Microfoundation of Inflation Persistence of a New Keynesian Phillips Curve

Marcelle Chauvet and Insu Kim
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Standard New Keynesian Phillips curve (NKPC) based on optimizing behavior of price setters in the presence of nominal rigidities. Mostly based on:


Framework used in analysis of monetary policy: price rigidity main transmission mechanism through which it impacts the economy:

- when firms face difficulties in changing some prices, they may respond to monetary shocks by changing instead their production and employment levels
Background

- Popular frameworks to derive the NKPC
  - Calvo (1983)’s staggered price setting: only a fraction of firms completely adjusts their prices to optimal level at discrete time intervals
  - Rotemberg (1982): firms set prices to minimize deviations from optimal price subject to quadratic frictions of price adjustment
- Both designed to model sticky prices:
  \[
  \text{Rotemberg} : \quad \frac{c}{2} (P_t - P_{t-1})^2 Y_t \rightarrow P_t = f^q(P_{t-1}, \ldots)
  \]
  \[
  \text{Calvo} : \quad P_t = \left[ (1 - \theta) \tilde{P}_t^{1/(1-\lambda_f)} + \theta P_{t-1}^{1/(1-\lambda_f)} \right]^{1-\lambda_f} \rightarrow P_t = f^c(P_{t-1}, \ldots)
  \]
- Rotemberg: \( \hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{a-1}{c} \hat{m}c_t \)
- Calvo: \( \hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \lambda \hat{m}c_t \)
- Calvo pricing related to the frequency of price changes
- Rotemberg pricing associated with size of price changes
Motivation: Phillips Curve

- Econometric Phillips curve: $\pi_t = \beta \pi_{t-1} + \lambda y_t$
- NKPC: $\pi_t = \beta E_t \pi_{t+1} + \lambda y_t$
  - Taylor (1980 JPE), Rotemberg (1982 JPE), and Calvo (1983 JME)
    - 1. Inflation persistence
    - 2. Delayed response of inflation to a monetary shock
    - 3. Delayed response of inflation to changes in output gap
    - 4. Costly disinflation - Disinflation Boom (Ball, 1994 AER)
- HNKPC: $\pi_t = \alpha^f E_t \pi_{t+1} + \alpha^b \pi_{t-1} + \lambda y_t$
  - CEE (2005 JPE): automatic indexation to past inflation
Motivation: Welfare Analysis

- $\pi_t = \beta E_t \pi_{t+1} + \lambda y_t$ - failure to explain the dynamics of inflation
  - $Loss_t = \sum \beta^k E_t [\pi_{t+k}^2 + \delta y_{t+k}^2]
- $\pi_t = \alpha^f E_t \pi_{t+1} + \alpha^b \pi_{t-1} + \lambda y_t$ - failure to explain individual price changes
  - $Loss_t = \sum \beta^k E_t [(\pi_{t+k} - \pi_{t+k-1})^2 + \delta y_{t+k}^2]

Figure 6. Price of Angel Soft Bathroom Tissue at Dominick's Finer Food and Price Implied by Backward Indexation

Source: Chari, Kehoe, and McGrattan(2009)
This Paper: Infrequent and Incomplete Price Adjustment

- Sticky price model that endogenously generates inflation persistence
- We consider that firms face two sources of price rigidities, related to both the inability to change prices frequently and to the cost of sizeable adjustments
  - although firms change prices periodically, they face convex costs that preclude optimal adjustment
- In essence, model assumes that price stickiness arises from both the frequency and size of price adjustments
This Paper

- Monetary policy shocks first impact economic activity, and subsequently inflation but with a long delay, reflecting inflation inertia.
- The model captures the joint dynamic correlation between inflation and output gap.
- The frequency and size of price changes.
Alternative New Keynesian models that can account for some of the empirical facts on inflation and output. Most popular ones are extensions of Calvo’s staggered prices or information:

- Sticky information
- Indexation Models
Sticky information (Mankiw and Reis 2002 QJE) - information is costly and, therefore, disseminates slowly:

- Prices adjust continuously but information does not
- Model is consistent with inflation persistence
- Empirical implication: prices change frequently, which contradicts widespread micro-data studies

Evidence found across countries and different data sources is that firms keep prices unchanged for several months:

- Fabiani et al (2005): Firms review their prices more often than the frequency of price adjustment.
Literature: Sticky Information

- Sticky Information Phillips Curve (Mankiw and Reis 2002 QJE)
  \[
  \pi_t = \left[\frac{\alpha \lambda}{1-\lambda}\right] y_t + \lambda \sum_{j=0}^{\infty} (1-\lambda)^j E_{t-1-j}(\pi_t + \alpha \Delta y_t)
  \]
  \[
  m_t = p_t + y_t \text{ and } \Delta m_t = 0.5 \Delta m_{t-1} + \epsilon_t
  \]
- Fuhrer (2009): \( \Delta m_t = 0.5 \Delta m_{t-1} + \epsilon_t \) versus \( \Delta m_t = 0.25 \Delta m_{t-1} + \epsilon_t \)

"In this model, one can see by inspection (and the authors verify) that inflation will inherit the persistence of the output process."
Indexation Models - Gali and Gertler (1999), Christiano, Eichenbaum, and Evans (2005), and Smets and Wouters (2003, 2007): a fraction of the firms adjust their prices by automatic indexation to past inflation:

- Models explain inflation inertia as they incorporate a lagged inflation term into the resulting hybrid NKPC
- Arbitrary role given to past inflation as at least some agents are backward-looking in the process of setting prices
  - firms do not reoptimize prices each given period
Literature

- Indexation models and Sticky Information models:
  - imply that prices are adjusted continuously
  - imply that the size of price adjustments is small
- Evidence not supported by microdata evidence of price stickiness
  - both infrequent, small and large price adjustments
- Continuously price updating is an implication of many NKPC models including Reis (2006), Christiano et al (2005), Smets and Woulters (2003, 2007), Rotemberg (1982), Kozicki and Tinsley (2002), among many others
This Paper: Infrequent and Incomplete Price Adjustment

- Proposes a microfounded theoretical model that endogenously generates inflation persistence as a result of optimizing behavior of the firms
- Combines staggered price setting (Calvo) and quadratic costs of price adjustment (Rotemberg) in a unified framework
This Paper

- Phillips curve derived from DSGE model, and relates current inflation to inflation expectations, lagged inflation, and real marginal cost or output gap.
- Lagged inflation term is endogenously generated in a forward-looking framework:
  - Agents remain forward-looking and follow an optimizing behavior.
In contrast to the general indexation models and sticky information models, in the proposed model:

- prices are not continuously adjusted and firms that are able to change prices do not fully adjust them due to convex costs of adjustment
- New Phillips curve based on dual stickiness nests the standard NKPC as a special case (Calvo pricing)
- Model as an alternative to ad-hoc hybrid NKPC and sticky information Phillips curve
This Paper

- Price stickiness
  - direct microeconomic evidence
  - firms' decisions (frequency and size of price changes)
Firms Face Two Problems

- When to change prices? - frequency of price changes
  - Physical menu costs
  - Implicit and explicit contracts (ranked the first and second in the EU area)
  - Coordination failure (ranked the first in the U.S.)

- How much to change prices? - size of price changes
  - Managerial costs (information gathering costs, decision making, and internal communication costs)
  - Customer costs (communication and negotiation costs)
  - Other costs – antagonizing customers
  - Zbaracki et al. (2004): These costs are sizable and greater than physical menu costs.
“the firm reacted to major changes in supply and demand conditions slowly and/or partially because of the convexity of costs [of price adjustment]...”

- Quantitatively, they show that managerial costs are 6 times, and customer costs are 20 times greater than the physical menu costs.

Firm investigated changes prices “once a year”

- “We can't change prices biannually, it is not the culture here.”
  - Pricing manager- (Source: Zbaracki et al. 2004)

- Implicit and explicit contracts matter.
Model

- Firms’ Problems and the Phillips Curve
- Two types of firms:
  - Representative final goods-producing firm
  - Continuum of intermediate goods-producing firms
Firms’ Problems and the Phillips Curve

- **Final Goods-Producing Firm**
  - The final goods-producing firm purchases a continuum of intermediate goods, $Y_{it}$, at input prices, $P_{it}$, indexed by $i \in [0, 1]$. The final good, $Y_t$, is produced by bundling the intermediate goods:

    $$Y_t = \left[ \int_0^1 Y_{it}^{1/\lambda_f} di \right]^{\lambda_f}$$

  - The final-good-producing firm chooses $Y_{it}$ to maximize its profit in a perfectly competitive market taking both input ($P_{it}$) and output prices ($P_t$) as given, solving the following problem:

    $$P_t \left[ \int_0^1 Y_{it}^{1/\lambda_f} di \right]^{\lambda_f} - \int_0^1 P_{it} Y_{it} di$$
Firms’ Problems and the Phillips Curve

- Final Goods-Producing Firm

\[ Y_{it} = \left( \frac{P_{it}}{P_t} \right)^{-\lambda_f / (\lambda_f - 1)} Y_t \]

where \( \lambda_f / (\lambda_f - 1) \) measures the constant price elasticity of demand for each intermediate good.

- The relationship between the prices of the final and intermediate goods can be obtained by integrating the equation:

\[ P_t = \left[ \int_0^1 P_{it}^{1/(1-\lambda_f)} \, di \right]^{1-\lambda_f} \]

- The final good price can be interpreted as the aggregate price index.
Firms’ Problems and the Phillips Curve

- Intermediate Goods-Producing Firm - Calvo pricing

\[ P_t = \left[ (1 - \theta)\tilde{P}_t^{1/(1-\lambda_f)} + \theta P_{t-1}^{1/(1-\lambda_f)} \right]^{1-\lambda_f} \]

where \( \tilde{P}_t \) denotes the optimal price set by the intermediate good-producing firms.
We assume that each intermediate goods-producing firm faces a quadratic adjustment cost of adjusting its price given by:

\[ QAC = \frac{c}{2} \left( \tilde{P}_t - \pi_t \tilde{P}_{t-1} \right)^2 Y_t \]

\[ QAC = \frac{c}{2} \left( \frac{\tilde{P}_t}{P_t} - \frac{\tilde{P}_{t-1}}{P_{t-1}} \right)^2 Y_t \]

It is costly for current individual price to deviate from past price level, which makes prices sticky.
Firms’ Problems and the Phillips Curve

\[
\frac{c}{2} (P_t - \pi_t P_{t-1})^2 Y_t \rightarrow \hat{\pi}_t = E_t \hat{\pi}_{t+1} + \frac{a - 1}{\beta c} \hat{m} c_t
\]

\[
(c/2) (P_t - \pi P_{t-1})^2 Y_t \rightarrow \hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{a - 1}{c} \hat{m} c_t
\]
Firms’ Problems and the Phillips Curve

The firm chooses $\tilde{P}_t$ to maximize

$$E_t \sum_{k=0}^{\infty} (\theta \beta)^k \left[ \frac{(\tilde{P}_t - mc_{t+k}P_{t+k})Y_{it+k}}{P_{t+k}} \right] - \frac{c}{2} \left( \frac{\tilde{P}_t}{P_t} - \frac{\tilde{P}_{t-1}}{P_{t-1}} \right)^2 Y_t$$

subject to the demand function

$$Y_{it} = \left( \frac{\tilde{P}_t}{P_t} \right)^{-\lambda_f/(\lambda_f-1)} Y_t.$$
Log-linearization of the first order condition gives rise to:

\[ E_t \sum_{k=0}^{\infty} (\theta \beta)^k [(\hat{p}_t + \hat{X}_{tk} - \hat{mc}_{t+k})] = \frac{c}{1 - a} (\hat{p}_t - \hat{p}_{t-1}) \]

where \( \hat{X}_{tk} \equiv 1/\pi_{t+1} \pi_{t+2} ... \pi_{t+k} \) and \( a \equiv \lambda_f / (\lambda_f - 1) \).
Firms’ Problems and the Phillips Curve

- The first order condition and Calvo pricing \( P_t = \left[ (1 - \theta) \tilde{P}_t^{1/(1 - \lambda_f)} + \theta P_{t-1}^{1/(1 - \lambda_f)} \right]^{1 - \lambda_f} \)
yield

\[ \pi_t = \Lambda_f E_t \pi_{t+1} + \Lambda_l \pi_{t-1} + \lambda mc_t \]
Firms’ Problems and the Phillips Curve

- **FOC:**
  \[
  E_t \sum_{k=0}^{\infty} (\theta \beta)^k \left[ (\hat{p}_t + \hat{X}_{tk} - \hat{m}_{c_{t+k}}) \right] = \frac{c}{1 - a} (\hat{p}_t - \hat{p}_{t-1})
  \]

- **Calvo pricing:**
  \[
  P_t = \left[ (1 - \theta) \tilde{P}_t^{1/(1-\lambda_f)} + \theta P_{t-1}^{1/(1-\lambda_f)} \right]^{1-\lambda_f} \rightarrow \hat{p}_t = \frac{\theta}{1 - \theta} \hat{\pi}_t
  \]

- \( \tilde{p}_t \equiv \tilde{P}_t / P_t \): \( \hat{p}_t \) denotes the log-deviation of \( \tilde{p}_t \) from its steady state value.

  \[
  E_t \sum_{k=0}^{\infty} (\theta \beta)^k \left[ (\hat{p}_t + \hat{X}_{tk} - \hat{m}_{c_{t+k}}) \right] = \frac{c}{1 - a} \frac{\theta}{1 - \theta} (\hat{\pi}_t - \hat{\pi}_{t-1})
  \]
Firms’ Problems and the Phillips Curve

- Intuition behind lagged inflation term:

\[
\frac{c}{2} \left( \frac{\tilde{P}_t}{P_t} - \frac{\tilde{P}_{t-1}}{P_{t-1}} \right)^2 \rightarrow P_t = f^q(P_{t-1}, \ldots)
\]

\[
P_t = \left[ (1 - \theta) \tilde{P}_t^{1/(1 - \lambda_f)} + \theta P_{t-1}^{1/(1 - \lambda_f)} \right]^{1 - \lambda_f} \rightarrow P_t = f^c(P_{t-1}, \ldots)
\]

\[
P_t = f^c(f^q(P_{t-1}, \ldots), \ldots) = f(P_{t-2}, \ldots)
\]
Phillips Curve

\[ \pi_t = \Lambda_f E_t \pi_{t+1} + \Lambda_f \pi_{t-1} + \lambda mc_t \]

If \( c = 0 \), the model collapses into the New Keynesian Phillips Curve.

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa mc_t \]
Properties of the Model - Coefficients
Properties of the Model - Coefficients

Slope of the Phillips Curve

Parameter c

θ = 0.5
θ = 0.66
θ = 0.75
θ = 0.85
The firm chooses $\tilde{P}_t$ to maximize

$$E_t \sum_{k=0}^{\infty} (\theta \beta)^k \left[ \frac{(\tilde{P}_t - mc_{t+k}P_{t+k})Y_{it+k}}{P_{t+k}} \right]$$

subject to the demand function

$$Y_{it} = \left( \frac{P_{it}}{P_t} \right)^{-\lambda_f/(\lambda_f - 1)} Y_t$$

The first order condition and

$$(P_t = \left[ (1 - \theta)\tilde{P}_t^{1/(1-\lambda_f)} + \theta(\pi_{t-1}P_{t-1})^{1/(1-\lambda_f)} \right]^{1-\lambda_f}) \text{ yield}$$

$$\pi_t = \alpha_f E_t \pi_{t+1} + \alpha_b \pi_{t-1} + \lambda mc_t$$
Households maximize the expected present discounted value of utility,

$$E_t \sum_{k=0}^{\infty} \beta^k \left( \frac{C_{t+k}^{1-1/\sigma}}{1 - 1/\sigma} - \frac{N_{t+k}^{1+\varphi}}{1 + \varphi} \right),$$

subject to the budget constraint,

$$C_{t+k} + \frac{B_{t+k}}{P_{t+k}} = \left( \frac{W_{t+k}}{P_{t+k}} \right)(N_{t+k}) + \exp(-\zeta_{t+k-1})(1 + i_{t+k-1})\left( \frac{B_{t+k-1}}{P_{t+k}} \right) + \Pi_{t+k},$$

where $C_t$ is the composite consumption good, $N_t$ is hours worked, $\Pi_t$ is real profits received from firms, and $B_t$ is the nominal holdings of one-period bonds that pay a nominal interest rate $i_t$. As in Smets and Wouter (2007), we include the risk premium shock, $-\zeta_{t-1}$. 
DSGE Model

- The IS curve is given by:

\[ y_t = E_t y_{t+1} - \sigma (i_t - E_t \pi_{t+1}) + \varepsilon_t^Y \]

- We interpret the disturbance term as the preference shock, \( \varepsilon_t^Y \equiv \sigma \xi_t \), which is assumed to follow the AR(1) process,

\[ \varepsilon_t^Y = \delta \pi \varepsilon_{t-1}^Y + \nu_t^Y, \]

with \( \nu_t^Y \sim \mathcal{N}(0, \sigma_y^2) \).
DSGE Model

\[ y_t = E_t y_{t+1} - \sigma (i_t - E_t \pi_{t+1}) + \varepsilon^y_t \]

\[ \pi_t = \alpha_f E_t \pi_{t+1} + \alpha_b \pi_{t-1} + \lambda mc_t + \varepsilon^\pi_t \]

\[ mc_t = \left( \frac{1}{\sigma} + \varphi \right) y_t \]

\[ i_t = \rho i_t + (1 - \rho) (\alpha_\pi E_t \pi_{t+1} + \alpha_y y_t) + \varepsilon^i_t \]
Cost-push Shock

- The shock $\varepsilon_t^\pi$ can be introduced into the model by considering an exogenous cost component ($e_t^\pi$) in the objective function of firms as follows:

$$E_t \sum_{k=0}^{\infty} (\theta \beta)^k \left[ \frac{(\tilde{P}_t - \exp(e_t^\pi)mc_{t+k}P_{t+k})Y_{it+k}}{P_{t+k}} \right] - \frac{c}{2} \left( \frac{\tilde{P}_t}{P_t} - \frac{\tilde{P}_{t-1}}{P_{t-1}} \right)^2 Y_t$$

$\varepsilon_t^\pi$ can be expressed as a linear function of $e_t^\pi$.

- The shock $\varepsilon_t^\pi$ can be also introduced into the model by allowing $\lambda_f$ to vary over time as in the literature.
## Estimation

<table>
<thead>
<tr>
<th>parameter</th>
<th>prior dist.</th>
<th>prior mean</th>
<th>prior st. dev.</th>
<th>posterior mean</th>
<th>95% of confidence interval</th>
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<td>0.10</td>
<td>0.76</td>
<td>[0.70, 0.82]</td>
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<td>30.0</td>
<td>167.3</td>
<td>[140.0, 195.6]</td>
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<tr>
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<td>invg</td>
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<td>$\infty$</td>
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<td>0.50</td>
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<td>beta</td>
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<td>0.2</td>
<td>0.95</td>
<td>[0.93, 0.98]</td>
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<td>$\sigma_{\pi}$</td>
<td>invg</td>
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<td>2</td>
<td>0.70</td>
<td>[0.63, 0.76]</td>
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<tr>
<td>$\sigma_y$</td>
<td>invg</td>
<td>0.1</td>
<td>2</td>
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</tr>
<tr>
<td>$\sigma_i$</td>
<td>invg</td>
<td>0.1</td>
<td>2</td>
<td>0.99</td>
<td>[0.89, 1.07]</td>
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</table>
Infrequent and Incomplete Price Adjustment
Model Parameters

- Evidence of intrinsic inflation persistence
  - Parameter estimates associated with the two types of price stickiness are highly significant, supporting the proposed model
Frequency of Price Changes

- Calvo parameter $\theta$, degree of nominal rigidity $= 0.76$. 1/4 firms reset prices to optimize profit. Average length of time between price changes is 4 quarters.
- Estimates closely match microeconomic evidence on price changes:
  - These research studies individual prices during the Great Moderation period.
  - Our estimates of $\theta$: 9 months $\sim$ 12.5 months for the Great Moderation period.
Magnitude of adjustment costs is large and statistically significant: quadratic adjustment cost, \( c = 167.3 \) with 95% confidence interval [140.0, 195.6]

Empirical findings: price changes are mostly smaller than the size of aggregate inflation (e.g. Dhyne et al. 2005, Alvarez et al. (2006), Klenow and Kryvstov 2008, etc.)

- Klenow and Kryvtsov (2008) - U.S. consumer price changes (absolute value):
  - 44% < 5% 25% < 2.5% 12% < 1%
- Vermeulen et al. (2007) - Euro area producer price: 25% <1% Mean price change only 4%
Impulse Response Functions

- Cost-push shock
- Preference shock
- Interest rate shock
Taylor (1999) considers as a yardstick of a success of monetary models their ability to generate the “reverse dynamic” crosscorrelation between output gap and inflation.
Dynamic Correlation Between the Output Gap and Inflation

![Graph showing correlation between CBO output gap and inflation](image1)

![Graph showing correlation between HP-filtered output gap and inflation](image2)
Dynamic Correlation Between the Output Gap and Inflation

Correlation(output gap(t), inflation(t+k) ) and Shocks

- interest rate shocks
- demand shocks
- cost shocks

Model: c=167.3

cost shocks
demand shocks
interest rate shocks

Correlation(output gap(t), inflation(t+k) ) and Price Adjustment Cost

- c=0
- c=25
- c=50
- c=100
- c=150
- c=167.3
## Average of the Absolute Values of Price Changes

<table>
<thead>
<tr>
<th></th>
<th>Calvo</th>
<th>our model</th>
<th>CEE</th>
<th>Rotemberg</th>
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<tr>
<td>1960:1-2008:4</td>
<td>15.51</td>
<td>12.42</td>
<td>5.98</td>
<td>3.18</td>
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</table>

- Klenow and Kryvtsov (2008) report that the mean (median) value of price changes in regular prices is **11 percent (10 percent)** in absolute value.
- In the Euro area, Dhyne et al. (2005) present that the average value of consumer price decrease (increase) is **10 percent (8 percent)**.
- These research studies individual prices during the Great Moderation period.
Distribution of Price Changes: Post-1980

- Calvo
- proposed model
- CEE
- Rotemberg

*bin range = 2%*

*bin range = 5%*

*bin range = 10%*
### Distribution of Price Changes: Post-1980

|                | $P(|\Delta p| < 5\%)$ | $P(|\Delta p| < 2.5\%)$ | $P(|\Delta p| < 1\%)$ |
|----------------|------------------------|--------------------------|------------------------|
| data           | 44%                    | 25%                      | 12%                    |
| model          | 47%                    | 25%                      | 11%                    |
| CEE            | 73%                    | 44%                      | 19%                    |
| Rotemberg      | 90%                    | 59%                      | 26%                    |
| Calvo          | 38%                    | 20%                      | 8%                     |

data: Klenow and Kryvtsov (2008) - U.S. consumer price changes (absolute value)
Distribution of Price Changes: Subsamples

pre-1980

post-1980

Rotemberg

CEE

proposed model

Calvo
## Model Comparison

### Table: Restriction on $c$: likelihood and estimates (1960:1~2008:4)

<table>
<thead>
<tr>
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<th>restriction on $c$ ($c = 0$)</th>
<th></th>
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<td>$\theta$</td>
<td>likelihood</td>
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<td>167.3</td>
<td>0.76</td>
<td>-1119.8</td>
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<tr>
<td></td>
<td>(140.0, 195.6)</td>
<td>(0.70, 0.82)</td>
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<td>(0.87, 0.91)</td>
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<tr>
<td>HP</td>
<td>-879.1</td>
<td>121.7</td>
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<td></td>
<td>(97.4, 146.3)</td>
<td>(0.63, 0.79)</td>
<td></td>
<td>(0.83, 0.88)</td>
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<tr>
<td>CF</td>
<td>-836.3</td>
<td>127.0</td>
<td>0.73</td>
<td>-1008.3</td>
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<tr>
<td></td>
<td>(102.5, 152.3)</td>
<td>(0.66, 0.81)</td>
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<td>(0.86, 0.90)</td>
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</table>
### Subsample Estimates: 1960:1~2008:4

<table>
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<tr>
<td></td>
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<tr>
<td>HP</td>
<td>(55.6, 107.0)</td>
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<td>CF</td>
<td>(70.8, 129.8)</td>
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### Cost Shocks $\sim$ ARMA(4,1): 1960:1$\sim$2008:4

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<td>140.1</td>
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Cost Shocks $\sim$ ARMA(4,1): 1960:1~2008:4

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<td>(61.4, 121.0)</td>
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</table>
Thank You.