Sunspots and the Nature of Price Rigidity

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Abstract

It is well known that increasing returns to scale and firms’ market power are two potential sources of sunspot expectations in Neoclassical models. In this paper we show that both the degree of returns to scale and market power can have fundamentally different implications for self-fulfilling expectations in New Keynesian models depending on the nature of price rigidity. As a result, the design of stabilization monetary policy can depend in a delicate way on precise knowledge about these real and nominal features of the economy. Thus, a clear understanding of the specific economic environment and its relevance to monetary policymaking for ensuring macroeconomic stability should be an integrated part of monetary policy practice.

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1. Introduction

The sources of sunspot expectations unrelated to economic fundamentals have been an important subject in macroeconomics. In Neoclassical models, two real features pertaining to production technology and market structure, namely, increasing returns to scale and firms’ market power, have been found as potential factors for real indeterminacy of equilibrium and sunspot-driven business cycles.¹ In New Keynesian models, the focus has been on implications of nominal features pertaining to monetary policy and price rigidity for multiple equilibria and sunspot-driven fluctuations.

In this paper, we study the interactions of these real and nominal features in inducing sunspot expectations. Specifically, we investigate the implications of returns to scale and market power for real indeterminacy in two sticky price models often used in the New Keynesian literature, the Rotemberg (1982) model of costly price adjustment and the Calvo (1983) model of partial price adjustment, with a flexible forward-inflation targeting policy under which the monetary authority sets nominal interest rate in response to variations in expected future inflation and current output. Forward-inflation targeting is justified on the ground of both policy effectiveness and central bank accountability and credibility,² while a flexible targeting rule which allows the nominal interest rate to respond not only to expected future inflation but also to other endogenous variables such as current output is crucial for avoiding policy-induced macroeconomic instability.³ This is why such a flexible inflation-forecast targeting procedure has been advocated by many researchers⁴ and has become a dominant framework for monetary policymaking at central banks around the world.⁵

Our main finding is that the two real features can have fundamentally different implications for macroeconomic stability in the Rotemberg model than in the Calvo model. Understanding these

¹For the former, see Benhabib and Farmer (1994, 1996), Farmer and Guo (1994), Wen (1998), and Benhabib and Wen (2004), among others. For the latter, see the survey by Schmitt-Grohé (1997).


⁵This policy practice started in the industrial and middle-income countries in the late 1980s (e.g., Leiderman and Svensson, eds., 1995; Bernanke and Mishkin, 1997; Bernanke, Laubach, Mishkin, and Posen, 1999), and spread to the transition and emerging-market economies in the late 1990s (e.g., Schaechter, Stone, and Zelmer, 2000; Roger and Stone, 2005; Jonas and Mishkin, 2005), with more moving toward this direction. Indeed, while the United States Federal Reserve, the European Central Bank, and the Bank of Japan are usually viewed as having followed some implicit inflation-forecast targeting procedures, estimated forward-looking interest rate feedback rules explain well the behavior of interest rates in the U.S., Germany, and Japan (e.g., Chinn and Dooley, 1997; Clarida and Gertler, 1997; Clarida, Gali, and Gertler, 1998; Orphanides, 2000; Orphanides and Williams, 2003; Carare and Stone, 2006).
implications is important for the monetary policy practices of central banks that aim to preempt sunspot expectations and fluctuations unrelated to economic fundamentals.

Our results are easy to summarize. First, in the Calvo model, increasing returns to scale is a potential source of sunspot expectations, as in the real models, whereas decreasing returns to scale or fixed factors is potentially a stabilizing factor for the economy. In the Rotemberg model, by contrast, it is decreasing returns to scale or fixed factors that is a potential source of self-fulfilling expectations, whereas increasing returns to scale may actually be a stabilizing factor for the economy. Second, in the Calvo model, firms’ market power is a potential source of sunspot-driven fluctuations, as in the real models, provided that production technology exhibits decreasing returns to scale or fixed factors, while such market power may actually be a stabilizing factor if production technology exhibits increasing returns to scale. In the Rotemberg model, by contrast, market power of firms is a stabilizing factor for the economy regardless of the degree of returns to scale in production technology. Finally, we show that these contrasts continue to hold in the presence of policy inertia in the form of interest-rate smoothing, which is considered an important ingredient in monetary policy practice (e.g., Sack, 1998; Kozicki, 1999; Clarida Galí, and Gertler, 2000; Orphanides, 2000; Levin, Wieland, and Williams, 2003; Levin, Onatski, Williams, and Williams, 2005; Christiano Eichenbaum, and Evans, 2005).

One lesson that we have learned from these results is that the design of stabilization monetary policy can depend in a delicate way on exact knowledge about some economic fundamentals, such as those concerning the degree of returns to scale or fixed factors in production technology, market power of firms, and, most important, the nature of nominal rigidity in the economy. This poses a challenge to monetary policymakers, as existing empirical studies do not always provide unambiguous answers with regard to these real or nominal features. The fact that technological changes, institutional or policy reforms may all lead to structural changes in these specifics further complicates the issue. For instance, globalization may increase competition and lower the market power of firms, while protectionism policy may lead to the opposite. Likewise, antitrust laws and regulatory policies implemented for a domestic economy, or lack thereof, may affect firms’ market power. For another example, technological changes may change the view on which factor input in the produc-

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6Xiao (2008) and Huang and Meng (2009) also use the Calvo model to analyze the implications of increasing returns to scale for macroeconomic stability under several interest rate rules. Weder (2006) conducts a similar analysis using a monetary model with flexible prices.
tion process may be regarded as virtually fixed over the business cycle frequency. By virtue of our results presented in this paper, a clear understanding of the specific economic environment in this changing world and its relevance to monetary policymaking for ensuring macroeconomic stability should be an integrated part of monetary policy design and practice at central banks.

The remaining of the paper is organized as follows. Section 2 describes the basic framework that abstracts from capital, with the baseline monetary policy rule, and with the nature of the Rotemberg price rigidity and that of the Calvo price rigidity explained. Section 3 derives the necessary and sufficient conditions for real determinacy of equilibrium for the Rotemberg model and for the Calvo model, and uses these conditions to establish our main results, which contrasts the implications of returns to scale in production technology and market power of firms for macroeconomic stability in the Rotemberg model and in the Calvo model. This section also provides some intuition for why the two real features of the economy can have fundamentally different implications for self-fulfilling expectations across the two sticky price models. Section 4 extends the results to settings with capital and with generalized monetary policy rules. Section 5 concludes.

2. The basic environment

We start by describing the basic environment that abstracts from capital. The economy consists of a representative household and a continuum of firms that produce differentiated intermediate goods indexed by \( j \in [0, 1] \). At each date \( t \), a representative distributor combines all differentiated goods \( \{Y_{jt}\}_{j\in[0,1]} \) into a composite goods \( Y_t = \left[ \int_0^1 Y_{jt}^{(\varepsilon-1)/\varepsilon} dj \right]^{\varepsilon/(\varepsilon-1)} \), where \( \varepsilon > 1 \) is the elasticity of substitution between the differentiated goods. The distributor takes the prices \( \{P_{jt}\}_{j\in[0,1]} \) of the individual goods as given and chooses the bundle of these intermediate goods to minimize the cost of fabricating a given quantity of the composite goods. It sells the composite goods to the household at its unit cost \( P_t = \left( \int_0^1 P_{jt}^{1-\varepsilon} dj \right)^{1/(1-\varepsilon)} \), which acts also as the price level of the economy. The demand for a type \( j \) goods is given by \( Y_{jt} = (P_{jt}/P_t)^{-\varepsilon} Y_t \).

At any date \( t \), the household seeks to maximize

\[ E_t \sum_{s=t}^{\infty} \beta^{s-t} \left( \frac{C_s^{1-\sigma} - 1}{1 - \sigma} - \psi \frac{N_s^{1+\eta}}{1 + \eta} \right), \quad \text{for } \psi > 0, \]
where $E_t$ denotes the expectation operator conditional on information up to date $t$, $\beta \in (0, 1)$ is a subjective discount factor, $C_s$ and $N_s$ are the household’s consumption and labor supply in period $s$, and $\sigma$ and $\eta$ denote its relative risk aversion in consumption and in labor, respectively, subject to a sequence of budget constraints,

$$C_s + \frac{E_s(D_{s,s+1}B_{s+1}) - B_s}{P_s} \leq w_s N_s + \Pi_s,$$

for $s \geq t$, where $B_s$ is the household’s holding of nominal bonds at the beginning of period $s$, $D_{s,s+1}$ denotes the stochastic discount factor from $s+1$ to $s$, thus the gross nominal interest rate in period $s$ is $R_s = (E_sD_{s,s+1})^{-1}$, $w_s$ is the real wage rate, and $\Pi_s$ is the real profit that the household receives from the firms in period $s$. The optimality conditions for the utility maximization problem give rise to an intratemporal consumption-labor relation

$$w_t = \psi C_t^\sigma N_t^\eta,$$

(1)

and an intertemporal consumption Euler equation

$$1 = \beta E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{R_t}{\pi_{t+1}} \right],$$

(2)

where $\pi_{t+1} = P_{t+1}/P_t$ denotes the gross inflation rate from $t$ to $t+1$. We shall abstract from trend inflation so the gross steady-state inflation rate is 1.

At any date $t$, intermediate goods $j$ is produced using labor as the only input according to

$$Y_{jt} = AN_{jt}^\theta,$$

for $\theta \in (0, \varepsilon/(\varepsilon - 1))$, where $A$ is the technology level, and the upper bound on $\theta$ ensures that the firm’s profit maximization problem has a well-defined interior solution.\(^7\) Cost minimization implies that $w_t = \theta AN_{jt}^{\theta-1}mc_{jt}$, where $mc_{jt}$ denotes firm $j$’s real marginal cost, thus its real average cost is given by $ac_{jt} = w_tN_{jt}/Y_{jt} = \theta mc_{jt}$. Define the average real marginal cost across all firms in the

\(^7\)See, for example, Benhabib and Farmer (1994), and Huang and Meng (2009) for proof of this condition.
economy by \( mc_t = \left[ \int_0^1 mc_jt^{1/(1-\theta) dj} \right]^{1-\theta} \), we can show that

\[
w_t = \theta AN_t^{\theta-1}mc_t, \tag{3}
\]

where \( N_t = \int_0^1 N_{jt}dj \) is the aggregate labor input used by all firms. It follows that

\[
\frac{mc_{jt}}{mc_t} = \left( \frac{P_{jt}}{P_t} \right)^{\frac{1-\theta}{\theta}}. \tag{4}
\]

### 2.1. Rotemberg’s costly price adjustment

In the Rotemberg (1982) model of costly price adjustment, price rigidity takes the form of a quadratic cost of nominal price adjustment faced by all firms \( j \in [0, 1] \),

\[
\gamma \left( \frac{P_{jt}}{P_{jt-1}} - 1 \right)^2 Y_t,
\]

where \( \gamma \) governs the magnitude of the cost. At any date \( t \), firm \( j \) seeks to maximize the expected present value of its real profit stream net of the price adjustment costs in all future periods

\[
E_t \sum_{s=t}^\infty D_{t,s} \left[ \left( \frac{P_{js}}{P_s} - ac_{js} \right) \left( \frac{P_{js}}{P_s} \right)^{-\varepsilon} - \frac{\gamma}{2} \left( \frac{P_{js}}{P_{js-1}} - 1 \right)^2 \right] Y_s,
\]

where \( D_{t,s} = \prod_{r=t}^{s-1} D_{t+r-1,t+r} \) is a \( s \)-period stochastic discount factor from \( t \) to \( s > t \), with \( D_{t,t} \equiv 1 \).

In a symmetric equilibrium, all firms make identical pricing decision and \( mc_{jt} = mc_t \) for all \( j \in [0, 1] \).

The optimality condition for profit maximization then yields

\[
\gamma (\pi_t - 1) \pi_t - \beta \gamma E_t Y_{t+1} (E_t \pi_{t+1} - 1) E_t \pi_{t+1} = 1 - \varepsilon + \varepsilon \theta mc_t. \tag{5}
\]

### 2.2. Calvo’s partial price adjustment

In the Calvo model of partial price adjustment, price rigidity takes the form of random staggered-price setting. At each date, each firm receives a random signal with a constant probability \( \upsilon \) which forbids it to reset price. The signal is identically and independently distributed across firms and dates. At a date \( t \), if a firm \( j \) gets the chance to reset price, it chooses \( P_{jt}^* \) for all \( s \geq t \), conditional
on that it will not get another chance to change the price in the future and thus have to stick to
the price it currently chooses, to maximize the expected present value of its real profit stream

\[ E_t \sum_{s=t}^{\infty} \upsilon^{s-t} D_{t,s} \left( \frac{P_{js}}{P_s} - ac_{js} \right) \left( \frac{P_{js}}{P_s} \right)^{-\varepsilon} Y_s, \]

giving rise to the following optimal pricing decision

\[ P_{jt}^* = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \sum_{s=t}^{\infty} \upsilon^{s-t} D_{t,s} P_1^{1+\varepsilon} \theta mc_{js}}{E_t \sum_{s=t}^{\infty} \upsilon^{s-t} D_{t,s} P_1^{1+\varepsilon} Y_s}. \]  

With the large number of firms, at each date there is a fraction \( \upsilon \) of randomly selected firms that
cannot reset prices and a fraction \((1 - \upsilon)\) that can. The price level at \( t \) is then

\[ P_t = \left[ \upsilon P_{t-1}^{1-\varepsilon} + (1 - \upsilon) P_{jt}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}. \]  

2.3. Monetary policy

Motivated by the theoretical justifications and policy practices at central banks around the world,
as discussed in the introduction section, especially, Footnotes, 2, 3, 4, and 5, we consider a flexible
forward-inflation targeting rule under which the monetary authority sets nominal interest rate
in response to the deviation of expected future inflation from the inflation target (equal to the
steady-state inflation rate) and variation in current output,

\[ R_t = R \left( E_t \pi_{t+1} \right)^{\tau} \left( \frac{Y_t}{Y} \right)^{\omega}, \]  

where \( R \) and \( Y \) denote respectively the steady-state nominal interest rate and real output. We
allow the possibility for the interest rate to respond also to the deviation of current output from
the steady state to capture the idea of flexible inflation targeting, as the variation in current output
may bring with it a pressure or relief on future inflation.
3. Contrast of the Rotemberg and Calvo models in stability analysis

It can be verified that both the Rotemberg model and the Calvo model have a unique steady state. For local determinacy analysis, we can thus examine a log-linearized system of equilibrium conditions around the steady state. Throughout the rest of the paper, we shall use a variable with a hat to denote the percentage deviation of that variable in level from its steady-state value.

We begin by deriving the log-linear version of (2)

$$\hat{R}_t - E_t \hat{\pi}_{t+1} = \sigma (E_t \hat{y}_{t+1} - \hat{y}_t),$$

(9)

and by combining the log-linear versions of (11) and (3) to get

$$\hat{m}_c = \left( \frac{\eta + 1}{\rho} + \sigma - 1 \right) \hat{y}_t,$$

(10)

where we have used the log-linear version of the market clearing condition for the composite goods, $$\hat{c}_t = \hat{y}_t$$, in rewriting (11) and (10). The log-linearized version of (8) is simply

$$\hat{R}_t = \tau E_t \hat{\pi}_{t+1} + \omega \hat{y}_t.$$

(11)

These log-linear equilibrium conditions hold for both the Calvo model and the Rotemberg model.

The log-linearized version of (5) gives rise to the New Keynesian Phillips curve for the Rotemberg model and the log-linearized versions of (6), (7), and (4) gives rise to the New Keynesian Phillips curve for the Calvo model, which share the same functional form,

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \lambda_i \hat{m}_c,$$

for $$i = R, C,$$

(12)

but what differentiates the two types of price rigidities lies with the Phillips curve slope that is linked to the deep parameters governing returns to scale in production technology and monopolistic power of firms due to market structure in fundamentally different ways in the Rotemberg model than in the Calvo model, or, specifically,

$$\lambda_R = \frac{\varepsilon - 1}{\gamma},$$
for the Rotemberg model, and

$$\lambda_C = \frac{(1 - \nu)(1 - \nu \beta)}{\nu} \cdot \frac{1}{1 + \varepsilon \frac{1 - \theta}{\theta}}.$$

for the Calvo model.

Equations (9)-(12) constitute a complete set of equilibrium conditions that can be used for local determinacy analysis. With some algebra, we can establish the necessary and sufficient conditions for real determinacy for the Rotemberg model and for the Calvo model as follows:

$$-(1 - \beta) \omega < B \lambda_i (\tau - 1) < (1 + \beta) (2\sigma + \omega),$$

where

$$B \equiv \frac{\eta + 1}{\theta} + \sigma - 1$$

measures the elasticity of real marginal cost with respect to real output in the light of (10). We shall consider the regular situation in which this elasticity is positive, so expansion in real output will put an upward pressure on real marginal cost, and vice versa. Given \(\varepsilon > 1\) and \(\theta \in (0, \varepsilon/(\varepsilon - 1))\), a sufficient condition for this to always be the case is for the inverse of the monopolistic markup \((\varepsilon - 1)/\varepsilon\) to be bounded below by \((1 - \sigma)/(1 + \eta)\), which is met for almost all empirically plausible values of the parameters. The elasticity of current inflation with respect to current real marginal cost, that is, the slope of the New Keynesian Phillips curve, is also positive in both the Rotemberg model and the Calvo model, that is, both \(\lambda_R > 0\) and \(\lambda_C > 0\).

3.1. The contrast

Examining the necessary and sufficient conditions for determinacy (13) and how \(B, \lambda_R, \) and \(\lambda_C\) depend on \(\theta\) and \(\varepsilon\) reveals the following contrasts. First, in the Calvo model, increasing returns to scale is a potential source of real indeterminacy of equilibrium, while decreasing returns or fixed factors may be a stabilizing factor for the economy. In the Rotemberg model, decreasing returns or fixed factors is a potential source of sunspot expectations, while increasing returns may help

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\(^8\)Recall that the upper bound on \(\theta\) is required in order to satisfy the second order condition for firms’ profit maximization problem.
stabilize the economy. Second, in the Calvo model, market power of firms, as is measured by the monopolistic markup $\varepsilon/(\varepsilon - 1)$ and thus is inversely related to $\varepsilon$, is a potential source of self-fulfilling expectations if production technology exhibits decreasing returns or fixed factors, while such market power may help stabilize the economy if production technology exhibits increasing returns to scale. In the Rotemberg model, market power of firms is a stabilizing factor for the economy regardless of the degree of returns to scale in production technology.

These contrasts demonstrate the fundamentally different implications of returns to scale in production technology and market power of firms for macroeconomic stability in the Rotemberg model than in the Calvo model. Understanding these differences is important for monetary policy practices at central banks that aim to preempt sunspot-driven fluctuations. In particular, a clear view on the nature of nominal rigidity in the economy should be elevated to the top of the central bank’s research agenda.

### 3.2. Some intuition

We provide here some intuition as to why the two real features of the economy can have different implications for self-fulfilling expectations across the two sticky price models that are workhorses in the New Keynesian literature. Our intuition rests upon the insight provided in Kurozumi and Zandweghe (2008) and Huang, Meng, and Xue (2009) that endogenous inflation inertia can make the forward-inflation targeting policy more effective in avoiding sunspot-driven fluctuations. Greater inflation inertia can follow from a smaller elasticity of inflation with respect to marginal cost or a smaller elasticity of marginal cost with respect to output, as each of which implies a smaller change in inflation in response to a given change in output.

We discuss first the effect of returns to scale on endogenous inflation inertia. We note that the parameter $\theta$ affects inflation inertia through two channels: its effect on the elasticity of marginal cost with respect to output, $B$, as shown by equation (10), and its effect on the elasticity of inflation with respect to marginal cost, or, the slope of the New Keynesian Phillips curve, $\lambda_i$, for $i = R, C$, as shown by equation (12). The first effect is the same across the two models, and a large $\theta$ tends to stabilize the economy through this channel by inducing a smaller elasticity of marginal cost to

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9We have also examined the case in which the interest rate is allowed to respond to expected future output changes and found similar contrasts between the implications of these real features of the economy for macroeconomic stability across the two sticky price models. We don’t present these results here in order to conserve space.
output, which tends to give rise to greater inflation inertia. The second effect is muted in the Rotemberg model, as \( \lambda_R \) is independent of \( \theta \). Thus, in the Rotemberg model, real indeterminacy of equilibrium is less likely to occur, the greater is \( \theta \). This explains why increasing returns may help stabilize the economy while decreasing returns or fixed factors is a potential source of sunspot expectations, if price rigidity is of the Rotemberg type. In the Calvo model, by contrast, the second effect is active and works against the first effect, as \( \lambda_C \) is an increasing function of \( \theta \). That is, in the Calvo model, a large \( \theta \) tends to destabilize the economy through the second channel by inducing a greater elasticity of inflation to marginal cost, which tends to give rise to smaller inflation inertia. This second effect dominates the first one, and this is why increasing returns is a potential source of self-fulfilling expectations while decreasing returns or fixed factors can be a stabilizing factor for the economy, if price rigidity is of the Calvo type.

We turn now to the effect of market power of firms on endogenous inflation inertia. Since \( B \) is independent of \( \varepsilon \), market power of firms affects inflation inertia solely through its effect on the elasticity of inflation with respect to marginal cost, \( \lambda_i \), for \( i = R, C \), as shown by equation (12). We note that \( \lambda_R \) is an increasing function of \( \varepsilon \). Thus, in the Rotemberg model, a greater market power of firms (corresponding to a smaller value of \( \varepsilon \)) acts to stabilize the economy by inducing a smaller elasticity of inflation with respect to marginal cost to generate greater inflation inertia. This explains why market power of firms can help preempt sunspot expectations if price rigidity is of the Rotemberg type. We note now that \( \lambda_C \) is increasing in \( \varepsilon \) if \( \theta > 1 \), but is decreasing in \( \varepsilon \) if \( \theta < 1 \). Thus, in the Calvo model, a greater market power of firms acts to stabilize the economy by inducing a smaller elasticity of inflation with respect to marginal cost to generate greater inflation inertia if production technology exhibits increasing returns, but it acts in the opposite way to destabilize the economy by inducing a greater elasticity of inflation with respect to marginal cost to generate smaller inflation inertia if production technology exhibits decreasing returns or fixed factors. This explains why market power of firms may help preempt self-fulfilling expectations under increasing returns to scale production technology, but may itself becomes a source of sunspot expectations with decreasing returns to scale or fixed factors, if price rigidity is of the Calvo type.
4. With capital

We now extend the baseline setting to include capital as another variable input, along with labor. With this extension, closed-form analytical results are hard to come by, but the numerical exercises below confirm our conclusions based on the analytical results obtained in the previous section for the case without capital.

To be specific about the extended setting, on the household side, any quantity of the composite goods purchased by the household can now be not only consumed but invested to accumulate capital stock that the household can rent to firms in a competitive capital market. The household’s budget constraint in period $t$ is modified as

$$C_t + I_t + \frac{E_t(D_{t,t+1}B_{t+1}) - B_t}{P_t} \leq w_tN_t + r_tK_t + \Pi_t,$$

where $I_t$ and $K_t$ are the household’s investment and capital supply, respectively, and $r_t$ is real capital rental rate. The law of motion for capital is

$$K_{t+1} = I_t + (1 - \delta) K_t,$$

where $\delta$ is the capital depreciation rate. The household’s utility maximization conditions now also include a capital Euler equation

$$r_{t+1} = E_t \left( \frac{\alpha}{1 - \alpha} \frac{N_{t+1}}{K_{t+1}^{1-\alpha}} w_{t+1} \right) = \frac{1}{\beta} - (1 - \delta). \quad (14)$$

On the firm side, we now have the following production function for intermediate goods $j$:

$$Y_{jt} = A \left( K_{jt}^\alpha N_{jt}^{1-\alpha} \right)^\theta,$$

where $\theta \in (0, \varepsilon/(\varepsilon - 1))$ still measures the degree of returns to scale at the individual firm level. Cost minimization by firm $j$ involves choosing $N_{jt}$ and $K_{jt}$ to minimize $w_tN_{jt} + r_tK_{jt}$ subject to the above production function, taking the real wage rate $w_t$ and real capital rental rate $r_t$ as given. The resulting factor demand conditions imply the following relations between firm $j$’s real marginal
cost, $mc_{jt}$, and its output, $Y_{jt}$, its factor inputs, $N_{jt}$ and $K_{jt}$, and factor prices, $w_t$ and $r_t$, 

$$mc_{jt} = \frac{1}{(1-\alpha)\theta} Y_{jt}/N_{jt} = \frac{1}{\alpha\theta} Y_{jt}/K_{jt}.$$  

Using these relations for the individual firms, we can also derive the following relations between the average real marginal cost across all firms, $mc_t$, and total output, $Y_t$, total factor inputs used by all firms, $N_t = \int_0^1 N_{jt}dj$ and $K_t = \int_0^1 K_{jt}dj$, and factor prices, $w_t$ and $r_t$, 

$$mc_t = \frac{1}{(1-\alpha)\theta} Y_t/N_t = \frac{1}{\alpha\theta} Y_t/K_t.$$  

(15)

It follows that 

$$\frac{mc_{jt}}{mc_t} = \left( \frac{P_{jt}}{P_t} \right)^{\frac{1-\theta}{\sigma}} = \left( \frac{Y_{jt}}{Y_t} \right)^{\frac{1-\theta}{\sigma}}.$$  

(16)

The profit maximization problems and the resulting optimal pricing equations in the two models remain the same as in the baseline setting.

The system of log-linearized equilibrium conditions that can be used for local determinacy analysis consists of the policy rule (11) and the New Keynesian Phillips curve (12), which all remain the same as in the baseline setting, plus a modified consumption Euler equation, a capital Euler equation, and a modified market clearing condition for the composite goods,

$$\hat{R}_t - E_t \hat{\pi}_{t+1} = \sigma (E_t \hat{c}_{t+1} - \hat{c}_t),$$  

(17)

$$\left( \delta - 1 \right) \sigma E_t \hat{c}_{t+1} - \tilde{\delta} k_{t+1} + \tilde{\delta} (1 + \eta) E_t \hat{n}_{t+1} = -\sigma \hat{c}_t,$$  

(18)

$$\hat{k}_{t+1} = -c_k \hat{c}_t + [\alpha \theta (\delta + c_k) + 1 - \delta] \hat{k}_t + (1 - \alpha) \theta (\delta + c_k) \hat{n}_t,$$  

(19)

where $\tilde{\delta} \equiv 1 - \beta (1 - \delta)$ and $c_k \equiv \left( \varepsilon \delta \right) / \left[ \alpha \theta \beta (\varepsilon - 1) \right] - \delta$.

In this extended setting with endogenous capital accumulation, the contrasts between the implications of returns to scale and firms’ market power for real indeterminacy of equilibrium across the Rotemberg model and the Calvo model presented in the baseline setting continue to hold. To get a quantitative feel about the contrasts, we now assign values to the fundamental parameters in the economy.
We set $\alpha$ to 0.33 so that the share of payment to capital in value-added productive factors is equal to one third, as in the National Income and Product Account. Given that one period in our model corresponds to one quarter of a year, we set $\beta = 0.99$ to be consistent with a steady-state annualized real interest rate of 4 percent, and we set $\delta$ to 0.02 to match the steady-state annual capital depreciation rate of 8 percent. These are standard parameter values used in the literature. While some studies in the literature suggest that relative risk aversion in consumption, $\sigma$, can be as low as 0 or as high as 30, the general consensus is that it lies between 1 and 10. Our results are robust to the choice of $\sigma$ in its empirically reasonable range, and thus we fix $\sigma$ at 2.

The inverse of $\eta$, the relative risk aversion in labor, corresponds to the Frisch elasticity of labor supply, which has been estimated in many empirical studies. The evidence obtained based on different sources of data, frequencies of time, sample periods, aggregation levels, substitution margins (intensive versus extensive), seasonality adjustments, and estimation procedures suggests that the elasticity lies between 0.05 and 2.\footnote{Many of these studies based on life-cycle data of hours worked (intensive margin) by men at annual or lower frequencies find an elasticity in the range of 0.05 to 0.3, as summarized by Pencavel (1986). Evidence on a low elasticity is also found by Altonji (1986), Ball (1990), and Card (1994). Using monthly data from the Denver Income Maintenance Experiment, MaCurdy (1983) finds that the (intensive) elasticity can be above 0.3 though it is below 0.7. Rupert, Rogerson, and Wright (2000) obtain similar estimates taking home production into account. Using tri-annual micro panel data of the Survey of Income and Program Participation, Kimmel and Kniesner (1998) estimate an hours worked elasticity (intensive margin) of 0.5 and an employment elasticity (extensive margin) of 1.5. Mulligan (1998) shows that the elasticity can lie in the 1-2 range. Many other of these studies, using multi-industry panel or macro-level data (as reported by Browning, Hansen, and Heckman, 1999), using industry-specific data (e.g., Treble, 1986; Carrington, 1996; Oettinger, 1999), and using data at several different frequencies (e.g., Abowd and Card, 1989) or taking into account seasonal variation in total hours worked (e.g., Barsky and Miron, 1989; Hall, 1999), also provide corroborating evidence that helps reach a consensus that the elasticity lies between 0.05 and 2.} This suggests a value for $\eta$ between 0.5 and 20. On the other side, a value of $\eta = 0$, corresponding to an infinite labor supply elasticity, is often assumed in many studies on indeterminacy (e.g., Carlstrom and Fuerst, 2005; Benhabib and Eusepi, 2005; Kurozumi and Van Zandwaghe, 2008). Our results are robust to the choice of $\eta$ in its empirically reasonable range, and thus we fix $\eta$ at 1, corresponding to a unit labor supply elasticity, as is standard in the macroeconomic literature.

A reasonable range for $\varepsilon$, the elasticity of substitution between differentiated goods, is from 4 to 20, in light of the empirical studies by Domowitz, Hubbard, and Petersen (1986), Shapiro (1987), Basu (1996), Rotemberg (1996), Rotemberg and Woodford (1997), Basu and Kimball (1997), Linnemann (1999), and Basu and Fernald (1994, 1995, 1997, 2000, 2002). We set our benchmark value of $\varepsilon$ to 10, as is common in the literature, while we examine how our indeterminacy results in the
two sticky price models are affected by varying $\varepsilon$ in its empirically relevant range. We also recall here that the profit maximization problem has a well-defined interior solution as long as the degree of returns to scale $\theta$ is bounded above by the steady-state monopolistic markup $\varepsilon/(\varepsilon - 1)$. The benchmark value that we choose for $\theta$ is 1, corresponding to the case with constant returns to scale in production technology, which meets the upper bound regardless of the value considered for $\varepsilon$. For the benchmark value of $\varepsilon$ we can consider a degree of increasing returns to scale as high as 1.1, while we can consider a degree of increasing returns to scale as high as 1.05 when examining the case with $\varepsilon = 20$, in order to guarantee a well-defined interior solution. We consider a possible extent of decreasing returns to scale, or fixed factors in production technology, equal to 0.33, similarly as in Chari, Kehoe, and McGrattan (2000). We shall examine how our indeterminacy results in the two sticky price models are affected by varying $\theta$ in a relevant range.

We set $\nu$, the hazard rate of price adjustment in the Calvo model, to 0.57, which lies in the middle of its empirical estimates (e.g., Galí and Gertler, 1999; Sbordone, 2002; Galí, Gertler, and Lopez-Salido, 2001, 2003; Bils, Klenow, and Kryvtsov, 2003; Bils and Klenow; 2004; Klenow and Krystov, 2005; Christiano, Eichenbaum, and Evans, 2005; Alvarez, et al., 2005; Dhyne, et al., 2005; Levin, Onatski, Williams, and Williams, 2005; Nakamura and Steisson, 2007), and which is close to the values used in studies on indeterminacy in the Calvo setting (e.g., Carlstrom and Fuerst, 2005; Sveen and Weinke, 2005, 2007; Kurozumi and Zandweghe, 2008). We set $\gamma$, the quadratic price adjustment cost parameter in the Rotemberg model, to 30.424, so that the elasticity of price to marginal cost in the Rotemberg model and in the Calvo model is the same under the benchmark values of the other parameters in the economy, and as is close to the values used in studies on indeterminacy in the Rotemberg setting.

With these values of the fundamental parameters in hand, we can start to search for the values of $\tau_\pi$ and $\tau_y$ that ensure a unique equilibrium. We search in the range $[0, 5]$ for $\tau_\pi$ and $[0, 1]$ for $\tau_y$, which cover most of the empirical estimates of the two policy parameters (e.g., Taylor, 1993; de Brouwer and Ellis, 1998; Kozicki, 1999; Clarida, Galí, and Gertler, 2000; Orphanides, 2000; Levin, Wieland, and Williams, 2003; Levin, Onatski, Williams, and Williams, 2005).

Our numerical exercises under endogenous capital accumulation confirm our analytical results obtained in Section 3 for the case that abstracts from capital. Results from these numerical exercises are reported in Figures 1-6. As before [see equation (13)], the determinacy region is characterized
by a lower bound and an upper bound on policy’s response to future inflation $\tau$ (the vertical axis) as a function of policy’s response to current output $\omega$ (the horizontal axis), while equilibrium is indeterminant in the area outside the determinacy region.

Figure 1 displays the determinacy (indeterminacy) regions for the Rotemberg model and the Calvo model with different degrees of returns to scale in production technology. As the figure illustrates, as we increase the degree of returns to scale from 0.67 to 1, and then to 1.1, while keeping the other parameters at their benchmark values, the determinacy region gets enlarged and the indeterminacy region shrinks if price rigidity is of the Rotemberg type (the left panel), while the opposite is true if price rigidity is of the Calvo type (the right panel). This confirms our original finding that increasing returns to scale is a potential source of sunspot expectations and decreasing returns to scale or fixed factors is potentially a stabilizing factor in the Calvo model, whereas it is decreasing returns to scale or fixed factors that is a potential source of self-fulfilling expectations and increasing returns to scale may actually be a stabilizing factor in the Rotemberg model.

Figure 2 plots the determinacy (indeterminacy) regions for the Rotemberg model and the Calvo model with varying extents of the market power of firms under different degrees of returns to scale in production technology. As the figure demonstrates, as we increase the elasticity of substitution between differentiated goods from 4 to 10, and then to 20, that is, as we decrease the steady-state monopolistic markup of price over marginal cost from 33% to 11%, and then to 5.26%, the determinacy region shrinks and the indeterminacy region gets enlarged in the Rotemberg model, regardless of whether production technology exhibits decreasing, constant, or increasing returns to scale (the left panel), while the same is true in the Calvo model if production technology exhibits increasing returns to scale but the opposite holds if production technology exhibits decreasing returns to scale (the right panel). The other parameters are maintained at their benchmark values. This confirms our original finding that firms’ market power is a stabilizing factor in the Rotemberg model regardless of the degree of returns to scale in production technology, while, in the Calvo model, firms’ market power is also a stabilizing factor under increasing returns to scale in production but it becomes a potential source of sunspot-driven fluctuations if production technology exhibits decreasing returns to scale or fixed factors.

As a further robustness check, we also examine how policy inertia affects the above contrasts. To model policy inertia, we replace the baseline policy rule by the following interest-rate rule.
$R_t = R_{t-1}^{\phi} R^{1-\phi} (E_t \pi_{t+1})^\tau (Y_t/Y)\omega$ and its log-linear version (11) by $\tilde{R}_t = \phi \tilde{R}_{t-1} + \tau E_t \tilde{\pi}_{t+1} + \omega \tilde{y}_t$, where $\phi$ measures the degree of interest-rate smoothing in the policy rule. With some algebra, which we skip here in order to conserve space, we show that the local determinacy analysis can be done by examining the following system of five first-order linear difference equations in three jump variables, $\tilde{c}_t$, $\tilde{n}_t$, and $\tilde{\pi}_t$, and two predetermined variables, $\tilde{k}_t$ and $\tilde{R}_{t-1}$,

$$
\begin{pmatrix}
\left(\tilde{\delta} - 1\right) \sigma & -\tilde{\delta} & \tilde{\delta} (1 + \eta) & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & \beta & 0 \\
\sigma & 0 & 0 & 1 & -1 \\
0 & 0 & 0 & -\tau & 1
\end{pmatrix}
\begin{pmatrix}
\tilde{E}_t \tilde{c}_{t+1} \\
\tilde{E}_t \tilde{n}_{t+1} \\
\tilde{E}_t \tilde{\pi}_{t+1} \\
\tilde{R}_t
\end{pmatrix}
= 
\begin{pmatrix}
-\sigma & 0 & 0 & 0 & 0 \\
-c_k & \alpha \theta (\delta + c_k) + 1 - \delta & (1 - \alpha) \theta (\delta + c_k) & 0 & 0 \\
-\lambda_i \sigma & \alpha \theta \lambda_i & -\lambda_i \left[ 1 + \eta - (1 - \alpha) \theta \right] & 1 & 0 \\
\sigma & 0 & 0 & 0 & 0 \\
0 & \alpha \theta \omega & (1 - \alpha) \theta \omega & 0 & \phi
\end{pmatrix}
\begin{pmatrix}
\tilde{c}_t \\
\tilde{k}_t \\
\tilde{n}_t \\
\tilde{\pi}_t \\
\tilde{R}_{t-1}
\end{pmatrix}
$$

for $i = R, C$, corresponding to the Rotemberg model and the Calvo model, respectively, where

$$c_k = \frac{\varepsilon}{\varepsilon - 1} \alpha \theta \beta - \delta.$$

Figures 3-6 display the determinacy (indeterminacy) regions for the Rotemberg model and the Calvo model with various degrees of interest-rate smoothing in the policy rule, firms’ market power, and returns to scale in production technology. As can be seen from the figures, the determinacy region in the face of policy inertia is once again characterized by a lower bound and an upper bound on policy’s response to future inflation $\tau$ (the vertical axis) as a function of policy’s response to current output $\omega$ (the horizontal axis), while equilibrium is indeterminant in the area outside the determinacy region. As is clear, the contrasts between these two models presented above in terms of the implications of the two real features of the economy for sunspot expectations and macroeconomic instability continue to hold in the face of policy inertia.
5. Conclusion

Increasing returns to scale and market power of firms are the two real features of the economy that have been identified as potential sources of sunspot expectations in Neoclassical models. In this paper we have found that both the degree of returns to scale or fixed factors in production technology and market power of firms can have fundamentally different implications for self-fulfilling expectations in New Keynesian models depending on the specific nature of price rigidity.

One message that one can take out of the results presented in this paper is that the design of stabilization monetary policy can depend in a delicate way on exact knowledge about some economic fundamentals, such as that concerning the degree of returns to scale or fixed factors in production technology, market power of firms, and the nature of nominal rigidity. This poses a challenge to monetary policymakers, as existing empirical studies do not always provide unambiguous answers with regard to these real or nominal features. The fact that technological changes, institutional or policy reforms may all result in structural changes in these specifics further complicates the issue. If policymakers misperceive these fundamental features of the economy, then policy intended to preempt sunspot expectations may lead instead to economic fluctuations unrelated to fundamentals. To meet this challenge, a clear understanding of the specific economic environment in this changing world and its relevance to monetary policymaking for ensuring macroeconomic stability should be an integrated part of monetary policy design and practice at central banks.

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Figure 1. Comparison of determinacy regions for the Rotemberg model and the Calvo model with different degrees of returns to scale in production:
Upper bound (solid line) and lower bound (broken line) for policy’s response to future inflation $\tau$ (vertical axis) as a function of policy’s response to current output $\omega$ (horizontal axis) – equilibrium is indeterminant in the areas outside the determinacy regions.
Figure 2. Comparison of determinacy regions for the Rotemberg model and the Calvo model with varying extents of firms’ market power under different degrees of returns to scale in production: Upper bound (solid line) and lower bound (broken line) for policy’s response to future inflation $\tau$ (vertical axis) as a function of policy’s response to current output $\omega$ (horizontal axis) – equilibrium is indeterminant in the areas outside the determinacy regions.
Figure 3. Determinacy region in the Rotemberg model with different degrees of returns to scale in production under varying degrees of interest rate smoothing in monetary policy: Upper bound (solid line) and lower bound (broken line) for policy’s response to future inflation $\tau$ (vertical axis) as a function of policy’s response to current output $\omega$ (horizontal axis) – equilibrium is indeterminant in the area outside the determinacy region.
Figure 4. Determinacy region in the Calvo model with different degrees of returns to scale in production under varying degrees of interest rate smoothing in monetary policy: Upper bound (solid line) and lower bound (broken line) for policy’s response to future inflation $\tau$ (vertical axis) as a function of policy’s response to current output $\omega$ (horizontal axis) – equilibrium is indeterminant in the area outside the determinacy region.
Figure 5. Determinacy region in the Rotemberg model with varying extents of firms’ market power under different degrees of returns to scale in production and policy inertia: Upper bound (solid line) and lower bound (broken line) for policy’s response to future inflation $\tau$ (vertical axis) as a function of policy’s response to current output $\omega$ (horizontal axis) – equilibrium is indeterminant in the area outside the determinacy region
Figure 6. Determinacy region in the Calvo model with varying extents of firms’ market power under different degrees of returns to scale in production and policy inertia: Upper bound (solid line) and lower bound (broken line) for policy’s response to future inflation $\tau$ (vertical axis) as a function of policy’s response to current output $\omega$ (horizontal axis) – equilibrium is indeterminant in the area outside the determinacy region.