Downtown Parking Supply, Work-Trip Mode Choice and Urban Spatial Structure

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May 25th, 2015

Abstract

This paper examines the effects of changes in downtown parking supply on urban welfare, modal choice decisions and urban spatial structure using a spatial general equilibrium model of a closed monocentric city with two transport modes, endogenous residential parking and a form of bottleneck congestion at the CBD. Our analysis shows that parking reforms at the CBD that increase delay congestion costs in the short-run such as parking supply limits can be welfare improving if other commuting externalities such as air pollution can be reduced. In addition, because parking limits can also change location decisions such as where to live and invest they may complement anti-sprawl policies efforts by leading to a more compact urban spatial structure in the long run. We also show that changes in downtown parking supply can have different spatial impacts on the market supply of residential parking by affecting urban residents’ location decisions. Finally, we discuss the role of parking pricing as a complementary tool of congestion pricing to combat congestion in central areas and investigate whether the self-financing theorem of transportation economics holds within the context of our spatial urban model.

Key words: Downtown Parking, Bottleneck Congestion, Urban Form, Modal Choice

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† I thank Ken Small, Richard Arnott and Antonio Bento for their comments and, especially Jan Brueckner for his very helpful comments and suggestions which improved the presentation. I also thank the financial support of a grant from the Multicampus Research Program and Initiative (MPRI) of the University of California (award number 142934). The views expressed in this paper are nevertheless solely mine. In addition, any errors or shortcomings, of course, are also mine.
1. Introduction

Traffic congestion is often seen as a sign of a city’s economic and social health and vitality. However, economists have long recognized that congestion tends to exceed economically efficient levels because auto commuters do not bear the full cost of their use of the roads and parking facilities. In heavily trafficked areas such as downtown areas, each additional vehicle causes additional traffic delay on all the other auto users. Yet no auto user is charged for this negative externality. The result is that traffic delay slows economic activity and can reduce some of the agglomeration benefits that characterize central business districts.¹

In recent years planners have given increasing attention to the possibility of reducing downtown use of cars by controlling parking—either by restricting the number of spaces available, or by charging users to park.² After all, empirical evidence has shown that parking prices and parking availability are two of the key factors in auto users’ decision on whether to drive to work or use public transportation (Shoup 2005). Among the possible benefits of these auto-restraint schemes are reductions in traffic delay costs, air pollution and energy consumption (STHC Report 2009; Kodransky and Hermann, 2011; Weinberger et al., 2010).

¹Shoup (2011a) found that, in a single year, drivers wasted 100,000 hours while cruising for underpriced curb parking in a 15-block business district in Los Angeles, CA, USA.
²While some European cities such as London and Stockholm have managed to implement congestion pricing to reduce auto use, more are turning to parking. For example, Hamburg, Zurich and Budapest have instituted caps on the total parking supply in the city center, reforming the building codes to freeze the existing inventory and ban any further increases. London and Paris have also abolished parking minimums and several other cities such as Zurich, Amsterdam and Strasbourg have established zone-based maximums. Antwerp and Zurich have also reduced parking maximums and minimums in locations proximate to transit facilities. Kodransky and Hermann (2011) examine ten different European cities on a variety of parking measures ranging from pricing mechanisms to regulatory measures (such as supply caps and parking minimums and maximums) to reduce vehicle kilometers travelled and shift travel from car trips to other mode transports. The case studies examined on the report highlight that expanding the supply of free, cheap, or excessive parking—once viewed as necessary to economic vitality and to increase mobility—has been reassessed across Europe. On the other hand, Weinberger et al. (2010) provide an overview of the best practices in parking management in the United States. The report emphasizes that many US cities still take a passive approach to managing parking and just a few cities (Chicago, New York City and San Francisco) are taking steps to align parking policy with the broader city goals of accessibility, economic development and better quality of life. Even though experiments in alternative parking approaches are still quite new in the US, the overall conclusion of the Weinberger et al. (2010) report is that dysfunction will continue as long as parking policy is viewed independent of transportation policy and as long curbside and off-street parking are treated independently.
Even though parking reform has been a hot topic in the policy arena in several parts of the world, there is remarkably little analytical work on how parking reform affects the urban form. Yet, parking pricing and supply policies may have a considerable impact on the transport and land use system within an entire urban region. The reason is because parking is a key link between transportation systems and land use. It is therefore quite surprising that this issue has been overlooked, given the current advocacy to move away from parking minimums to parking maximums and to erase employer transportation subsidies as ways to reduce solo driving and to achieve less dispersed urban spatial structure (Shoup 2005; Shoup 2011; STHC Report 2009; Kodransky and Hermann, 2011; Weinberger et al., 2010).

Most of the theoretical studies on parking policies focus on (i) the efficiency of second-best pricing of parking spots in the absence of congestion tolls (Arnott et al. 1991; Glazer and Niskanen 1992; Verhoef et al. 1995; Arnott and Rowse 1999; Anderson and de Palma 2004) and a cordon system (Calthrop et al. 2000), (ii) on the effects of parking and transit subsidies on the CBD size (Voith 1998), (iii) on the effects of curbside parking fees on cruising for parking in downtown areas (Arnott and Inci, 2006), (iv) on the optimal on-street parking policy in the presence of an off-street parking market (Calthrop and Proost, 2006), or (v) on the effects of road-tolls schemes and parking fees on the traffic patterns of morning and evening commutes (Zhang et al. 2008). Other analytical studies have focused either on the role of employer-paid parking for the desirability of congestion tax reform and for the relative efficiency of recycling instruments (De Borger and Wuyts 2009) or on the optimal curbside parking capacity in downtown areas in first and second best settings (Arnott et al. 2013).³

³ For a very nice brief review on the economics of parking see Arnott et al. (2013).
All this previous research has definitely offered valuable insights not just on how parking policy affects commuters’ short run decisions such as trip frequency, mode, parking location or scheduling but it has also improved our understanding of the potential efficiency gains of supplementing road pricing with parking regulation to combat road and CBD traffic congestion. Still, because these studies do not develop a spatial general equilibrium model (with an explicit representation of space) that allows consideration of interactions between residential choices, modal choices, residential parking and floor-space supplies and congestion, their frameworks are not suitable to capture long run adjustments of location and building stock changes and thus, examine how changes in downtown auto restraints influence land use and urban form.

One exception is the work by Anderson and De Palma (2007). The authors integrate parking land and parking congestion (cruising) into a simple monocentric city framework in which households and parking operators compete for land. The study shows that closest to the CBD are parking lots while residential use locates further out. It is also shown that the social optimum configuration is identical to the market equilibrium provided that parking lots are monopolistically priced. As the number of parking operator goes to infinity, each parking operator lot sets a per-user price equal to the congestion externality-cost.

The goal of this paper is twofold. First, we develop a spatial closed city model that allows one to examine how changes in downtown parking supply when parking is underpriced affect the urban spatial structure, welfare, modal choice and residential parking supply outside the CBD. Then, we discuss the optimal value of a budget-balancing parking fee when parking capacity is chosen to maximize the equilibrium utility of urban residents and explore the implications of our results for the self-financing theorem.
In this sense, our approach is complementary to the model of Anderson and De Palma (2012) in that we look at a spatial urban equilibrium with parking provision in the presence of auto externalities and at the interactions between optimal pricing and optimal parking capacity at the CBD, whereas they focus solely on the relation between market performance and the optimum.

In addition, our model is based on residential location and includes two competitive transport modes. Housing is portrayed as a commodity with floor space and parking spaces, which are both choice variables of the housing developer. Residential parking is assumed to consume a fixed amount of land per parking space. All urban residents are car owners and commute to job sites in a congested downtown district. Auto commuting generates air pollution, which affects both auto and transit users equally. Downtown parking supply is exogenous. Expansions of the congested downtown parking capacity are fully paid with parking fees that only cover infrastructure costs. Within this framework we then examine how changes in CBD parking supply affect residential land rents, residential parking supply, modal choice, urban welfare, share of auto-commuters, population densities and city size. All the impacts are expressed in terms of behavioral elasticities. This allows for an easy interpretation, while elasticities can be linked to empirical estimates obtained from observed behavior. Furthermore, we examine how the optimal choice of CBD parking supply affects the value of a budget-balancing parking fee.

Our analysis shows that when the auto-travel elasticity with respect to parking capacity is inelastic and financing relies on budget-balancing parking fees, an expansion in downtown parking supply in the presence of underpriced parking tends to decrease overall parking congestion at the CBD. However, increased parking supply also increases automobile commuting and reduces transit rides and, as a result contributes to an increase in air pollution. Provided that the primary effects dominate the effects from induced demand, an increase in CBD parking supply is welfare
improving. In addition, the overall decrease in congestion delay costs at the CBD makes automobile-dependent locations in the long run more attractive to urban residents, potentially leading to an expansion in the city size.

Another interesting finding of our analysis is that when downtown parking capacity is chosen to maximize the equilibrium level of urban utility, a balanced-budget parking fee coincides with the optimal congestion toll despite the presence of environmental spillovers. Moreover, the self-financing equality (where revenues from congestion tolls cover capacity costs) remains valid even if an environmental tax is levied to internalize the air pollution externality from auto commuting.

The remainder of the paper is organized as follows. In the next section, we develop our theoretical model and discuss the optimality conditions. Section 3 analyses the spatial behavior of choice variables, and section 4 discusses the urban equilibrium conditions. Section 5 presents the market equilibrium comparative statics of a change in downtown parking supply and section 6 focuses on the interactions between pricing and parking capacity choices, and between parking revenues and capacity costs. Finally, section 7 discusses some policy implications while our last section offers conclusions.

2. Model

Suppose a linear city exists extending from a Central Business District (CBD), located at zero, to the urban fringe \( x \). The city consists of \( N \) urban residents who are assumed to be renters, car owners and to have identical tastes and income. For simplicity, urban residents own the same type of vehicle. Urban residents reside outside the CBD and commute to work either by car or public transit. All car commuters park at work.
The use of automobiles generates two types of external effects in the urban area. The first is that each additional parker adds to parking congestion costs at the workplace, increasing commuting time for all drivers. The other external effect is that driving automobiles causes air pollution. The level of pollution for any urban resident at a particular location is assumed to be a function of the number of residents commuting by car, and hence, of the transport and residential location choices.

Let \( x \) denote distance from the place of residence to the CBD, \( N^c \) the total number of residents driving to work and \( N^b \) the number of residents using public transit. To the extent that all residents either take the public transit or drive to work in the CBD, the urban population satisfies the condition

\[
N = N^b + N^c. 
\]  

(1)

Land is owned by absentee landowners and \( r_a \) is the exogenous rural land rent.

**Transportation Costs**

The cost of auto commuting to work is represented by travel time costs and monetary costs

\[
[t_c + \theta_c y]x + \left(\frac{N^c}{C} y + f_c \right)
\]

(2)

where \( t_c x \) denotes a part of cost varying proportionally with distance such as fuel costs, \( \theta_c x \) is the time spent in regular car travel, \( C \) is parking supply at the CBD (which we assume to be exogenous) and \( f_c \) represents a parking fee at the CBD. The auto commuter faces a delay time
cost at the CBD because of local traffic congestion, which is represented by $\frac{N^c}{C}$. Car travel 
time and delay time costs are valued at the individual wage rate, $y$.

On the other hand, commuting to work by public transit costs

$$f_b + \theta_{by}$$  

(3)

where $f_b$ denotes the fixed costs such as fares and $\theta_{by}$ represents the time cost to an urban 
resident using public transit.

We assume that $\theta_b > \theta_c$ and $f_c > f_b$. Commuting by car is faster but entails a higher fixed 
cost. In addition we assume the variable cost of using the car is lower than that of using public 
transit: $t_c + \theta_{cy} < \theta_{by}$.

Under the cost functions (2) and (3), an urban resident chooses between the two transport 
modes according to his location and takes the number of car commuters and parking supply at the 
CBD as given. That is, an urban resident located $x$ miles from the CBD chooses commuting to 
work by public transit if

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4 Workers who drive to work in a central business district have a wide range of alternative parking options such as multi-story or underground parkades where stalls are rented to the public, un-metered on-street locations and in some cases parking options arranged by their employers. Parking location choice at the CBD can be influenced not only by money cost and proximity to final destination as well as by whether or not it is likely to spend some time searching/waiting for a stall. Since our goal is to understand how changes in downtown parking supply affect mode choice behavior and the urban form, we have assumed that auto-commuters just face delay time costs due to congestion in downtown. Even though delay costs may result from cruising and road traffic at the CBD, for simplicity we have assumed that our congestion function in downtown depends on the overall parking capacity and number of drivers to the CBD. For a more detailed representation of the downtown parking problem see Arnott and Rowse (1999) and Arnott and Inci (2006). It is also worth mentioning that our bottleneck congestion function is different from the traditional bottleneck congestion setup in the literature (see for example Arnott et al. 1993). Trips to work are the most essential trips for most people. Shoup (1993) has reported that commuters who worked in downtown Los Angeles responded to parking price rises only by choosing a different mode for their trips, not by changing the number of trips they made or the time at which they travelled. But for other trip purposes, travelers would respond to a congestion toll not only by changing the transport mode but also shifting their travel time and destination. Since in our model we are just modeling work trips, for simplicity we have disregarded the possibility of workers choosing alternative times to commute by car to the CBD. Arnott et al. (1993) present a bottleneck model of downtown but without an endogenous urban form.
\[ (t_c + \theta_c y)x + f_c + \frac{N^c}{C} y > f_b + \theta_b y x. \]  \hspace{1cm} (4)

Therefore, there is a cutoff distance, denoted the modal boundary (\( \hat{x} \)), where urban residents are indifferent between using the car and public transit and which satisfies the following conditions

\[
\hat{x} = \frac{N^c}{C} y + f_c - f_b \\
\frac{\theta_b y - \theta_c y - t_c}{\theta_b y - \theta_c y - t_c}
\]

and

\[ 0 < \hat{x} < \bar{x}. \] \hspace{1cm} (6)

Conditions (5) and (6) imply that both modes are used in the city and that close to the CBD urban residents will always commute by public transit. In addition, condition (5) also helps to explain the demand for CBD parking since the demand for parking depends on how many people want to drive to downtown. The decision of driving downtown, in turn, hinges, as seen in (4) and (5), on the fraction of people who choose to drive rather than use mass transit to the CBD, parking capacity, and quality of alternative means of transportation to the CBD as well as the costs associated with driving downtown, including parking.

If transit services to downtown are widely available (low \( \theta_b y \)), of high quality (low \( \theta_b \)), and reasonable priced (low \( f_b \)), urban residents will use transit as a viable alternative to cars to go to the CBD and, hence, will lower the demand for parking. If a parking fee must be paid in downtown (which could be captured by \( f_c \)) or if congestion is too severe at the CBD, urban residents may choose to use public transportation. In this case, demand for CBD parking is lower because driving costs to work are high.

*Residential Bid Rent Functions*
Residents’ tastes are represented by \( U(q, \alpha, m, E) = \omega(q) + \phi(\alpha) + m + E \) where \( m \) is consumption of a numeraire non-housing commodity, \( \alpha \) is the number of parking spaces per dwelling and \( q \) is consumption of housing, measured in square feet of floor space. We assume that \( \omega_q > 0, \omega_{qq} < 0 \) and \( \phi_a > 0, \phi_{aa} < 0 \). In addition, urban residents also receive utility from environmental quality, \( E \). Environmental quality deteriorates with air pollution, which is directly related to the number of urban residents driving to work, \( E(N^c) \) with \( E_N^c < 0 \).

While the price of the composite good is assumed to be the same everywhere in the city (taken to be unity for simplicity), the rental price per square foot of housing floor space, denoted \( R \), varies with location.

Since urban residents are identical, the urban equilibrium must yield identical utility levels for all individuals. Spatial variation in \( R \) allows equal utilities throughout the city. In particular, the price per square foot of housing varies over space so that the highest utility level attainable at each location equals a constant level of utility \( U \). Given residents’ choice of transport modes implied by (5) and (6), the maximum amount an urban resident living at distance \( x \) from the CBD would be willing to pay for a dwelling of size \( q \) with \( \alpha \) parking spaces at a given utility level, income and downtown congestion levels satisfies

\[
R(q, \alpha, x; \varphi) = \begin{cases} 
\omega(q) + \phi(\alpha) - U + E + y - f_b - \theta_b y x & \text{for } 0 < x \leq \hat{x} \\
\omega(q) + \phi(\alpha) - U + E + y - f_c - [t_c + \theta_c y] x - \frac{N^c}{C} y & \text{for } \hat{x} < x.
\end{cases}
\]

where \( \varphi = \varphi(U, E, y, f_b, \theta_b) \) when \( 0 < x \leq \hat{x} \) and \( \varphi = \varphi(U, E, y, f_c, \theta_c, t_c, N^c, C) \) when \( \hat{x} < x \).

The bid function is increasing and concave in the housing attributes, and decreasing in the given level of utility. Note, from (4), that urban residents do not consider the environmental and congestion consequences of their mode choices and as a result, driving to work is underpriced.
The Costs of Residential Parking

For simplicity, we assume the only type of residential parking provided in the city is surface parking. The cost per residential parking space can be represented as

\[ i\bar{K} + r\bar{l} \]  \hspace{1cm} (8)

where \( r \) and \( i \) are the prices of land and capital, \( \bar{K} \) is the fixed amount of capital per surface parking space and \( \bar{l} \) is the fixed amount of land per surface parking space. While the price of capital is assumed to be exogenous and uniform across space, the price of land is endogenously determined and varies over space.

Housing Developers

The amount of floor space in a developer’s complex is given by \( H(K,L) \), where \( K \) is the capital input and \( L \) is the amount of building land. We assume that \( H \) is strictly concave and homogenous of degree one. The intensive form of this production function is written as \( h(S) \), where \( S \) is capital per unit of covered land or structural density and \( h \) satisfies \( h_s > 0 \) and \( h_{ss} < 0 \). \( h(S) \) represents residential total floor space per unit of building land. Since \( q \) is floor space per dwelling, it follows that the number of dwellings in a complex is given by \( \frac{H(K,L)}{q} \), which can be written as \( \frac{Lh(S)}{q} \).

Given the preceding discussion, the developer’s profit equals

\[ L \left\{ \frac{h(S)}{q} \left[ R(q,\alpha,x) - \alpha[r\bar{l} + i\bar{k}] - iS - r \right] \right\} \]  \hspace{1cm} (9)
where the expression in brackets in (9), denoted \( \pi \), is profit per acre of building land and \( R(q, \alpha, x, \cdot) \) is defined by (7). For fixed \( L \), developers choose \( q \), \( \alpha \) and \( S \) to maximize (9) and competition bids up land rent \( r \) until maximized profit per acre equals zero. Since total profit is zero regardless of the value of \( L \), the scale of the developer’s building is indeterminate.

Assuming an interior solution, the first-order conditions for choice of structural density, dwelling size and parking spaces per dwelling that must be met are respectively\(^5\)

\[
\frac{\partial \pi}{\partial S} = \frac{h_s(S)}{q} [R(q, \alpha, x, U, E) - \alpha[r \tilde{l} + i \tilde{k}]] - i = 0
\]  

(10)

\[
\frac{\partial \pi}{\partial q} = \frac{h(S)}{q} \left[ R_q - \frac{R(q, \alpha, x, U, E) - \alpha[r \tilde{l} + i \tilde{k}]}{q} \right] = 0
\]  

(11)

\[
\frac{\partial \pi}{\partial \alpha} = \frac{h(S)}{q} [R_\alpha - r \tilde{l} - i \tilde{k}] = 0
\]  

(12)

and the zero profit condition is

\[
\pi = \frac{h(S)}{q} [R(q, \alpha, x, U, E) - \alpha[r \tilde{l} + i \tilde{k}]] - iS - r = 0.
\]  

(13)

Equation (10) says that structural density is expanded until the marginal increase in revenue per acre of building land equals the marginal increase in cost from the extra capital plus the marginal increase in parking land cost required to hold parking spaces per dwelling fixed.

\(^5\) While the utility level \( U \) and \( N^C \) are ultimately endogenous, they are viewed as parametric at this stage in the analysis. If we consider the case where surface parking spaces per dwelling equals zero (a corner solution), then condition (12) would be replaced by the following three Kuhn Tucker conditions: \( \alpha \geq 0 \), \( \frac{h(S)}{q} [R\tilde{l} - i\tilde{k}] \leq 0 \) and \( \alpha \cdot \frac{h(S)}{q} [R_\alpha - r \tilde{l} - i \tilde{k}] = 0 \).
Equation (11) says that dwelling size is expanded until the marginal decrease in revenue per acre of building land equals the marginal decrease in parking land cost from holding the number of parking spaces per dwelling fixed.

Finally, equation (12) says that the number of parking spaces per dwelling should be increased until the net increase in revenue per acre of building land equals zero.

The Hessian matrix of \( \pi \) evaluated at the solution to (10)-(12) may be written

\[
D = \begin{bmatrix}
\frac{h_{ss}}{q} [R - \alpha (r\bar{I} + i\bar{K})] & 0 & 0 \\
0 & \frac{h}{q} R_{qq} & 0 \\
0 & 0 & \frac{h}{q} R_{\alpha\alpha}
\end{bmatrix}
\] (14)

The negative definiteness of \( D \) required by the second-order condition is guaranteed by \( h_{ss} < 0 \) and the strict concavity of \( R \), where \( R_{qq} < 0 \) and \( R_{\alpha\alpha} < 0 \). Thus, \( |D_1| < 0 \), \( |D_2| > 0 \) and

\[
|D| = \frac{h_{ss} R_{qq} R_{\alpha\alpha} h^2}{q^3} [R - \alpha (r\bar{I} + i\bar{K})] < 0.
\]

We now proceed to examine how the main endogenous variables vary over space.

3. Spatial Behavior of the Main Endogenous Variables

Spatial Behavior of \( S \), \( q \), \( \alpha \) and \( \bar{I}\alpha + q/h \)

We now focus on the spatial behavior of structural density (\( S \)), dwelling size (\( q \)), parking spaces per dwelling (\( \alpha \)) and land per dwelling (\( \bar{I}\alpha + q/h \)).

Differentiating (13) with respect to \( x \) while taking into account (7) and (10)-(12) yields, after some manipulations,
According to (15) residential land rent decreases with distance from the CBD but exhibits a kink at the boundary between the two transport modes. At the modal boundary $\hat{x}$, structural density, dwelling size and the number of parking spaces per dwelling are the same regardless of which mode the urban resident selects since transportation costs are the same at $\hat{x}$.

Equation (15) shows that the residential land rent associated with the automobile is less steep at $x = \hat{x}$.

Totally differentiating (10) - (12) taking account of the dependence of $r$ on $x$ and solving for $S_x, q_x$ and $\alpha_x$ using Cramer’s rule gives, after simplifying, the following results:

$$S_x = \frac{-h_x r_x q}{h_{ss} h[R - \alpha[rI + iK]]} < 0$$

(16)

$$q_x = \frac{r_x}{hR_{qq}} > 0$$

(17)

$$\alpha_x = \frac{\bar{I}r_x}{R_{\alpha\alpha}} > 0.$$ 

(18)

Let $\delta = \bar{a}^2 + q/h$. Taking advantage of the results thus far, it is possible to derive the spatial behavior of the amount of land per dwelling in the following way:

$$\delta_x = \alpha_x \bar{I} + \frac{q_x h - h_x q}{q^2} > 0.$$ 

(19)

Note that $r(x)$ is not differentiable at $x = \hat{x}$ because $R(x)$ is not differentiable at $x = \hat{x}$, but left and right differentiable at that point.
Moreover, the amount of land at location $x$ is given by $N(x)\delta(x)$ where $N(x)$ is the number of dwellings. Thus, $\frac{N(x)}{N(x)\delta(x)} = \frac{1}{\delta(x)}$ represents population density at location $x$ which, exhibits a spatial behavior described by

$$\left(\frac{1}{\delta}\right)_x = -\frac{\delta_x}{\delta^2} < 0.$$  \hspace{1cm} (20)

According to (16)-(18), buildings have fewer storeys farther from the CBD and dwellings are bigger closer to the edge of the city. Moreover, the number of residential parking spaces per dwelling increases with distance from the CBD, implying that bigger houses are bundled with more parking spaces. This pattern is actually consistent with the fact that per capita vehicle ownership and travel tend to be higher in automobile-dependent suburban areas while public transit travel tend to be higher in urban areas (Litman 2013). Equations (19) and (20) also reveal that urban residents consume more land as we move away from downtown (where land is typically more expensive) and population density decreases with distance from the CBD.

4. Urban Equilibrium Conditions

Next, we turn to the analysis of the urban equilibrium under the assumption of a closed city, where population ($N$) is viewed as fixed, while the urban utility level is determined within the system. The first spatial equilibrium condition requires that urban land rent at the edge of the city, $\bar{x}$, must equal the agricultural land rent:

$$r(\bar{x},U,N,C,E,f_c) = r_a.$$  \hspace{1cm} (21)
The second equilibrium condition is that population must fit inside the city. The population condition must also reflect mode choice:  

\[
\int_0^1 \frac{1}{\delta_b(x)} \, dx + \int_{\hat{x}}^\infty \frac{1}{\delta_c(x)} \, dx = N \tag{22}
\]

or

\[
r(\hat{x},U,E)[\theta_b y - \theta_c y - t_c] + r(0,0,0)[t_c + \theta_c y] = \theta_b y [N[t_c + \theta_c y] + r_a]. \tag{23}
\]

Since \(1/\delta(x)\) represents population density and the city is linear with unit width, the integrals in (22) aggregate total residents out to the urban boundary and equate it to \(N\). Finally, the number of workers commuting by car is determined as

\[
N^c = \int_{\hat{x}}^\infty \frac{1}{\delta(x)} \, dx = \frac{r(\hat{x},U,E) - r_a}{t_c + \theta_c y}. \tag{24}
\]

Another condition that must be met is a balance-budget parking condition stating that parking infrastructure costs are covered through the use of a parking fee

\[
f_c N^c = iC \tag{25}
\]

where \(i\) represents the exogenous unitary cost of providing one unit of parking capacity and \(C\) represents the (exogenous) supply of parking in the CBD.

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7 We define \(\delta_i\) with \(i = b, c\) as the total amount of land per dwelling under mode \(i\).

8 Recall that land size at location \(x\) is fixed and equal to 1. From (15) we get \(1/\delta_b(x) = -\frac{r_b x}{\theta_b y}\) and \(1/\delta_c(x) = -\frac{r_c x}{t_c + \theta_c y}\) which, we insert back into (22). Then, integrating while taking into account (21) and that \(\eta_b(\hat{x}) = r_b(\hat{x})\) yields, after some manipulations, (23).

9 The supply of parking in a CBD depends on several factors such as production costs namely land prices as well as on maintenance and operation costs and regulations. Since our goal is to understand how changes in parking supply in a CBD affect urban welfare and the urban spatial structure, for simplicity we have not explicitly modeled the problem of parking suppliers in downtown or distinguished between curbside and off-street parking. Note also that in our model, the number of auto commuters using the parking capacity equals the share of urban residents choosing to drive to work. Therefore the number of auto commuters represents the demand for CBD parking.
The left-hand side of (25) represents total revenues generated with parking fee $f_c$, while the right-hand-side represents the cost of providing downtown parking capacity. Since this cost is linear, parking capacity is produced under constant returns to scale. In addition, note that capacity congestion, $\frac{N^c}{C} y$, is homogenous of degree zero in traffic volume and capacity, conditions that lead to the self-financing theorem of transportation economics. Under these conditions, congestion tolls should exactly cover the cost of an optimal-size congested transportation facility.

Suboptimal Parking Fee

Before proceeding further, it is convenient at this point to draw attention to an assumption that provides useful interpretations when examining the impacts of a change in the supply of downtown parking on mode choice for the work trip and on the urban spatial structure.

The parking price that should be charged when a facility is congested should be equal to the marginal infrastructure costs plus an additional fee which is equal to the marginal external cost of congestion. This guarantees that individual users of the parking facility pay for the specific marginal costs they cause. The private cost of travel by automobile is given by (2). Note from (2) that travel by car is subsidized when downtown parking is underpriced that is, $f_c < \frac{N^c}{C} y$.

Since the individual parking-congestion cost is given by $\frac{N^c y}{C}$, the effect of an extra parker on this cost is the derivative $\frac{y}{C}$, while the effect on all parkers (the external cost) is $\frac{y}{C} N^c$, which is the optimal congestion charge. On the other hand, from (25) the balanced-budget parking fee is given by $\frac{i C}{N^c}$. Therefore, the auto travel subsidy is measured by $\frac{y}{C} N^c - \frac{i C}{N^c}$. 
The relationship between the balanced-budget parking fee and the optimal congestion charge depends nevertheless on whether \( C \) is set optimally. For what follows, we assume that capacity is initially fixed at some level \( C \) such that the balanced-budget parking fee (user fee) falls short of the optimal congestion charge, implying that \( \frac{N^C y}{C^2} - \frac{i}{N^C} > 0 \) holds, which in turn yields a nonzero auto commuting subsidy.

Next we explore the effects of changing the CBD parking capacity \( C \) when parking is underpriced on the optimal values of \( U \), \( \bar{x} \), \( \hat{x} \) and \( N^C \). Later, in section 6, we show that when \( C \) is chosen to maximize the equilibrium urban utility, the budget-balancing parking fee and the optimal congestion charge coincide. Thus, the self-financing theorem holds in the context of our model. In other words, when parking capacity is chosen optimally, the congestion toll and the balanced-budget parking fee coincide (even in the presence of environmental spillovers), so that aggregate total parking revenue just covers capacity costs.

5. The effects of a change in the supply of CBD parking

Together (5), (21), (23), (24) and (25) constitute a system of equations that can be solved for the unknowns \( U \), \( \bar{x} \), \( \hat{x} \) and \( N^C \). We now use the framework just developed to examine the effects of a marginal increase in \( C \) when parking is underpriced on the long run optimal values of

\[10\] From (21) and (23) we get the solutions for \( U \) and \( \bar{x} \) conditional on \( N^C \) and \( f_c \). These solutions are then substituted into (24) and (25), which then determine \( N^C \) and \( f_c \) as functions of the underlying parameters \( r_a, N, y, \theta_b, \theta_c, t_c, f_b, i \) and \( C \), with the latter parameter being the one of interest. Finally, the resulting solutions for \( N^C \) and \( f_c \) are substituted back into (21) and (23), which then determine \( U \) and \( \bar{x} \) as functions of the mentioned parameters. In order to make the exposition clear, all the parameters not relevant for this paper analysis will be suppressed in the equilibrium solutions.
structural density, dwelling size, residential parking supply, commuting mode choice, city size and urban welfare.

**Impacts on the Modal Boundary**

Differentiating equation (5) with respect to $C$ yields

$$
\frac{d\hat{x}}{dC} = \frac{i}{C} \left( \frac{N_c^e}{y - N_c^e} \right) - \frac{dN_c^e}{N_c^e} \left[ \frac{N_c^e}{y - N_c^e} \right] < 0
$$

(26)

since it can be shown that

$$
\frac{dN_c^e}{dC} = \frac{y}{C^2} \left[ \left( \frac{1}{0} \right) \left[ \delta(0) \theta_b y + t_c + \theta_c y \right] \left[ \delta(x) - \delta(0) \right] \right] < 1
$$

(27)

with $\frac{y}{C^2} > 0$, $\delta(x) > \delta(0)$ and $\theta_b y > \theta_c y + t_c$ implying that $0 < \frac{dN_c^e}{dC} \frac{C}{N_c^e} < 1$.

The sign of (26) captures the net effect of a change in the supply of CBD parking on modal substitution. The sign of (27) represents the net impact on the equilibrium urban residents driving to work.\(^{11}\)

To understand the signs of (26) and (27), note that, initially, an increase in $C$ generates a gain from decreased congestion delay costs in downtown $(\frac{N_c^e}{C^2} y)$ and also an increase in the balanced-

\(^{11}\)Totally differentiating (24) with respect to $C$ and using (26) and (28), yields after simplifying (27). The fact that the parking elasticity (27) is inelastic is consistent with empirical studies that have shown that the elasticity measure of auto use for work trip with respect to parking costs is around -0.31 (Gillen 1997). The reason for such low parking cost elasticity is that urban residents faced with increased parking costs (such as for example parking fees or reduction in parking supply which increase their parking time costs) are able to substitute between money and time costs by switching modes.
budget parking fee to cover the cost from the additions to parking supply \( \left( \frac{i}{N^c} \right) \). We denote the difference between the gain from reduced congestion and the increase in the parking fee as the \textit{primary (or partial equilibrium) effect} on the price of driving of a marginal increase in \( C \). The \textit{primary effect} is captured by term \( I \) in (26). Since the \textit{primary effect} is positive, there is a decrease in the price of driving to work.\(^{12}\) The magnitude of the \textit{primary effect} depends nevertheless on the level of the parking subsidy.

However, by relieving the bottleneck congestion and thus reducing the generalized cost of driving to work, the increase in \( C \) also induces more urban residents to drive downtown which in turn rebounds on CBD congestion delay costs, pushing the price of driving to work up.\(^{13}\) We denote this negative effect on driving costs from induced driving as the \textit{rebound effect} of a marginal increase in \( C \). The \textit{rebound effect}, captured by term \( II \) in (26), is the product of the \textit{primary effect} and the induced driving demand measured by the driving demand elasticity of parking capacity expansion.

It is worth mentioning that the magnitude of the \textit{rebound effect} depends on the value of this elasticity, which represents how sensitive auto travel activity is to a change in the price of driving.\(^{14}\) Whenever there is a change in \( C \), there is also a change in time costs and thus in the cost (price) of driving to work.

\(^{12}\) In equilibrium the marginal social benefit from the capacity expansion \( N^c \frac{N^c}{C^2} y \) should be equal to the marginal social cost \( i + \frac{N^c}{C} y \). Because parking is underpriced, there is a gap between \( N^c \frac{N^c}{C^2} y \) and \( i \) which corresponds to \( \frac{N^c}{C} y \). Since \( \frac{N^c}{C} y > 0 \), the \textit{primary effect} is positive.

\(^{13}\) Note that it is not the capacity expansion itself that generates travel but the reduction in congestion delays.

\(^{14}\) Stated differently, this elasticity is the price elasticity of demand of the number of cars that commuters drive to (and park) at work.
To the extent that this elasticity is inelastic, that is, has a value lower than unity, the rebound effect is small. Since the rebound effect pushes the modal boundary outwards while the primary effect works in the opposite direction, the rebound effect partially offsets the primary effect of a change in $C$ on the modal boundary. Yet, because the driving demand elasticity of parking capacity expansion is less than unity, the rebound effect is actually outweighed by the primary effect and the modal boundary shortens ($\frac{d\hat{x}}{dC} < 0$), meaning that the enlarging of the existing CBD parking capacity has actually decreased congestion delay costs in downtown. Together with (27) we may conclude that downtown will be carrying more vehicles, that is, more urban residents will be driving to work after the parking capacity expansion ($\frac{dN^c}{dC} > 0$). This theoretical outcome is also evidenced by Mildner et al. (1997) who show that increased parking supply tends to increase automobile commuting and reduce transit and ridesharing.

Impact on Utility level

Totally differentiating (23) with respect to $C$ while substituting (26) and (27), yields the impact of an increase in downtown parking capacity on residents’ welfare as

$$
\frac{dU}{dC} = \delta(0)\theta_y \left[ 1 - \frac{dN^c}{dC} \frac{C}{N^c} \left( \frac{N^c}{C^2} y - \frac{i}{N^c} \right) \right] + \frac{\partial E}{\partial N^c} \frac{N^c}{E} \left[ \frac{dN^c}{dC} \frac{C}{N^c} \right] \frac{E}{C} \geq 0.
$$

(28)

where $\frac{\partial E}{\partial N^c} \frac{N^c}{E}$ represents the elasticity of environmental quality (air pollution) with respect to auto travel to the CBD. The decomposition of the welfare sources in (28) shows that the induced
driving demand from an increase in \( \frac{dN^c}{dC} \), given by \( \frac{dN^c}{dC} \frac{C}{N^c} \), also impacts the city environmental quality by affecting traffic air pollution.

Since \( 0 < \frac{dN^c}{dC} \frac{C}{N^c} < 1 \) it is likely that the indirect rebound effects caused by the parking capacity expansion would be small. In this case, it may be possible that the direct benefits of the capacity expansion that would occur with no change in travel behavior (that is, holding the number of drivers, \( N^c \), unchanged) dominate and an increase in utility occurs. To the extent that these same diseconomies pull utility down as auto commuting increases and depend on the elasticity of environmental quality, the net effect of an increase in downtown parking supply on welfare cannot be ascertained a priori.

**Impacts on Residential Land Rent**

The impacts of an increase in \( C \) on residential landowners’ welfare can be described by the change on the residential land rent profile.

\[
\frac{dr(0)}{dC} = -\frac{1}{\delta(0)} \left[ \frac{dU}{dC} - \frac{\partial E}{\partial N^c} \frac{dN^c}{dC} \right] = \\
\theta_b y \left[ 1 - \frac{dN^c}{dC} \frac{C}{N^c} \left( \frac{N^c}{C^2 y} - \frac{i}{N^c} \right) \right] \\
= -\frac{\delta(0) \theta_b y [\theta_c + \theta_c y \delta(\hat{\delta}) - \delta(0)]}{\delta(0) \theta_b y + [\theta_c + \theta_c y \delta(\hat{\delta}) - \delta(0)]} < 0
\]

and
The above comparative static analysis reveals that the impact of an increase in downtown parking on residential land rent is not the same everywhere in the city. Urban residents make tradeoffs between transportation costs (time and money) and locations decisions. It is therefore likely that changes in vehicle commuting costs will affect the desirability of automobile-dependent locations and therefore the amount of urban fringe development that occurs.

Because the primary effect outweighs the rebound effect, residential land rent near the CBD decreases while residential land rent at locations farther from the business district increases. An increase in downtown parking supply decreases driving costs to work in the long run because the net effect is to decrease congestion delay costs in downtown. Since auto travel to work is less expensive, some of the urban residents who originally were commuting by mass transit will switch to the car mode. Since they will use the car, they are motivated to move farther away from their jobs. On the other hand, for those urban residents who keep using mass transit, the income net of transport cost increases. These two effects together bid up housing bid rents in the area of the city where the car is the transportation mode to commute to work, while depressing housing bid rents near the CBD. Two notes are now in order.

First, the rise in auto-commuting increases air pollution throughout the city, affecting all urban residents the same way. The reason is that air pollution is assumed to be a global externality and environmental damages to be the same in every location. Consequently, residential bid rents experience the same decrease everywhere in the city due to increased vehicle air emissions. This
decrease in housing rents in turn increases residents’ urban welfare. In equilibrium, these two effects cancel out. However, because the net effect of the capacity expansion is to decrease congestion costs in downtown and increase the number of residents commuting by car, there is an overall increase in aggregate air pollution which pushes down the equilibrium urban utility. This effect is captured by the second component of (28).

Second, since attributes of mass transit did not change, workers already commuting by transit must enjoy a higher utility. This is actually consistent with the lower housing rents everywhere in their residential area. The reason for lower rents in central areas where transit is the work mode of transportation is the increase in the desirability of farther locations because parking congestion costs at the workplace decreased after the capacity expansion. This, in turn, results in lower residential land rents in central areas and higher residential land rents farther away from the downtown district (where there is an excess demand for residential land) as some urban residents relocate from central to suburban residential areas.

Impact on the urban boundary

Totally differentiating (21) with respect to C while substituting (27) and (28) yields,

$$\frac{d\bar{x}}{dC} = \frac{\left[ \delta(\bar{x}) - \delta(0) \right] \left[ 1 - \frac{dN^C}{dC} \frac{C}{N^c} \left[ \frac{N^c y - i}{N^C} \right] \right]}{\theta_b y \delta(0) + \left[ \theta_c y + t_c \right] \delta(\bar{x}) - \delta(0)} > 0$$

(31)

since \( \delta(\bar{x}) > \delta(0) \).

The key observation regarding (31) is that an increase in CBD parking supply can actually cause the urban spatial structure to become less compact. Since the rebound effect on delay congestion costs is dominated by the primary effect, the city size increases. Further intuition can be gained by examining the impacts on population densities.
Impact on structural density, dwelling size and residential parking spaces per dwelling

Totally differentiating (12) with respect to $C$ while taking into account the dependence of the land rent on $C$, yields

$$\frac{d\alpha(0)}{dC} = \frac{\tilde{I}}{R_{\alpha\alpha}} \frac{dr(0)}{dC} > 0 \quad \text{and} \quad \frac{d\alpha(\hat{x})}{dC} = \frac{\tilde{I}}{R_{\alpha\alpha}} \frac{dr(\hat{x})}{dC} < 0$$

(32)

given (29) and (30) and that $R_{\alpha\alpha} < 0$. According to (32), the number of residential parking spaces per dwelling increases in central locations and decreases at farther locations from the CBD following an expansion in $C$. This suggests that changes in CBD parking supply also influence residential parking supply throughout the city because of its heterogeneous impact on land rents throughout the urban area. This in turn means that limited parking in downtown areas influences developers’ investment decisions in other parts of the urban area.

The impacts on structural density at the CBD and at the modal boundary can also be examined by first computing $S_u$ and $S_E$ and then using (28) and (26) to evaluate, yields\(^{15}\)

---

\(^{15}\)The partial effect of $S_u$ was determined by totally differentiating the system of equations (10)-(12) evaluated at the initial equilibrium with respect to the exogenous parameter of interest while taking into account the dependence of $r$ on $U$ and then using Cramer’s rule to find the result.
\[
\frac{dS(0)}{dC} = S_u(0) \frac{dU}{dC} + S_E(0) \frac{dE}{dC} = \frac{h_s q}{h_{ss} \delta(0) h[R(0) - \alpha r - \alpha i k]} \frac{dU}{dC} - \frac{h_s q}{h_{ss} \delta(0) h[R(0) - \alpha r - \alpha i k]} \frac{dE}{dC} = \]
\[
\frac{h_s q}{h_{ss} \delta(0) h[R(0) - \alpha r - \alpha i k]} \left[ \frac{dU}{dC} - \frac{dE}{dC} \right] \]
\[
= \frac{h_s q \theta_{b,y} \left[ 1 - \frac{dN^c}{dC} \frac{C}{N^c} \left( \frac{N^c}{C^2} \frac{y - i}{N^c} \right) \right]}{h_{ss} h[R(0) - \alpha r - \alpha i k]} \left[ \delta(0) \theta_{b,y} + \left[ t_c + \theta_{c,y} \right] \delta(\hat{x}) - \delta(0) \right] \]
\[
= \frac{h_s q}{h_{ss} h[R(0) - \alpha r - \alpha i k]} \frac{dr(0)}{dC} < 0
\]

and
\[
\frac{dS(\hat{x})}{dC} = S_x(\hat{x}) \frac{d\hat{x}}{dC} + S_u(\hat{x}) \frac{dU}{dC} + S_E(\hat{x}) \frac{dE}{dC} = \frac{h_s q \theta_{b,y} \left[ t_c + \theta_{c,y} \right] \left[ 1 - \frac{dN^c}{dC} \frac{C}{N^c} \left( \frac{N^c}{C^2} \frac{y - i}{N^c} \right) \right]}{h_{ss} h[R(\hat{x}) - \alpha r - \alpha i k]} \left[ \delta(0) \theta_{b,y} + \left[ t_c + \theta_{c,y} \right] \delta(\hat{x}) - \delta(0) \right] \left[ \theta_{b,y} - \theta_{c,y} - t_c \right] \]
\[
= \frac{h_s q}{h_{ss} h[R(\hat{x}) - \alpha r - \alpha i k]} \frac{dr(\hat{x})}{dC} > 0.
\]

By a similar method, we can find the effects on dwelling size at the city center and at the modal boundary by determining the sign of\( q_u \).

---

\(q_u\) was determined by totally differentiating the system of equations (10)-(12) evaluated at the initial equilibrium with respect to the exogenous parameter of interest while taking into account the dependence of \(r\) on \(U\) and then using Cramer’s rule to find the result.

---

16 The partial effect of \(q_u\) was determined by totally differentiating the system of equations (10)-(12) evaluated at the initial equilibrium with respect to the exogenous parameter of interest while taking into account the dependence of \(r\) on \(U\) and then using Cramer’s rule to find the result.
\[
\frac{dq(0)}{dC} = q_u \frac{dU}{dC} + q_E \frac{dE}{dC}
\]

\[
= -\frac{1}{h(0)\delta(0)R_{qq}} \left[ \frac{dU}{ds} - \frac{dE}{dC} \right]
\]

\[
= -\frac{\theta_{b,y}}{h(0)R_{qq}} \left[ \delta(0)\theta_{b,y} + \left[ t_c + \theta_{c,y} \right] \delta(\hat{x}) - \delta(0) \right]
\]

\[
= \frac{1}{h(0)R_{qq}} \frac{dr(0)}{dC} > 0
\]

and

\[
\frac{dq(\hat{x})}{dC} = q_x \frac{d\hat{x}}{dC} + q_u \frac{dU}{dC} + q_E \frac{dE}{dC}
\]

\[
= -\frac{\theta_{b,y}}{\delta(\hat{x})hR_{qq}} \frac{d\hat{x}}{dC} - \frac{1}{h\delta(\hat{x})R_{qq}} \frac{dU}{dC} + \frac{1}{h\delta(\hat{x})R_{qq}} \frac{dE}{dC}
\]

\[
= -\frac{1}{\delta(\hat{x})h(\hat{x})R_{qq}} \left[ \frac{dU}{dC} + \theta_{b,y} \frac{d\hat{x}}{dC} - \frac{dE}{dC} \right]
\]

\[
= \frac{1}{h(\hat{x})R_{qq}} \frac{dr(\hat{x})}{ds} < 0.
\]

The effects on structural density and dwelling size of an increase in downtown parking supply are consistent with the impacts already discussed on residential land rents. Residential land rents near downtown fall while suburban residential land rents increase. This induces smaller dwellings and taller buildings in the suburbs which increases suburban population. The opposite occurs in locations near the CBD. Since there is a fixed number of urban residents that must be housed in the city, this can also explain the expansion of the city size after an increase in \( C \).

\textit{Impact on the parking subsidy}
Let \( s = \frac{y}{C} N^c - \frac{iC}{N^c} \). Recall that we assumed downtown parking to be underpriced. The impact of a change in the CBD parking capacity on the automobile travel subsidy from underpriced parking is captured by

\[
\frac{ds}{dC} = \frac{1}{C} \left[ \frac{yN^c}{C} + \frac{iC}{N^c} \right] \left[ \frac{dN^c}{dC} \frac{C}{N^c} - 1 \right] < 0 \tag{37}
\]

Since \( 0 < \frac{dN^c}{dC} \frac{C}{N^c} < 1 \) holds, the sign of (37) is negative. An increase in parking capacity on the one hand increases the budget-balancing parking fee but on the other hand it decreases the marginal external cost of congestion. The net result of these two countervailing effects is to decrease the level of the parking subsidy. Unless the expansion in \( C \) is such that the new capacity leads the budget-balancing parking fee to coincide with the external congestion costs of the parking capacity usage, there is still a positive (yet lower) auto subsidy.

Table 1 summarizes our comparative statics results.

<table>
<thead>
<tr>
<th>( N^c )</th>
<th>( \hat{x} )</th>
<th>( \bar{x} )</th>
<th>( S(0) )</th>
<th>( S(\hat{x}) )</th>
<th>( q(0) )</th>
<th>( q(\hat{x}) )</th>
<th>( \alpha(0) )</th>
<th>( \alpha(\hat{x}) )</th>
<th>( r(0) )</th>
<th>( r(\hat{x}) )</th>
<th>( U )</th>
</tr>
</thead>
<tbody>
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<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
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<td>-</td>
<td>+</td>
<td>+/-</td>
</tr>
</tbody>
</table>

The comparative static results from table 1 yield two interesting conclusions.

**Proposition 1:** When parking at the CBD is underpriced and its financing relies on budget-balancing parking fees, an expansion in downtown parking supply tends to decrease overall parking congestion at the CBD. However, increased parking supply also tends to increase automobile commuting and reduce transit rides and as a result, contributes to an increase in air pollution. Provided that the primary effect dominates the rebound effect and the increase in air pollution, an increase in CBD parking supply is welfare improving. In addition, the overall decrease in congestion delay costs at the CBD makes automobile-dependent locations in the long run more attractive to urban residents, potentially leading to an expansion in the city size.
Proposition 1 implies that parking supply policies that increase the amount of downtown parking supply in the presence of suboptimal parking fees may actually work at cross purposes with anti-sprawl policies. In contrast, caps on the amount of downtown parking, while reducing total auto travel as well as city size, can be welfare decreasing because they exacerbate (rather than alleviate) the overall parking congestion in centralized workplaces. With this mind we may further conclude that

**Proposition 2:** When parking is underpriced, increases in parking capacity tend to satisfy unpriced demand and to reduce the severity of congestion costs from parking underpricing. However, over the long run, such supply decisions contribute to a self-reinforcement cycle of automobile dependency and sprawl.

6. Optimal Parking Capacity and Self-Financing

In this section we relax the assumption that $C$ is exogenously set and discuss how optimal parking capacity provision affects the comparative static results of section 5 as well as its implication for the self-financing of a congested facility.

*Optimizing capacity*

Substituting (27) into (28) yields after some manipulations

$$
\frac{dU}{dC} = \Theta \left[ \frac{N^c y}{C^2} - \frac{i}{N^c} \right]
$$

(38)

where $\Theta = \Omega + \Delta$ with

$$
\Delta = \frac{\delta(0)\theta_b y}{\delta(0)\theta_b y + \theta_c y [\delta(\tilde{x}) - \delta(0)]} \quad \text{and}
$$
Now suppose that downtown parking capacity is chosen to maximize residents’ equilibrium utility. It can be shown that when parking capacity is chosen such that

\[
\frac{N^c y}{C^2} = i
\]

is satisfied then \( \frac{dU}{dC} = 0 \), which is a condition that must be satisfied for the optimal capacity choice.

Under capacity rule (40), parking capacity is chosen until the total time-cost savings from an increase in downtown parking capacity (given by the left-hand-side of (40)) equals the marginal cost of adding capacity.

**Self-financing of capital cost**

However, if the investment rule (40) is satisfied, then the following marginal-cost pricing rule also emerges given (25)

\[
\frac{N^c y}{C} = \frac{iC}{N^c} \iff \frac{N^c y}{C} = f^c.
\]

The parking fee derived in (41) may be viewed as a charge for the use of the parking capacity in the downtown area. This parking fee also acts as a congestion fee since it captures the parking-congestion damage from an extra auto commuter to the CBD.\(^\text{17}\) Since this parking fee satisfies the

\(^\text{17}\) In this paper we abstract from the case of second-best capacity. For a study that has examined optimal capacity in the presence of unpriced congestion see Wilson (1983). The author shows that whether the optimal capacity of a road is affected by a toll fixed below its optimal value depends on the value of the price elasticity of travel demand at the second-best optimum. If this price elasticity is sufficiently high, the pricing constraint lowers the optimal capacity. However, if the price elasticity is less than the ratio of the private price of travel to the private congestion cost at the
balanced budget constraint (25), it further follows that parking capacity costs equal the congestion toll-revenue (the self-financing result).

Summarizing yields

**Proposition 3:** When downtown parking capacity is chosen to maximize the equilibrium level of urban utility, a balanced-budget parking fee coincides with the optimal congestion toll albeit the presence of environmental spillovers (air pollution).

The reason why the result in proposition 3 occurs is because air pollution from auto commuting is a global externality. Therefore, when capacity in downtown is chosen to internalize CBD congestion, it also maximizes utility in our setting. This optimal utility is never the less a second best result because too much car commuting still occurs since air pollution from auto usage is not being fully internalized.

It is also interesting to note that the equality from the self-financing result still applies even if other charges (say a pollution tax) are levied to cover other auto commuter externalities (for example air pollution). The intuition is as follows.

Remember that commuting to work by car generates two externalities. On the one hand it creates parking congestion (in the form of time delay costs) in the downtown area and on the other hand, it generates air pollution which in turn affects equally all households in the urban area. For the sake of illustration, let the environmental effect of an extra auto commuter be constant and equal to $e$. Then the external effect on all urban households is given by $Ne$, which corresponds to the optimal environmental charge. If it is the case that this environmental tax is charged to every auto commuter, the cost of auto commuting defined by (2) changes to

---

second-best optimum, then a suboptimal toll raises the optimal capacity. Within the context of our model, we can nevertheless observe from (28) that the optimal capacity when parking is underpriced implies further increases in capacity as long as the environmental impacts are small. This in turn suggests that there is a need to compensate for lack of pricing by building or providing more parking. Moreover, financial balance is not a guarantee of efficiency since there are several combinations of capacity and parking fee that satisfy (25).
\[(t_c + \theta_c y)x + \frac{N^c}{C} y + f_c + Ne. \quad (42)\]

While the presence of an environmental tax set at \(Ne\) changes (expands) the optimal modal boundary \(\hat{x}\), it does not change the user parking cost \(\frac{N^c}{C} y\) or the parking capacity cost \(iC\) functions, so the relationships established for congestion remain valid although they will be associated with a different (lower) traffic volume (that is, number of auto commuters). As a result, the comparative static outcomes described in section 5 remain valid and both the investment rule (40) and pricing rule (41) hold. We may then infer that

**Proposition 4:** The self-financing result remains valid even if an environmental tax is levied to internalize an air pollution externality from commuting by car in a closed urban area.

Finally, when CBD parking capacity is set under the investment rule (40), the parking subsidy and therefore all the derivatives in section 5 become zero. Subsequently, we conclude that

**Proposition 5:** When parking is underpriced \(f_c < \frac{N^c}{C}\), an expansion of the CBD parking capacity generates indirect costs due to induced demand which derives from both traffic creation and traffic deviation from public transit to road. These indirect costs may partially or completely offset the direct benefits of an expansion in the parking capacity. In contrast, when the parking subsidy is removed and therefore \(f_c = \frac{N^c}{C}\), further increases in parking capacity do not induce further driving \(\frac{dN^c}{dC} = 0\) and \(\frac{d\hat{x}}{dC} = 0\) nor impact the urban spatial structure \(\frac{d\hat{x}}{dC} = 0\) or the equilibrium utility level \(\frac{dU}{dC} = 0\).

7. Further Discussion

**Parking Elasticities**
The described impacts on welfare and urban spatial structure of changes in downtown parking supply hinge crucially on the presence of a balanced-budget parking fee that falls short of the external congestion cost and on the magnitude of the elasticity of driving demand with respect to parking supply (thus, auto price). Given the low probability that existing parking fees are actually set optimally, estimates that capture how auto price changes affect auto travel activity are important information when evaluating transport problems and possible solutions.

Empirical evidence on transport elasticities is surveyed by Litman (2013), and he argues that the evidence demonstrates that auto users are sensitive to parking prices. The studies revisited in the author’s literature review indicate that the elasticity of vehicle trips with regard to parking prices is typically in the -0.1 to -0.3 range, with significant variation depending on demographics, geographic, travel choice and trip characteristics (work versus shopping). Some caution should nevertheless be taken when using these estimates in economic analyses of parking policies in downtown areas, especially in North America cities. The reason rests on the fact that most workers commuting by car to centralized workplaces park free at work. Moreover, our analysis also reveals that the magnitude of the elasticity of driving demand with respect to parking supply is determined by the level of parking underpricing.

Possible sources for free parking in CBD areas include employer-paid parking and zoning policies such as minimum parking requirements (Willson and Shoup 1990, Shoup 2005).\(^\text{18}\) While the main goal of minimum parking requirements is to address problems associated with an

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\(^\text{18}\) Under the U.S. Internal Revenue Code section 132(a), the qualified transportation benefits is one of the eight types of statutory employee benefits (also known as fringe benefits) that are excluded from gross income. As of 2014, the two types of qualified transportation benefits are transit passes and vanpooling (up to $130/mo), and parking (up to $250/mo). Bike commuters can also be reimbursed for certain expenses (up to $20/mo). The commuter benefits program offers then employers the possibility to enhance their benefit package with an incentive that can be used to attract and retain qualified employees. In addition, employers who provide the benefit as a tax-free fringe benefit (paid by the employer) save on payroll taxes because the employer does not need to include the amount of the fringe benefit in the employee’s gross income.
undersupply of parking, especially in central areas, it is increasingly recognized that they can create others. Minimum parking requirements (MPRs) are generally designed to satisfy peak demand for free parking (EPA 1999; Shoup 2002). They are not designed to accurately reflect the need for parking, nor are they intended to optimize land usage. Thus MPRs can create an oversupply of parking spaces (Cutter and Franco, 2012). But as our comparative results reveal, as parking availability increases, fewer people use public transportation. As the number of cars increases and mass transit use decreases, air quality decreases. Decreased air quality creates and/or aggravates health problems for urban residents. Individuals who suffer from pulmonary diseases such as asthma and bronchitis suffer as the increased number cars decreases air quality (EPA 1999; Shoup 2002).

Several cities in the United States have already begun to supplement minimum parking requirements with maximum parking requirements in an attempt to balance land use management (VTPI 2013; Wittenberg 2003). However, the rule to set up these maximum parking requirements works in the same way as the rule for MPRs. Numerical limits set by municipalities for particular land uses (such as for example office, commercial or industrial uses) usually come from either the parking generation manual published by the Institute of Transportation Engineers, and/or from limits other municipalities have instituted (Shoup 2002). If parking maximums are set too low, then this parking supply policy may end up creating an undersupply of parking. In very congested areas, as our comparative results have shown, a decrease in parking supply can actually be welfare.

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19 For example, San Francisco (CA) limits parking downtown to 7% of the building's floor area. Seattle (WA) allows a maximum of one parking space per 1,000 square feet of office space downtown, and is considering extending this limit to areas outside of downtown as well. Redmond (WA), a suburban community, allows a minimum of 4 and a maximum of 5 spaces per 1,000 square feet of floor area for most uses in the Neighborhood, Retail, and General commercial zones. Helena (MT) establishes maximum parking ratios as a percent above the minimum parking ratio (e.g. no more than 110% of the minimum for parking lots of more than 51 spaces).
decreasing when the auto travel elasticity with respect to parking supply is inelastic due to the increase in congestion costs associated with auto users not being able to find a parking spot.

Furthermore, the studies in Litman (2013) also reveal that transport elasticities tend to increase over time as consumers incorporate price changes in more long-term decisions such as vehicles purchased and home locations. However, evidence of rising price elasticities is still preliminary and deserves more research. If in indeed there is growing evidence that transport elasticities are high, then parking policy reforms such as caps on parking supply in downtown areas may be able to provide higher benefits in terms of congestion and emission reductions than previously thought.

**Other Policy Implications**

Our analysis also reveals that changes in downtown parking supply influence residential parking supply throughout the city. A reduction in downtown parking supply leads in the long run to an increase in the number of residential parking spaces per dwelling near the CBD and to a reduction in central-suburban and suburban locations. This is an interesting interaction because if increases in residential parking ultimately lead to increases in residential parking demand, this may in turn affect the amount local authorities require of off-street residential parking in different parts of the city.  

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20 Guo and Ren (2013) have examined the residential parking supply in London before and after the minimum off-street parking standard was replaced by a maximum in 2004. It is reported that before the parking reform, developers created 94 percent of the required minimum; after the reform, they created just 52 percent of the old minimum. But what it is surprising about this study is the finding that the actual parking supplied after the reform was higher in Central London, where density and access are greatest, compared to adjacent outer areas. The authors suspect that local authorities may want to keep a high maximum (and therefore allow more spaces) to avoid a parking spillover onto already crowded streets in Central London. Another explanation is that the market simply wants more spaces there: people who can afford to live downtown are willing to pay a premium for parking. Thus, the authors conclude that the market-oriented approach to parking regulation can reduce excessive parking, but it depends on the particular sub-markets.
Recent empirical evidence (see for example Weinberger 2015) has shown that guaranteed parking at home, whether it is in a driveway or in a garage, increases the propensity to use the automobile for work trips even between origin and destination pairs that are reasonably well served by public transit. Together with our theoretical results, we may infer that limits on parking supply in downtown areas may end up increasing auto-use trips from new areas with high on-site residential parking even if these areas are well served by public transportation.

Another possible by product of this type of parking reform may be an increase in congestion from parking spillover into other parts of city, outside but close to the CBD and where parking may become more accessible. For instance, Hensher and King (2001) model the price elasticity of CBD parking, and predict how an increase in parking prices in one location will shift cars to park at other CBD locations and drivers to public transit. The study reports that a 10% increase in prices at preferred CBD parking locations will cause a 5.41% reduction in demand there, a 3.63% increase in park & ride trips, a 2.91% increase in public transit trips and a 4.69% reduction in total CBD trips.

*Parking Fees and Congestion Tolls as Complements*

Finally, we discuss how parking and congestion pricing policies may reinforce each others’ goal of reducing congestion in urban areas. Parking pricing and congestion pricing are often seen as alternative means to reduce traffic congestion, with parking pricing usually seen as a potentially useful but rather blunt tool when compared to the precision of congestion tolls. However, parking pricing and congestion pricing can be viewed as complementary tools to reduce congestion. If parking subsidies at trip destinations reduce the cost of automobile use, congestion tolls will have to be set higher than if parking is not subsidized. The reason is because congestion tolls would
need to charge auto users for the use of roads during the time motorists are cruising for parking (search time). Thus, charging for parking and charging for road use should be seen as complementary policies.  

For the sake of illustration let’s assume there is a fixed parking capacity $C$ in downtown and that parking is financed with budget-balancing user fees. Under this scenario, the private parking price is given by the sum of the budget-balancing parking fee and the individual congestion time cost

$$ p = \frac{iC}{N^c} + \frac{N^c}{C} y. \quad (43) $$

An interesting feature of (43) is that this price is not a flat price, independent of the number of car commuters in downtown. In particular it can be shown that

$$ \frac{\partial p}{\partial N^c} = -\frac{iC}{N^c} + \frac{y}{C} \frac{yN^c}{C} - iC^2 > 0. \quad (44) $$

Let’s assume that $y > i$ and that there is a shortage of parking spaces in downtown such that $N^c > C$. In this case, as the number of auto users increases, parking price also rises because downtown becomes more congested. Conversely, if there is plenty of parking spaces available (  

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21 Pierce and Shoup (2013) report that ten studies conducted in eight cities (six American cities and two European cities) between 1927 and 2011 found that on average 34% of traffic congestion in downtown areas was due to drivers cruising for an open parking space. Relatively inexpensive parking can lead to increased demand for spaces which can lead to a longer amount of time being spent searching for parking spaces. As these vehicles cruise for parking, they must remain in the traffic system and travel at a lower speed and thereby slow the entire system until they find a space. Thus, where curb parking is underpriced and congested, cruising for parking can greatly increase traffic congestion (Arnott and Rowse, 1999; Arnott and Inci, 2006; Pierce and Shoup, 2013).

22 If (43) is positive then $\frac{N^c}{C^2} y > i$, which implies that the marginal benefit of capacity (that is, the value of aggregate travel time saving) exceeds its marginal cost of the parking capacity. Capacity should thus expand until the marginal benefit equals marginal cost. However, when $C$ expands $N^c$ also increases. Because $0 < \frac{dN^c}{dC} \frac{C}{N^c} < 1$ holds, as
\( N^C < C \), the parking price may be reduced as the number of auto users entering downtown increases because infrastructure costs would be split among more motorists. This suggests that higher parking rates can be charged during peak hours (when demand is high) and lower rates would be charged when demand is less. In this sense, demand-based pricing could reduce congestion in downtown areas.\(^{23}\)

Since demand-based parking programs may be more politically feasible and easier to implement than a congestion tax or even caps on the number of parking spaces, parking pricing and congestion pricing should be used as complementary tools to combat downtowns’ congestion. In a very recent study Pierce and Shoup (2013) provide an evaluation of pricing parking by demand in San Francisco. To address the problems associated with distorted prices (either too low or too high) for curb parking, San Francisco has established in 2011 a program, called SFpark, that adjusts parking prices to achieve a target parking availability of one or two open spaces on each block. Pierce and Shoup (2013) conclude from their first evaluation of the program that since SFpark managers began adjusting meter prices in August 2011, the elasticity of parking demand has varied across different locations and times of day (possibly due to different trip purposes), and drivers changed their behavior most profoundly after the second price adjustment, possibly due to the increased in the program awareness. Even though the SFpark program is relatively new, the capacity expands, the equilibrium volume-capacity ratio falls. Note also that if \( \frac{N^C^2}{C^2} > 1 \) occurs then

\[
\frac{N^C}{C} \cdot y > \frac{iC}{N^C} = f^C
\]

and the capacity that is initially set is not only suboptimal but it is such that the budget-balancing parking fee is set below the optimal congestion toll.

\(^{23}\)The mechanism just described replicates the behavior of multi-space parking meters installed in several European cities and in San Francisco, CA, USA. Multi-space metered pay stations are automated kiosks that replace all of the individual parking meters on a block. They accept coins but also accept credit and debit cards, making the purchase of parking more convenient. In addition, users may pay for any multi-space metered parking space at any multi-space pay station.
optimistic results achieved so far actually show the potential of parking pricing to reduce congestion in certain parts of an urban area, in particular the central city and areas of high retail and employment activity.

8. Conclusions

A natural conjecture is that commuting subsidies amplify urban sprawl. However, when downtown congestion delay costs and financing based on budget-balancing user fees are included in the standard monocentric city model with two transport modes, the result is less clear. Our analysis shows that an increase in CBD parking capacity may alleviate downtown congestion delay costs and reduce the generalized cost of driving in the short-run. However, over the long-run induced travel may increase congestion delay costs as transport systems and land use patterns become more car dependent. The net effect on the urban spatial structure and welfare thus depends on the driving demand elasticity with respect to parking supply. Low elasticities are shown to generate a low rebound effect and as a result the impact of an increase in downtown parking capacity would be welfare improving but lead to an increase in city size. Conversely, a decrease in downtown parking supply is welfare reducing but leads to a less dispersed urban spatial structure.

Our analysis also shows that the impacts on residential parking supply from changes in downtown parking capacity are not homogenous in space. In particular, caps on CBD parking supply lead to an increase in the number of parking spaces per dwelling in central locations outside the CBD. This can have important implications on how parking requirements maybe established for a particular zone. If the supply of more residential parking spaces in turn generates an increase in parking demand, this may lead local authorities to require more off-street parking in residential

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development projects in some parts of the city. Future research on the interconnections between nonresidential and residential parking requirements would be of interest.

A third interesting outcome from our analysis is that in a closed monocentric city model with two transportation modes and under the conditions of the self-financing theorem from transportation economics, the budget-balancing parking fee that emerges from a parking capacity that maximizes urban welfare coincides with the optimal congestion toll even in the presence of environmental spillovers. Changes in auto travel price from changes in parking price affect transportation mode choices and in turn the amount of air pollution.

Air pollution affects urban residents both commuting by car and by public transit. It is natural then to expect that population distribution is inefficient unless air pollution from vehicle use is effectively internalized by a corrective policy. While an increase in air pollution creates an externality that decreases urban welfare, it also decreases residential rents, which is welfare improving. In equilibrium, the two effects cancel and the potential distortion in air pollution from a change in CBD parking capacity is eliminated. This result requires nevertheless that full capitalization of air quality into residential rents occurs and moving costs are low. It should be noted also that in the current model the level of air pollution at each location depends on aggregate auto usage. It would be interesting to examine how the results would change if instead air pollution at a given location is a function of the number of commuters traveling through that location. It is likely that this type of air pollution function would imply that unpriced pollution from automobiles would exacerbate the increase in city size induced by lower congestion costs at the CBD from an increase in CBD parking capacity. Instead of having an homogeneous decrease in housing rents at each location, it is expected that households bid less for housing in more polluted locations (downtown) and more for housing in the less polluted suburbs. Since the decrease in the CBD
congestion externality from an expansion in parking capacity gives the same results, the two effects would reinforced each other, leading to a bigger expansion of the city size. Future work could then explore whether the self-financing theorem still holds when these types of locational interdependencies exist.

In this study urban residents are homogeneous. However, heterogeneous residents may react differently to the exact same change in price when that change in price occurs in different components of the total driving price. Future work should also explore how workers heterogeneity may affect the impacts and benefits of alternative CBD auto restrain policies. Another avenue for future research would be to examine the efficiency and distributional effects of parking supply restrictions in the presence of a competing suburban center. Finally, given that multiple types of parking subsidies may co-exist at the CBD, another interesting exercise would be to evaluate how pre-existing employer-paid subsidies interact with the impacts of off-street parking limits.

References


Senate Transportation and Housing Committee (2009), Summary Report: Reducing Congestion and Greenhouse Gas Emissions through Parking Policy.


