The Equity Premium and the Financial Accelerator

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February 27, 2017

Abstract

This paper investigates the amplification mechanism of the financial accelerator on the equity premium in a production economy. To accomplish this, I incorporate the Gertler and Karadi (2011) type financial accelerator into a medium-scale New Keynesian model with generalized recursive preferences. I find that the financial accelerator is a very plausible and new amplification mechanism for risk premia in the model. For the baseline calibration, the financial accelerator increases the equity premium by 46 basis points and produces fourfold greater response to shocks than the model-implied equity premium without financial frictions. I also show two channels by which the financial accelerator affects the equity premium. The first channel increases the variability of the stochastic discount factor, and the second channel affects interest rates and inflation through the Taylor rule and marginal cost, respectively. Finally, increasing the adjustment costs of investment does not improve the asset pricing performance in the model.

JEL classification: E32, E44, G12

Keywords: Financial accelerator, Epstein-Zin preferences, Equity premium, DSGE models

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1 Introduction

Macroeconomic models with the financial accelerator have received substantial attention after the Great Recession. The financial accelerator introduces a friction between lenders and borrowers that amplifies business cycle fluctuations in macroeconomic models (e.g., Bernanke, Gertler, and Gilchrist, 1999; Kiyotaki and Moore, 1997; and Gertler and Karadi, 2011). With a negative shock, the amplification mechanism is driven by the disruption of asset value that reduces lending (or borrowing) capacity for financial intermediaries (or non-financial firms). These macroeconomic models, however, do not seek to account for asset prices and risk premia, and are not particularly good at matching financial market variables.\(^1\)

In the present paper, I investigate the amplification effect of the financial accelerator on the equity premium in a general equilibrium framework. This attempt has important implications for the macrofinance literature. Traditionally, there are two different approaches in the asset pricing literature to capturing sufficiently large risk premia: one is increasing risk in the model by introducing uncertainty in the model (e.g., Weitzman, 2007; and Barillas, Hansen and Sargent, 2009), long-run risk (e.g., Bansal and Yaron, 2004; and Rudebusch and Swanson, 2012), or rare disaster (e.g., Rietz, 1988; and Gourio, 2012) and the other is using heterogeneous agents (e.g., Constantinides and Duffie, 1996; and Schmidt, 2015). I therefore make an attempt to expand the understanding of the interaction between the macroeconomy and financial markets by analyzing the role of financial frictions on the risk premium.

To accomplish this, I incorporate the Gertler and Karadi (2011) type financial accelerator and generalized recursive preferences into a medium-scale New Keynesian DSGE model. In addition to the financial accelerator, generalized recursive preferences also have a salient role because they allow model not only to match the basic macroeconomic behaviors but also to generate substantial risk premia. Thus, in this paper, generalized recursive preferences are exploited rather than expected utility preferences or habit preferences.\(^2\) I then compare the results of my model with those of the standard DSGE model without the financial accelerator.

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\(^1\) For example, Mehra and Prescott (1985), Rouwenhorst (1995), Lettau and Uhlig (2000), and Rudebusch and Swanson (2008) find a large discrepancy between the model-implied risk premium and the actual data.

\(^2\) As Lettau and Uhlig (2000) and Rudebusch and Swanson (2008) point out, habit-based DSGE models cannot fit the term premium in a production economy because habit preferences generate “super” consumption smoothing.
My results are as follows. First, the financial accelerator amplifies both the size and the response of the equity premium. For the baseline calibration, the financial accelerator increases the equity premium by 46 basis points and produces fourfold greater response to shocks than the model-implied equity premium without financial frictions. Second, the impact of the financial accelerator on the equity premium has two different channels. The first channel increases the variability of the stochastic discount factor, and the second channel affects interest rates and inflation through the Taylor rule and marginal cost, respectively. Finally, increasing the adjustment costs of investment does not improve the asset pricing performance in the model.

The intuition for the amplification mechanism of the financial accelerator is simple. During recessions, the marginal productivity of capital decreases and this leads to a lower return of capital. The net worth of financial intermediaries then declines because the return of capital is the only source of profits for the bank in the model. Because net worth is reduced, financial intermediaries are less likely to issue security, which reduces capital. As capital demand declines, so does the price of capital, which again reduces capital returns and lowers the bank’s net worth. Through this cycle, even small shock can have an amplified impact on the economy, and the volatility of consumption and the stochastic discount factor increases.

The financial accelerator also reduces the output more and increases inflation less when there is a negative technology shock. As the financial accelerator causes the price of capital to fall further and offsets the rise in marginal cost, the inflation rises slightly. The central bank thus is tempted to respond more to the economic downturn rather than inflation although it depends on the central bank’s stance to monetary policy. If this is true, the risk-free return may fall and the equity premium increases more with the financial friction.

Finally, the equity premium is not sensitive to the investment adjustment cost. There are two main reasons for this. The first is that capital producers own all the capital stock in this model. As in Bernanke, Gertler, and Gilchrist (1999), my model introduces capital producers to easily determine the capital price endogenously. Therefore, the inelastic supply of capital has little effect on the consumption of households and the equity premium. Second, households do not have habit preferences. The habit preferences in a production economy focus on achieving consumption smoothing at any cost. In

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Note that consumption smoothing requires more cost for households with the high elasticity of investment adjustment costs in the standard DSGE model. Accordingly, the stochastic discount factor fluctuates more and this increases the equity premium as in Jermann (1998).
contrast, households without habit preferences do not cost more for consumption smoothing, even as the investment adjustment costs increase.

This paper is closely related with two strands of literature. First, Gertler and Karadi (2011) and Gertler and Kiyotaki (2015) construct quantitative DSGE models with financial intermediaries that face endogenously determined balance sheet constraints so that they amplify shocks to the macroeconomy. In contrast to these papers, the model here matches not only features of the real economy but also the equity premium. Second, Tallarini (2000), Campanale, Castro, and Clementi (2010), and Swanson (2016) document that generalized recursive preferences allow models to generate substantial risk premium without distortion of their ability to match the macroeconomic facts. Relative to them, the model here considers financial market disruptions and a sharp contraction of the real economy due to the financial accelerator, and its effect on the equity premium.

The present paper is also linked to the three papers that study the implication of financial frictions on asset pricing fluctuations. Gomes, Yaron and Zhang (2003, henceforth GYZ) find that the mean of the equity premium is significantly higher with financial market imperfections. There are several important differences between my paper and GYZ. First, in my model, generalized recursive preferences have a dominant effect on the equity premium compared to that of the financial frictions. Instead, their study uses standard expected utility preferences that replicate a very small equity premium. Second, I employ conventional asset pricing theory as in Cochrane (2009) and solve the model nonlinearly to reflect the risk of the model. On the other hand, their work defines the equity premium as the spread between the return of capital and the risk-free rate, which they solve using a standard first order solution. Thus, their equity premium is a risk-neutral external finance cost rather than a risk premium. Finally, the financial frictions in their model are more traditional: their friction is between the entrepreneurs and banks, and an entrepreneur’s ability to raise funds relies on its capital. In contrast, in my model, the agency problem is between financial intermediaries and households to reflect some features of the recent financial crisis.\footnote{Adrian, Colla and Shin (2013) find that the supply of intermediated credit is sharply reduced in the recent financial crisis.}\footnote{In other words, the present paper focuses on the supply side of financial service, while GYZ analyze the demand side of finance.}

In terms of the model, perhaps Bigio (2012) is the closest to the present paper. Bigio (2012) incorporates Kiyotaki and Moore (2008) type liquidity shocks into a real business cycle model with Epstein-Zin preferences. Relative to my paper, he documents that...
the liquidity constraint do not generate a substantial increase of the equity premium. This occurs because Bigio (2012) uses the stochastic discount factor of the saving-type entrepreneur who increases consumption to the liquidity shocks. By contrast, my paper exploits the household’s stochastic discount factor which is countercyclical. Moreover, due to the lack of explicit financial system in that paper, the liquidity shocks have so small effects on both asset pricing and the macroeconomy. The other related paper is He and Krishnamurthy (2013). In that paper, financial intermediaries are considered as marginal investors in asset pricing, and a more sophisticated structure could be established to calibrate risk-premia. However, their model is relatively weak in explaining the linkage with the macroeconomy, as an overlapping generation model is used in an endowment economy.

As the goal of this paper is illustrating the underlying mechanisms how the financial accelerator affects the risk premium, I want to keep the simplicity of the model by considering technology shock only. This is not an unreasonable assumption. According to Rudebusch and Swanson (2012), the response of the term premium to the technology shock shows greater response by a factor of 250 and 625 than monetary policy shock and government spending shock, respectively. Thus, Tallarini (2000), GYZ, and Swanson (2016) did not consider any exogenous shock other than the technology shock.

The organization of the paper is as follows. Section 2 presents the model with the financial accelerator and generalized recursive preferences. Section 3 lays out the calibration results. Sections 4 concludes. An appendix to the paper provides additional details of how the model is solved.

2 The Model

In this section, I begin by outlining a medium-scale New Keynesian DSGE model and use it to price equity. The model has two important ingredients: the financial accelerator (as in Gertler and Karadi, 2011) and generalized recursive preferences (as in Tallarini, 2000; and Swanson, 2016). The Gertler-Karadi type financial accelerator introduces frictions between financial intermediaries and households and allows the model to have the feedback between the financial market and the economy. Generalized recursive preferences allows the model to match both macroeconomic and financial stylized facts.

There are four types of agents in the model: households, financial intermediaries, non-financial

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6 The model-implied equity premium is 0.23% with the liquidity constraint, while it is 0.224% without the friction.
firms, and capital producers. The latter are required to make the endogenous capital price tractable as suggested by Bernanke, Gertler, and Gilchrist (1999). Figure 1 displays the building blocks of the model. In order to produce output, non-financial firms purchase capital and hire labor from capital producers and households, respectively. Firms issue security claims, \( S_t \), to buy capital, \( K_{t+1} \), and pay gross return of capital, \( R^k_{t+1} \), to financial intermediaries. Households give funds to financial intermediaries as deposits, \( D_t \), and receive a risk-free return, \( e^{r_{t+1}} \). Finally, the price of capital, \( Q_t \), is endogenously determined by capital demand from non-financial firms and supply from capital producers.

2.1 Households

There is a unit continuum of identical households. Each household is endowed with generalized recursive preferences as in Epstein and Zin (1989) and Weil (1989). For simplicity, the model in the present paper employs the additive separability assumption for period utility following Woodford (2003).\(^7\)

\[
u(c_t, l_t) \equiv \log c_t - \chi_0 \frac{l_t^{1+\chi}}{1+\chi},
\]

\(^7\)van Binsbergen et al. (2012) uses Cobb-Douglas preferences since they consider consumption and leisure as composite good.

\[
u(c_t, l_t) = (c_t^{\nu} (1 - l_t)^{1-\nu})^{\frac{1-\gamma}{\nu}}
\]

In this case, the stochastic discount factor is more complicated since consumption and leisure form a composite good. On the other hand, the additive separability assumption facilitates a simpler stochastic discount factor which is affected by the growth of consumption only rather than the composite good.
where $c_t$ is household consumption, $l_t$ is labor in period $t$, and $\chi_0 > 0$ and $\chi > 0$ are parameters. Moreover, assuming logarithmic period utility for consumption allows a balanced growth path and unit intertemporal elasticity of substitution as in King and Rebelo (1999). Households deposit to financial intermediaries to earn the continuously-compounded default free interest rate, and provide labor to non-financial firms to receive their wages. Using continuous compounding is convenient for equity pricing and comparison with the finance literature. Hence, the household’s budget constraint is given by:

$$c_t + \frac{d_{t+1}}{P_t} = w_t l_t + e^{\gamma_t} \frac{d_t}{P_t} + \Pi_t,$$

where $d_t$ is deposits, $P_t$ is the aggregate price level (to be defined later), $w_t$ is the real wage, $e^{\gamma_t}$ is the nominal gross risk-free return from deposits, and $\Pi_t$ is the household’s share of profits in the economy.

Following Hansen and Sargent (2001) and Swanson (2016), I assume that households have multiplier preferences. In every period, the household faces the budget constraint (2) and maximizes lifetime utility with the no-Ponzi game constraint. The household’s value function $V^h(d_t; \Theta_t)$ satisfies the Bellman equation:

$$V^h(d_t; \Theta_t) = \max_{c_t, l_t} \left[ (1 - \beta) u(c_t, l_t) - \beta \alpha^{-1} \log \left( E_t \exp \left( -\alpha V^h(d_{t+1}; \Theta_{t+1}) \right) \right) \right],$$

where $\Gamma$ is the choice set for $c_t$ and $l_t$, $\Theta_t$ is the state of the economy, $\beta$ is the household’s time discount factor, and $\alpha$ is a parameter. Risk aversion is closely related to the Epstein-Zin parameter $\alpha$ which amplifies risk aversion by including the additional risk for the lifetime utility of households.\footnote{Rudebusch and Swanson (2012) use a generalized form of Epstein-Zin-Weil specification with nonnegative period utility:

$$V_t = u(c_t, l_t) + \beta \left( E_t V^{1-\alpha}_{t+1} \right)^{\frac{1}{1-\alpha}},$$

which is similar to expected utility preferences except “twisted” and “untwisted” by the factor $1 - \alpha$. Note that the expected utility preferences are the special cases of generalized recursive preferences when $\alpha = 0$, and the household’s intertemporal elasticity of substitution is the same as that of the expected utility preferences case, but risk aversion can be amplified or attenuated by the additional curvature parameter $\alpha$ when $\alpha \neq 0$. Although this form is convenient, an Epstein-Zin-Weil specification depends on the sign of period utility $u(\cdot)$. Therefore, Hansen and Sargent (2001) and Swanson (2016) consider multiplier preferences as they are free from the sign of period utility. Multiplier preferences can be obtained when $\rho \rightarrow 0$ from the specification in Epstein and Zin (1989):

$$U_t = \left[ \tilde{u}(c_t, l_t)^{\rho} + \beta \left( E_t U^{\frac{\alpha}{1-\alpha}}_{t+1} \right)^{\frac{1}{1-\alpha}} \right]^{\frac{1}{\rho}}.$$

Precisely, $R^e = \alpha + \left( 1 + \frac{\alpha}{\chi} \right)^{-1}$ for the case with period utility as (1). This closed form expression considers both consumption and labor which provides additional cushion to the household against the negative shock.}
The household’s stochastic discount factor is given by\(^\text{10}\)
\[
m_{t+1} = \beta \frac{c_t}{c_{t+1}} \frac{\exp \left( -\alpha V^h (d_{t+1}; \Theta_{t+1}) \right)}{E_t \exp \left( -\alpha V^h (d_{t+1}; \Theta_{t+1}) \right)}.
\] (4)

The first order necessary conditions for deposit and labor are given by:
\[
d_{t+1} : \ 1 = E_t \left( m_{t+1} e^{\pi_{t+1}^{-1}} \right),
\] (5)
\[
l_t : \ \chi_0 l_t^X \left( \frac{1}{c_t} \right)^{-1} = w_t.
\] (6)

Then, the one-period continuously-compounded risk-free real interest rate, \(r_{t+1}\), is
\[
e^{-r_{t+1}} = E_t m_{t+1},
\] (7)

since \(e^{r_{t+1}} \equiv e^{\pi_{t+1}^{-1}}\).

### 2.2 Financial Intermediaries

There is a unit continuum of bankers, and each risk neutral banker runs a financial intermediary. The financial intermediaries lend funds to non-financial firms by using their own net worth or issuing deposits to households. As suggested by Gertler and Karadi (2011), I introduce two key assumptions to ensure that there is always financial friction between financial intermediaries and households. First, financial intermediaries have to borrow from households each period in the form of deposits. This assumption prevents the financial intermediary from lending funds to non-financial firms with their own capital alone. Thus, its balance sheet constraint is given by:

\[
\text{Balance Sheet Constraint (BC)} : \ Q_t s_t = n_t + d_{t+1},
\] (8)

where \(Q_t\) is the relative price of financial claims on firms that the bank holds, \(s_t\) is the quantity of each claim, and \(n_t\) is the banker’s net worth. The asset of the financial intermediary, \(Q_t s_t\), is composed of equity capital (or net worth), \(n_t\), and debt, \(d_{t+1}\) as in (8). To keep the number of bankers stable\(^\text{10}\)The household’s optimization problem with generalized recursive preferences can be solved using the standard Lagrangian method. See Rudebusch and Swanson (2012) for more detail.
and to prevent the accumulation of net worth, there is an \( i.i.d. \) survival probability \( \sigma \) for the fraction who can remain in the financial industry in the next period. So, \((1 - \sigma)\) fraction of bankers retire and consume their net worth when they leave. Second, households are willing to deposit in financial intermediaries. There is a moral hazard problem between depositors and financial intermediaries: a financial intermediary may divert a portion of its assets after deposits are collected. Consequently, the incentive constraint must hold in order to avoid households punishing diverting bankers by ceasing to supply deposits:

\[
\text{Incentive Constraint (IC)}: \quad V^b_t \geq \psi Q_t s_t,
\]

where \( \psi \) is a fraction when the banker diverts the assets of the financial intermediary, and \( V^b_t \) is the bank’s franchise value (defined below). As long as the banker is constrained due to the financial friction, the risk neutral banker’s objective is to maximize its consumption at the exit period:

\[
\max V^b_t = E_t \left[ \sum_{j=1}^{\infty} \beta^j (1 - \sigma) \sigma^{j-1} n_{t+j} \right].
\]

Observe that the financial intermediary’s terminal wealth, \( n_{t+j} \), is the banker’s consumption, \( c^b_{t+j} \), in the exit period.\(^{11}\) (10) can be written in the first-order recursive form:

\[
V^b_t = E_t \left[ \beta (1 - \sigma) n_{t+1} + \beta \sigma V^b_{t+1} \right].
\]

The net worth of a surviving financial intermediary in the next period, \( n_{t+1} \), is simply the gross return of the asset net of the cost of debts:

\[
n_{t+1} = R^k_{t+1} Q_t s_t - e^{r_{t+1}} d_{t+1}
\]

\[
= \left( R^k_{t+1} - e^{r_{t+1}} \right) Q_t s_t + e^{r_{t+1}} n_t,
\]

where \( R^k_{t+1} \) is the gross return of capital. Then, the growth rate of net worth is

\[
\frac{n_{t+1}}{n_t} = \left( R^k_{t+1} - e^{r_{t+1}} \right) \phi_t + e^{r_{t+1}},
\]

\(^{11}\)The bankers discount net worth with \( \beta \) since they are risk neutral. The basic results of the model do not change even if the bankers use the stochastic discount factor of households as in Gertler and Karadi (2011).
where $\phi_t \equiv \frac{Q_s n_t}{n_t}$ is the “leverage multiple.” Note that the growth rate of net worth is increasing in the leverage multiple when the spread, $R_{t+1}^k - e^{r_{t+1}}$, is positive.

Since (8) and (11) are constant returns to scale, (11) is equivalent to\(^{12}\)

$$\frac{V^b_t}{n_t} = E_t \left[ \beta \left( (1 - \sigma) + \sigma \frac{V^b_{t+1}}{n_{t+1}} \right) \frac{n_{t+1}}{n_t} \right]$$

$$= \mu_t \phi_t + \nu_t,$$

where $\mu_t \equiv \beta E_t \Omega_{t+1} \left( R_{t+1}^k - e^{r_{t+1}} \right)$ is the excess marginal value of assets over deposits, $\nu_t \equiv \beta E_t \Omega_{t+1} e^{r_{t+1}}$ is the marginal cost of deposits, and $\Omega_{t+1} \equiv (1 - \sigma) + \sigma \frac{V^b_{t+1}}{n_{t+1}}$ is the weighted average of the marginal values of net worth to exiting and to continuing bankers at $t + 1$.

Combining (9) and (14) yields the leverage multiple:

$$\phi_t = \frac{\nu_t}{\vartheta - \mu_t},$$

if incentive constraint binds and $\mu_t \in (0, \vartheta)$. Since the leverage multiple is a common factor as in (15), the aggregate leverage constraint is

$$Q_t S_t = \phi_t N_t,$$

where $S_t$ is the aggregate quantity of claims and $N_t$ is the aggregate net worth.

The aggregate net worth consists of two components. The first is the net worth of surviving financial intermediaries. With the survival probability, $\sigma$, the banker remains in the banking sector, in which case the banker earns the net revenue, $R_t^k Q_{t-1} S_{t-1} - e^{r_t} D_t$. The second corresponds to seed money, $\omega Q_{t-1} S_{t-1}$, that a new banker receives in every period from their respective household. This seed money is a small fraction, $\omega$, of the value of the exiting financial intermediary’s assets. Accordingly, the aggregate net worth of the entire banking sector is

$$N_t = \sigma \left( R_t^k Q_{t-1} S_{t-1} - e^{r_t} D_t \right) + \omega Q_{t-1} S_{t-1},$$

where $D_t$ is the aggregate amount of deposits.

\(^{12}\)Gertler and Kiyotaki (2015) calls the franchise value per unit of net worth, $\frac{V^b_t}{n_t}$, as Tobin’s Q.
Lastly, aggregate consumption of exiting bankers is the fraction \((1 - \sigma)\) of net earnings of assets:

\[
C^b_t = (1 - \sigma) \left[ R^k_t Q_{t-1} S_{t-1} - e^r D_t \right],
\]

where \(C^b_t = c^b_t\) denotes aggregate consumption demanded by bankers.

### 2.3 Firms

#### 2.3.1 Non-Financial Firms

There is a single final good which is produced using a continuum of intermediate goods indexed by \(f \in [0, 1]\) with the following production function:

\[
Y_t = \left( \int_0^1 y_t(f)^{\frac{1}{1+\sigma}} df \right)^{1+\theta}, \tag{19}
\]

where \(y_t(f)\) is an intermediate good, and \(\theta > 0\) is a parameter captures the equilibrium markup. The final goods firms are perfectly competitive and maximize profits subject to the production function. This implies a downward sloping demand curve for each intermediate good:

\[
y_t(f) = \left( \frac{p_t(f)}{P_t} \right)^{-\frac{1+\theta}{\theta}} Y_t, \tag{20}
\]

where \(P_t\) is the CES aggregate price of the final good:

\[
P_t = \left( \int_0^1 p_t(f)^{-\frac{1}{\theta}} df \right)^{-\theta}, \tag{21}
\]

which can be derived from the zero profit condition.

The economy contains a continuum of monopolistically competitive intermediate goods firms indexed by \(f \in [0, 1]\). Firms purchase capital goods from capital producers and hire labor from households. They also issue claims, \(s_t\), to financial intermediaries in order to obtain financing. Firms have identical Cobb-Douglas production functions:

\[
y_t(f) = A_t k_t(f)^{1-\eta} l_t(f)^\eta, \tag{22}
\]
where \( k_t(f) \) and \( l_t(f) \) are firm \( f \)'s capital and labor inputs, and \( \eta \in (0,1) \) denotes the firm’s output elasticity with respect to labor. \( A_t \) is a technology which follows an exogenous AR(1) process:

\[
\log A_t = \rho_A \log A_{t-1} + \epsilon^A_t, \tag{23}
\]

where \( \rho_A \in (-1,1] \), and \( \epsilon^A_t \) follows an i.i.d. white noise process with mean zero and variance \( \sigma^2_A \). I set \( \rho_A = 1 \) for comparability to the asset pricing literature (e.g., Tallarini, 2000; and Swanson, 2016).

Subject to the demand function and the production function, the intermediate goods firm chooses labor, \( l_t(f) \), and capital, \( k_t(f) \). The first order necessary conditions are:

\[
\begin{align*}
\text{l}_t(f) : & \quad w_t P_t = \varphi_t(f) \eta A_t \left( \frac{k_t(f)}{l_t(f)} \right)^{1-\eta}, \\
\text{k}_t(f) : & \quad R_k^k P_t Q_{t-1} - Q_t P_t (1-\delta) = \varphi_t(f) (1-\eta) A_t \left( \frac{k_t(f)}{l_t(f)} \right)^{-\eta},
\end{align*} \tag{24}
\]

where \( \varphi_t(f) \) is the Lagrange multiplier of the cost minimization problem, and \( \delta \) denotes the depreciation rate of capital. Note that \( Q_t P_t (1-\delta) \) in (25) is the value of the remained capital stock from the previous period. Combining these conditions yields the capital-labor ratio:

\[
\frac{k_t(f)}{l_t(f)} = \frac{1-\eta}{\eta} \frac{w_t}{R_k^k Q_{t-1} - Q_t (1-\delta)}. \tag{26}
\]

Since the capital-labor ratio is a common factor as in (26), it is the same to the aggregate ratio:

\[
\frac{k_t(f)}{l_t(f)} = \frac{K_t}{L_t}, \tag{27}
\]

where \( K_t \) is aggregate capital and \( L_t \) is the aggregate quantity of labor. Moreover, every firm hires capital and labor in the same way, so marginal cost is also the same across firms. Let \( mc_t(f) \equiv \frac{\varphi_t(f)}{P_t} \) be the real marginal cost. Then, \( mc_t(f) = MC_t \) for all \( f \) since \( \varphi_t(f) \) is not an individual firm-specific factor either:

\[
MC_t = \frac{1}{A_t} \frac{w_t^{\eta}}{l_t^{\eta}} \left( R_k^k Q_{t-1} - (1-\delta)Q_t \right)^{1-\eta} \left( \frac{1}{\eta} \right)^{\eta} \left( \frac{1}{1-\eta} \right)^{1-\eta}. \tag{28}
\]
Therefore, the demand functions for capital and labor are:

\[
R^k_{t+1} = \frac{MC_{t+1} (1 - \eta) A_{t+1} \left( \frac{K_{t+1}}{L_{t+1}} \right)^{-\eta} + (1 - \delta) Q_{t+1}}{Q_t}.
\] (29)

\[
w_t = MC_t \eta A_t \left( \frac{K_t}{L_t} \right)^{1-\eta}.
\] (30)

Each intermediate goods firm sets the new contract price \( p_t(f) \) to maximize the firm’s lifetime profit according to Calvo contracts: only a fraction, \( 1 - \xi \), can adjust its price each period. Hence, the value of the firm is given by:

\[
\max_{p_t(f)} E_t \sum_{j=0}^{\infty} m_{t,t+j} (P_t/P_{t+j})^{\xi_j} \left[ p_t(f) e^{\xi_j y_{t+j}(f)} - mc^n_{t+j}(f) y_{t+j}(f) \right],
\] (31)

where \( m_{t,t+j} \equiv \Pi_{i=1}^{j} m_{t+i} \) is the stochastic discount factor of household from period \( t \) to \( t + j \), \( \bar{\pi} \) is steady-state inflation rate, and \( mc^n_{t+j}(f) \) is firm-specific nominal marginal cost.

The first order necessary condition of (31) with respect to \( p_t(f) \) yields the standard New Keynesian price optimality condition:

\[
p_t^*(f) = \frac{(1 + \theta) E_t \sum_{j=0}^{\infty} m_{t,t+j}^{1/\theta} P_{t+j}^{1/\theta} Y_{t+j}}{E_t \sum_{j=0}^{\infty} m_{t,t+j}^{1/\theta} P_{t+j}^{1/\theta} Y_{t+j}^{1/\theta}}.
\] (32)

Note that the optimal price \( p_t^*(f) \) is a markup over a weighted average of current and expected future marginal costs.

### 2.3.2 Capital Producers

Lastly, there is a continuum of representative capital producers. They sell new capital to intermediate goods firms at price \( Q_t \), and produce it using the input from the final output at price unity subject to convex (quadratic) investment adjustment cost.\(^{13}\) The capital producer chooses new capital, \( I_t \), in order to maximize expected discounted profits over her lifetime:

\(^{13}\)While there are multiple ways to introduce the investment adjustment cost, this paper follows along the lines of Gertler, Kiyotaki, and Queralto (2012).
\[
\max_{t, I_t} \sum_{j=0}^{\infty} m_{t,t+j} \left\{ (Q_{t+j} - 1) I_{t+j} - \frac{\kappa}{2} \left( \frac{I_{t+j}}{I_{t+j-1}} - 1 \right)^2 I_{t+j} \right\},
\]

where \( \kappa \) denotes the elasticity of the investment adjustment costs. Observe that with zero investment adjustment costs, \( \kappa = 0 \), the firms would produce infinite capital if \( Q_t > 1 \). A large elasticity of the investment adjustment costs \( \kappa \) implies that the capital producer cannot change her supply easily.

The first order necessary condition with respect to \( I_t \) yields:

\[
Q_t = 1 + \frac{\kappa}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 + \kappa \left( \frac{I_t}{I_{t-1}} - 1 \right) \left( \frac{I_t}{I_{t-1}} \right) - E_t m_{t,t+1} \kappa \left( \frac{I_{t+1}}{I_t} - 1 \right) \left( \frac{I_{t+1}}{I_t} \right)^2,
\]

which is the supply of new capital.

2.4 Aggregate Resource Constraints and Monetary Policy

Combining the downward sloping demand curve and the production function yields the aggregate output:

\[
Y_t = \Delta_t^{-1} A_t K_t^{1-\eta} L_t^\eta,
\]

where \( \Delta_t \equiv \int_0^1 \left( \frac{p_t(f)}{f} \right) -1/\pi \ df \) denotes the cross-sectional price dispersion.

A monetary authority in the model determines the one-period nominal interest rate, \( i_t \), by a simple Taylor-type rule with interest-rate smoothing:

\[
i_t = \rho_i i_{t-1} + (1 - \rho_i) \left[ r + \pi_t + \phi_\pi (\pi_t - \bar{\pi}) + \frac{\phi_y}{4} (y_t - \bar{y}_t) \right],
\]

where \( \rho_i \in (0,1) \) is the smoothing parameter, \( r = \log(1/\beta) \) is the continuously compounded real interest rate in steady state, \( \pi_t \equiv \log(P_t/P_{t-1}) \) is the inflation rate, \( \bar{\pi} \) is the target inflation of the monetary authority, \( y_t \) is the log of output \( Y_t \),

\[
\bar{y}_t = \rho_y \bar{y}_{t-1} + (1 - \rho_y) y_t,
\]

is a trailing moving average of \( y_t \), and \( \phi_\pi, \phi_y \in \mathbb{R} \) and \( \rho_y \in [0,1] \) are parameters. As suggested by
Swanson (2016), the term \((y_t - \bar{y}_t)\) in (36) is an empirically motivated measure of the output gap. In practice, the central bank adjusts the short term nominal interest rate when the output deviates from its recent history. Since monetary policy also affects the real risk-free return according to the Fisher equation, setting the output gap with (37) helps to generate the risk premium consistent with the actual data.

Finally, the economy-wide resource constraint is given by:

\[
Y_t = C_t + C^{b}_t + \left\{ 1 + \frac{\kappa}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right\} I_t,
\]

(38)

where \(C_t = c_t\) denotes aggregate consumption of households.

### 2.5 The Equity Premium

Once I obtain the stochastic discount factor for the household, it is straightforward to calculate the equity price (Cochrane, 2009). I model stocks as a levered claim on the aggregate consumption for simplicity. This interpretation of dividends is standard in the asset pricing literature (e.g., Abel, 1999; and Campbell, Pfueger, and Viceira, 2014). In every period, the levered equity pays the consumption stream \(C^v_t\). Note that \(v\) is the degree of leverage which captures a broad leverage in the economy, including operational and financial leverage. Therefore, the price of an equity security in equilibrium is given by:

\[
p^e_t = E_t \left( m_{t+1} (C^v_{t+1} + p^e_{t+1}) \right),
\]

(39)

where \(p^e_t\) denotes the ex-dividend price of an equity at time \(t\).

Let \(R^e_{t+1}\) be the ex-post gross return on equity, \(R^e_{t+1} \equiv \frac{c^v_{t+1} + p^e_{t+1}}{p^e_t} - 1\). Then, (39) is equivalent to

\[
1 = E_t \left( m_{t+1} R^e_{t+1} \right),
\]

(40)

which is the same form as the intertemporal Euler equation.

Let \(\psi^e_t\) denote the equity premium, \(\psi^e_t \equiv E_t R^e_{t+1} - e^{r_{t+1}}\). Using the definition of covariance, (40) is equivalent to

\[
E_t \left( m_{t+1} R^e_{t+1} \right) = Cov_t \left( m_{t+1}, R^e_{t+1} \right) + E_t m_{t+1} E_t R^e_{t+1}.
\]

(41)
Using (5) and (41), and dividing both sides by $E_t m_{t+1}$ yields,

$$
\psi_t = E_t R^e_{t+1} - e^{r_{t+1}} = \frac{1}{E_t m_{t+1}} - \frac{\text{Cov}_t (m_{t+1}, R^e_{t+1})}{E_t m_{t+1}} - e^{r_{t+1}}
$$

$$
= -\frac{\text{Cov}_t (m_{t+1}, R^e_{t+1})}{E_t m_{t+1}}
= -\text{Cov}_t \left( \frac{m_{t+1}}{E_t m_{t+1}}, R^e_{t+1} \right).
$$

(42)

Intuitively, (42) shows why the equity is a very long-lived asset. Recall that the household’s stochastic discount factor is comprised of the consumption and the value function, $V^h_t$, that is the infinite sum of discounted future period utilities. The equity premium is thus sensitive to any changes in the consumption, even at a distant period.

### 2.6 Solution Method

I solve the model above using a third-order perturbation method based on the algorithm of Swanson, Anderson, and Levin (2006). I use this solution method for three reasons. First, I have eight state variables: $A_{t-1}, \Delta_{t-1}, D_{t-1}, I_{t-1}, i_{t-1}, K_{t-1}, r_{t-1}, \bar{y}_{t-1}$ and one shock $\epsilon^A_t$. Due to high dimensionality, projection methods are computationally not feasible. Second, a third-order perturbation shows almost the same performance as projection methods for models with generalized recursive preferences, but with much faster computing time (Caldera, Fernández-Villaverde, Rubio-Ramírez, and Yao, 2012). The model incorporates the financial accelerator which has the amplification structure containing many state variables. Thus, computation time is also important because a third-order accurate solution may take considerable time to compute. Lastly, a third-order perturbation is necessary to capture the dynamic of the risk premia, such as the impulse-response analysis of the equity premium.

### 3 Quantitative Results

I calibrate the model rather than estimate the parameters since the main objective of this study is to illuminate the role of the financial accelerator on asset pricing. As can be seen in Table 1, the baseline calibration is fairly standard for both macroeconomics and finance variables.
Table 1: BASELINE CALIBRATION

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.992</td>
<td>Discount rate</td>
<td></td>
</tr>
<tr>
<td>$\chi_0$</td>
<td>0.79</td>
<td>Relative utility weight of labor</td>
<td>To normalize $L = 1$</td>
</tr>
<tr>
<td>$\chi$</td>
<td>3</td>
<td>Inverse Frisch elasticity of labor supply</td>
<td>Del Negro et al. (2015)</td>
</tr>
<tr>
<td>$R^c$</td>
<td>60</td>
<td>Relative risk aversion</td>
<td></td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.6</td>
<td>Labor share</td>
<td></td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.025</td>
<td>Depreciation rate</td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.1</td>
<td>Monopolistic markup</td>
<td>Smets and Wouter (2007)</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.8</td>
<td>Calvo contract parameter</td>
<td>Altit et al. (2010)</td>
</tr>
<tr>
<td>$\rho_A$</td>
<td>1</td>
<td>Persistence of technology</td>
<td>Tallarini (2000)</td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>0.007</td>
<td>Standard deviation of technology shocks</td>
<td>King and Rebelo (1999)</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>3</td>
<td>Elasticity of investment adjustment cost</td>
<td>Del Negro et al. (2015)</td>
</tr>
<tr>
<td>$\vartheta$</td>
<td>0.19</td>
<td>Seizure rate</td>
<td>Gertler and Kiyotaki (2015)</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.002</td>
<td>Proportional transfer to new bank</td>
<td>Gertler and Karadi (2011)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.95</td>
<td>Survival probability of bank</td>
<td>Gertler and Kiyotaki (2015)</td>
</tr>
<tr>
<td>$\rho_i$</td>
<td>0.73</td>
<td>Smoothing parameter of monetary policy</td>
<td>Rudebusch (2002)</td>
</tr>
<tr>
<td>$\phi_{\pi}$</td>
<td>0.53</td>
<td>Response of monetary policy to inflation</td>
<td>Rudebusch (2002)</td>
</tr>
<tr>
<td>$\phi_y$</td>
<td>0.93</td>
<td>Response of monetary policy to output</td>
<td>Rudebusch (2002)</td>
</tr>
<tr>
<td>$\bar{\pi}$</td>
<td>0.008</td>
<td>The monetary authority’s inflation target</td>
<td>Swanson (2016)</td>
</tr>
<tr>
<td>$\rho_y$</td>
<td>0.9</td>
<td>Coefficient of trailing moving average</td>
<td>Swanson (2016)</td>
</tr>
<tr>
<td>$\upsilon$</td>
<td>3</td>
<td>Degree of leverage</td>
<td>Abel (1999)</td>
</tr>
</tbody>
</table>

For the household’s discount factor, $\beta$, the depreciation rate, $\delta$, and the elasticity of output with respect to labor, $\eta$, I use conventional values. I also set the relative utility weight of labor, $\chi_0 = 0.79$, to normalize the steady state labor, $L = 1$. I use relatively high risk aversion $R^c = 60$ for simplicity and comparability to the asset pricing literature. This high value is common in the macro-finance literature, and is due to the small amount of uncertainty in the simple model.\(^{14}\) As Bloom (2009) shows, the real economy has many uncertainties. In contrast, agents in the model perfectly know all parameter values and equations, so the quantity of risk is very small. Barillas, Hansen, and Sargent (2009) document that increasing the uncertainty of the model could lower risk aversion. Another method is to increase the quantity of risk by introducing additional shocks such as long-run risk, heterogeneous agents, or

\(^{14}\)For instance, Piazzesi and Schneider (2006) estimate risk aversion to 57, and Tallarini (2000), Rudebusch and Swanson (2012) and Swanson (2016) use baseline calibration of risk aversion as 100, 110, and 60, respectively.
rare disaster.

For the rest of the macroeconomic parameters, I use estimates from previous studies. The inverse Frisch elasticity of labor supply, $\chi$, is set to 3 as in Del Negro, Giannoni, and Schorfheide (2015). The calibrated value of the Calvo contract parameter, $\xi = 0.8$, implies that the lifetime of the contract is five quarters as in Altig, Christiano, Eichenbaum, and Linde (2010) and Del Negro, Giannoni, and Schorfheide (2015). I also set the elasticity of investment adjustment costs $\kappa = 3$ as in Del Negro, Giannoni, and Schorfheide (2015). I set firm’s steady state markup, $\theta$, as 10 percent, consistent with the estimates in Smets and Wouter (2007). The persistence of technology, $\rho_A$, is set to 1 as in Tallarini (2000), and the standard deviation of technology shocks, $\sigma_A$, is set to 0.007, consistent with the estimates in King and Rebelo (1999). These calibrated values generate high enough risk in the model so that the equity premium in the model is sufficiently large.

Turning to the financial sector parameters, I set the fraction of capital that can be diverted to $\vartheta = 0.19$, as in Gertler and Kiyotaki (2015). Proportional transfer to a new financial intermediary, $\omega$, is set to 0.002 as in Gertler and Karadi (2011). Lastly, I set survival probability in banking industry, $\sigma = 0.95$, implying twenty quarters of the expected lifetime for financial intermediary as in Gertler and Kiyotaki (2015).

I set the smoothing parameter of monetary policy, $\rho_i = 0.73$, the response of monetary policy to output, $\phi_y = 0.93$, and the response of monetary policy to inflation, $\phi_\pi = 0.53$, as in Rudebusch (2002). The monetary authority’s inflation target, $\bar{\pi}$, is set to 0.008 which implies 3.2 percent per year in the nonstochastic steady state as in Swanson (2016). I set the coefficient of trailing moving average of output to $\rho_y = 0.9$, which implies that the central bank considers the whole history of past output levels, because $y_t$ is an infinite moving average of past $y_t$.\footnote{Note that the average historical lag is about 10 quarters.} Finally, I calibrate the degree of leverage as $v = 3$, similar to that in Abel (1999) and Bansal and Yaron (2004).

\subsection{Macroeconomic Implications}

Figure 2 depicts the impulse response functions to a one-standard-deviation (0.7 percent) negative technology shock for the third-order solution of the models.\footnote{As the value of standard deviation of technology shocks increases, differences between the third-order impulse response functions for macroeconomic variables and their linear counterparts are not negligible.} These are computed by the period-by-period difference between two scenarios: (i) given nonstochastic steady state values of state variables,
I simulate out the variables in the absence of a shock and (ii) I repeat the same process in the presence of one standard deviation to the shock in the first period.\textsuperscript{17} The horizontal axes are periods (quarters) and the vertical axes are percentage deviations from the nonstochastic steady state.

To better highlight the role of the financial accelerator, I compare the model responses to those of a model without the financial accelerator. This alternative model is a medium-scale New Keynesian DSGE model with generalized recursive preferences, and the calibration is the same as the baseline model. Moreover, in the alternative model, the arbitrage condition between the return of capital and the real gross risk-free return holds due to the absence of financial frictions. The solid blue lines in each panel plot impulse response functions to the baseline model, and the dashed orange lines plot the impulse response functions for the alternative model.

Figure 2 shows the impact responses of the main macroeconomic variables to the negative technology shock. The model with the financial accelerator shows generally amplified and persistent responses compared to the model without financial frictions. When the technology falls, the marginal productivity of capital decreases and the return of capital decreases. As a result, the capital price is reduced and the lending capacity of the bank is declined, thereby triggers the financial accelerator.\textsuperscript{18}

As the top right panel illustrates, the response of marginal cost is attenuated in the model with the financial accelerator. Since the price of capital is further reduced by the financial accelerator, the production cost of intermediate goods firm is reduced. This can be found more formally from (28). Negative technology shock increases marginal cost, but the capital price decreases more with financial frictions, which moderately offset marginal cost increases. As a result, inflation, which is the discounted weighted average of current and future marginal costs, also rises less in the model with the financial accelerator. Although the mechanisms are different, the behavior of inflation is similar to that of Gilchrist, Schoenle, Sim, and Zakrajšek (forthcoming). The authors find through micro-level data that only intermediary goods firms that are bound by financial constraints raise their prices in recession and explain why inflation has remained low since the Great Recession.

Due to the financial accelerator, inflation is less increased and output is decreased further, so the

\textsuperscript{17}There are many other alternatives. For example, I draw random numbers for the technology shock $\epsilon_t^A$ from its distribution using a random number generator and use these values for the simulation. There is, however, no large difference in the results between these two methods because agents in the model economy do not have perfect foresight.

\textsuperscript{18}In models without financial frictions, the impulse response of the spread is not completely zero, because the figure is the third-order impulse response. On the other hand, in the first-order impulse response, the spread always shows a zero response as in Gertler and Karadi (2011).
Figure 2: IMPACT OF THE FINANCIAL ACCELERATOR ON MACRO VARIABLES

Note: The figure plots third-order impulse response functions of the return of capital, $R^k_t$, price of capital, $Q_t$, marginal cost, $MC_t$, inflation rate, $\pi_t$, the net risk-free return, $r_{t+1}$, the spread, $E_tR^k_{t+1} - e^{-r_{t+1}}$, consumption, $C_t$, investment, $I_t$, and output, $Y_t$, to a one-standard-deviation (0.7 percent) negative technology shock. The solid blue lines are from the baseline model and dashed orange lines are from the model without the financial accelerator. See text for details.

Thus, as can be seen from the middle center panel, the risk-free return is further reduced in the presence of financial frictions. This can also
Table 2: COMPARISON OF EQUITY PREMIUM

<table>
<thead>
<tr>
<th>Risk aversion $R^c$</th>
<th>Shock persistence $\rho_A$</th>
<th>Smoothing parameter $\rho_i$</th>
<th>Equity premium without FA</th>
<th>Equity premium with FA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>0.73</td>
<td>0.98</td>
<td>1.06</td>
</tr>
<tr>
<td>30</td>
<td>1</td>
<td>0.73</td>
<td>3.01</td>
<td>3.24</td>
</tr>
<tr>
<td>60</td>
<td>1</td>
<td>0.73</td>
<td>6.06</td>
<td>6.52</td>
</tr>
<tr>
<td>90</td>
<td>1</td>
<td>0.73</td>
<td>9.12</td>
<td>9.80</td>
</tr>
<tr>
<td>Panel B</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>0.995</td>
<td>0.73</td>
<td>2.56</td>
<td>2.77</td>
</tr>
<tr>
<td>60</td>
<td>0.99</td>
<td>0.73</td>
<td>1.46</td>
<td>1.58</td>
</tr>
<tr>
<td>60</td>
<td>0.98</td>
<td>0.73</td>
<td>0.70</td>
<td>0.75</td>
</tr>
<tr>
<td>60</td>
<td>0.95</td>
<td>0.73</td>
<td>0.22</td>
<td>0.23</td>
</tr>
<tr>
<td>Panel C</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>1</td>
<td>0</td>
<td>6.66</td>
<td>6.43</td>
</tr>
<tr>
<td>60</td>
<td>1</td>
<td>0.2</td>
<td>6.56</td>
<td>6.44</td>
</tr>
<tr>
<td>60</td>
<td>1</td>
<td>0.6</td>
<td>6.24</td>
<td>6.48</td>
</tr>
<tr>
<td>60</td>
<td>1</td>
<td>0.8</td>
<td>5.93</td>
<td>6.56</td>
</tr>
</tbody>
</table>

Note: The equity premium implied by the model in annualized percentage points with different values of relative risk aversion, $R^c$, persistent of technology shock, $\rho_A$, and smoothing parameter of monetary policy, $\rho_i$. Model with Financial Accelerator is a New Keynesian DSGE model with generalized recursive preferences and the financial accelerator, while Model without Financial Accelerator has no financial frictions. See the text for more details.

...affect the equity premium, which I will discuss in Section 3.3 in detail. Lastly, the similarity in impulse response functions for key macroeconomic variables in this model and these in the literature allow us to focus on implications for the equity premium.

### 3.2 Equity Premium Results

Table 2 reports the equity premium, $\psi_t$, implied by the model, solved to third order, holding other parameters of the model set at their benchmark values. For comparison purposes, I also provide the results from the model with various values of risk aversion, $R^c$, and persistence of technology, $\rho_A$. The equity premium increases monotonically with risk aversion and persistence of technology since they raise the volatility of the stochastic discount factor as in Swanson (2016). The equity premium
responds more sensitively to changes in technology persistence, because equity is long-lived asset and the equity premium is related with the household’s value function as shown in (42).

For the baseline model, the model-implied equity premium, $\psi^e_t = 6.52$, matches its empirical estimate (typically about 3 to 6.5 percent for quarterly excess returns at an annual rate).\textsuperscript{19} Thus, the model with the financial accelerator and generalized recursive preferences generates a sufficiently large equity premium. Table 2 also reports the results from the model without financial frictions. As can be seen in the last column of Panel A and B, the financial accelerator increases the equity premium in all cases. For instance, when the risk aversion is 60, the equity premium is increased about 46 basis points above the prediction of the model without the financial accelerator.\textsuperscript{20}

Why does the financial accelerator increase the equity premium? It is because the financial accelerator increases the volatility of the stochastic discount factor in the model. I compute unconditional standard deviations of the stochastic discount factors from two models using logarithmic deviation with a Hodric-Prescott detrending. The standard deviation of the stochastic discount factor is 64.72 percentage points in the presence of the financial accelerator and 62.22 percentage points without financial frictions. For the sake of argument, consider a negative technology shock. When the technology declines, the return of capital, $R_t^k$, decreases as the marginal productivity of capital contracts. The decrease in the return of capital lowers the aggregate net worth of the banks, $N_t$, because the decline in return of capital reduces the marginal value of assets for intermediated finance. The reduction in net worth of the banks decreases their lending capacity, making the quantity of claims, $S_t$, and capital, $K_{t+1}$, decrease. Then, the price of capital, $Q_t$, falls as demand for capital decreases. In turn, the lower capital price further pushes down the return of capital, the net worth of the financial intermediary, and the price of capital repeatedly. As a consequence, the amplification mechanism of the financial accelerator increases the volatility of the stochastic discount factor.

I also provide a dynamic analysis for a more detailed examination. Figure 3 plots the impulse response functions to a one-standard-deviation (0.7 percent) negative technology shock, computed by

\textsuperscript{19}Of course, even with lower risk aversion values, the equity premium predicted by the model can be increased by incorporating other shocks, such as monetary policy shock or fiscal policy shock.

\textsuperscript{20}Even though the financial accelerator increases the equity premium, it does not increase as much as GYZ who document that costly external finance increases the equity premium by a factor of 10 to 20. This is likely due to the model-implied equity premium in GYZ being very small (0.022 percent in the presence of financial friction), so financial frictions may seem to play a relatively large role. In contrast, in my model, generalized recursive preferences play a significant role in generating the substantial equity premium.
Figure 3: Impact of The Financial Accelerator on Financial Variables

Note: Third-order impulse response functions for the stochastic discount factor, $m_t$, the equity price, $p^e_t$, and the equity premium, $\psi^e_t$, to a one-standard-deviation (0.7 percent) negative technology shock. The solid blue lines in each panel plot impulse response functions with financial accelerator (the baseline model), and the dashed orange lines plot impulse response functions without the financial accelerator. See the text for more details.

the same method as the nonlinear impulse response functions in Figure 2. The blue solid lines and the orange dashed lines in each panel report the impulse response functions for the model with the financial accelerator and the model without financial frictions, respectively. The left-hand panel reports the impulse response function for the stochastic discount factor, $m_t$, to the shock. The stochastic discount factor jumps about 65 percent in response to the negative technology shock for the baseline model, while it jumps about 62 percent for the frictionless model. This is consistent with the intuition that the financial accelerator increases the volatility of the stochastic discount factor.

Since the equity premium is conditional covariance between the stochastic discount factor and the return of the equity as in (42), it is worth noting not only the dynamic of the stochastic discount factor but also the equity price. The center panel shows the impulse response function for the equity price, $p^e_t$. It immediately drops downward about 2.5 percent to the shock and gradually converges to its new nonstochastic steady state. For the frictionless model, the equity price drops less in response to the shock and shows a less volatile response. The reason of this can be found in the impulse response of consumption in Figure 2. Because the dividend is a levered claim on consumption, the equity price is highly correlated with consumption. Therefore, the equity premium shows fourfold greater and much more persistent response when the model has the financial accelerator as in the right-hand panel.
3.3 Smoothing Parameter of Monetary Policy

The financial accelerator increases inflation a little and amplifies the decrease in output as in Section 3.1. This affects how the central bank sets the interest rate which has a significant impact on the equity premium. For example, if the central bank responds more strongly to lowering the inflation gap, the risk-free return goes up and, in turn, it reduces the equity premium. Thus, the size of the response of the equity premium varies depending on the stance of the central bank regarding output.
and inflation. This section analyzes the impact on the equity premium by controlling the smoothing parameter, \( \rho \), rather than only the inflation coefficient or the output gap coefficient separately.

Panel C in Table 2 reports the model-implied equity premiums calculated with various smoothing parameters. In the model with the financial accelerator, the smaller the interest rate inertia, the less the equity premium. On the other hand, the model without financial frictions increases the equity premium as interest rate inertia weakens. The equity premium generated by the model with the financial accelerator is even smaller when the smoothing parameter is as small as 0.2.\(^{21}\) Why do they give opposite results? In the model without the friction, the risk-free rate increases as the interest rate inertia decreases. This reduces the inflation gap, but it does not help overcome the recession. Consumption therefore becomes more unstable, and this increases the equity return. Since the rise in the equity return is greater than the increase in the risk-free rate, the equity premium increases.

By contrast, in the presence of the financial friction, the recession is more severe, and this limits the increase in the risk-free rate. Consumption and the equity price are relatively less volatile, so the equity return is not sensitive to changes in the smoothing parameter. As a consequence, the equity premium decreases as the risk-free rate increases.

For more detailed explanation, Figure 4 reports the impulse response functions for the risk-free return, the equity price, and the equity premium to 0.7 percent negative technology shock with different interest rate inertia values. The solid blue lines in each panel plot impulse response functions for a stronger smoothing parameter, the dashed orange lines plot the impulse response functions for weaker inertia, and the dash-dot yellow lines plot the impulse response functions for no interest rate smoothing. The interest rate responds more to the inflation gap as interest rate inertia gets smaller. This is because the coefficient of the inflation gap, \( \phi_\pi \), is 1.53, while the coefficient of the output gap, \( \phi_y \), is about 0.23 per quarter, so the central bank is more concerned with the increased inflation gap. However, in the presence of the financial accelerator, because the recession is more severe, the risk-free return always responds negatively even if there is no inertia. The response of consumption and equity price are therefore not as volatile as in the frictionless model, and there is not much effect of interest rate smoothing on equity returns. So the response of the equity premium is moderately reduced as in the

\(^{21}\)The estimates for the interest rate inertia are between 0.6 and 0.8 (e.g., Rudebusch, 2002; Smets and Wouter, 2007; and Del Negro, Giannoni, and Schorfheide, 2015). So, 0.2 is an unrealistically small number, but I calculated it for thoroughness of the paper. For the same reason, I find that both models produce 6.45 of the equity premium when the smoothing parameter is 0.35.
Table 3: INVESTMENT ADJUSTMENT COSTS

<table>
<thead>
<tr>
<th>Model without Financial Accelerator</th>
<th>Model with Financial Accelerator</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \kappa = 3 )</td>
<td>( \kappa = 3 )</td>
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<td>( \sigma(m_t) )</td>
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<td>62.22</td>
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<td>62.31</td>
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<td>62.30</td>
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<td>( \sigma(m_t^C) )</td>
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<td>( \sigma(m_t^V) )</td>
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<td>61.68</td>
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<td>61.70</td>
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<td>6.06</td>
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<td>6.44</td>
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<td>6.24</td>
<td>6.40</td>
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Note: Model-based unconditional moments of the stochastic discount factor, \( \sigma(m_t) \), and the equity premium, \( \psi_t^e \). \( \sigma(m_t^C) \) denotes the standard deviation of the stochastic discount factor due to consumption growth, and \( \sigma(m_t^V) \) denotes the additional standard deviation of the stochastic discount factor due to generalized recursive preferences. All numbers are in percentage points.

right panel at (a). On the other hand, the case is different in the financial frictionless model. Since inflation is relatively higher and output is less decreased, the central bank raises the interest rate, when there is no interest rate inertia as in the left panel at (b). In turn, the responses of consumption and the equity price are further reduced and more volatile. As a consequence, even if the risk-free return increases, the equity premium responds more positively as the equity return increases.

3.4 Investment Adjustment Costs

Jermann (1998) and GYZ astutely note that increasing the adjustment costs of capital improves the asset pricing performance by raising both the volatility of consumption and stock returns. Table 3 shows the model-implied equity premium for various investment adjustment costs. The equity premium is not very sensitive to \( \kappa \), with or without the financial accelerator. For example, in the frictionless model, the equity premium increases only 18 basis points, even if the investment adjustment cost is raised unrealistically to 30 from 3.\(^{22}\) There are two key reasons why investment adjustment costs do not significantly affect the equity premium in my model. First, households do not own capital. Since the models in this paper introduce capital producers to easily determine the capital price endogenously, the volatility of the capital price does not affect the consumption or the stochastic discount factor of households. Second, households do not have habit preferences. If the household owns capital, consumption smoothing can be affected greatly by habit, as in Jermann (1998). At the same time,

\(^{22}\)This is a big difference from GYZ where the equity premium increases by a factor of four in respect to a similar change.
the equity premium decreases as the investment adjustment cost gets larger in the model with the financial accelerator. For instance, the equity premium is reduced about 12 basis points by increasing the elasticity of investment adjustment costs to 30 from 3. As $\kappa$ increases, the variability of capital decreases, which reduces the volatility of the bank’s net worth. Accordingly, it is less costly to smooth consumption for the households although the impact is very small.

Finally, for a more detailed examination, I decompose the standard deviation of the stochastic discount factor into two parts. The first is a marginal rate of substitution of consumption, so it is the same as the stochastic discount factor from the expected utility preferences. The second is the additional volatility of the stochastic discount factor due to generalized recursive preferences. The result tells us that the volatility of the stochastic discount factor varies due to the additional part, rather than from changes of consumption growth in two models. In the model with the financial accelerator, the adjustment costs of investment have a small impact on the household’s consumption smoothing; therefore, it does not change the equity premium substantially.

4 Conclusion

This paper examines the effect of the financial accelerator on the equity premium with two modifications to a medium-scale New Keynesian DSGE model: first, the Gertler-Karadi type financial accelerator which allows the model to have interaction between the financial market and macroeconomy, and second, generalized recursive preferences to generate a substantial risk premium without compromising stylized macroeconomic facts. A quantitative analysis of the model shows that the financial accelerator is a very plausible and new amplification mechanism for asset pricing in the model because it increases the volatility of the stochastic discount factor and lowers the risk-free return. The financial accelerator increases the price of the risk in the model since it makes consumption more volatile. Moreover, the financial accelerator generates more severe recessions but lower inflation to a negative technology shock. Due to the lower inflation, the central bank has more incentive to lower the risk-free interest rate in a recession and this increases the equity premium—the difference between the expected return to equity and the risk-free rate.

In a nutshell, the model makes progress on the task of consolidating the analysis of asset prices and
macroeconomics with financial frictions. However, there is a lot of potential for improvement because the model has been simplified to understand the underlying mechanism of the financial accelerator and its impact on the equity premium. In future research, it may be useful to extend the role of lenders to taking into account the collateral of the borrower. Because housing finance was particularly a big issue in the Great Recession, and household consumption was dampened due to the subprime mortgage crisis. Financial intermediaries in this paper however consider only corporate finance and household deposits. Extending the model to incorporate housing prices such as Iacoviello (2005) and Liu, Wang, and Zha (2013) would be an interesting idea since the financial intermediary could have a greater influence on household consumption and the stochastic discount factor.
A Appendix: Model Equations

The following equations show how I incorporate generalized recursive preferences and the financial accelerator into a medium-scale New Keynesian DSGE model.

Householder

\[ V_t = (1 - \beta) \left( \log C_t - \chi_0 \frac{L_t^{1+\chi}}{1+\chi} \right) - \beta \alpha^{-1} \log V_{\text{exp}} \]  
(A.1)

\[ V_{\text{exp}} = E_t \exp (-\alpha V_{t+1}) \]  
(A.2)

\[ 1 = E_t \left( \beta \frac{C_t}{C_{t+1}} \exp \left( -\alpha V_{t+1} \right) \right) \]  
(A.3)

\[ \chi_0 L_t^{\chi} \left( \frac{1}{C_t} \right)^{-1} = \frac{w_t}{P_t} \]  
(A.4)

Banking sector

\[ Q_t K_{t+1} = \phi_t N_t \]  
(A.5)

\[ Q_t K_{t+1} = N_t + D_{t+1} \]  
(A.6)

\[ \phi_t = \frac{\beta E_t ((1 - \sigma) + \sigma \phi_{t+1}) e^{r_{t+1}}}{\nu - \mu_t} \]  
(A.7)

\[ \mu_t = \beta E_t ((1 - \sigma) + \sigma \phi_{t+1}) \left( R^k_{t+1} - e^{r_{t+1}} \right) \]  
(A.8)

\[ N_t = \sigma \left[ R^k_t Q_{t-1} K_t - e^{r_{t}} D_t \right] + \omega Q_t K_t \]  
(A.9)

\[ C^b_t = (1 - \sigma) \left[ R^k_t Q_{t-1} K_t - e^{r_{t}} D_t \right] \]  
(A.10)

Capital Producer

\[ Q_t = 1 + \frac{\kappa}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 + \kappa \left( \frac{I_t}{I_{t-1}} - 1 \right) \left( \frac{I_t}{I_{t-1}} \right) - E_t \beta \frac{C_t}{C_{t+1}} \exp \left( -\alpha V_{t+1} \right) \frac{\kappa}{2} \left( \frac{I_{t+1}}{I_t} - 1 \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \]  
(A.11)

\[ K_{t+1} = (1 - \delta) K_t + \left\{ 1 - \frac{\kappa}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right\} I_t \]  
(A.12)
Intermediate goods sector

\[ R_{t+1}^k = \frac{MC_{t+1} (1 - \eta) A_{t+1} \left( \frac{K_{t+1}}{L_{t+1}} \right)^{-\eta} + (1 - \delta) Q_{t+1}}{Q_t} \]  
(A.13)

\[ \text{spread}_t = E_t R_{t+1}^k - e^{\pi_{t+1}} \]  
(A.14)

\[ \frac{w_t}{P_t} = MC_t \eta A_t \left( \frac{K_t}{L_t} \right)^{1-\eta} \]  
(A.15)

\[ zn_t = (1 + \theta) MC_t Y_t + \xi E_t \beta \frac{C_t}{C_{t+1}} \exp \left( -\alpha V_{t+1} \right) \left( e^{\pi_{t+1}-\bar{\pi}} \right)^{\frac{1+\theta}{\theta}} zn_{t+1} \]  
(A.16)

\[ zd_t = Y_t + \beta E_t \beta \frac{C_t}{C_{t+1}} \exp \left( -\alpha V_{t+1} \right) \left( e^{\pi_{t+1}-\bar{\pi}} \right)^{\frac{1}{\theta}} zd_{t+1} \]  
(A.17)

\[ \frac{p_t^*}{P_t} = \frac{zn_t}{zd_t} \]  
(A.18)

\[ \left( e^{\pi_{t+1}-\bar{\pi}} \right)^{-\frac{1}{\theta}} = (1 - \xi) \left( \frac{p_t^*}{P_t} \right)^{-\frac{1}{\theta}} \left( e^{\pi_{t}-\bar{\pi}} \right)^{-\frac{1}{\theta}} + \xi \]  
(A.19)

Final goods sector

\[ Y_t = \Delta_t^{-1} A_t K_t^{1-\eta} L_t^{\eta} \]  
(A.20)

\[ \log A_t = \rho_A \log A_{t-1} + \epsilon_t^A \]  
(A.21)

\[ \Delta_t = (1 - \xi) \left( \frac{p_t^*}{P_t} \right)^{-\frac{1+\theta}{\theta}} + \xi \Delta_{t-1} \]  
(A.22)

Policy rule

\[ i_t = \rho_i i_{t-1} + (1 - \rho_i) \left[ \ln \frac{1}{\beta} + \pi_t + \phi_\pi (\pi_t - \bar{\pi}) + \frac{\phi_y}{4} \log \left( \frac{Y_t}{\bar{Y}_t} \right) \right] \]  
(A.23)

Aggregate

\[ Y_t = C_t + C_t^b + \left\{ 1 + \frac{\kappa}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right) \right\} I_t \]  
(A.24)

For the alternative model, there is no banking sector and the arbitrage condition \( E_t(m_{t+1} e^{\pi_{t+1}}) = E_t(m_{t+1} R_{t+1}^k) \) holds.
References


