambda$, it can be increasing or decreasing. Given $\theta$, $\exists i > 0$ such that $\frac{\partial z}{\partial \lambda} \leq 0
\forall i \leq i$, and given $i$, $\exists \theta > 0$ such that $\frac{\partial z}{\partial \lambda} \geq 0 \forall \theta \leq \hat{\theta}$.

Interpreting $p_j$ as price, as makes sense when $q$ is unobserved quality, this means
prices can rise or fall with $\lambda$. Alternatively, interpreting $P_j = \frac{p_j}{q_j}$ as price, $q_j$ and
$p_j$ co-move with $z$, but examples show $P_j$ can also rise or fall. To understand this,
intuitively, note that when $\lambda$ increases buyers may bring more money (this is the case,
e.g., when $\theta$ is low). When buyers have more money sellers can charge higher prices. It
is even easier to see how higher $\lambda$ need not reduce price dispersion: $\lambda = 0$ implies no
dispersion since there are only tourist shops; $\lambda = 1$ implies no dispersion since there
are only local shops; and $\lambda \in (0, 1)$ has dispersion whenever both submarkets are open.
Hence price dispersion is nonmonotone in $\lambda$.14

4.2 Endogenous Information

Now suppose buyers are informed iff they pay a cost $s > 0$. Then the two types of
buyers, informed and uninformed, generally choose different real balances, say $z_I$ and

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13One might say online shopping does not use much cash, but for us monetary transactions include
check, debit and paypal. All these payment methods are monetary in the sense that one must work to
get purchasing power before spending, and purchasing power held as demand deposits, like cash, bears
approximately zero interest. This is distinct from credit, where one first spends, then works to pay it off.

14Lester (2011) also shows prices can rise or fall with $\lambda$, but only with small numbers of agents; our
result applies to large markets and hinges on monetary considerations. The result on dispersion and $\lambda$
does not hinge on money and holds, e.g., in any Burdett-Judd model.
Also, to simplify the presentation and clarify the insights, in this application we rule out all credit, and assume \( i \) is not too big. This allows us to focus on the effects of an information externality: uninformed buyers benefit when there are more informed buyers because that reduces the number of tourist shops.

As in the baseline model, local shops post terms to attract informed buyers. Since buyers choose information and money in the CM, informed buyers only bring enough cash to trade at local shops, \( z_I = p_L \), and now these shops believe that they can affect \( z_I \) by posting \( p_L \) (similar to Rocheteau and Wright (2005)). In particular, a local shop’s problem is now

\[
\max_{q_L, n_L, p_L} \left\{ \frac{\alpha(n_L)}{n_L} [u(q_L) - p_L] - ip_L \right\} \text{ st } \alpha(n_L)(p_L - q_L) = k. \tag{16}
\]

As \( i \) rises, one can check using the methods in Choi (2015) that \( q_L \) and \( p_L \) go down while \( n_L \) goes up. The terms of trade in submarket \( T \) then solve

\[
z_U - q_T = (1 - \theta) [u(q_T) - q_T] \text{ and } \alpha(n_T)(z_U - q_T) = k. \tag{17}
\]

At \( i = 0 \) buyers carry enough cash to get \( q^* \) for sure, \( z_I^* = (1 - \eta)u(q^*) + \eta q^* \) and \( z_U^* = (1 - \theta)u(q^*) + \theta q^* \). Hence, \( \theta < \eta \) implies \( z_U \geq z_I \) for \( i = 0 \), and, by continuity, also for \( i \approx 0 \). Similarly, buyers strictly prefer not to bargain at local shops for \( i \approx 0 \).

It follows that \( z_U \) maximizes the expected DM payoff,

\[
V(z_U) = W(z_U + \tau) + \hat{\omega}_L \frac{\alpha(n_L)}{n_L} [u(q_L) - p_L] + \hat{\omega}_T \frac{\alpha(n_T)}{n_T} [u(q_T) - p_T]
\]

where now the chance of entering each submarket is

\[
\hat{\omega}_L = \frac{N_L}{N_L + N_T} = \lambda \frac{n_T}{(1 - \lambda)(n_L - n_T)} \quad \text{and} \quad \hat{\omega}_T = 1 - \hat{\omega}_L. \tag{18}
\]

Since \( z_U \geq p_L \), uninformed buyers benefit from a marginal dollar only when they enter the submarket \( T \). Using the bargaining solution, the FOC for \( z_U \) is

\[
i = \hat{\omega}_T \frac{\alpha(n_T)}{n_T} \frac{\theta[u'(q_T) - 1]}{1 + (1 - \theta)[u'(q_T) - 1]} \tag{19}
\]

---

\(^{15}\) As a referee suggested, it would be interesting to have persistent heterogeneity in money holdings. An easy way to get this is to add a one-time cost to acquiring information; that does not affect the results too much. A more involved way is to have multiple rounds of DM trade before reconvening the CM (Molico 2007 is a limiting case where the CM never convenes); that changes a few quantitative results (in the direction one should expect from Molico 2007) but not a lot. See the Supplemental Appendix for details.
However, if $i$ were large, uninformed buyers only carry enough cash to trade at local shops, $z_U = z_I = p_L$, and (19) does not hold. Also, $\lambda \in (0, 1)$ implies the buyers’ indifference condition:

$$\frac{\alpha(n_L)}{n_L}[u(q_L) - z_I] - iz_I - s = \hat{w}_L\frac{\alpha(n_L)}{n_L}[u(q_L) - p_L] + \hat{w}_T\frac{\alpha(n_T)}{n_T}[u(q_T) - z_U] - iz_U. \quad (20)$$

**Definition 2** A stationary monetary equilibrium with endogenous $\lambda$ (and no credit) is a nonnegative list $\langle \lambda, \Gamma_L, \Gamma_T, z_I, z_u \rangle$ solving the above conditions with $z_U, z_I > 0$.

In Fig. 3, in region $B_1$ the cost $s$ is so high that no buyer becomes informed and only submarket $T$ is open, $\lambda = N_L = 0$. In $B_2$ both submarkets are open and $z_U > z_I$. In $B_3$ both submarkets are open, but because $i$ is high, uninformed buyers only carry $z_U = z_I$.

Note that submarket $T$ is always open, since for all $s > 0$ we have $\lambda < 1$, for the usual reason: $\lambda = 1$ implies all shops are the same, so it not a best response to pay $s$. The next result establishes existence; it also gives a sufficient condition for uniqueness, although it is restrictive, and more generally it is possible for welfare-ranked multiple equilibria to arise for parameters in $B_2$.\footnote{Here is the intuition: if there are more informed buyers, uninformed buyers are less likely to enter submarket $T$ and hence carry less cash; when uninformed buyers carry less cash, profit in submarket $T$ falls, so $n_T$ rises, and hence it worse to be uniformed; then more buyers acquire information. While other information-based theories also display multiplicity, this is novel, and again relies on interactions between information and money. Still, we downplay it to focus on other issues.}

**Proposition 8** Monetary equilibrium with endogenous $\lambda$ exists and it is unique if $i$ is not too big and $\alpha(n)/n$ is sufficiently inelastic.
Next we derive comparative statics for equilibrium in $B_2$, which is the interesting region, because information and money are substitutes in the sense that informed buyers carry less cash (results for $B_1$ and $B_3$ are in the Appendix).

**Proposition 9** If monetary equilibrium with endogenous $\lambda$ is unique, in $B_2$, as $i$ rises, $q_T$, $q_L$, $z_U$, $z_I$ and $\dot{\varphi}_T$ fall, and $n_T$ and $n_L$ rise. Moreover, as $s$ rises, $q_T$, $z_U$ and $\dot{\varphi}_T$ rise, $n_T$ falls, and $q_L$, $n_L$ and $\partial z_I$ remain unchanged.

Now consider welfare. Using the envelope condition one can show
\[
\frac{\partial \Omega}{\partial i} \bigg|_{i=0} = (1 - \lambda)(z_U^* - z_I^*) > 0.
\]
Hence, $i = 0$ is not optimal. This is because higher $i$ makes buyers carry less cash and hence more willing to pay more to avoid tourist shops. While raising $i$ from 0 reduces real balances, the loss is second-order by the envelope theorem, while higher $\lambda$ implies fewer tourist shops and that is a first-order gain. This is relevant because people consider it a puzzle that the Friedman rule is optimal in many models and yet central banks rarely if ever target $i = 0$. While not the first model to provide a reason why $i > 0$ may be desirable, we think our effect is novel and compelling.\footnote{The result is reminiscent of Benabou (1992a) and Head and Kumar (2005), but the economic idea is different. They say inflation increases price dispersion, which increases buyer search effort and hence seller competition, which is desirable. Here inflation directly discourages inefficient high-price sellers, which is desirable irrespective of whether it raises price dispersion or search effort.}

**Proposition 10** In monetary equilibrium with endogenous $\lambda$ optimal monetary policy has $i > 0$.

The welfare impact of $s$ is given by
\[
\frac{\partial \Omega}{\partial s} = -1 + i \frac{\partial [\lambda z_I + (1 - \lambda) z_U]}{\partial s}.
\]

The first term on the RHS is the direct effect, while the second captures the idea that higher $s$ makes buyers carry more cash and that leads to more output. Proposition 15 in the Appendix provides a sufficient condition, always satisfied when $u$ is not too concave,
for net effect to be positive. In terms of second-best theory, there are two inefficiencies: (1) the usual result that agents do not acquire enough cash; and (2) a novel effect that they do not acquire enough information. Due to these considerations, the above results show higher $s$ or $i$ can be desirable.

4.3 Noisy Competitive Search

Until now informed buyers see every posted $(p, q)$. Suppose instead they see exactly two random draws, in the spirit of what Burdett and Judd (1983) call noisy search (it is straightforward to let them see $h$ draws, where $2 \leq h < \infty$). To motivate this, first, it is a reasonable alternative way to capture a situation between two extremes: if $\lambda \in (0, 1)$ is the fraction of buyers with 2 observations, as $\lambda \to 0$ search becomes random, and as $\lambda \to 1$ the outcome becomes the same as competitive search. Second, as shown below, $\lambda \in (0, 1)$ yields a continuous distribution of posted terms, different from and perhaps more realistic than the baseline model. Third, it permits a robustness check on the results, including our numerical findings.\(^{18}\)

While sellers can in principle post any $(p, q)$, without loss of generality we can assume they simply post their surplus, $R = p - q$. To see this, define the bilateral frontier by letting $\Sigma(R)$ be a buyer’s maximal surplus given a seller’s surplus is $R$:

$$
\Sigma(R) \equiv \max_{p,q} \{u(q) - p - \gamma(p - z)\} \quad \text{st} \quad p - q = R.
$$

(22)

Sellers only post $(p, q)$ on this frontier, along which each point corresponds to a unique $R$; then given $R$ one can find the implied $p = p(R)$ and $q = q(R)$. For $z < R + q^*$, the solution is given by $p = R + q$ and the first-order condition

$$
u'(q) = 1 + \gamma'(R + q - z),
$$

(23)

which indicates that $q(R)$ and $\Sigma(R)$ decrease while $p(R)$ increases with $R$. For $z \geq R + q^*$, the solution is $q = q^*$ and $p = R + q^*$.

\(^{18}\)Acemoglu and Shimer (2000) employ a similar setup but without the details we need – e.g., divisible goods, money or costly credit. Still, some of the results are similar, including the result that the endogenous distribution of posted terms can have a mass point, then a gap, then a density above that – which is different from the standard Burdett-Judd model with no mass points or gaps.
As in any Burdett-Judd-type model, sellers posting a higher $R$ have a lower probability of trade. If $n(R)$ is market tightness for sellers that post $R$, the free entry condition

$$\alpha [n(R)] R = k$$  \hspace{1cm} (24)$$

holds for each $R$ posted in equilibrium. Buyers visit the seller that offers the highest expected payoff $\alpha [n(R)] \Sigma (R)/n(R)$. Let $R_L = p_L - q_L$ be a seller’s trade surplus with directed search, as at local shops in the benchmark model. Then Lemma 4 in the Appendix shows buyers’ expected payoff is strictly quasi-concave and maximized at $R_L$. As a result, no seller posts any $R < R_L$, because $R_L$ attracts more buyers and yields more surplus per trade. Therefore, only $R \geq R_L$ will be posted.

Let $F(R)$ be the CDF of posted $R$. When $\lambda$ is large, competition creates a mass point at $R_L$. Thus, there exists $\lambda^*$ such that $\lambda \geq \lambda^*$ implies all sellers post $R_L$ and thus $F(R_L) = 1$. For $\lambda < \lambda^*$, some sellers post $R > R_L$ to take advantage of the uninformed buyers, and $F(R)$ has an atomless part in an interval $[R, \bar{R}]$, plus potentially a mass point at $R_L \leq \bar{R}$. In equilibrium $\bar{R} = R_T$, where $R_T = p_T - q_T$ is a seller’s surplus under bargaining, as at a tourist shop in the benchmark model, because: if a seller posts $\bar{R} > R_T$ buyers opt to bargain; if he posts $\bar{R} < R_T$ he can profitably deviate to $R_T$. Since sellers that post $R_T$ only trade with uninformed buyers, the buyer-seller ratio is $n_T = (1 - \lambda)/N$. By free entry, $\alpha(n_T)R_T = k$, the measure of sellers is $N = (1 - \lambda)/\alpha^{-1}(k/R_T)$.

The size of the mass point at $R_L$ can be derived from free entry, $\alpha(n_L)R_L = k$. For a seller that posts $R_L$, tightness is

$$n_L = \frac{1 - \lambda}{N} + \frac{2 \lambda}{N} [1 - F(R_L)] + \frac{2 \lambda}{N} \frac{F(R_L)}{2}.$$  \hspace{1cm} (25)$$

The first term on the RHS is the measure of uninformed buyers that sample this seller; the second is the measure of informed buyers who sample him plus one other seller, where the other one posts $R > R_L$; and the third is the measure of informed buyers who sample him plus one other seller, where the other one posts $R_L$. It follows that the size
of the mass point $\mu$ is
\[
\mu \equiv F(R_L) = \max \left\{ 2 - \frac{(1 - \lambda)}{\lambda} \left[ \frac{\alpha^{-1}(k/R_L)}{\alpha^{-1}(k/R_T)} - 1 \right], 0 \right\}, \tag{26}
\]
where the max operator takes care of the case when $F$ is atomless.

As is standard, one can easily show $F(R)$ is continuous and atomless for $R \in [R, \bar{R}]$.\(^\text{19}\) Different from the usual specification, a seller posting $R \in [R, \bar{R}]$ here can be interpreted as facing a buyer-seller ratio
\[
n(R) = \frac{1 - \lambda}{N} + \frac{2\lambda}{N} [1 - F(R)]. \tag{27}
\]
By (24), (27) and $F(R_T) = 1$, we have
\[
F(R) = 1 - \frac{1 - \lambda}{2\lambda} \left( \frac{\alpha^{-1}(k/R)}{\alpha^{-1}(k/R_T)} - 1 \right). \tag{28}
\]
The lower support of the atomless part $\underline{R}$ solves $F(\underline{R}) = \mu$, or
\[
\underline{R} = \frac{k}{\alpha(\alpha^{-1}(k/R_T)[2\lambda(1 - \mu) + 1])}. \tag{29}
\]

We summarize as follows:

**Proposition 11** For $\lambda \geq \lambda^* \equiv 1 - n_T/n_L$, all sellers post $R_L$. For $\lambda < \lambda^*$, a fraction $\mu$ of sellers post $R_L$ where $\mu$ is given by (26). Other sellers post $R \in [R, \bar{R}]$ where $\bar{R} = R_T$ and $\underline{R} \geq R_L$ solves (29). For $R \in [R, \bar{R}]$, $F$ is given by (28).

With these results in hand, consider money demand. Let $\tilde{F}$ be the distribution function of the lowest of two draws from $F$, $\tilde{F}(R) = (1 - \lambda)F(R) + \lambda \{1 - [1 - F(R)]^2\}$.

Then
\[
V(z) = W(z + \tau) + \int \frac{\alpha[n(R)]}{n(R)} \left\{ u[q(R)] - p(R) - \gamma [p(R) - z] \right\} d\tilde{F}(R).
\]
The FOC wrt $z$ is given by
\[
i = \int \frac{\alpha[n(y)]}{n(R)} \gamma'(p(R) - z) d\tilde{F}(R). \tag{30}
\]
\(^{19}\)If $F$ has a mass point at $R' \in [R, \bar{R}]$, a seller posting $R'$ has a profitable deviation to $R' - \varepsilon$ for $\varepsilon > 0$. If $F$ has a flat spot between $R'$ and $R'' > R'$, a seller posting $R'$ has a profitable deviation to $R''$.\(^{20}\)
Definition 3 With noisy search, a stationary monetary equilibrium is a list \( (F, z) \) where \( F \) is characterized by Proposition 11 and \( z > 0 \) solves (30).

Proposition 12 Stationary monetary equilibrium with noisy search exists and is unique if \( i \) is not too big.

Proposition 13 In stationary monetary equilibrium with noisy search \( \partial p(R) / \partial R > 0 \), \( \partial q(R) / \partial R \leq 0 \) and \( \partial n(R) / \partial R < 0 \) \( \forall R \in [R_L, R_T] \).

As regards welfare, similar to (15), we have
\[
\Omega = N \int_R^R \alpha [n(R)] \Sigma(R)dF(R) = \int_R^R \alpha \frac{n(R)}{n(R)} \Sigma(R)dF(R). \tag{31}
\]
Among other things, the next result implies that in any equilibrium with price dispersion the Friedman rule is once again suboptimal:

Proposition 14 Assume \( \lambda < \lambda^* \). In stationary monetary equilibrium with noisy search near \( i = 0 \), as \( \lambda \) or \( i \) rise, \( z \) falls, the \( p \) distribution falls in the sense of first-order stochastic dominance, and \( \Omega \) rises.

As \( \lambda \) rises, price competition among sellers gets stronger and prices fall, so buyers are better off. As \( i \) rises, buyers carry less money and that creates two effects. First, buyers use more credit, but the cost of this is second-order near \( i = 0 \). Second, sellers post lower prices, and that improves welfare. Altogether, \( \Omega \) is maximized at \( i > 0 \), similar to Proposition 5, but now this holds whenever price dispersion arises. This is because \( F \) is continuous around \( R_T \). In the baseline model, raising \( i \) from 0 hurts the entire tourist submarket, but now it only impacts on sellers posting \( R_T \) and charging the highest price. The measure of such sellers is 0, so this loss is second order, when the \( R \) distribution is continuous. In what follows it is shown numerically that these effects can be important.

5 Quantitative Results

Here we quantify three key aspects of the theory: (1) the monetary wedge; (2) information frictions; and (3) credit conditions.
5.1 Calibration

The model is calibrated using observations on households’ demand for money and credit, firms’ market power, and dispersion in prices and markups. For different versions – exogenous information, endogenous information and noisy search – we use the same calibration strategy and match the same moments. The period length is set to a year, but this does not actually matter much. Then we set $\beta = 1 / (1 + r)$ with $r = 0.03$. The CM and DM utility functions are $U(x) = \log(x)$ and $u(q) = Bq^{1-b} / (1 - b)$, with $(b, B)$ set to match aggregate money demand, i.e. the empirical relationship between nominal interest rates and a measure of money scaled by output, $M/PY$. With $U(x) = \log(x)$, real CM output is $x^* = 1$ (a normalization), while DM output in the same units from submarket $j$ is $\alpha(n_j)N_j[p_j - \gamma(p_j - z)] - N_jk$. Aggregate output sums these, while $M/P$ is given by $z$.

Our measure of money is $M1$. The rationale is that checks and debit cards are about as liquid as currency, and they are backed by deposits that must be accumulated before expenditures, while credit means buying now and paying later. The best $M1$ data is the $M1J$ series from Lucas and Nicolini (2015), which augments the usual series with money market accounts after regulatory amendments in 1980 made them about as liquid as checking accounts. They provide annual observations from 1919 to 2008 and show the empirical money demand relationship is stable over the sample using the nominal rate on T-bills. To fit the data with $u(q) = Bq^{1-b} / (1 - b)$, intuitively, changing $B$ shifts the curve up or down and is set to match a mean $M/PY$ of $0.27$, while $b$ captures the elasticity and is set to minimize the sum of squared residuals between model and data.

---

20To see why, consider the simplest job search model where $V_0$ and $V_1(w)$ are the values of unemployment and employment at wage $w$, $\alpha$ is the arrival rate of jobs, and $\kappa$ is a search cost. Then $rV_0 = \alpha [V_1(w) - V_0] - \kappa$. To change from, e.g., a weekly to a monthly model, we can simply multiply $r$, $\alpha$, $\kappa$ and $w$ by 4 without changing payoffs or observables like the unemployment rate and hourly wages – the only caveat is we must respect $\alpha \leq 1$, which is not a problem when moving to higher frequencies. The same idea applies here. In particular, with shorter periods agents get to rebalance $z$ more often, but the lower arrival rates imply they hold cash for just as long on average, so the impact of the inflation tax is similar. One still might take issue with an annual model because it means households make at most one DM purchase per year – but if that is problematic, simply interpret a household as a collection of many buyers, as in Shi (1997). This is similar to macro-labor, where each vacancy hires at most one worker, but a firm is interpreted as a collection of many vacancies.
The cost of credit function is $\gamma(d) = Cd^c$, where willingness to substitute between money and credit is captured by $c$, and the share of purchases made with credit by $C$. Letting $D$ be credit averaged across all buyers, in both submarkets, and emulating the procedure for money demand, we set $(c, C)$ to match the relationship between $D/Y$ and $i$. The data is on consumer credit for household, family and other personal expenditures, exclusive of loans secured by real estate, which is appropriate because such credit largely supports retail trade and provides a long enough time series to capture the substitutability between money and credit.\textsuperscript{21} We do not calibrate to micro payment data, but it turns out the model matches this well: in equilibrium about 30% of DM transactions by value are made with credit, right in the middle of the range of numbers in the Boston Fed and Bank of Canada surveys discussed, with references to primary sources, by e.g. Liu et al. (2015).

Estimated money and credit demand curves are shown in Fig. 4. Note that the demand for credit is increasing in $i$ because this is the nominal, not the real, interest rate, and that is effectively the opportunity cost of using cash. For money demand (top-left panel) the relationship looks stable and the fit is good, consistent with Lucas and Nicolini (2015). For credit demand (top right) the result is reasonable given there is apparently a structural break in the 1990s. Understanding this break is beyond the scope of the paper, but it is important to say that including or excluding observations between 1996 and 2008 does not change the results much (see the Supplemental Appendix). Hence, since it does not matter much for the conclusions, we omit data after 1996.

In any case, the crucial factor for us is substitutability between money and credit, depending on $i$, and that is captured very well, as shown by the relationship between $D/z$ and $i$ (bottom panel). This relationship is reasonably stable and displays clear movement out of cash and into credit as $i$ rises. While we over predict somewhat credit demand for high nominal rates in the early 1980s, the fit is otherwise quite good. We also mention that the aggregate cost of credit is small, with $\gamma/Y = 0.2\%$ and $\gamma/D = \ldots$\textsuperscript{21}See FRB’s G.19 consumer credit release, FRED Series: TOTALSL. This data is also nice because the series is long.
2.0%. For what it’s worth, this is roughly in line with interchange fees on credit cards.
All things considered, therefore, we conclude that the model does reasonably well at accounting for the data on the use of money and credit.

As in many search models of money, for simplicity the DM matching technology is $\alpha(n) = n/(1 + n)$. Buyers’ bargaining power is $\theta$, and their effective bargaining power in local shops is $\hat{\theta} = u'(q)/(u'(q) + n)$. Price dispersion depends on the difference, $\theta - \hat{\theta}$, which also depends on the measure of informed buyers, $\lambda$. We set $\theta$ to match a relative standard deviation in retail prices of 15.5% that Kaplan et al. (2016) derive from the Kilts-Nielsen Consumer Panel, although using the alternative 19% in Kaplan and Menzio (2015) delivers similar results. This implies $\theta = 0.72$ and $\hat{\theta} = 0.84$, which means that even at tourist shops the uninformed still get a considerable surplus. When
\( \lambda \) is exogenous, we set it to match the average DM markup. When \( \lambda \) is endogenous, we set the cost \( s \) to match the same target, which implies that resources spent acquiring information in equilibrium amount to about 2% of total output.

In retail survey data the average ratio of gross margins to sales from 1992-2008 is 0.28, implying an average markup of \( 1 + 0.28/(1 - 0.28) = 1.39 \), which in the model comes from markups of 1.3 and 1.7 in submarkets \( L \) and \( T \).\(^{22}\) Then, finally, the entry cost \( k \) is set to get an aggregate markup across the CM and DM of 1.1, standard in macro going back to Basu and Fernald (1997). Since our DM and CM markups are 1.39 and 1.0, \( k \) gets the size of the DM to match the average trade-weighted markup. To be clear, we do not calibrate the size of the CM and DM – like the size of the DM submarkets \( L \) and \( T \), they are implied by observable targets – but it turns out the DM contributes about 1/4 of \( Y \), with around 1/2 from tourist shops and 1/2 from local shops.

<table>
<thead>
<tr>
<th>Description</th>
<th>Exog. ( \lambda )</th>
<th>Endog. ( \lambda )</th>
<th>Noisy Search</th>
<th>Source/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>DM utility curvature, ( b )</td>
<td>0.66</td>
<td>0.76</td>
<td>0.60</td>
<td>((z/Y, i)) relationship</td>
</tr>
<tr>
<td>DM utility level, ( B )</td>
<td>0.57</td>
<td>0.52</td>
<td>0.58</td>
<td>( \text{avg. } z/Y )</td>
</tr>
<tr>
<td>Credit cost curvature, ( c )</td>
<td>5.30</td>
<td>5.80</td>
<td>6.05</td>
<td>((D/Y, i)) relationship</td>
</tr>
<tr>
<td>Credit cost level, ( C )</td>
<td>12.72</td>
<td>24.81</td>
<td>50.60</td>
<td>( \text{avg. } D/Y )</td>
</tr>
<tr>
<td>Informed buyers, ( \lambda )</td>
<td>0.38</td>
<td>—</td>
<td>0.64</td>
<td>retail markup 40%</td>
</tr>
<tr>
<td>Cost of Information, ( s )</td>
<td>—</td>
<td>0.027</td>
<td>—</td>
<td>retail markup 40%</td>
</tr>
<tr>
<td>Sellers’ entry cost, ( k )</td>
<td>0.020</td>
<td>0.012</td>
<td>0.014</td>
<td>agg. markup 10%</td>
</tr>
<tr>
<td>Bargaining power, ( \theta )</td>
<td>0.72</td>
<td>0.82</td>
<td>0.36</td>
<td>avg. price dispersion</td>
</tr>
</tbody>
</table>

Table 1: Calibrated parameter values

Table 1 gives the results for three models: exogenous \( \lambda \); endogenous \( \lambda \); and noisy search. A Supplemental Appendix, summarized in Section 5.5, provides robustness

\(^{22}\)These data can be found at https://www.census.gov/retail. Note that our range for markups from 1.3 to 1.7 is actually quite close to the data, where at the low end are Warehouse Clubs, Superstores, Automotive Dealers and Gas Stations, while at the high end are Specialty Foods, Clothing, Footwear and Furniture. Although we do not push this, it would not be a big stretch to think these low-markup (high-markup) stores in a loose sense capture our local (tourist) shops – e.g., at least some of us buy gas all the time and have a good feel for the prices at different vendors, but buy footwear much less often and hence search randomly. Again we do not push the idea here, but understanding different markets for different goods seems worth further study.
checks. For one, we cut in half the targets for price dispersion and markups, since in the
data these in part may come from factors outside our theory. As the results do not change much, we are confident the findings are not overly sensitive to measurement issues along these lines. We also note that there are not very many parameters, considering the theory has a lot of ingredients, and they are all tied down by reasonable targets.

5.2 Inflation

To obtain welfare numbers, it is standard to first compute the equilibrium payoff $\Omega$ at a given inflation rate $\bar{\pi}$, typically 10%, then compute the percentage reduction in total consumption agents would accept to reduce $\bar{\pi}$ to the value $\pi^0$ consistent with the Friedman rule $i = 0$, which is $\pi^0 = -0.029$ in our calibration. Figs. 5 and 6 below show this for $\bar{\pi}$ varying from $\pi^0$ to 15%. However, while this is natural when $\pi^0$ is optimal, as it is in many models, $\pi^0$ is not optimal in any of our specifications at their calibrated parameters. Hence, Table 2 below also reports other measures, e.g., the cost of having the average $\hat{\pi}$ in the data rather than the optimal $\pi^*$. But, to begin with the typical exercise of computing the cost of having 10% inflation rather than $\pi^0$, in our benchmark specification with exogenous $\lambda$, the result is 1.1% of consumption, much lower than models with only random search, and close to those with only directed search, as discussed above.
Figure 6: Cost of inflation: model comparisons.

One might guess this comes from a calibrated $\lambda$ close to 1; in fact, it is only 0.38. To understand this, Fig. 5 gives results for the baseline model at the calibrated $\lambda$, plus versions with $\lambda = 0$ and $\lambda = 1$.\textsuperscript{23} Even at $\lambda = 0$ our results are lower than Lagos and Wright (2005) for the following reasons: rather than Nash we use Kalai bargaining (it has several advantages); we have entry; we allow some credit; and our calibration targets differ. Eliminating the first three differences, the results get closer: at $\lambda = 0$, with no credit or entry and using Nash bargaining, we get 5.6%, compared to 6.9% in Lagos and Wright (2005), with the residual due to our calibration targets. Using Kalai instead of Nash bargaining, the cost falls from 5.6% to 4.0%. Allowing entry brings it down further to 1.6%, and using $\lambda = 0.38$ instead of 0 lowers it to 1.1%, our baseline number. This decomposes how each component in theory contributes to the result.\textsuperscript{24}

In addition to the baseline model, Fig. 6 shows results for the specification with endogenous $\lambda$ and the one with noisy search. As mentioned above, in all cases the\textsuperscript{23} The models are re-calibrated using a similar procedure, except when $\lambda \in \{0, 1\}$ we drop the target for price dispersion, and when $\lambda = 1$ we drop the retail markup.

\textsuperscript{24} We also mention that changing the value of $\lambda$ exogenously can have nonmonotone effects on the cost of inflation. Low $\lambda$ implies submarket $T$ is large, and since inflation discourages tourist shops it is beneficial on this dimension. However, inflation also reduces the surplus in both submarkets. As a result, at the calibrated $\lambda = 0.38$, moderate inflation hurts welfare more than when $\lambda$ is 0 or 1. Hence, while our setup in a sense averages over existing models, with random and directed search, its predictions are not averages of existing results.
welfare cost is negative for $\pi$ just above $\pi^0$, which means the optimal policy is $\pi^* > \pi^0$. While this is true with exogenous $\lambda$, it is hard to see in the chart because $\pi^*$ is very close to $\pi^0$. It is easier to see in the formulation with endogenous $\lambda$, where at calibrated parameters $\pi^* = -1.7\%$, and easiest to see with noisy search, where $\pi^* = 0.9\%$ is not only above $\pi^0$ it is above 0. Interestingly, to the extent that one takes this specification seriously – and we don’t see why one would take it any less seriously than the reduced-form models currently used by central bankers – this is below but not far off their actual inflation targets.

In any case, for the typical experiment of going from 10% to $\pi^0$, the result comes in at 0.7% with $\lambda$ endogenous and 0.3% with noisy search. In especially the latter specification, the number is really quite low. Perhaps this is no surprise since, after all, we do not want to go to $\pi^0$ in that version, and in fact do not want to go below $\pi^* = 0.9\%$. One consideration is that calibration in the noisy search model delivers a fairly low bargaining power for buyers, $\theta = 0.36$, compared to $\theta = 0.72$ in the baseline. That means high-price sellers in this version have much larger markups than in the other two models, and since inflation drives them out, the gain is bigger and the cost of inflation is smaller.

The left panel of Fig. 7 shows the effects of $\pi$ on markups and price dispersion in the model with $\lambda$ exogenous (the other models are similar). At calibrated parameters, it shows that DM markups rise with $\pi$, counter to some empirical findings. However, since the DM shrinks with $\pi$, the markup averaged over the CM and DM falls, although the quantitatively the effect is not big. The right panel of Fig. 7 shows DM price dispersion increases with $\pi$ in all three models. To understand this, first note that inflation increases all sellers’ markups, because both $p$ and $q$ fall but $q$ falls by more. One can show the increase in tourist shops exceeds that in local shops, and so dispersion goes up. Hence, these models can match some of the facts deemed important in the literature

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25In the noisy search model, bargaining power disciplines the top of the distribution of prices and markups, and a large mass of transactions (even for uninformed buyers) occur well below this point. In order to match the data, therefore, the markup at the top of the distribution must be quite high and the bargaining power of buyers quite low.
5.3 Information

The left panel of Fig. 8 shows the cost, or benefit if negative, of changing \( \lambda \) in the different models (with endogenous \( \lambda \), we change the cost of information \( s \) and map that to \( \lambda \)). There are two forces at work. As \( \lambda \) increases, the probability of trading at a tourist shop falls. Then, since local shops are less expensive, buyers carry less cash, as shown in the right panel. Proposition 4 says this lowers the surplus in both submarkets, suggesting welfare may fall with \( \lambda \), but the net effect is positive at calibrated parameters. To be precise, increasing \( \lambda \) from 0 to 1 is worth between 3.5% and 15.7% of consumption, depending on the specification. However, note that we do not have to go all the way to \( \lambda = 1 \): with exogenous \( \lambda \) or endogenous \( \lambda \), welfare peaks at around \( \lambda = 0.51 \), which is sufficient to drive all tourist shops out of the market and eliminate the information problem; with noisy search, this occurs at around \( \lambda = 0.79 \).

Instead of starting from \( \lambda = 0 \) we can start from the calibrated \( \lambda \). Then increasing \( \lambda \) by enough to eliminate the information friction is worth between 1.0% to 1.9% of consumption, depending on the specification. These numbers are driven by the effects.
on the mix between high- and low-markup firms in the DM. In Fig. 9, as $\lambda$ increases the average DM markup falls due to changing market composition, and the aggregate markup falls as the DM shrinks. Also, as regards price dispersion, for low (high) values of $\lambda$ it increases (decreases) with $\lambda$. Hence, as mentioned above, while either positive or negative net effects of information on dispersion are possible in theory, at the calibrated parameters dispersion falls with $\lambda$ in all the models.
5.4 Credit conditions

Credit provides an alternative to cash, and that matters for the impact of inflation. Symmetrically, money matters when discussing the changes in credit conditions. Fig. 10 shows the effects of changing $C$ in $\gamma(d) = Cd^c$. Lower $C$ raises the use of credit, crowding out real balances, but less than one for one (top-right panel). With exogenous $\lambda$, the effect on the seller’s surplus differs in submarkets $L$ and $T$. For high $C$ credit is not used much in submarket $L$, while lowering $C$ makes $q_T$ increase and $p_T$ decrease, with a net negative effect. Then as credit becomes cheaper, tourist shops enter and the share of local shops falls (bottom). As a result, $Y$ and $\Omega$ decrease (top-left). For lower $C$ more credit is used, and further decreases raise $p_T$ and $q_T$ but lower $p_L$ and $q_L$, with a net negative effect on profit in each submarket. This makes buyers in submarket $T$ better off and those in submarket $L$ worse off. Total welfare decreases monotonically as the use of credit rises.

To see how money matters, consider changing credit conditions in nonmonetary equilibrium with exogenous $\lambda$ (the other versions are similar). For this we keep the parameters in Table 1 fixed, but add a costless credit limit $\bar{d}$ equal to $z$ from monetary equilibrium, to make the models more comparable. Without money, tighter credit is bad for welfare: raising $C$ to get a 20% reduction in debt has a welfare cost of 0.1% and reduces output by 0.9%. This contrasts with monetary equilibrium, where raising $C$ raises $Y$ and $\Omega$. These results provide a word of caution to those trying to measure the importance of credit in environments with no money. Of course this should not be controversial – it is like trying to measure the importance of taxis in environments with no Uber – but it is not necessarily easy to model money and credit seriously.

5.5 Robustness

The Supplementary Appendix contains several alternatives to the calibration and modeling details that we now briefly review. As mentioned earlier, one check is to recalibrate targeting only half the standard deviation of the price distribution and/or only
half the markup; that does not change the results too much.\textsuperscript{26} Another check is to allow agents’ information type to be permanent, not determined each period; that makes buyers’ money holdings persistently heterogeneous, but otherwise does not have a big impact. Another is to analyze a version with multiple rounds of DM trade before reconvening the DM; that reduces the cost of inflation, as one should expect based on Molico (2007), but not by much. We also tried a formulation where better informed agents get easier access to credit, based on the idea that they are known at local shops, in addition to knowing the prices at local shops, and again the findings were similar.

\textsuperscript{26}The finding that the markup does not matter too much (at least over a reasonable range) may be surprising, but it is consistent with the results in Aruoba et al. (2011). Intuitively, it matters a lot whether, e.g., the markup is 0 or 5\% (think envelope theorem) but not so much whether it is 5\% or 10\%.
A quick summary is that the quantitative results are fairly robust to all the alternative specifications. In the baseline model, e.g., the welfare cost of 10% inflation varies between 0.8% and 1.1%; the gain to moving from the calibrated value to \( \lambda = 1 \) varies between 1.1% and 1.8%; and the impact of changes in credit conditions is roughly the same. The biggest changes come from having multiple rounds of DM trade, but even that is not very big. To be clear, we are not saying the results are the same in the three models – i.e., the versions with exogenous \( \lambda \), with endogenous \( \lambda \), and with noisy search – but that in each model the results are similar after reasonable changes in the calibration and modeling details.

### 6 Conclusion

This paper developed a framework with a decentralized market designed to resemble retail, and used it to analytically and quantitatively study monetary policy, credit conditions and information frictions. The model is quite tractable, delivering sharp results on existence, uniqueness, efficiency and comparative statics. Many theoretical results are novel, e.g., the possibility of better information raising prices in monetary economies, and perhaps especially the optimality of deviations from the Friedman rule. The specification was amenable to calibration based on observations on the use of money and credit as function of the nominal rate, markups and price dispersion.

<table>
<thead>
<tr>
<th>Model</th>
<th>( \pi^* )</th>
<th>( \bar{\pi} )</th>
<th>WC of Inflation (( \pi ))</th>
<th>WC of Info. (( \lambda ))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \pi^* ) to ( \bar{\pi} )</td>
<td>( \pi^* ) to 10%</td>
<td>( \pi^* ) to 0% to ( \bar{\pi} )</td>
<td>1 to ( \lambda )</td>
</tr>
<tr>
<td>Exogenous ( \lambda )</td>
<td>-2.8%</td>
<td>38%</td>
<td>0.3%</td>
<td>1.1%</td>
</tr>
<tr>
<td>Endogenous ( \lambda )</td>
<td>-1.7%</td>
<td>63%</td>
<td>0.1%</td>
<td>0.7%</td>
</tr>
<tr>
<td>Noisy Search</td>
<td>0.9%</td>
<td>48%</td>
<td>&lt; 0.01%</td>
<td>0.3%</td>
</tr>
</tbody>
</table>

*Notation: \( \pi^* \)=optimal inflation rate, \( \bar{\pi} \)=highest inflation rate consistent with monetary equilibrium, \( \hat{\pi} \)=inflation rate (data), \( \lambda \)=calibrated value, WC = % annual consumption equivalent welfare cost.*

Table 2: Welfare Cost (WC) of Inflation and Information - Summary
Table 2 summarizes a few quantitative findings in the three specifications, including the optimal inflation rate $\pi^*$, the rate at which the economy demonetizes $\bar{\pi}$, and different ways to measure the cost of inflation or information frictions. Perhaps the most surprising result is that inflation is worth considerably less in all these models than those with pure random search and bargaining. Did those models overestimated the negative impact of inflation? Not exactly. Those papers were about the cost of inflation, and correctly pointed out that reduced-form papers underestimated that. None of those papers identified the benefit of inflation emphasized here, namely, its positive impact on the composition of sellers in the market. This effect can be sizable – e.g., with noisy search the optimal inflation rate $\pi^* = 0.9\%$, close to the target of some central banks. We think these results constitute progress, and are hopeful that more can be done with the framework in future work.
Appendix: Proofs

Lemma 1: In monetary equilibrium, the FOC holds at equality, \( 1 + i = V'(z) \). Hence, \( i > 0 \) implies either \( \gamma'(d_L) > 0 \) or \( \gamma'(d_T) > 0 \) by (32). This implies either \( d_L > 0 \) or \( d_T > 0 \), and so \( z < \max \{ p_L, p_T \} \). Then we can rewrite (11) using \( u(q_T) - p_T - \gamma(d_T) = \theta [u(q_T) - q_T - \gamma(d_T)] \), from Kalai bargaining, as

\[
V'(z) = 1 + \omega_L \frac{\alpha(n_L)}{n_L} \gamma'(d_L) + \omega_T \frac{\alpha(n_T)}{n_T} \theta [u'(q_T)q_T' - q_T' - \gamma'(d_T) (p_T' - 1)]
\]

\[
= 1 + \omega_L \frac{\alpha(n_L)}{n_L} \gamma'(d_L) + \omega_T \frac{\alpha(n_T)}{n_T} \theta \gamma'(d_T) (q_T' - p_T' + 1)
\]

\[
= 1 + \omega_L \frac{\alpha(n_L)}{n_L} \gamma'(d_L) + \omega_T \frac{\alpha(n_T)}{n_T} \frac{\theta \gamma'(d_T)}{1 + (1 - \theta) \gamma'(d_T)}.
\] (32)

The second line uses \( u'(q_T) = 1 + \gamma'(d_T) \) by (8). The third line uses \( q_T' - p_T' + 1 = 1/[1 + (1 - \theta) \gamma'(d_T)] \) which comes from differentiating (9) wrt \( z \). Since \( \omega_j \) and \( n_j \) are independent of \( z \) and \( \gamma \) is convex, \( V''(z) \leq 0 \) as long as \( d_j = p_j - z \) decreases in \( z \) for \( j = L, T \), and \( d_L \) falls in \( z \), because \( p_L \) is not a function of \( z \). Next, \( d_T \) falls in \( z \) by (8) and (9). Finally, \( V''(z) < 0 \) if either \( d_L \) or \( d_T \) falls strictly in \( z \), and \( d_j \) falls strictly in \( z \) as long as it is strictly positive, and we already established \( d_L > 0 \) or \( d_T > 0 \). \( \square \)

The next lemma is useful for proving Proposition 2 and 4. Recall from (27), (4) and (7) that \( N_j \) and \( \Gamma_j \) can be solved as a function of \( z \). Lemma 2 shows that \( \Gamma_j \) is differentiable in \( z \) and characterizes how it varies with \( z \).

Lemma 2 Given \( q_j < q^* \), \( \Gamma_j \) is differentiable wrt \( z \) with \( \partial p_j / \partial z, \partial q_j / \partial z > 0, \partial n_j / \partial z, \partial d_j / \partial z < 0 \), and \( \partial P_j / \partial z \), ambiguous for \( j = L, T \).

Proof: Consider first tourist shops. If \( p^*_T \leq z \), then credit is not needed and all derivatives wrt \( z \) are 0. If \( p^*_T > z \), we use the Implicit Function Theorem on (8)-(9) to define \( p_T(z) \) and \( q_T(z) \) as functions of \( z \). Differentiating wrt \( z \), we get

\[
p_T'(z) = 1 - \left\{ 1 - \frac{\gamma''(d_T)}{u''(q_T)} \left[ 1 + (1 - \theta) \gamma'(d_T) \frac{\gamma''(d_T) - u''(q_T)}{\gamma'(d_T)} \right] \right\}^{-1} \in [0, 1]
\]

\[
q_T'(z) = [p_T'(z) - 1] \frac{\gamma''(d_T)}{u''(q_T)} > 0.
\] (33)

Since \( p_T'(z) \in [0, 1] \), we have \( d_T'(z) = p_T'(z) - 1 < 0 \). Next, the RHS of (9) rises in \( z \), since \( q_T'(z) > 0 \) and \( d_T'(z) < 0 \), so \( R_T'(z) > 0 \). Therefore \( n_T'(z) < 0 \) by free entry.
Now consider local shops’ problem (4). By a change of variable \( y \equiv p - z - q \), using the constraint to eliminate \( p \) and \( z \) from (4), we have

\[
\max_{n,q,y} \left\{ \frac{\alpha(n)}{n} [u(q) - q - \gamma(y + q)] - \frac{k}{n} \right\} \quad \text{st} \quad \alpha(n)(y + z) \geq k. \tag{35}
\]

We can first choose \( q \) independent of \( n \). Define \( G(y) \equiv \max_q \{u(q) - q - \gamma(y + q)\} \).

The solution for \( q \) satisfies \( u'(q) - 1 = \gamma'(y + q) \). Thus, \( q \) decreases continuously and is differentiable in \( y \). Then \( G''(y) = -\gamma'(y + q) < 0 \) and \( G''(y) = \gamma''(y + q)u''(q)/[\gamma''(y + q) - u''(q)] < 0 \). Eliminating \( y \) using the constraint from (35), we get

\[
\max_n F(n, z) \equiv \max_n \left\{ \frac{\alpha(n)}{n} G \left[ \frac{k}{\alpha(n)} - z \right] - \frac{k}{n} \right\}.
\]

Since \( F(0, z) = -\infty \), \( n = 0 \) is not a solution. Since \( \alpha(n) = 1 \) and \( \alpha'(n) = 0 \) \( \forall n > \hat{n} \), the solution is \( n(z) \leq \hat{n} \), as otherwise we can lower \( n \) to increase the objective function. Thus \( n(z) \in (0, \hat{n}) \). If \( n(z) < \hat{n} \), then \( \partial F(n, z)/\partial n \|_{n=n(z)} = 0 \); otherwise \( n(z) = \hat{n} \).

We now verify the SOC’s and show \( n(z) \) is unique. Consider

\[
\frac{\partial F(n, z)}{\partial n} = \left[ \frac{\alpha'(n)}{n} - \frac{\alpha(n)}{n^2} \right] G \left[ \frac{k}{\alpha(n)} - z \right] - \frac{k}{n^2} \alpha'(n) G' \left[ \frac{k}{\alpha(n)} - z \right] + \frac{k}{n^2}.
\]

\[
= \frac{1}{n^2} \left\{ [n\alpha'(n) - \alpha(n)] G \left[ \frac{k}{\alpha(n)} - z \right] - \frac{k}{n^2} \alpha'(n) G' \left[ \frac{k}{\alpha(n)} - z \right] + k \right\}.
\]

This derivative vanishes at an interior solution. For the SOC’s, differentiate the expression in braces wrt \( n \) to get

\[
\alpha''(n) n G \left[ \frac{k}{\alpha(n)} - z \right] - \frac{k\alpha''(n) n}{\alpha(n)} G' \left[ \frac{k}{\alpha(n)} - z \right] + \frac{\alpha'(n)^2 nk^2}{\alpha(n)^3} G'' \left[ \frac{k}{\alpha(n)} - z \right].
\]

At \( n = n(z) \), \( G[k/\alpha(n) - z] > 0 \) and this is strictly because \( \alpha''(n), G', G'' < 0 \).

Hence the SOC’s hold and the optimizer \( n(z) \) is unique. Next we show it falls with \( z \). Since

\[
\frac{\partial^2 F(n, z)}{\partial z \partial n} = \frac{\alpha'(n) k}{\alpha(n) n} G'' \left[ \frac{k}{\alpha(n)} - z \right] + \alpha(n) \left[ 1 - \frac{\alpha'(n) n}{\alpha(n)^2} \right] G' \left[ \frac{k}{\alpha(n)} - z \right] \leq 0,
\]

any interior \( n(z) \) is differentiable with \( n'(z) \leq 0 \). Given \( q_L < q^* \), we have \( G'[k/\alpha(n) - z] < 0 \) and \( \partial^2 F(n, z)/\partial z \partial n < 0 \) and \( n'(z) < 0 \).

Finally, we show \( q \) increases with \( z \). Write (35) as a Lagrangian

\[
\max_{n,y} \left\{ \frac{\alpha(n)}{n} G(y) - \frac{k}{n} + \zeta [\alpha(n)(y + z) - k] \right\},
\]

36
with ζ the multiplier for free entry. Taking the FOC’s wrt \( n \) and \( y \), and eliminating ζ, we get

\[
0 = \frac{1}{n} \left[ \alpha'(n) - \frac{\alpha(n)}{n} \right] G(y) + \frac{k}{n^2} - \frac{G'(y)}{n} \alpha'(n) (y + z).
\] (36)

Since \( n'(z) \) exists, \( y'(z) \) and \( q'(z) \) exist. To show \( q'(z) > 0 \), it is sufficient to show \( y'(z) < 0 \) because the solution for \( q \) falls and is differentiable in \( y \). Consider (36).

Since the elasticity of \( \alpha \) is less than 1, the term in square brackets is negative. Since \( G', G'' < 0 \), the RHS of (36) rises strictly with \( z \) or \( y \), so \( y'(z) < 0 \) by the Implicit Function Theorem.

**Proposition 2:** We first show \( T(z) \) is continuous. By Lemma 2, \( q_j(z), n_j(z), N_j(z) \) and \( p_j(z) \) are continuous, and thus (13) and (14) are continuous in \( z \). When \( \lambda = 1 - n_T(z)/n_L(z) \), (13) and (14) are identical, so \( T(z) \) is continuous. At \( i = 0 \), by (3) the solution for \( z \) is big enough to sustain equilibrium with \( N_T + N_L > 0 \). Since \( T(z) \) decreases in \( z \), a unique monetary equilibrium exists when \( i \) is small by continuity.

**Proposition 3:** From \( N_L > 0 \) we know \( \lambda > 0 \). Then \( (27) \) and \( N_L, \lambda > 0 \) imply \( n_L > n_T \).

Since \( n_L \) solves (4), \( n_L \leq \hat{n} \) where \( \alpha(\hat{n}) = 1 \). For \( n_L > \hat{n} \) the objective function in (4) can be increased by lowering \( n_L \). Since \( \hat{n} \leq n_L > n_T \), \( \alpha(n_L) > \alpha(n_T) \). Hence \( \Pi_L = \Pi_T \) implies \( p_L - q_L < p_T - q_T \). The results for \( P \) and \( R \) are obvious once we check \( q \) and \( p \), so it remains to show \( q_L \geq q_T \) and \( p_L < p_T \). There are different cases. First suppose \( p_L, p_T \geq z \). By (5)-(8), if \( q_j \) is big then \( p_j \) is small, so \( p_L - q_L < p_T - q_T \) implies \( p_L < p_T \) and \( q_L > q_T \). Second suppose \( p_L < z \). By (5)-(8), \( q_L = q^* \geq q_T \). Third suppose \( p_L \geq z > p_T \). By (5)-(8), \( q_T = q^* \geq q_L \), but then \( \Pi_L = \Pi_T \) implies \( p_T \geq p_L \), contradicting the supposition \( p_L \geq z > p_T \); so this case cannot occur. Finally suppose \( p_L, p_T < z \). Then \( q_L = q_T = q^* \), so \( \Pi_L = \Pi_T \) implies \( p_L < p_T \). Hence \( p_L < p_T \) and \( q_L \geq q_T \) in all cases. If \( q_T < q^* \), then either the first or the second case holds, and \( q_L > q_T \).

**Proposition 4:** Since \( T(z) \) is continuous and \( T(z) \) decreases in \( z \), the solution for \( T(z) = i \) falls continuously in \( i \). If \( T'(z) \) exists, the results follow from Lemma 2.

For \( N_L, N_T > 0 \), by (32), (13) can be written

\[
T(z) = \omega_L \alpha(n_L) \gamma'(p_L - z) + \omega_T \alpha(n_T) \frac{\theta \gamma'(d_T)}{1 + (1 - \theta) \gamma'(d_T)}.
\]
Clearly \( T'(z) \) exists because \( \gamma'' \) exists and \( \{n_j,p_j,d_j,\omega_j\} \) is differentiable in \( z \) by Lemma 2. For \( N_L > N_T = 0 \), \( T'(z) \) exists by (14). So \( T'(z) \) exists except when (13) equals (14), i.e., when \((i, \lambda)\) is on the border between \( A_2 \) and \( A_3 \).

**Proposition 5:** When \( \lambda = 0 \), as in Aruoba et al. (2007), \( \Omega \) is maximized at \( i = 0 \). Also, as in Rocheteau and Wright (2005), \( \Omega \) is maximized when \( i = 0 \) when \( \lambda = 1 \). By continuity, for \( \lambda \) near 0 or 1, \( \Omega \) falls with \( i \).

It remains to show \( \Omega \) rises with \( i \) for \((\lambda, i) \in A^*_1 \). Rewrite (15) as

\[
\Omega = \frac{\omega_L \alpha(n_L)}{n_L} [u(q_L) - p_L - \gamma(p_L - z)] + \frac{\omega_T \alpha(n_T)}{n_T} [u(q_T) - p_T - \gamma(p_T - z)],
\]

where the entry cost \( k \) does not show up since the terms in brackets are the buyer’s (not total) surplus, and \( k \) cancels with the seller’s surplus. Using (6), (9) and free entry, then using (1) and (27), we get

\[
\Omega = \frac{\omega_L}{n_L} \left( \frac{\eta_L k}{1 - \eta_L} \right) + \frac{\omega_T}{n_T} \left( \frac{\theta k}{1 - \theta} \right) = \frac{\lambda}{n_L - n_T} \left( \frac{\eta_L k}{1 - \eta_L} \right) + \frac{(1 - \lambda)n_L - n_T}{n_T(n_L - n_T)} \left( \frac{\theta k}{1 - \theta} \right).
\]

The RHS depends on \( i \) only through \( n_T \) and \( n_L \). Since \( \partial n_T / \partial i > \partial n_L / \partial i = 0 \) in \( A_1 \), \( \partial \Omega / \partial i \) has the same sign as \( \partial \Omega / \partial n_T \). Differentiating the RHS wrt \( n_T \) yields

\[
\frac{\partial \Omega}{\partial n_T} = \frac{k}{(n_L - n_T)^2} \left\{ \frac{\lambda n_L}{n_T} + \frac{\theta}{1 - \theta} \left[ (1 - \lambda) \frac{n_L}{n_T} \left( 2 - \frac{n_L}{n_T} \right) \right] \right\}.
\]

(37)

The sign of the RHS depends on the term in braces, call it \( \Phi \). An equilibrium in \( A_1 \) is in \( A^*_1 \) iff \( \Phi \geq 0 \). At the intersection point of \( A_1 \) and \( A_3 \), \( N_T = 0 \iff (1 - \lambda)n_L = n_T \).

In this situation, inflation enhances welfare:

\[
\Phi = \frac{\lambda n_L}{1 - \eta_L} - \frac{\theta}{1 - \theta} \left[ (1 - \lambda) \frac{n_L}{n_T} \left( 2 - \frac{n_L}{n_T} \right) \right] = \frac{\lambda n_L}{1 - \eta_L} - \frac{\theta \lambda}{1 - \theta} \frac{n_L}{n_T} = \frac{\lambda n_L}{k} \left\{ \frac{\alpha(n_L)}{n_L} [u(q_L) - p_L - \gamma(d_L)] \right\} > 0.
\]

The second equation uses \( (1 - \lambda)n_L = n_T \). The third uses (6), (9) and free entry. The inequality holds because a buyer receives a higher expected payoff in submarket \( L \) than submarket \( T \). By continuity, there is an interval for \( \lambda \) where \( \Phi > 0 \) at \( i = 0 \). Therefore, by continuity there is a non-empty region \( A^*_1 \) in the lower-right part of \( A_1 \) such that \( \Omega \) increases with \( i \). □
Lemma 3 If $\gamma_2$ is more costly than $\gamma_1$, then $\gamma_2(\gamma_2'^{-1}(a)) < \gamma_1(\gamma_1'^{-1}(a)) \forall a \geq 0$.

Proof: Since $\gamma_2^{-1}(b)$ is flatter than $\gamma_1^{-1}(b) \forall b > 0$,

$$\frac{\partial \gamma_2^{-1}(b)}{\partial b} < \frac{\partial \gamma_1^{-1}(b)}{\partial b} \iff \frac{1}{\gamma_2(\gamma_2^{-1}(b))} < \frac{1}{\gamma_1(\gamma_1^{-1}(b))} \iff \gamma_1'(\gamma_1^{-1}(b)) < \gamma_2'(\gamma_2^{-1}(b)).$$

Since $\gamma'(d)$ and $\gamma^{-1}(b)$ are increasing, $\gamma'(\gamma^{-1}(b))$ rises with $b$. Thus, the last inequality implies $\forall b_1, b_2$, $\gamma_2'(\gamma_2^{-1}(b_2)) = \gamma_1'(\gamma_1^{-1}(b_1)) \Rightarrow b_1 > b_2$. In other words, $a = \gamma_2'(\gamma_2^{-1}(b_2)) = \gamma_1'(\gamma_1^{-1}(b_1)) \Rightarrow \gamma_2(\gamma_2'^{-1}(a)) = b_2 < b_1 = \gamma_1(\gamma_1'^{-1}(a))$. Therefore, given $a \geq 0$, $\gamma(\gamma'^{-1}(a))$ falls strictly as $\gamma^{-1}$ grows flatter. \qed

Proposition 6(a). To prove part (a), we consider in turn (i) $\lambda = 0$ and (ii) $\lambda = 1$.

(i) $\lambda = 0$. Since $\gamma(d)$ is strictly increasing, strictly convex and differentiable $\forall d \geq 0$, $\gamma'(d)$ exists and rises in $d$. For any given $a = \gamma'(d)$, one can write $\gamma(d)$ as an implicit function $\gamma[\gamma'^{-1}(a)]$. When $\lambda = 0$, only submarket $T$ exists and by (13) and (32)

$$i = \frac{\alpha(n_T)}{n_T} \frac{\theta \gamma'(d)}{1 + (1 - \theta)\gamma'(d)} = \frac{\alpha(n_T)}{n_T} \frac{\theta a}{1 + (1 - \theta)a}. \tag{38}$$

By (8), $q_T$ falls continuously in $\gamma'(d)$. Thus, we can also write $q_T$ as an implicit function $q(a) \equiv u'^{-1}(1 + a)$ where $a = \gamma'(d)$, and the inverse function exists since $u''(q) < 0$. This also implies $q(a)$ rises strictly in $a$. It is easy to verify $q(0) = q^*$. By (9) and free entry,

$$\frac{k}{(1 - \theta)\alpha(n_T)} = u(q(a)) - q(a) - \gamma(\gamma'^{-1}(a)). \tag{39}$$

Using (38) we define an implicit function $n_T = n_1(a)$ for any $a$ because $\alpha(n_T)/n_T$ falls strictly in $n_T$. Similarly, we can define $n_T = n_2(a)$ by (39). Any $a$ solving $n_1(a) = n_2(a)$ determines the terms of trade in equilibrium. Since equilibrium is unique by Proposition 2, the solution for $n_1(a) = n_2(a)$ is unique. To characterize it, note that at $a = 0$, $n_1(0) = 0$ by (38) and $n_2(0) = \alpha^{-1}(k/(1 - \theta)[u(q^*) - q^*]) > 0$ by (39). Thus $n_2(a)$ cuts $n_1(a)$ from above once as $a$ rises.

As the cost of credit increases, the implicit function $\gamma(\gamma'^{-1}(a))$ falls $\forall a$ by Lemma 3. In this case, $n_2(a)$ falls $\forall a$ by (39). This implies $a$ and $n$ fall, so there are more sellers and $\alpha(n_T)/n_T$ rises. Also, $q_T(a)$ rises as $a$ falls because $u'(q_T(a)) = 1 + a$ and $u$ is concave. Moreover, the surplus for sellers $p_T - q_T$ rises by free entry, and thus $p_T$ rises.
The surplus for buyers $u(q_T) - q_T - \gamma(d_T)$ rises in $p_T - q_T$, by the bargaining solution. So buyers get a larger surplus per transaction and a higher matching probability, therefore welfare $\Omega$ rises. Finally, as the cost of using credit rises, the total expenditure on credit $\gamma(d_T) = \gamma(\gamma^{-1}(a))$ falls because $a$ falls and $\gamma(\gamma^{-1}(a))$ falls $\forall a$. Then debt $d_T$ falls because $\gamma(d)$ rises $\forall d > 0$ and $\gamma(d_T)$ falls in equilibrium. Since $p_T$ rises and $d_T$ falls, $z = p_T - d_T$ rises.

(ii) $\lambda = 1$. With pure directed search trade $(n, q, p)$ solves

$$\max_{n,q,p,z} \left\{ \frac{\alpha(n)}{n} [u(q) - q - \gamma(p - z)] - \frac{k}{n} - iz \right\} \text{ st } k = \alpha(n)(p - q).$$

Now we make several changes of variables. First, let $a = \gamma'(p - z)$ so $p = \gamma^{-1}(a) + z$. Second, since the solution satisfies (5), $q$ solves $q(a) \equiv u^{-1}(1 + a)$. Third, by free entry $k = \alpha(n)(p - q)$, $z = p - \gamma^{-1}(a) = q(a) - \gamma^{-1}(a) + k/\alpha(n)$. Fourth, from the FOC $i = \gamma'(p_L - z)\alpha(n)/n$, one can express $n$ as an implicit function $n(a)$ where $i = a\alpha(n(a))/n(a)$. Substitute $a, q(a), n(a)$ and $z = q(a) - \gamma^{-1}(a) + k/\alpha(n)$ into the problem to get

$$\max_{a} \left\{ \frac{i}{a} [u(q(a)) - q(a) - \gamma(\gamma^{-1}(a))] - \frac{k}{\alpha(n(a))} - i \left[ q(a) - \gamma^{-1}(a) + \frac{k}{\alpha(n(a))} \right] \right\}.$$

Now we argue the solution for $a$ falls strictly as $\gamma$ becomes more costly. Let $\gamma_2$ be more costly than $\gamma_1$, so that $\gamma_2^{-1}$ is weakly flatter than $\gamma_1^{-1}$. Let $F_j(a)$ be the function in braces when the cost function is $\gamma_j$ for $j = 1, 2$. Differentiate $F_j$ to get

$$\frac{\partial F_2(a)}{\partial a} - \frac{\partial F_1(a)}{\partial a} = \frac{i}{a^2} \left[ \gamma_2(\gamma_2^{-1}(a)) - \gamma_1(\gamma_1^{-1}(a)) \right] < 0.$$

The last inequality uses Lemma 3. So $a$ falls strictly in $j$ by standard monotone comparative statics. It follows that $n$ falls as $\gamma$ becomes more costly by (38). The rest of the proof is identical to the last part of (i).

Finally, we verify that for $\lambda \in \{0, 1\}$, $\Omega$ is maximized when credit is not used. Define $\bar{\gamma}(d) \equiv \gamma(bd)$ for $b > 1$, so $\bar{\gamma}^{-1}$ is flatter than $\gamma^{-1}$. As $b$ rises, $\bar{\gamma}$ grows more costly and $\Omega$ rises by (i) and (ii) from above. As $b \to \infty$, $\bar{\gamma}(d) \to \infty \forall d > 0$ and the equilibrium converges to one without costly credit.

**Proposition 6(b).** We first show that as $\gamma$ becomes more costly, $\Gamma_L$ stays the same, $p_T, q_T$ and $\omega_T$ rise, and $n_T$ falls. By (1) and (27), $\omega_T = 1 - \lambda n_L/(n_L - n_T)$. Substitute
this into (32) and let \( a = \gamma(d_T) \) to get

\[
i = \left(1 - \frac{\lambda n_L}{n_L - n_T}\right) \frac{\alpha(n_T)}{n_T} \frac{\theta a}{1 + (1 - \theta)a}.
\]

Any \((n_T, a)\) solving (39) and (40) characterizes \( \Gamma_T \). Now we argue that if \((\lambda, i) \in A_1\) we stay in \( A_1 \) as \( \gamma \) grows more costly. To show this, we assume we stay in \( A_1 \) as \( \gamma \) grows more costly, then verify it. If we stay in \( A_1 \), buyers have enough money to purchase \( q^* \) in submarket \( L \), and \( \Gamma_L \) is constant. Using the logic from part (i) of Proposition 6(a), define \( n_1(a) \) and \( n_2(a) \) by (40) and (39), so \( \Gamma_T \) is characterized by the solution \( n_1(a) = n_2(a) \).

One can use part (i) of the proof of Proposition 6(a) to show that \( n_T \) falls and \( p_T, q_T \) and \( z \) rise as \( \gamma \) grows more costly. Since \( z \) rises, buyers have enough \( z \) to get \( q^* \) in submarket \( L \), and the equilibrium stays in \( A_1 \). Then \( \omega_T = 1 - \lambda n_L/(n_L - n_T) \) rises since \( n_T \) falls and \( n_L \) is constant.

Finally, if the equilibrium is in \( A_1^* \subset A_1 \), then \( n_T \) rises as \( \gamma \) grows less costly as argued in the previous paragraph. Also, \( \Omega \) rises in \( n_T \) when equilibrium is in \( A_1^* \) by (37). Therefore \( \Omega \) rises as \( \gamma \) grows less costly in \( A_1^* \).

\[\]

**Proposition 7.** If \( N_T = 0 \) then \( T(z) \) is given by (14) and \( \lambda \) has no effect on \( z \), so \( \partial z/\partial \lambda = 0 \). If \( N_T > 0 \) then \( T(z) \) is given by (13). Eliminating \( N_j \) using (27) and differentiating, we get

\[
\frac{\partial T(z)}{\partial \lambda} \propto \frac{\alpha(n_L)}{n_L} \gamma'(p_L - z) - \frac{\alpha(n_T)}{n_T} [u'(q_T) q_T - p_T' - \gamma'(p_T - z)(p_T' - 1)].
\]

The RHS is continuous in \( z \) and independent of \( \lambda \). By \( T(z) = i \) and the Implicit Function Theorem, \( T'(z) \partial z/\partial \lambda + \partial T(z)/\partial \lambda = 0 \). Therefore, \( \partial z/\partial \lambda \) and \( \partial T(z)/\partial \lambda \) have the same sign because \( T'(z) < 0 \). When the RHS is 0, \( \partial T(z)/\partial \lambda = 0 \Rightarrow \partial z/\partial \lambda = 0 \), so the RHS stays 0 as \( \lambda \) increases further. Hence, \( \partial z/\partial \lambda \) never changes sign as \( \lambda \) increases. When \( \lambda = 0 \) there is \( \hat{i} > 0 \) such that \( \forall i \leq \hat{i} \) an extra dollar is redundant in local shops, \( z > p_L^* \). Thus, \( \forall i < \hat{i}, \partial T(z)/\partial \lambda|_{\lambda = 0} \leq 0 \Rightarrow \partial z/\partial \lambda|_{\lambda = 0} \leq 0 \). By the earlier analysis, \( \partial z/\partial \lambda \leq 0 \forall \lambda \) and \( i \leq \hat{i} \).

Next, consider the effects of \( \lambda \) on \( z \) when \( \theta \) is small. If \( i = 0 \), then buyers bring enough money to buy \( q^* \) in both markets, namely \( z = \max\{p_T^*, p_L^*\} \) and thus \( z \) is independent of \( \lambda \). Assume \( i > 0 \). Use (32) to rewrite the displayed equation as

\[
\frac{\partial T(z)}{\partial \lambda} \propto \frac{\alpha(n_L)}{n_L} \gamma'(p_L - z) - \frac{\alpha(n_T)}{n_T} \frac{\theta \gamma'(p_T - z)}{1 + (1 - \theta) \gamma'(p_T - z)}.
\]

\(41\)
Fix $\lambda = 0$. If $\theta = 0$, then the buyers bring $z = 0$ because there is no chance of meeting a local shop and no benefit from bringing money to a tourist shop. In this case $\gamma'(p_L - z) > 0$ and $\partial T(z)/\partial \lambda > 0$ by (41). By continuity, there is a $\tilde{\theta}$ such that if $\theta \leq \tilde{\theta}$, then $\partial T(z)/\partial \lambda \geq 0$, and thus $\partial z/\partial \lambda \geq 0$ at $\lambda = 0$. Since we already show $z$ is monotone in $\lambda$, $\partial z/\partial \lambda \geq 0$ at any $\lambda \in [0, 1]$ when $\theta \leq \tilde{\theta}$.

**Proposition 8.** In area $B_1$, $B_2$ and $B_3$, the terms of trade in the local submarket $\{n_L, q_L, z_I\}$ are given by the solution of (16) and hence are independent of $s$.

In area $B_1$, namely when $\hat{\omega}_T = 1$, the terms of trade $\{n_T, q_T\}$ must solve the free entry condition $\alpha(n_T)(1 - \theta)(u(q_T) - q_T)) = k$ and buyers’ first-order condition

$$i = \frac{\alpha(n_T)}{n_T} \left[ \frac{\theta(u'(q_T) - 1)}{1 + (1 - \theta)(u'(q_T) - 1)} \right].$$

(42)

It is easy to check that if $\alpha(n)/n$ is sufficiently inelastic, then there is a $\{n_T, q_T\}$ solving the first-order condition and free entry condition. The money holding $z_U$ can then be derived by (17). Given $\{n_L, q_L, z_I, n_T, q_T, z_U\}$, we can define $\bar{s}$ as the value of information when $\hat{\omega}_T = 1$, namely

$$\bar{s} \equiv \frac{\alpha(n_L)}{n_L}(u(q_L) - z_I) - iz_I - \frac{\alpha(n_T)}{n_T}(u(q_T) - z_U) + iz_U.$$

It follows that an equilibrium with $\hat{\omega}_T = 1$ exists if $s \geq \bar{s}$.

In area $B_2$, by the free entry condition we can write $n_T$ as an implicit function $n_T(q_T)$. Then, eliminate $\hat{\omega}_T$ from (20) using (19) to get

$$s = q_T - \frac{u(q_T) - q_T}{u'(q_T) - 1} - z_I + \frac{\frac{\alpha(n_L)}{n_L}[u(q_L) - p_L]}{\frac{\alpha(n_T(q_T))}{n_T(q_T)} \theta(u'(q_T) - 1)}.$$

(43)

Since $q_L$, $p_L$ and $n_L$ do not depend on $q_T$, this condition alone determines the value of $q_T$ in $B_2$. The matching probability $\alpha(n_T(q_T))/n_T(q_T)$ falls in $q_T$ while the rest of the RHS rises in $q_T$. If $\alpha(n)/n$ is sufficiently inelastic, then $\alpha(n_T(q_T))/n_T(q_T)$ falls slowly in $q_T$ and thus the RHS rises in $q_T$. In this case there is a unique $q_T$ solving (43), and hence there is a unique solution for $\{q_T, n_T, z_U, \hat{\omega}_T\}$ by (17) and (19).

An equilibrium in $B_2$ exists as long as the induced $\hat{\omega}_T \leq 1$ and $z_U > z_I$. By Proposition 9, $\hat{\omega}_T$ and $z_U$ rise in $s$ in $B_2$. It is easy to check that $\hat{\omega}_T = 1$ when $s = \bar{s}$.
by (43). Define \( s \) such that \( z_U = z_I \) when \( s = \bar{s} \). Therefore, a unique equilibrium in \( B_2 \) exists as long as \( s \in [\underline{s}, \bar{s}] \).

In area \( B_3 \), using buyers’ indifference condition, sellers’ free entry condition and \( z_U = z_I \) one can solve for a unique \( \{q_T, n_T, \lambda, \hat{\omega}_T\} \). To make sure \( z_U = z_I \) is optimal for uninformed buyers, we need

\[
i \geq \hat{\omega}_T \frac{\alpha(n_T)}{n_T} \left[ \frac{\theta(u'(q_T) - 1)}{1 + (1 - \theta)(u'(q_T) - 1)} \right]. \tag{44}
\]

By Proposition 9, as \( s \) rises, \( \hat{\omega}_T \) rises while \( q_T, q_L, z_U, z_I, n_L \) and \( n_T \) remain unchanged. Therefore, the right-hand side rises in \( s \). By the definition of \( \bar{s} \) the inequality binds when \( s = \bar{s} \). Therefore, an equilibrium in \( B_3 \) exists when \( s < \bar{s} \). \( \Box \)

**Proposition 9.** We prove a more general version of the proposition: When equilibrium with endogenous information is unique: (i) In \( B_1 \), as \( i \) rises \( q_T \) and \( z_U \) fall while \( n_T \) rises; a change in \( s \) has no effect. (ii) In \( B_2 \), as \( i \) rises \( q_T, q_L, z_U, z_I, n_T \) and \( n_L \) rise; as \( s \) rises, \( q_T, z_U \) and \( \hat{\omega}_T \) rise, \( n_T \) falls, while \( q_L, n_L \) and \( z_I \) are unchanged. (iii) In \( B_3 \), as \( i \) rises, \( q_T, q_L, z_U \) and \( z_I \) fall, while \( n_T \) and \( n_L \) rise; as \( s \) rises, \( \hat{\omega}_T \) rises while \( q_T, q_L, z_U, z_I, n_L \) and \( n_T \) are unchanged.

**Proof** In \( B_1 \), the market equilibrium is the same as that of a pure random search market. Write \( n_T \) as an implicit function \( n_T(q_T) \) by the free entry condition. Then the right-hand side of (42) must fall in \( q_T \) provided that equilibrium is unique. Therefore \( q_T \) falls in \( i \). Then by the free entry condition \( n_T \) rises in \( i \) and by the bargaining solution (17) \( z_U \) falls in \( i \). It is obvious that a change in \( s \) has no effect on the terms of trade.

In \( B_2 \), assume the right-hand side of (43) increases in \( q_T \) such that the equilibrium is unique (if the right-hand side is non-monotone, then multiple equilibria necessarily arises). It immediately implies that \( q_T \) rises in \( s \). Since \( q_T \) rises, \( n_T \) falls by the free entry condition. Since \( q_T \) rises in \( s \) and the first term in the right-hand side of (43) falls in \( q_T \), the second term must rise in \( s \). It follows that \( \hat{\omega}_T \) rises in \( s \) by (19). The terms of trade in the local submarket is not affected by \( s \) because (17) is independent of \( s \).

As \( i \) rises, naturally \( q_L, z_I \) fall and \( n_L \) rises, see Choi (2017) for a proof. It follows that, fixing \( q_T \), the right-hand side of of (43) rises in \( i \) by the envelope theorem. Moreover, the left-hand side of (43) falls as \( i \) rises. Since we have assumed the right-hand
side rises in $q_T$, $q_T$ falls in $i$. Since $z_U = (1-\theta)u(q_T) + \theta q_T$, we know $z_U$ falls in $i$. Next, $n_T$ rises in $i$ by the free entry condition. Finally, as $i$ rises, the left-hand side of (20) falls by $-z_I$ by the envelope theorem. Fixing $n_T$, the last two terms on the right-hand side falls by $-z_U$ as $i$ rises. Since $z_U \geq z_I$, the right-hand side falls by a larger magnitude than the left-hand side. Moreover, as $i$ rises, $n_T$ rises and the first term on the right-hand side of (20) falls (because $\{n_L, q_L, p_L\}$ changes), both changes make the right-hand side lower. Therefore, $\hat{\omega}_T$ must fall in $i$ to balance the equation.

In $B_3$, as $i$ rises, $q_L$ and $z_I$ fall and $n_L$ rises as in area $B_2$. Since $z_U = z_I$, $z_U$ falls in $i$. Since $z_U$ falls, $q_T$ falls and $n_T$ rises by (17). As $s$ rises, $q_T$, $q_L$, $z_U$, $z_I$, $n_L$ and $n_T$ remain unchanged. To balance (20), $\hat{\omega}_T$ must rise in $s$. \hfill \square

The next result provides a sufficient condition such that welfare $\Omega$ rises in the cost of information $s$.

**Proposition 15** In monetary equilibrium with endogenous information, with $\lambda \in (0, 1)$, if the RHS of (43) is sufficiently flat when $(i, s)$ is on the border between $B_2$ and $B_3$ then $\partial \Omega / \partial s > 0$.

**Proof.** In $B_2$, if $|u''(q_T)|$ is sufficiently small, then the right-hand side of (43) is flat in $q_T$. In this case a small increase in $s$ leads to a large increase in $q_T$ and thus $z_U$ also increases by a large magnitude. Since $z_I = z_U$ when $(i, s)$ lies at the border between $B_2$ and $B_3$, a change in $\lambda$ has no impact on $\lambda z_I + (1 - \lambda) z_U$. Altogether, $\partial (\lambda z_I + (1 - \lambda) z_U) / \partial s$ in (21) can be arbitrarily large when the right-hand side of (43) is sufficiently flat in $q_T$. \hfill \square

**Lemma 4** Buyers’ expected payoff $\Sigma(R)\alpha(n(R))/n(R)$ is strictly quasi-concave in $R$ and maximized at $R_L$.

**Proof.** Since buyers prefer a smaller $R$ in equilibrium, their expected payoff must fall in $R$ or equivalently

$$\frac{d}{dy} \left\{ \Sigma(R)\alpha \left[ \frac{n(R)}{n(R)} \right] \right\} \leq 0.$$  

Since $n(R)$ falls in $R$, this inequality holds if and only if

$$\frac{d}{dn} \max_p \left\{ u[p - k/\alpha(n)] - p - \gamma(p - z) \right\} \frac{\alpha(n)}{n} \geq 0.$$
By the envelope theorem, the inequality holds if and only if \( \forall R \in [\bar{R}, \bar{\bar{R}}] \):

\[
\frac{R}{\Sigma(R)} \geq \frac{1 - \eta}{u'(q(R))\eta}
\]  

(45)

where \( \eta \equiv \alpha'(n)n/\alpha(n) \) is the elasticity of the matching function. If this inequality binds at some \( R' \), then the share of surplus that buyers get is the same as that implied by a pure directed search market by (6). This implies \( \{q(R'), p(R'), n(R')\} \) solve the same equations that pin down the terms of trade \( \{q_L, p_L, n_L\} \) of the local submarket (i.e. free entry (4), efficiency (5), and surplus sharing (6)). But the solution to these equations is unique by the proof of Lemma 2. Therefore \( R' = R_L \equiv p_L - q_L \). This implies the inequality (45) binds exactly once at \( R_L \). For \( R = R_T \), (45) is satisfied because

\[
\frac{R_T}{\Sigma(R_T)} = \frac{1 - \theta}{\theta} \geq \frac{1 - \eta}{u'(q(R_T))\eta}.
\]

The equation uses the definition of the Kalai bargaining solution. The inequality is true because \( u'(q(R_T)) \geq 1 \) and we have assumed \( \theta \leq \eta \). It follows that (45) is satisfied for all \( R \in [R_L, R_T] \) and violated for all \( R < R_L \).

**Proposition 11.** It remains to show \( \lambda^* \equiv 1 - n_T/n_L \). Suppose all sellers post \( R_L \), the free entry condition implies \( N = 1/n_L \). If a seller deviates to post any \( R > R_L \), then the most profitable deviation is \( R_T \). Since only uninformed buyers would visit the deviating seller, the buyer-to-seller ratio is \( (1 - \lambda)n_L \). If \( n_T \geq (1 - \lambda)n_L \) or equivalently \( \lambda \geq 1 - n_T/n_L \), then no seller would deviate to post \( R_L \). As discussed in the main text, it is never optimal for sellers to post \( R < R_L \), therefore all sellers posting \( R_L \) is an equilibrium iff \( \lambda \geq 1 - n_T/n_L \).

If \( \lambda \geq 1 - n_T/n_L \), then the competitive search outcome is the only equilibrium. Suppose \( F(R_L) < 1 \), then the sellers that post \( R > R_L \) can deviate to post \( R_L \) and the buyer-to-seller ratio would exceed \( n_L \) by (25) and \( \lambda \geq 1 - n_T/n_L \).

**Proposition 12.** Existence and Uniqueness: When \( i = 0 \), all sellers produces \( q^* \) and \( R_T = (1 - \theta)(u(q^*) - q^*) \) and \( R_L = (1 - \eta_L)(u(q^*) - q^*) \). By Proposition 11 there is a unique solution for the distribution \( F \). The money holding is determined by the seller that posts \( R_T \), therefore \( z^* = p(R_T) = (1 - \theta)u(q^*) + \theta q^* \). In this case the equilibrium is clearly unique. Since one can derive \( F \) for any given \( z \), we can consider the right-hand
side of (30) as a function of $z$. Since it is weakly positive and continuous in $z$, it must fall strictly in $z$ around the FR (i.e. when $z \approx (1 - \theta)u(q^*) + \theta q^*$). It follows that there is a unique solution for $z$ in (30), and thus equilibrium is unique near the FR. □

**Proposition 14.** Change in $i$: As mentioned above, the right-hand side of (30) falls in $z$ when $i \approx 0$. Therefore $z$ falls in $i$. Next, we argue that the distribution of prices falls in the first-order stochastic dominance (FOSD) sense as $z$ falls through two channels. First, given $R$, the price $p$ falls as $z$ falls by equation (23) and the envelope theorem. Second, the distribution of $R$ falls in the FOSD sense as $z$ falls. To see this, note that when $i = 0$ buyers carry $z = p_T > p_L$. In this case a small decrease in $z$ affects the terms of trade with high price sellers but do not affect the terms of trade at the local shops. Therefore the surplus $R_T$ falls as $z$ falls by the Kalai bargaining solution while $R_L$ remains constant as $z$ falls. It follows that the distribution $F$ falls in the FOSD sense by equation (26) and (28). The distribution of prices falls in the FOSD sense because $p$ falls as $R$ falls by (23).

As $i$ rises, buyers carry less money and it affects welfare $\Omega$ through three channels:

$$
\frac{d\Omega}{di} = \frac{\partial z}{\partial i} \left( \frac{\partial \Omega}{\partial z} + \frac{\partial R_T}{\partial z} \frac{\partial \Omega}{\partial R_T} + \frac{\partial R_L}{\partial z} \frac{\partial \Omega}{\partial R_L} \right).
$$

Since $\partial \Omega / \partial z = i$ by (30), it vanishes at the FR. Similarly, $\partial R_T / \partial z$ and $\partial R_L / \partial z$ vanishes at the FR by the envelope theorem. Altogether, $d\Omega / di = 0$ at $i = 0$.

Now consider $i \approx 0$. As discussed in the first paragraph, $\partial R_L / \partial z = 0$. Therefore

$$
\frac{d\Omega}{di} = \frac{\partial z}{\partial i} \left( \frac{\partial \Omega}{\partial z} + \frac{\partial R_T}{\partial z} \frac{\partial \Omega}{\partial R_T} \right).
$$

Differentiating $\Omega$ and $R_T$ with respect to $z$ yields

$$
\frac{d\Omega}{di} = \frac{\partial z}{\partial i} \gamma'(p(R) - z) d\tilde{F}(R) + \frac{(1 - \theta)\gamma'(p(R_T) - z)}{[(1 - \theta)\gamma'(p(R_T) - z) + 1]} \frac{\partial \Omega}{\partial R_T}.
$$

Define an implicit function $\tilde{R}(z)$ by $p(\tilde{R}(z)) = z$. Therefore buyers do no use costly credit when they meet a seller who posts $R < \tilde{R}(z)$ or equivalently $\gamma'(p(R) - z) = 0$ for all $R \leq \tilde{R}(z)$. We can rewrite the displayed inequality as

$$
\frac{d\Omega}{di} = \frac{\partial z}{\partial i} \gamma'(p(R_T) - z) \left( \int_{\tilde{R}(z)}^{R} \gamma'(p(R) - z) d\tilde{F}(R) + \frac{(1 - \theta)}{[(1 - \theta)\gamma'(p(R_T) - z) + 1]} \frac{\partial \Omega}{\partial R_T} \right).
$$
When \( i = 0 \), buyers do not use costly credit and thus \( p(R_T) = z \), or equivalently \( \bar{R}(z) = R_T = \bar{R} \). It follows that the integral in the large bracket is 0 at \( i = 0 \). The second term is strictly negative at \( i = 0 \) because (i) the fraction reduces to \( 1 - \theta > 0 \) at \( i = 0 \) and (ii) the derivative \( \partial \Omega / \partial R_T < 0 \) when \( i = 0 \) because \( \bar{F}(R) \) strictly falls in the FOSD sense in \( R_T \) and the integrand in (31) falls strictly in \( R \). Therefore the expression in the large bracket is strictly negative at \( i = 0 \), and thus is strictly negative for \( i \approx 0 \) by continuity. Since \( \gamma'(p_T - z) > 0 \) for \( i > 0 \), \( d \Omega / d i > 0 \) when \( i \) is close to 0.

Change in \( \lambda \): The distribution \( F \) decreases in \( \lambda \) in the strict FOSD sense by (26) and (28) and thus so does the distribution \( \bar{F} \) in (30). It follow that the right-hand side of (30) falls in \( \lambda \) because \( n(R) \) rises and \( p(R) \) falls in \( R \). Since the right-hand side falls in \( z \) near the FR, \( z \) falls in \( \lambda \) to balance (30). Next, since \( z \) falls and \( \lambda \) rises, \( \bar{F} \) falls in the strict FOSD sense, and so does the distribution of prices \( p(R) \), because \( p(R) \) falls in \( R \) by (23). Moreover, fixing \( R \), \( p(R) \) falls as \( z \) falls by (23).

Finally, we argue that welfare \( \Omega \) rises in \( \lambda \). As \( \lambda \) rises, buyers carry less money and we can write the direct and indirect effects of \( \lambda \) as:

\[
\frac{d \Omega}{d \lambda} = \frac{\partial \Omega}{\partial \lambda} + \frac{\partial z}{\partial \lambda} \frac{\partial \Omega}{\partial z}.
\]

The first term is strictly positive because \( \bar{F} \) falls strictly in the FOSD sense in \( \lambda \) as argued above and the integrand in (31) falls strictly in \( R \). The second term vanishes at \( i = 0 \) because \( \partial \Omega / \partial z = 0 \) at the FR as discussed above. Altogether, \( \Omega \) rises in \( \lambda \) at \( i = 0 \). \( \square \)
References


A Results for Nonmonetary Equilibrium

Existence and Uniqueness: Given $z = 0$, from (8)-(9), the bargaining solution $q_T$ and $p_T$ are unique. If $p_T - q_T > k$ then submarket $T$ is active. In this case there is a unique $n_T > 0$ solving $\alpha(n_T)(p_T - q_T) = k$, and submarket $T$ is active at $\Gamma_T = (p_T, q_T, n_T)$. One can similarly show $n_L$ and $q_L$ are unique (see the proof of Lemma 2). By free entry $p_L$ is also unique, and submarket $L$ is active at $\Gamma_L$ if the surplus for consumers is nonnegative. Since the terms of trade are efficient in submarket $L$ and not submarket $T$, if the latter is active so is the former. Hence, a unique non-monetary equilibrium exists.

Comparison of $\Gamma_T$ and $\Gamma_L$: From $N_L > 0$, $\lambda > 0$ (there are no local shops if everyone is uninformed). Immediately (27) and $N_L, \lambda > 0$ imply $n_L > n_T$. Since $n_L$ solves (4), $n_L \leq \hat{n}$ where $\alpha(\hat{n}) = 1$. For $n_L > \hat{n}$ the objective function in (4) can be increased by lowering $n_L$. Since $\hat{n} \geq n_L > n_T$, $\alpha(n_L) > \alpha(n_T)$. Hence $\Pi_L = \Pi_T$ implies $p_L - q_L < p_T - q_T$. The results for $P$ and $R$ are obvious once we check $q$ and $p$, so it remains to show $q_L > q_T$ and $p_L < p_T$. By (5)-(8), if $q_j$ is big then $p_j$ is small, so $p_L - q_L < p_T - q_T$ implies $p_L < p_T$ and $q_L > q_T$. \hfill \Box

B Heterogenous Buyers

Here we sketch a model with heterogenous buyers, first with exogenous $\lambda$, then endogenous $\lambda$. To make the point more clearly, let us consider the case $i = \theta = 0$. Then suppose there are buyer types $j = 1, 2$, where $\rho_j$ is the measure of type-$j$ and $\epsilon_j u(q)$ is their DM utility function, with $\epsilon_1 < \epsilon_2$. Define $q_j^*$ by $\epsilon_j u'(q_j^*) = 1$. Since $i = 0$, all buyers carry enough money to purchase $q_j^*$.

When a type-$j$ buyer is in a tourist shop, the price is given by bargaining with $\theta = 0$, which is $p_T^j = \epsilon_j u(q_j^*)$. Then market tightness is determined by free entry,

$$\alpha(n_T) \frac{(1 - \lambda_1)\rho_1(p_T^1 - q_T^1) + (1 - \lambda_2)\rho_2(p_T^2 - q_T^2)}{(1 - \lambda_1)\rho_1 + (1 - \lambda_2)\rho_2} = k$$

(46)

where $\lambda_j$ is the fraction of type-$j$ buyers that are informed, and the denominator is the to-
tal measure uninformed buyers. If a local shop caters to type-\(j\) buyers then \(\{q^j_L, n^j_L, p^j_L\}\) solves
\[
\max_{q^j_L, n^j_L, p^j_L} \left\{ \frac{\alpha(n^j_L)}{n^j_L} \left[ \epsilon_j u(q^j_L) - p^j_L \right] \right\} \text{ st } \alpha(n^j_L)(p^j_L - q^j_L) = k.
\]

Given \(\{n_T, n^1_L, n^2_L\}\) and the measures of uninformed and informed buyers, we can solve for the measure of all sellers \(N\) and local sellers \(N^j_L\) from
\[
N = \frac{(1 - \lambda_1)\rho_1 + (1 - \lambda_2)\rho_2}{n_T}, \quad N^j_L = \frac{\lambda_j\rho_j}{n^j_L}.
\] (47)

The measure of tourist shops is then \(N_T = N - N^1_L - N^2_L\). The probability of an uninformed buyer entering the tourist submarket is \(N_T/N\) and the probability of entering a local shop catering to type-\(j\) buyers is \(N^j_L/N\).

Define \(U^*_j\) as the expected payoff of type-\(j\) conditional on entering the correct local submarket, namely
\[
U^*_j \equiv \frac{\alpha(n^j_L)}{n^j_L} \left[ \epsilon_j u(q^j_L) - p^j_L \right].
\]

The expected payoff for a type-1 uninformed buyer in the DM is
\[
\frac{N_T}{N} 0 + \left( 1 - \frac{N_T}{N} \right) \left( \frac{N^1_L}{N} U^*_1 + \frac{N^2_L}{N} U^*_2 \right);
\] (48)
type-2 is similar. An insight from this extension is that the information externality can be negative:

**Proposition 16** An increase in \(\lambda_2\) raises \(N^1_L\) and \(N^2_L\), lowers \(N_T\) and \(N\), and raise the expected payoff of both types of uninformed buyers. An increase in \(\lambda_1\) raises \(N_T\), \(N\), \(N^1_L/N\) and \(N^2_L/N\), provided \(\alpha(n)\) is sufficiently inelastic, and lowers the expected payoff of both types uninformed buyers.

**Proof:** By (46), a small increase in \(\lambda_1\) reduces \(n_T\). If \(\alpha(n)\) is sufficiently inelastic, then the drop in \(n_T\) would be arbitrarily large such that \(N\) in (47) rises by an arbitrary amount. Since \(N^1_L\) increases at the same rate as \(\lambda_1\) by (47) and \(N^2_L\) remains constant, \(N^1_L/N\) and \(N^2_L/N\) fall and \(N_T/N\) rises provided that \(\alpha(n)\) is sufficiently inelastic. Since \(N_T/N\) can rise by an arbitrarily amount, a type 1 uninformed buyer’ payoff drops by (48) (the ratio \(N^1_L/N_L\) and \(N^2_L/N_L\) change in \(\lambda_1\) but their effect would be dominated by
that of $N_T/N$). One can use the same proof logic to show a type-2 uninformed buyer’

Intuitively, higher $\lambda_1$ reduces the fraction of type-1 uninformed buyers and thus

increases tourist shops’ expected surplus. By (46) $n_T$ falls and this effect is particularly

strong when $\alpha(n)$ is inelastic. By (47), an increase in $n_T$ raises $N_T$, so there are more

tourist shops and uninformed buyers are more likely to enter the tourist submarket.

Now suppose information is endogenous. We focus on equilibrium where informed

buyers can always find the relevant local shops. In other words, we ignore the trivial

equilibrium where buyers do not acquire information because they think submarket $L$

is empty, and submarket $L$ is empty because sellers think there is no informed buy-

ers. Then stationary equilibrium is unique. Depending on the size of $s$, there are four

possible cases:

**Proposition 17** There are four types of equilibrium

1. If $s \geq U_2^*$, then $\lambda_1 = \lambda_2 = 0$.

2. If $U_2^* > s > U_2^* \left( \frac{U_1^* - U_2^*}{U_2^* - U_1^*} \right)$, then $\lambda_2 > \lambda_1 = 0$.

3. If

$$U_2^* \left( \frac{U_1^* - U_2^*}{U_2^* - U_1^*} \right) \geq s \geq \frac{(U_1^* - U_2^*)(U_2^* - U_1^*)}{U_1^2 + U_2^1},$$

then $\lambda_1, \lambda_2 > 0$.

4. If

$$s < \frac{(U_1^* - U_2^*)(U_2^* - U_1^*)}{U_1^2 + U_2^1},$$

then $\lambda_1 = \lambda_2 = 1$.

**Proof:** We consider each case in turn.

**Case 1:** $\lambda_1 = \lambda_2 = 0$. When $s$ is large, no buyer acquires information and only

the tourist submarket is open. Since $\theta = 0$, buyers’ DM payoff is 0. Then the value

of information to a type-$j$ buyer is $U_j^*$. If $s \geq U_j^*$ for $j = 1, 2$, no buyer acquires

information. It is easy to show $U_2^* \geq U_1^*$. Thus, equilibrium with $\lambda_1 = \lambda_2 = 0$ exists if

and only if $s \geq U_2^*$.
Case 2: $\lambda_2 > \lambda_1 = 0$. For $s \leq U^*_2$, some type-2 buyers acquire information. Since $\lambda_2 \in (0, 1)$, they are indifferent between acquiring information or not. Therefore,

$$U^*_2 - s = \frac{N_T}{N}0 + \left(1 - \frac{N_T}{N}\right)U^*_2 \implies \frac{N_T}{N} = \frac{s}{U^*_2}.$$  

Given $N_T/N$, one can solve for $\lambda_2$ by (47).

To sustain $\lambda_1 = 0$, we need to make sure type-1 buyers do not acquire information. Letting $U^*_j$ be the payoff of a type $j$ buyer at a local shop catered to a type $\ell$ agent, we need

$$U^*_1 - s < \frac{N_T}{N}0 + \left(1 - \frac{N_T}{N}\right)U^*_1 \implies s > U^*_1 - \left(1 - \frac{N_T}{N}\right)U^*_1.$$  

Using $N_T/N = s/U^*_2$, one can show this equilibrium exists when

$$U^*_2 > s > U^*_2 \left(\frac{U^*_1 - U^*_2}{U^*_2 - U^*_1}\right).$$

Case 3: $\lambda_2, \lambda_1 > 0$. When $s$ is small, both types acquire information. The expected DM payoff for a type 1 uninformed buyer is

$$\frac{N_T}{N}0 + \left(1 - \frac{N_T}{N}\right)\left(\frac{N_L^1}{N_L}U^*_1 + \frac{N_L^2}{N_L}U^*_2\right).$$

It follows that the indifference conditions are

$$U^*_1 - s = \left(1 - \frac{N_T}{N}\right)\left(\frac{N_L^1}{N_L}U^*_1 + \frac{N_L^2}{N_L}U^*_2\right)$$

$$U^*_2 - s = \left(1 - \frac{N_T}{N}\right)\left(\frac{N_L^2}{N_L}U^*_2 + \frac{N_L^1}{N_L}U^*_1\right).$$

Using these we can solve for

$$\frac{N_T}{N} = \frac{(U^*_2 + U^*_1)s - (U^*_1 - U^*_2)(U^*_2 - U^*_1)}{U^*_2(U^*_1 + U^*_2) - (U^*_2 - U^*_1)(U^*_2 - U^*_1)}$$ \hspace{1cm} (49)$$

$$\frac{N_L^1}{N_L} = \frac{(U^*_1 - U^*_2) - (U^*_2 - U^*_1)\frac{N_T}{U^*_2}}{U^*_1 + U^*_2}. $$ \hspace{1cm} (50)$$

As $s$ decreases, $N_T/N$ falls and $N_L^1/N_L$ rises. An equilibrium exists when $N_L^1/N_L, N_T/N > 0$. One can show that this holds when

$$U^*_2 \left(\frac{U^*_1 - U^*_2}{U^*_2 - U^*_1}\right) \geq s \geq \frac{(U^*_1 - U^*_2)(U^*_2 - U^*_1)}{U^*_2 + U^*_1}.$$  

Case 4: $\lambda_2 = \lambda_1 = 1$. When $s$ is sufficiently small, all buyers are informed and the tourist submarket vanishes. By (49), $N_T/N < 0$ when

$$s < \frac{(U^*_1 - U^*_2)(U^*_2 - U^*_1)}{U^*_2 + U^*_1}. \hspace{1cm} \Box$$
C  Quantitative Robustness

Here we consider alternative models or calibration strategies to check robustness of the main results. The results are reported in Figure 11 and Table 3.

**Markup and Standard Deviation Targets** Consider a calibration where we target half of the observed markup and standard deviation in prices, i.e., a 20% DM markup and a standard deviation of 7.5%. Other targets stay the same. This yields similar values for the utility and cost functions: $b = 0.67$, $B = 0.645$, $c = 5.4$ and $C = 12.5$. The lower standard deviation implies $\theta = 0.87$, rather than 0.72 as in the baseline calibration, and the lower markup implies $\lambda = 0.19$, rather than 0.38 as in the baseline. The markups in the local and tourist submarket come out as 13.7% and 27.6%, respectively.

The welfare cost of inflation falls slightly – going from 10% to $\pi^*$ is worth 0.9% of consumption rather than 1.1%. The welfare gain of going to $\lambda = 1$ from the calibrated value is 1.2% rather than 1.0%, while the gain of going to $\lambda = 1$ from 0 is 1.9% rather than 3.5%. The impact of changing credit conditions is essentially unchanged.

**Heterogeneous Credit Costs** Supposed informed agents know the terms at all shops, and also know the owners enough to get a lower cost of credit. Thus the credit cost function is $\gamma_j(d) = C_j d^c$, where $j = I, U$ and $C_I < C_U$. To avoid adding a new moment, let $C_I = C_U / 3$. Then calibration yields $C_U = 33$ and $C_I = 11$. While 1/3 is arbitrary, to give a sense of magnitude, it implies this: informed agents use credit for 70% of their expenditures while the uninformed the number is 42%. The remaining parameters remain essentially unchanged.

The welfare cost of inflation, information, and credit conditions change little in this version, despite large heterogeneity in the use of debt between informed and uninformed agents. The cost of 10% inflation is around 0.8%. The gain of going to $\lambda = 1$ from 0 is 1.8% and the gain of going to $l = 1$ from the calibrated value is 4.2%. The impact of changing credit conditions remains small.
Figure 11: Robustness: (from top-left to bottom-right) money demand, credit demand, and the welfare cost of, respectively, \( i \), \( \lambda \), and \( C \) across models.
Multiple DM’s  Consider next a version of the model with multiple – here, two – rounds of DM trade. Let informed buyers be informed in both rounds and the rest uninformed in both rounds. Here λ is exogenous (we also tried endogenous λ, with similar results). For simplicity, each buyer makes only 1 purchase per period – so if he trades in DM1 then he skips DM2. Hence, there are exactly two types of buyers (informed and uninformed) in each DM.

This builds in persistence in money holdings and information (across DM’s) that has consequences for the distribution of prices and, importantly, entry of tourist shops. In DM1, buyers have an outside option to trade in DM2, which lowers the surplus of high-priced sellers. Hence, equilibrium tightness in the tourist submarket is higher in DM1 than DM2, and in fact, for calibrated parameters tourist shops are open in DM2 and not in DM1. The calibrated utility and cost parameters are similar to the baseline: $b = 0.69$, $B = 0.55$, $c = 5.4$ and $C = 14.9$. Also, $\theta = 0.65$ and $\lambda = 0.37$.

The measure of tourist shops falls from 42% in the baseline specification to 7%.
However, their markup increases from 65% to 100%. So while buyers are less likely to run into tourist shops, they get worse deal when they do. The cost of 10% inflation does not change much, falling only to 0.9%. The gain of going to $\lambda = 1$ from 0 is 1.2% and of going to $\lambda = 1$ from the calibrated value is 0.2%. While more work could be done on this class of models, that is relegated to future work.

**Fixed Permanent Information Types**  Suppose information types are permanent. Of course, buyers choose money given their type. However, now changes in inflation or variables do not alter $\lambda$. Calibration yields $b = 0.76$, $B = 0.52$, $c = 5.8$, $C = 24.81$, $\theta = 0.81$ and $\lambda = 0.40$. The cost of inflation and incomplete information are essentially unchanged.