Corporate Cash Savings: 
Precaution versus Liquidity

Martin Boileau and Nathalie Moyen*

August 2010

Abstract

Cash holdings as a proportion of total assets of North American corporations have roughly doubled between 1971 and 2006. Prior research attributes the large cash increase to a rise in firms’ cash flow volatility. We investigate two mechanisms by which increased volatility can lead to higher cash holdings. The first is the precautionary motive inducing firms to be prudent about their future prospects. The second mechanism is the liquidity motive requiring firms to meet their current liquidity needs. We find that the liquidity motive best explains how the increased volatility nearly doubled cash holdings.

Keywords: Dynamic capital structure, cash holdings, precautionary savings, corporate liquidity.

JEL Classifications: G31, G32, G35, E21, E22

*Boileau and Moyen are from the University of Colorado at Boulder. We thank Gian Luca Clementi, Adlai Fisher, Rick Green, Burton Hollifield, Lars-Alexander Kuehn, Toni Whited, as well as participants at the American Finance Association meetings, Carnegie-Mellon University, the Northern Finance Association meetings, the University of British Columbia Summer Finance Conference, the University of Colorado at Boulder, the University of Wisconsin at Milwaukee, and Wilfrid-Laurier University for helpful comments.

Boileau: Department of Economics and CIRPÉE, University of Colorado, 256 UCB, Boulder Colorado 80309, United States. Tel.: 303-492-2108. Fax: 303-492-8960. E-mail: boileau@colorado.edu.

Moyen: Leeds School of Business, University of Colorado, 419 UCB, Boulder Colorado 80309, United States. Tel.: 303-735-4931. Fax: 303-492-5962. E-mail: moyen@colorado.edu.
Corporate Cash Savings:
Precaution versus Liquidity

Abstract

Cash holdings as a proportion of total assets of North American corporations have roughly doubled between 1971 and 2006. Prior research attributes the large cash increase to a rise in firms’ cash flow volatility. We investigate two mechanisms by which increased volatility can lead to higher cash holdings. The first is the precautionary motive inducing firms to be prudent about their future prospects. The second mechanism is the liquidity motive requiring firms to meet their current liquidity needs. We find that the liquidity motive best explains how the increased volatility nearly doubled cash holdings.

Keywords: Dynamic capital structure, cash holdings, precautionary savings, corporate liquidity.
JEL Classifications: G31, G32, G35, E21, E22
1 Introduction

Cash holdings as a proportion of total assets of North American COMPUSTAT firms have increased dramatically from 1971 to 2006. Figure 1 documents this large and steady increase for the sample average of North American firms. Bates, Kahle, and Stulz (2009) first noted the dramatic increase in cash holdings. In their empirical work, they find that the main determinant of the large increase is the concurrent increase in cash flow volatility. As a result, Bates, Kahle, and Stulz (2009) conclude that the large cash increase is attributable to a precautionary motive, in the general sense that higher cash flow risk generates higher cash holdings.

In what follows, we investigate two precise mechanisms by which increased volatility can lead to higher cash holdings. The first mechanism is the more standard precautionary (or prudence) motive. It arises because firms face various taxes, as well as adjustment and issuing costs, which may lead to convex marginal payouts. For example, firms may act prudently and save more to self-insure against future adverse shocks, thereby avoiding large issuing costs. With a precautionary motive, an increase in firms’ idiosyncratic risk underscores the need to self-insure and therefore increases cash holdings. This mechanism is similar to that discussed in Leland (1968) and Carroll and Kimball (2006), where a convex marginal utility yields a precautionary motive for consumers.

The second mechanism is the liquidity motive. It arises because it is more costly for firms to alter their investment, dividend, debt and equity policies than to accumulate cash to meet their liquidity requirements. For example, firms can transfer funds between interest-earning assets and cash at the beginning of each period to meet their liquidity requirements during the year with accumulated cash. With a liquidity motive, an increase in firms’ income volatility may escalate the need for liquidity and therefore increases required cash holdings. Such a liquidity motive is featured in the seminal work of Miller and Orr (1966), where firms must manage cash inventories to face liquidity needs generated by income fluctuations. The liquidity mechanism is also similar to that discussed in Telyukova and Wright (2008), where liquidity needs yield a motive for consumers to

\footnote{Cash refers to COMPUSTAT Mnemonic CHE and is composed of cash (CH) and short-term investments (IVST). It includes, among others, the following items: cash in escrow; government and other marketable securities; letters of credits; time, demand, and certificates of deposit; restricted cash.}
accumulate money.  

Our analysis is derived from a standard dynamic model of a firm’s investment and financial decisions augmented by both a precautionary motive and a liquidity motive to hold cash. Importantly, our model recognizes that cash is a dominated security in terms of return. We show that the model offers a reasonable description of firms’ behavior, and can be used to study the large increase in cash holdings. We focus our analysis on two extreme periods: the first third of our sample period from 1971 to 1982 and the last third from 1995 to 2006. We find that both the precautionary and liquidity motives play important roles in explaining cash holdings in the first period, but that only the liquidity motive explains cash holdings in the last period. The astounding increase in cash holdings from 1971 to 2006 is attributable to the liquidity motive, where a large increase in firms’ income volatility raises required cash holdings.

Our model does not consider other motives to hold cash. Our neoclassical framework does not feature the agency motive to hold cash. We recognize that the relationship between higher cash holdings (or lower cash value) and higher agency costs is well documented in other contexts. For example, see Dittmar and Mahrt-Smith (2007), Dittmar et al. (2003), Faulkender and Wang (2006), Harford (1999), Harford et al. (2008), Nikolov and Whited (2010), and Pinkowitz et al. (2006). In contrast, Opler et al. (1999) find little evidence that cash holdings lead to larger capital investments, and Bates, Kahle, and Stulz (2009) fail to support the agency problem explanation for the overall increase in cash holdings of U.S. firms.

Our model does not feature the tax motive to hold cash. We recognize that Foley et al. (2007) show convincingly that U.S. multinationals accumulate cash in their subsidiaries to avoid the tax costs associated with repatriated foreign profits. However, Bates, Kahle, and Stulz (2009) note that the large increase in cash holdings is observed also for domestic firms.

Our model does not feature a fixed transaction cost motive to hold cash, as in Baumol (1952) and Miller and Orr (1966). Mulligan (1997) provides empirical evidence consistent with a fixed cost of converting non-liquid assets into cash. Rather than imposing a fixed cost on asset conversions into cash, we recognize that changes in capital, debt, and equity trigger higher adjustment costs than
cash accumulation. On a related topic, there are a number of papers examining the relationship between firms’ financial constraints and cash hoarding, including Acharya, Almeida, and Campello (2007), Almeida, Campello, and Weisbach (2004), Han and Qiu (2007), Nikolov (2008), Riddick and Whited (2008).

The model of Gamba and Triantis (2008) is closely related to our model in that a firm can finance its investment through debt issues, equity issues, and internal funds, where only internal funds do not trigger transaction costs. Their focus is on valuing financial flexibility, while we explore the influence of the precautionary and liquidity motives behind the large increase in holdings.

Bolton, Chen, and Wang (2009) present another closely related model, and like ours, it recognizes that cash is dominated in return. They propose a dynamic model of investment decisions for financially constrained firms with constant returns to scale in production. Among other contributions, they find that Tobin’s marginal q may be inversely related to investment, and that financially constrained firms may have lower equity betas because of precautionary cash holdings. Our paper departs from the assumption of constant returns to scale in production. As Figure 1 shows, the increase in cash holdings is accompanied by a steady decline in firms’ capital shares, and we are interested in exploring the effect of the declining capital share on the relative strengths of the precautionary and liquidity motives to hold cash.

The paper is organized as follows. Section 2 presents the model, and provides some analytical results characterizing the behavior of cash holdings. The model however does not possess an analytical solution. Section 3 discusses the calibration of the model. Section 4 presents our simulation results, where we first ensure that the model broadly reproduces observed facts. We then use the model to study the increase in cash holdings. Finally, we study the sensitivity of our results to a number of extensions. Section 5 offers some concluding remarks.

2 The Model

We study how a firm manages its cash holdings in an otherwise standard dynamic model of financial and investment decisions. The firm does not consider cash as a perfect substitute for debt: cash is
not negative debt. Instead, cash savings can serve two purposes. They may provide self-insurance against future adverse shocks, and they may provide liquidity to meet current adverse shocks.

To operationalize both motives, we assume that the firm faces shocks to its revenues and expenses. Knowing the current realization of the revenue shock, the firm chooses how much to invest, how much cash to save, how much debt to issue, how much dividend to pay out (or how much equity to raise). During the year, however, the firm may face expenses that turn out to be either lower or higher than expected back at the beginning of the year. When expenses are higher than expected, the firm cannot scale back its investment commitments, take back its distributed dividend, or go back to external markets with more favorable issuing conditions. Unexpectedly high expenses are met with cash. As is explained in details below, the firm holds sufficient cash to pay expenses that may occur during the year. The firm may also hold additional cash as a precaution against future adverse shocks to revenues.

2.1 The Firm

The firm, acting in the interest of shareholders, maximizes the discounted expected stream of payouts $D_t$ taking into account taxes and issuing costs. When payouts are positive, shareholders pay taxes on the distributions according to a tax schedule $T(D)$. The schedule recognizes that firms can minimize taxes for smaller payouts by distributing them in the form of share repurchases. Firms, however, have no choice but to trigger the dividend tax for larger payouts. Following Hennessy and Whited (2007), the tax treatment of payouts is captured by a schedule that is increasing and convex:

$$T(D_t) = \tau_D D_t + \frac{\tau_D}{\phi} \exp^{-\phi D_t} - \frac{\tau_D}{\phi},$$  \hspace{1cm} (1)

where $\phi > 0$ controls the convexity of $T(D)$ and $0 < \tau_D < 1$ is the tax rate. When payouts are negative, shareholders send cash infusions into the firm as in the case of an equity issue. The convex schedule $T(D)$ also captures the spirit of Altinkilic and Hansen (2000), where equity issuing costs are documented to be increasing and convex. Figure 2 displays the schedule $T(D)$ for different values of $\tau_D$ and $\phi$. 

5
Net payouts are

\[ U(D_t) = D_t - T(D_t). \]  

This function is increasing \( U'(D) = 1 - \tau_D + \tau_D \exp(-\phi D) > 0 \), concave \( U''(D) = -\phi \tau_D \exp(-\phi D) < 0 \), and its third derivative is positive \( U'''(D) = \phi^2 \tau_D \exp(-\phi D) > 0 \). As a result, the net payout function ensures that the firm is risk averse and has a precautionary motive. In this context, the parameter \( \phi \) is the coefficient of absolute prudence: \( \phi = -U'''(D)/U''(D) \).

The firm faces two sources of risk. The first source of risk comes from stochastic revenues. Revenues \( Y_t \) are generated by a decreasing returns to scale function of the capital stock \( K_t \):

\[ Y_t = \exp(z_t) \Gamma K_t^\alpha, \]  

where \( z_t \) is the current realization of the shock to revenues, the parameter \( 0 < \alpha < 1 \) denotes the capital share, and \( \Gamma > 0 \) is a scale parameter. The revenue shock follows the autoregressive process

\[ z_t = \rho_z z_{t-1} + \sigma_z \epsilon_{zt}, \]  

where \( \epsilon_{zt} \) is the innovation to the revenue shock, and \( 0 < \rho_z < 1 \) denotes the persistence of the revenue shock, and \( \sigma_z \) is the volatility of the revenue innovations. The innovations \( \epsilon_{zt} \) are independent and identically distributed random variables drawn from a standard normal distribution: \( \epsilon_{zt} \sim N(0, 1) \).

The second source of risk comes from stochastic expenses \( F_t \). The firm’s expenses are given by

\[ F_t = \bar{F} + f_t, \]  

where \( \bar{F} \geq 0 \) is the predictable level of expenses and \( f_t = \sigma_F \epsilon_{ft} \) is the innovation to the expense shock, where \( \sigma_F > 0 \) denotes the volatility of the expense innovation. The expense innovations are assumed to be independent of the revenue innovations, and drawn from a uniform distribution: \( \epsilon_{ft} \sim U[-1, 1] \).

The firm chooses how much to invest \( I_t \), how much cash to hold \( M_{t+1} \), how much debt to raise \( B_{t+1} \), how much to pay out (or how much equity to issue) \( D_t \). The sources and uses of funds defines
the cash holdings at the end of the year:

\[ M_{t+1} = Y_t - F_t - I_t + B_{t+1} - (1 + r)B_t - D_t + (1 + \iota)M_t - T^C_t - \Omega^K_t - \Omega^B_t, \]  

(6)

where \( B_t \) and \( M_t \) are the beginning-of-the-year stocks of debt and cash, \( T^C_t \) represents corporate taxes, and \( \Omega^K_t \) and \( \Omega^B_t \) denote costs to capital and debt. The constant \( r \) and \( \iota \) are the real interest rates applied to debt and cash.

Capital accumulates as follows:

\[ K_{t+1} = I_t + (1 - \delta)K_t, \]  

(7)

where \( 0 < \delta < 1 \) denotes the depreciation rate. The firm faces quadratic capital adjustment costs:

\[ \Omega^K_t = \frac{\omega_K}{2} \left( \frac{I_t}{K_t} - \delta \right)^2 K_t, \]  

(8)

where \( \omega_K \geq 0 \) is the capital adjustment cost parameter.

Debt issuance is given by \( \Delta B_{t+1} = B_{t+1} - B_t \). The firm faces quadratic costs to varying the debt level away from its long-run level \( \bar{B} > 0 \):

\[ \Omega^B_t = \frac{\omega_B}{2} (B_{t+1} - \bar{B})^2, \]  

(9)

where \( \omega_B \geq 0 \) is the debt cost parameter.

Cash accumulation is given by \( \Delta M_{t+1} = M_{t+1} - M_t \). In contrast to investment, debt issuance, and equity issuance, the firm does not incur any cost when changing its cash holdings.

Finally, corporate taxes are imposed on revenues after depreciation, interest payment, and interest income:

\[ T^C_t = \tau_C (Y_t - F_t - \delta K_t - rB_t + \iota M_t), \]  

(10)

where \( 0 < \tau_C < 1 \) is the corporate tax rate.

The firm faces a constant discount rate \( r \). The after-tax discount factor is \( \beta = 1/(1 + (1 - \tau_r)r) \), where \( \tau_r \) is the personal tax rate on interest income. Because individuals pay taxes on their interest income at a lower rate than the rate at which corporations deduct their interest payment, debt
financing is tax-advantaged. To counter this benefit of debt financing, the convex cost in Equation (9) bounds the debt level. In this sense, deviation costs play a role similar to a collateral constraint.

Similar to the constant discount rate $r$, the interest rate received on cash holdings $\iota$ is assumed to be constant over time. Moreover, cash is a dominated security in terms of return: $\iota < r$. In practice, a firm holding cash incurs a real return loss equal to the inflation rate.

### 2.2 The Intertemporal Problem

At the beginning of the year, the firm makes decisions knowing the current realization of the revenue shock, but not the current realization of the expense shock. During the year, the firm reacts to expense realizations that are different from the expected value. The expense shock triggers a need for liquidity. Of course, foreseeing all this, the firm may have already been cautious and invested less, paid out less in dividends, or raised more funds externally so that its accumulated cash could cover a large expense realization. As a result, the stock of cash at the end of the year, $M_{t+1}$, is equal to the firm’s choice of cash savings $S_t$ at the beginning of the year less the after-tax expenses:

$$M_{t+1} = S_t - (1 - \tau_C) f_t,$$

where the beginning-of-the-year cash saving is

$$S_t = (1 - \tau_C) \left( Y_t - \bar{F} - \delta K_t - r B_t + \iota M_t - \Delta K_{t+1} + \Delta B_{t+1} + M_t - \Omega^K_t - \Omega^B_t - D_t \right)$$

and $\Delta K_{t+1} = K_{t+1} - K_t$.

The firm’s intertemporal problem can be described by the following dynamic programming problem. At the beginning of the year, the firm’s problem is to choose payouts $D_t$, cash savings $S_t$, next period’s debt level $B_{t+1}$ and capital stock $K_{t+1}$ to solve

$$V(B_t, K_t, M_t; z_t, f_{t-1}) = \max_{\{D_t, S_t, B_{t+1}, K_{t+1}\}} U(D_t) + \beta E_t [V(B_{t+1}, K_{t+1}, M_{t+1}; z_{t+1}, f_t)]$$

subject to Equations (1) to (4), (8), (9), (11), and (12), as well as the non-negativity constraints $B_{t+1} \geq 0$, $K_{t+1} \geq 0$, and $M_{t+1} \geq 0$. Note that the conditional expectation is taken on an information set $\Phi_t$ that includes all lagged variables as well as the current values of the capital
stock $K_t$, debt level $B_t$, cash holding $M_t$, and revenue shock $z_t$, but not the year-end realization of the expense shock $f_t$. We denote this conditional expectation by $E_t$.

The juxtaposition of the end-of-the-year cash saving Equation (11) and the non-negativity constraint $M_{t+1} \geq 0$ requires that $S_t \geq (1 - \tau_C) f_t$. Because the firm makes its cash saving decision without knowledge of the realization of the expense shock $f_t$, the firm must save enough to cover all possible realizations. This imposes a liquidity constraint on the firm’s problem:

$$S_t \geq (1 - \tau_C) \sigma_F. \quad (14)$$

The problem of the firm can be restated as choosing payouts $D_t$, cash savings $S_t$, next period’s debt level $B_{t+1}$ and capital stock $K_{t+1}$ to solve (13) subject to Equations (1) to (4), (8), (9), (14), and the relevant non-negativity constraints. The necessary optimality conditions include the complementary-slackness conditions:

$$\lambda_t \geq 0, \quad S_t \geq (1 - \tau_C) \sigma_F, \quad \text{and} \quad \lambda_t [S_t - (1 - \tau_C) \sigma_F] = 0, \quad (15)$$

where $\lambda_t$ is the multiplier associated with the liquidity constraint (14).

In addition, the optimality conditions include Euler equations that describe the cash saving, debt, and capital decisions. The beginning-of-the-period cash saving decision is characterized by

$$U'(D_t) - \lambda_t = \beta E_t [U'(D_{t+1}) (1 + (1 - \tau_C) \iota)]. \quad (16)$$

Debt financing is characterized by

$$U'(D_t) [1 - \omega \beta (B_{t+1} - \bar{B})] = \beta E_t [U'(D_{t+1}) (1 + (1 - \tau_C) r)]. \quad (17)$$

Finally, investment is characterized by

$$U'(D_t) \left[1 + \omega_K \left(\frac{K_{t+1}}{K_t} - 1\right)\right] = \beta E_t \left[U'(D_{t+1}) \left\{1 + (1 - \tau_C) (\alpha \exp(z_{t+1}) \Gamma K_{t+1}^{(\alpha-1)} - \delta) + \frac{\omega_K}{2} \left[\left(\frac{K_{t+2}}{K_{t+1}}\right)^2 - 1\right]\right\}\right]. \quad (18)$$
2.3 Cash Holdings

The two motives to hold cash are related to the complementary slackness conditions (15). When \( \lambda_t > 0 \), the firm holds enough cash only to satisfy the liquidity constraint with equality. That is, all cash holdings are driven by the liquidity motive. When \( \lambda_t = 0 \), the firm may hold more cash than required by the liquidity motive. We summarize these findings in the following proposition.

**Proposition 1** When \( \lambda_t > 0 \), \( S_t = (1 - \tau_C) \sigma_F \) so that the firm holds cash only as a safeguard against the year-end expense shock realization. When \( \lambda_t = 0 \), \( S_t \geq (1 - \tau_C) \sigma_F \) so that the firm may hold more cash.

Cash holdings are characterized by Equation (16), rewritten as

\[
U'(D_t) - \lambda_t = \beta R^M \mathbb{E}_t [U'(D_{t+1})],
\]

(19)

where \( R^M = 1 + (1 - \tau_C) \iota \) is the net return to cash. Note that \( r > \iota \) and \( \tau_C > \tau_r \) imply \( \beta R^M < 1 \). This reflects the impatience embedded in cash holdings. In the special case of a risk neutral firm, the Euler equation reduces to \( 1 - \lambda_t = \beta R^M < 1 \). Thus, \( \lambda_t > 0 \) and the risk neutral firm would never hold more cash than necessary to meet its liquidity needs. More generally, the impatience ensures that Equation (19) naturally yields a stationary distribution of payouts over time when cash holdings are entirely driven by the liquidity motive (\( \lambda_t > 0 \)).

Obtaining a stationary distribution of payouts when \( \lambda_t = 0 \) requires that the firm be risk averse and that the marginal net payout function \( U'(D) \) be convex. To see this, note that Equation (19) implies that \( U'(D_t) < \mathbb{E}_t [U'(D_{t+1})] \) when \( \lambda_t = 0 \). By Jensen’s inequality, this is satisfied when \( U'(D) \) is convex. In the model, \( U'(D) \) is indeed convex because of the schedule of taxes and equity issuing costs \( T(D) \). The convexity of the marginal net payout function may therefore generate precautionary savings, so that cash holdings may be larger than required by the liquidity motive. We summarize these findings in the following proposition.

**Proposition 2** When \( \lambda_t = 0 \) and \( U'(D_t) \) is convex, \( S_t \geq (1 - \tau_C) \sigma_F \) and the firm may hold cash as a safeguard against the year-end expense shock realization and as a precaution against future shocks.
The relative strengths of the precautionary and liquidity motives depend on the firm’s decisions with respect to debt and investment. To see the effect of the debt decision on cash holdings, Equation (17) is rewritten as

\[ E_t [m_{t+1}] R_t^B = 1, \]  

(20)

where \( m_{t+1} = \beta U'(D_{t+1})/U'(D_t) > 0 \) and the return to debt net of taxes and deviation costs is \( R_t^B = [1 + (1 - \tau_C) r] / [1 - \omega_B (B_{t+1} - \bar{B})] \). We also rewrite Equation (19) in terms of \( m_{t+1} \) as:

\[ E_t [m_{t+1}] R^M + \lambda_t / U'(D_t) = 1. \]  

(21)

A comparison of Equations (20) and (21) yields

\[ \lambda_t / U'(D_t) E_t [m_{t+1}] = R_t^B - R^M. \]  

(22)

Equation (22) shows that whether the liquidity motive explains all cash holdings (\( \lambda_t > 0 \) for all \( t \)) or the precautionary motive also plays a role (\( \lambda_t = 0 \) for some \( t \)) depends on the extent to which cash is dominated in terms of net returns. That is, whether \( R_t^B - R^M > 0 \) or \( R_t^B - R^M = 0 \). The condition \( R_t^B - R^M \geq 0 \) can be expressed as \( (1 - \tau_C)(r - \iota) + \omega_B (B_{t+1} - \bar{B}) R^M \geq 0 \). The first term, \( (1 - \tau_C)(r - \iota) \), denotes the extent to which cash is dominated in after-tax return. All else equal, an increase the discount rate \( r \) or a decrease in the interest rate on cash \( \iota \) raises \( \lambda_t \) and it becomes more likely that the firm will hold no more cash than necessary to satisfy the liquidity constraint. The second term, \( \omega_B (B_{t+1} - \bar{B}) R^M \), denotes the cost to converting debt financing into cash. All else equal, making debt conversion into cash more costly raises \( \lambda_t \) and it becomes more likely that cash holdings are driven by the liquidity motive.

Conversely, the firm may hold cash to safeguard against both the current expense shock and future shocks (\( \lambda_t = 0 \)) when cash is not too dominated in after-tax rates of return and the firm finds debt conversions less costly. These results are summarized in the following proposition.

**Proposition 3** When \( R_t^B - R^M > 0 \), \( \lambda_t > 0 \) so that the firm holds cash only as a safeguard against the year-end expense shock realization. When \( R_t^B - R^M = 0 \), \( \lambda_t = 0 \) so that the firm may hold cash as a safeguard against the year-end expense shock realization and as a precaution against future shocks.
The relative strengths of the precautionary and liquidity motives also depend on the capital investment decision. To see this, Equation (18) is rewritten as

\[ E_t \left[ m_{t+1} R^K_{t+1} \right] = 1, \]  

(23)

where

\[ R^K_{t+1} = \frac{1 + (1 - \tau_C) \left[ \alpha \exp(z_{t+1}) \Gamma_k^{(\alpha-1)} - \delta \right] + \frac{\omega_K}{2} \left[ \left( \frac{K_{t+2}}{K_{t+1}} \right)^2 - 1 \right]}{1 + \omega_K \left( \frac{K_{t+1}}{K_t} - 1 \right)} \]  

(24)

is the return to capital net of adjustment costs and taxes. As is standard, a decomposition of Equation (23) yields

\[ E_t [m_{t+1}] E_t[R^K_{t+1}] + \text{Cov}_t[m_{t+1}, R^K_{t+1}] = 1. \]  

(25)

A comparison of Equations (21) and (25) reveals that

\[ \lambda_t U'(D_t) E_t[m_{t+1}] = E_t[R^K_{t+1}] - R^M + \frac{\text{Cov}_t[m_{t+1}, R^K_{t+1}]}{E_t[m_{t+1}]}. \]  

(26)

Equation (26) describes whether the liquidity motive explains all cash holdings (\( \lambda_t > 0 \) for all \( t \)) or the precautionary motive also plays a role (\( \lambda_t = 0 \) for some \( t \)). The first term of Equation (26), \( E_t[R^K_{t+1}] - R^M \), represents the extent to which cash is on average dominated in net return. All else equal, an increase in the extent to which cash is on average dominated raises \( \lambda_t \) and it becomes more likely that the firm will hold no more cash than necessary to satisfy its liquidity constraint. The second term, \( \text{Cov}_t[m_{t+1}, R^K_{t+1}] / E_t[m_{t+1}] \), represents covariance risk. All else equal, an increase in the covariance raises \( \lambda_t \) and it becomes more likely that cash holdings are strictly driven by the liquidity motive. A positive covariance implies that the return to capital is high when payouts are low, so that capital provides insurance in terms of higher returns when future payouts are low. With a large covariance, the firm may be able to self-insure against future adverse shocks without accumulating precautionary cash savings.

Conversely, the firm may hold cash to safeguard against both the current expense shock and future shocks (\( \lambda_t = 0 \)) when cash is not too dominated on average, and when capital does not provide insurance. These results are summarized in the following proposition.
Proposition 4 When \( E_t \left[ R_{kt+1}^K \right] - R^M + \text{Cov}_t \left[ m_{t+1}, R_{kt+1}^k \right] / E_t \left[ m_{t+1} \right] > 0, \lambda_t > 0 \) so that the firm holds cash only as a safeguard against the year-end expense shock realization. When \( E_t \left[ R_{kt+1}^K \right] - R^M + \text{Cov}_t \left[ m_{t+1}, R_{kt+1}^k \right] / E_t \left[ m_{t+1} \right] = 0, \lambda_t = 0 \) so that the firm may hold cash as a safeguard against the year-end expense shock realization and as a precaution against future shocks.

3 Data

The data comes from the North American COMPUSTAT file and covers the period from 1971 to 2006. To explain the large change in cash holdings, the data is split in two extreme time periods: the first third of the sample period from 1971 to 1982 and the last third from 1995 to 2006. The COMPUSTAT sample includes firm-year observations with positive values for total assets (COMPUSTAT Mnemonic AT), property, plant, and equipment (PPENT), and sales (SALE). Our measure of cash holdings is COMPUSTAT Mnemonic CHE, and it is composed of cash (CH) and short-term investments (IVST).

The sample includes firms from all industries, except utilities and financials, with at least five years of consecutive data. The data is winsorized to limit the influence of outliers at the 1 percent and 99 percent tails. The final sample contains 53,067 firm-year observations for the 1971 to 1982 period and 67,720 firm-year observations for the 1995 to 2006 period.

We seek to explain the large increase in cash holdings using model-simulated data. The model does not possess an analytical solution and is solved numerically. Once the model is solved, the policy functions for capital \( K_{t+1} \), debt \( B_{t+1} \), cash \( M_{t+1} \), and the equity value \( V_t \) of Equation (13) are simulated from random outcomes of the innovations to revenues \( \epsilon_{zt} \) and expenses \( \epsilon_{ft} \). These simulated series serve to build other series including dividends \( D_t \), operating income \( OI_t = Y_t - F_t \).

---

2The cash (CH) item includes: bank and finance company receivables; bank drafts; bankers’ acceptances; cash on hand; certificates of deposit included in cash by the company; checks; demand certificates of deposit; demand deposits; letters of credit; and money orders. The short-term investments (IVST) item includes accrued interest included with short-term investments by the company; cash in escrow; cash segregated under federal and other regulations; certificates of deposit included in short-term investments by the company; certificates of deposit reported as separate item in current assets; commercial paper; gas transmission companies’ special deposits; good faith and clearing house deposits for brokerage firms; government and other marketable securities (including stocks and bonds) listed as short term; margin deposits on commodity futures contracts; marketable securities; money market fund; real estate investment trusts’ shares of beneficial interest; repurchase agreements (when shown as a current asset); restricted cash (when shown as a current asset); time deposits and time certificates of deposit, savings accounts when shows as a current asset; treasury bills listed as short term.
net income \( NI_t = (1 - \tau_C)(Y_t - F_t - \delta K_t - rB_t + \iota M_t) \), and the total firm value \( A_t = V_t + (1 + (1 - \tau_r)r)B_t - B_{t+1} \). For each of the two time periods, we construct five panels that have roughly the same number of firm-year observations as observed in the COMPUSTAT panel.

The numerical method requires values for all parameters. To this end, a number of parameters are estimated directly from the data, and others are estimated in a moment matching exercise. The appendix provides an extensive description of the numerical and estimation methods, as well as a complete discussion of the parameter estimates. The resulting parameter values appear in Tables 1, 2, and 3.

Table 1 presents parameters whose values are estimated directly from the data. The capital share of revenues \( \alpha \), the scale of revenues \( \Gamma \), the persistence of the revenue shock \( \rho_z \), and the volatility of its innovations \( \sigma_z \) are estimated directly from the revenue Equation (3) and the autoregressive process (4). The table also presents parameters whose values are observed directly from the data, including the corporate tax rate \( \tau_C \), the interest income tax rate \( \tau_r \), the dividend tax rate \( \tau_D \), the real interest rate \( r \), and the real interest rate on cash holdings \( \iota \).

The remaining parameters are set to values chosen to ensure that simulated series from the model replicate important features of the data. Table 2 shows the parameter values and the target moments for the period of 1971 to 1982, while Table 3 does so for the period of 1995 to 2006. The parameters are the depreciation rate \( \delta \), the capital adjustment cost \( \omega_K \), the constant debt level \( \bar{B} \), the debt deviation cost \( \omega_B \), the average expense level \( \bar{F} \), the expense volatility \( \sigma_F \), and the coefficient of absolute prudence \( \phi \). The target moments are based on the average and the standard deviation of the investment-to-total assets ratio, the average and standard deviation of the leverage ratio, the average operating income-to-total assets ratio, and the standard deviation of the net income-to-total assets ratio.

Of special interest, the coefficient of absolute prudence \( \phi \), which dictates the strength of the firm’s precautionary motive, is estimated in the first time period from 1971 to 1982 so that the average of cash holdings-to-total assets matches the data. This choice ensures that we have the right starting point to investigate the increase in cash holdings. We are interested in predicting
average cash holdings during the 1995-2006 period without changing the coefficient of prudence from its 1971-1982 parameter value.

The average of COMPUSTAT firms’ cash holdings-to-total assets $M_{t+1}/A_t$ was 8.90 percent during the 1971-1982 years. Matching this moment requires a convexity parameter estimate of $\phi = 0.0019$. Note that the estimated value is much smaller than the values ranging from 0.73 to 0.83 estimated in the different environment of Hennessy and Whited (2007). In that sense, our explanation for cash holdings does not rely on a large coefficient of absolute prudence.

4 Results

4.1 Do Simulated Financial Policies Behave as in the Data?

Before studying cash holdings in detail, it is important to verify that the model provides a reasonable description of firms’ observed behavior. Admittedly, the moment matching exercise ensures that specific simulated moments match a number of targeted moments in the data. In what follows, the analysis moves on to controversial moments related to dividend smoothness and debt countercyclicality. These results appear at the bottom of Tables 2 and 3.

It has long been recognized that firms smooth dividends (see Lintner, 1956). In our COMPUSTAT data, payout policies are smooth in the first period. The average standard deviation of payouts is only 3.23 percent during the 1971 to 1982 period, and rises to 10.46 percent during the 1995 to 2006 period. The model replicates the smooth payout policies in the first period and the rise in volatility in the last period. The standard deviation is only 4.05 percent for the first period calibration, and rises to 16.58 percent for the last period calibration. We note however that the model overestimates the volatility of payouts in the last period.

In the model, firms smooth payouts to avoid large taxes on payouts and large equity issuing costs. Specifically, the net payout function $U(D)$ recognizes the convexity of taxes and issuing costs and therefore describes firms as risk averse. Firms smooth payouts well in the first period. Unavoidably the observed increase in net income volatility in the last period substantially affects the volatility of payouts.
It has also been recognized that firm leverage is countercyclical (see Choe, Masulis, and Nanda, 1993, Covas and Den Haan, 2010, and Korajczyk and Levy, 2003). In our COMPUSTAT data, the correlation between debt levels and revenues is $-0.3006$ for the 1971 to 1982 period and $-0.0411$ for the 1995 to 2006 period. In the model, the corresponding correlations are $-0.1867$ and $-0.1025$. Both in the data and in the model, the debt level is countercyclical, and the negative correlation attenuates in the recent time period. A similar pattern is observed for the correlation between changes in debt levels and revenues. The COMPUSTAT data correlation between debt issues and revenues is $-0.2415$ for the 1971 to 1982 period and $-0.1877$ for the 1995 to 2006 period. In the model, the correlations are $-0.1235$ and $-0.0482$.

The countercyclicality of debt in the model is surprising because standard dynamic capital structure models with a tax benefit of debt generate procyclical debt. In these models, firms take on more debt in persistent good times to benefit from the tax advantage because their strong abilities to repay the debt. In our model, the risk averse firm chooses to smooth the effect of adverse revenue shocks on payouts by issuing more debt.

4.2 Do Simulated Cash Policies Behave as in the Data?

The analysis of cash holdings begins by verifying that the model provides a reasonable overall description of cash holdings. The results of this analysis appear in Table 4.

In COMPUSTAT data, cash represents 8.90 percent of total assets in the 1971-1982 period. That number dramatically rises to 17.09 percent in the 1995-2006 period. In the model, the average ratio of cash holdings-to-total assets is specifically targeted by our calibration using the convexity parameter $\phi$ for the first period. Holding $\phi$ constant, the model predicts cash holdings of 17.93 percent of total assets in the last period. Interestingly, as dramatic as the observed cash increase has been in the data, the model predicts even more cash than observed in the data (17.09 percent). This result suggests that firms have become less prudent over time.

Cash policies are relatively smooth in COMPUSTAT data. The standard deviation of cash-to-total assets is only 4.96 percent during the 1971 to 1982 period and rises to 8.48 percent during the 1995 to 2006 period. In the model, the average standard deviation is 6.26 percent for the first
period and rises to 17.56 percent for the last period. As in the data, the model predicts that cash policies have become more volatile, although we note that the model overstates the increase in volatility.

4.3 Precaution Versus Liquidity

Given that the model replicates the large increase in cash holdings, as well as several other moments, the analysis now turns to identifying which motive to hold cash is responsible for the large increase.

To gain some intuition about the model, Figures 3, 4, and 5 show different aspects of the cash decision. The figures are constructed by first drawing one series of revenue shock innovations ($\epsilon_{zt}$) and one series of expense shock innovations ($\epsilon_{ft}$). The same series of innovations are then used to describe the firm behavior in both periods.

Figure 3 shows that the model-simulated cash holdings are on average higher and much more volatile with the last period parametrization than with the first period parametrization. This is consistent with the higher means and standard deviations of the simulated cash holdings presented in Table 4.

Figures 4 and 5 display the relative strengths of the two motives to hold cash. Figure 4 plots the multiplier $\lambda_t$ of the liquidity constraint (14) under both parameterizations. Recall that cash saving decisions are entirely driven by the liquidity motive when $\lambda_t > 0$, and that cash saving decisions may also be driven by the precautionary motive when $\lambda_t = 0$. The figure shows that strictly positive multiplier values ($\lambda_t > 0$) occur in both parameterizations of the model. In fact, the multiplier is always strictly positive in the last period, which means that the precautionary motive to hold cash is no longer relevant in explaining cash savings.

Figure 5 shows how movements of the multiplier translate into actual cash savings. The figure graphs the cash saving decision scaled by mean total assets $S_t/\bar{A}$. Standardizing each observation by the overall mean of total assets $\bar{A}$ rather than standardizing each observation by the corresponding total assets $A_t$ focuses attention on the variations in cash savings $S_t$ while maintaining the appropriate scale. Figure 5 shows that, for both parameterizations, cash savings are bound below by a threshold. The threshold corresponds to the lowest cash savings required to meet the
liquidity constraint. That is, when cash saving $S_t$ is at the lower threshold, cash holdings $M_{t+1}$ are entirely generated by the liquidity motive. When cash saving is above the threshold, cash holdings may also be driven by the precautionary motive.

In the last period, the firm saves more but cash savings are bound to the lower threshold. Together, Figures 3 to 5 suggest that firms hold more cash in the last period than in the first period, and that this decision is related to an increase in the relative strength of the liquidity motive.

4.4 What Drives the Large Increase in Cash Holdings?

To obtain a better understanding of the mechanisms responsible for the large cash increase and the recent dominance of the liquidity motive over the precautionary motive, Table 5 presents the results of a sensitivity analysis. The sensitivity analysis proceeds on the basis of the first period parametrization. In turn each parameter is reset from its first period value to its last period value, leaving all other parameters to their first period values. The results of the sensitivity analysis focuses on three groups of parameters.

4.4.1 Cash Policy Parameters

Propositions 1 and 2 state that the firm always holds enough cash during the year to meet its liquidity threshold $(1 - \tau_C)\sigma_F$. The firm’s cash saving decision directly depends on the volatility of the expense shock $\sigma_F$ and the corporate tax rate $\tau_C$. Over the two periods, the expense shock volatility (standardized by mean total assets) grows from 0.1234 to 0.2743, while the corporate tax rate decreases from 0.4733 to 0.35. The associated increase in the cash-saving threshold stimulates the liquidity motive because it forces the firm to save more cash to meet its current liquidity needs.\footnote{Carroll and Kimball (2001) show that liquidity constraints by themselves can also induce prudence. In our context, this means that an increase in the expense threshold may induce the firm to accumulate more precautionary savings. However, our numerical results show that this does not happen.}

Table 5 shows that the large increase in volatility $\sigma_F/\bar{A}$ is the most important change explaining the dramatic increase in cash holdings. By changing only the expense shock volatility to its last period value, the calibration otherwise based on the first period increases its predicted cash holdings
from 8.90 percent of total assets to 17.41 percent, explaining almost all of the increase observed in the data. Table 5 also shows that the reduction in the corporate tax rate increases cash holdings.

Equation (19) suggests that the coefficient of absolute prudence $\phi$ and the impatience regarding cash holdings $\beta R^M$ are important factors in assessing the strength of the precautionary motive. The coefficient of absolute prudence $\phi$ controls the convexity of the marginal net payout function $U'(D)$. Because the coefficient remains constant over the two periods, it cannot explain the increase in cash holdings or the diminished strength of the precautionary motive. As for the impatience factor $\beta R^M$, it grows from 0.9679 in the first period to 0.9755 during the last period. Because firms become more patient, it should be easier for the cash Euler Equation (16) to hold with $\lambda_t = 0$. That is, the strength of the precautionary motive should be increasing, rather than decreasing as is the case in the simulated cash savings of the last period. When the firm becomes more patient, it can afford to hold more cash.

Disentangling the effect of the impatience factor $\beta R^M = (1 + (1 - \tau_C)\iota)/(1 + (1 - \tau_r)r)$, Table 5 shows that the increase in the interest rate $r$ and the decrease in the interest income tax rate $\tau_r$, which depress the discount factor $\beta$ and therefore the impatience factor $\beta R^M$, should reduce the average cash holdings, but interestingly they do not. A more detailed analysis reveals that the increase in the interest rate $r$ does reduce cash holdings, but it also reduces total assets, such that the cash holdings-to-total asset ratio increases. Total assets decrease because they are valued at a higher discount rate $r$. As for the reduction of the interest income tax rate $\tau_r$, it does not significantly affect cash holdings.

Similarly, Table 5 shows that the large increase in average expenses $\bar F/\bar A$ does not significantly affect cash holdings.

The final cash policy parameter to discuss is the dividend tax rate $\tau_D$. Figure 2 graphs the schedules of taxes and equity issuing costs in the first period (when $\tau_D = 0.3948$ and $\phi = 0.0019$) and in the last period (when $\tau_D = 0.2325$ and $\phi = 0.0019$). When $\tau_D$ decreases in the last period, the firm faces lower taxes and lower issuing costs. The firm therefore has less incentive to save. Accordingly Table 5 reports lower cash holdings.
4.4.2 Debt Policy Parameters

Proposition 3 states that debt decisions affect cash holdings via two terms. The first term, \((1 - \tau_C)(r - \iota)\), describes the extent to which cash is dominated in return by debt. In the data, the extent to which cash is dominated in return declines from 3.11 percent in the first period to 2.31 percent in the last period. All else equal, this should increase cash holdings and stimulate the precautionary motive. As discussed above, all changes in parameter values for the corporate tax rate \(\tau_C\), the discount rate \(r\), and the interest rate on cash \(\iota\) have indeed increased the ratio of simulated cash holdings-to-total assets.

The second term, \(\omega_B(B_{t+1} - \bar{B})R^M\), is related to debt costs. In the simulation, firms on average adopt a conservative debt policy where leverage \(B_{t+1}/A_t\) is near the target \(\bar{B}/\bar{A}\). Table 5 shows that cash holdings are not significantly affected by changes in the debt cost parameter \(\omega_B\) and target leverage \(\bar{B}/\bar{A}\).

4.4.3 Capital Policy Parameters

Proposition 4 suggests that capital decisions affect cash holdings via two terms: the first is the extent to which cash is on average dominated in return, \(E_t[R^K_{t+1}] - R^M\), and the second is the covariance risk, \(\text{Cov}_t[m_{t+1}, R^K_{t+1}] / E_t[m_{t+1}]\). An analysis of the conditional moments reveal that the second term is negligible: the covariance varies between -0.0001 and -0.0004. In the first period, the extent by which cash is dominated in return fluctuates wildly. For some realizations, cash is dominated only slightly by expected returns on capital such that \(\lambda_t = 0\) and the firm may save as a precaution against future shocks. In the last period, the extent by which cash is dominated in return fluctuates much less, and cash is dominated by a large margin.

The effects of parameter changes on the conditional average net return to capital \(E_t[R^K_{t+1}]\) are difficult to study analytically because they cause an endogenous investment reaction. To gain some insight, we examine the deterministic steady state value of the net return to capital. Equation (23) reduces to \(\beta R^K = 1\) so that \(R^K = 1/\beta = 1 + (1 - \tau_r)r\). The deterministic net return to capital has increased from 1.0042 in the first period calibration to 1.0122 in the last period calibration.
However, the extent to which capital dominates cash in steady state return $R^{K^*} - R^M$ has fallen from 0.0322 in the first period to 0.0248 in the last period. This suggests that precautionary cash savings should have increased, which is not the case.

The important parameter effects therefore reside with the stochastic behavior of $E_t \left[ R_{t+1}^{K} \right] - R^M$ rather than its steady state. Its volatility is directly linked to the conditional volatility of the revenue shock innovation $\sigma_z$: the increase in the standard deviation of the innovation to the revenue shock, from 0.2104 in the first period calibration to 0.3093 in the last period calibration, increases cash holdings and stimulates the precautionary motive. In contrast, the reduction in the adjustment costs to capital $\omega_K$ makes it easier to use capital to smooth dividends rather than cash holdings, but the effect is small. In addition, Table 5 shows that changes in the depreciation rate $\delta$ and the persistence $\rho_z$ do not significantly affect cash holdings.

Interestingly, the most important parameter effects are related to the scale parameter $\Gamma$ and the capital share parameter $\alpha$. An increase in the scale parameter $\Gamma$ substantially increases cash holdings, while the reduction in the capital share $\alpha$ substantially reduces cash holdings. Moreover, the reduction in the capital share $\alpha$ is associated with the largest reduction in cash holdings.

The scale parameter $\Gamma$ and the capital share $\alpha$ directly affect the scale of revenues, raising the possibility that $\Gamma$ and $\alpha$ affect cash holdings not via the return to capital, but via the overall volatility of revenues. The volatility of revenues is increasing in the scale of the firm: the larger scale of revenues raises the extent to which revenue shocks affect the firm. With a precautionary motive, the more volatile revenues generate higher cash holdings.

To illustrate this, note that the deterministic steady state revenues are given by $Y^* = \Gamma K^{*\alpha}$. In the first period calibration, revenues $Y^*$ are 35.4677. All else equal the increase in $\Gamma$, from 2.2731 in the first period to 3.1799 in the last period, increases steady state revenues to 78.6556. Conversely, the reduction in $\alpha$, from 0.5785 in the first period to 0.4233 in the last period, lowers steady state revenues to 7.8553. These changes in revenues are large.

Moving from the deterministic steady state to the stochastic environment does not alter the intuition. The larger scale $\Gamma$ yields higher and more volatile revenues as well as higher cash holdings,
while the smaller capital share $\alpha$ yields lower and less volatile revenues as well as lower cash holdings. In the first period calibration, the average of revenues-to-total assets $Y_t/A_t$ is 0.1496, its standard deviation is 0.0290, and the resulting average of cash holdings-to-total assets is 0.0890. All else equal, the increase of $\Gamma$ raises all three moments to: 0.3400, 0.0656, and 0.1361. Conversely, the reduction of $\alpha$ lowers all three moments to: 0.0322, 0.0063, and 0.0654.

Figures 6 and 7 provide another way to see the results. The figures show the impact of $\Gamma$ and $\alpha$ on the benchmark cash savings presented in Figure 4. Figure 6 adds the cash saving decisions for two additional parameterizations. The first maintains the first period parametrization except for the scale $\Gamma$ that is set to its last period (higher) value. The second maintains the last period parametrization except for the scale $\Gamma$ that is set to its first period (lower) value. The figure shows that raising $\Gamma$ in the first period parametrization increases cash savings, and ensures that the savings are above the lower threshold more frequently. Lowering $\Gamma$ in the last period parametrization does not affect cash savings, as cash savings remain bound to the lower threshold. The figure therefore suggests that the larger scale $\Gamma$ may lead to higher cash holdings and more precautionary savings.

Figure 7 presents a similar analysis for the capital share. The first calibration maintains the first period parametrization except for the capital share $\alpha$ that is set to its last period (lower) value. The second maintains the last period parametrization except for the capital share $\alpha$ that is set to its first period (higher) value. The figure shows that lowering the capital share $\alpha$ in the first period parametrization reduces the firm’s cash savings decision, and binds them to the lower threshold. The figure also shows that raising the capital share $\alpha$ in the last period parametrization substantially increases cash savings, and ensures that they are frequently above the threshold. The figure thus confirms that the lower capital share $\alpha$ leads to lower cash holdings and less precautionary savings.

4.4.4 Discussion

Overall the sensitivity analysis documents that the increase in cash holdings is mostly attributable to a large increase in the calibrated value of the expense shock volatility set to match the large increase observed in the net income volatility. In that sense, the model confirms the empirical result of Bates, Kahle, and Stulz (2009). The sensitivity analysis also documents that the reduction in
the relative strength of the precautionary motive is mostly attributable to the reduction in the capital share $\alpha$ so that the associated volatility of revenues no longer generates much precautionary savings.

These two conclusions offer two testable predictions. First, industries that have witnessed the largest increase in net income volatility should also exhibit the largest cash increase. Second, changes in revenue volatility should be unrelated to changes in cash holdings. The aggregate data suggest that the two testable predictions may well hold. The increase in cash holdings and the increase in net income volatility have been substantial over time, while the increase in the volatility of revenues has been less dramatic. Cash holdings-to-total assets have nearly doubled from 0.0890 in the 1971-1982 period to 0.1709 in the 1995-2006 period; the average standard deviation of net income-to-total assets has more than doubled from 0.0665 to 0.1795; but the average standard deviation of revenues-to-total assets has increased from 0.2470 to only 0.2968.

Admittedly, a full test of these predictions is outside the scope of this paper. As an illustration of the evidence, consider a simple regression of the increase in average cash holdings within an industry between the first period 1971-1982 and the last 1996-2005 on a constant, the industry change in the standard deviation of net income between the two periods, and the industry change in the standard deviation of revenues between the two periods, using the cross-section of industries defined in Fama and French (1997). The first prediction requires that the estimated coefficient on the change in net income volatility be positive and significant, and it is: the cross-sectional estimate of the coefficient (standard deviation) is 0.7873 (0.2230). The second prediction requires that the estimated coefficient on the change in revenue volatility be zero, and it is not significantly different from zero at the five percent confidence level: the cross-sectional estimate of the coefficient (standard deviation) is -0.0095 (0.1687). The regression $R^2$ is 0.4354.

4.5 Extensions

Table 6 reports the sensitivity of cash holdings to three extensions. The extensions we consider are: introducing asymmetric costs to changing capital and debt, using the coefficient of absolute prudence $\phi$ to match the volatility of payouts, and introducing a credit line as an additional risk
management tool that the firm can use to respond to its liquidity needs.

4.5.1 Introducing Asymmetric Costs

The model assumes quadratic and thus symmetric costs to changing capital and debt. However, it is not uncommon to consider asymmetric costs. For example, Gamba and Triantis (2008) assume that a reduction in the capital stock (negative investment) is more costly than a symmetric increase. They also assume that issuing new debt is more costly than retiring it.

These considerations are important insofar as the costs to changing capital and debt directly affect the precautionary motive. If the firm cannot easily sell assets or issue more debt, it may depend more heavily on cash savings to self-insure against future adverse shocks. To study whether these asymmetric costs are quantitatively important, the capital adjustment cost function is changed to

$$\Omega^K_t = \frac{\omega_K}{2} \left( \frac{I_t}{K_t} - \delta \right)^2 K_t \left[ 1 - 1(I_t/K_t < \delta) \right] + \frac{\omega_K^a}{2} \left( \frac{I_t}{K_t} - \delta \right)^2 K_t 1(I_t/K_t < \delta),$$

(27)

where $1(I_t/K_t < \delta)$ is an indicator function that takes a value of one when the investment-to-capital ratio is less than the depreciation rate and zero otherwise. For simplicity we assume that $\omega_K^a = 2\omega_K$. Similarly, the debt issuance cost function is changed to

$$\Omega^B_t = \frac{\omega_B}{2} \left( B_{t+1} - \bar{B} \right)^2 \left[ 1 - 1(B_{t+1} > \bar{B}) \right] + \frac{\omega_B^a}{2} \left( B_{t+1} - \bar{B} \right)^2 1(B_{t+1} > \bar{B}),$$

(28)

where $1(B_{t+1} > \bar{B})$ is an indicator function that takes a value of one when debt is above its long-run level and zero otherwise. For simplicity we assume that $\omega_B^a = 2\omega_B$.

Table 6 reports the sensitivity of the average cash holdings to introducing asymmetric adjustment costs, keeping all other parameters at their benchmark values. The result of this experiment suggests that asymmetric costs do not significantly affect cash holdings. Cash holdings increase, but the increase is small. Average cash holdings grow from 8.90 percent of total assets to 0.0914 percent in the first period, but remain bound to the liquidity threshold at 17.93 percent of total assets in the last period.
4.5.2 Matching Payout Volatility

The coefficient of absolute prudence was set to match average cash holdings in the 1971-1992 period ($\phi = 0.0019$). The coefficient value was kept constant in the 1995-2006 period so that the model could generate an out-of-sample prediction on cash holdings. As a result, the simulated cash holdings of 17.93 percent of total assets for the last period does not exactly match the observed cash holdings of 17.09 percent.

As it turns out, it is not feasible to estimate the coefficient of absolute prudence $\phi$ to match the observed cash holdings in 1995-2006. Reducing the value of $\phi$ weakens the precautionary motive, which in turn reduces cash holdings up to a point. Cash holdings are bound below by the threshold of the liquidity constraint. Experimentation suggests that no precautionary cash holdings are generated for coefficients $\phi$ less or equal to 0.0045, which corresponds to the all-liquidity-driven cash holdings of 17.93 percent of total assets. Further lowering $\phi$ does not affect cash holdings.

It is however feasible to estimate the coefficient of absolute prudence $\phi$ to match another moment: the volatility of payouts. The coefficient $\phi$ affects not only prudence but also the firm’s degree of risk aversion and therefore its desire to smooth payouts.

Table 6 reports average cash holdings under the alternative calibration of $\phi$, where the model is estimated to match the full set of moments. As expected, matching payout volatility requires higher values of $\phi$. To match an average standard deviation of payouts of 0.0323 in the first period requires $\phi = 0.0031$ (compared to the benchmark value of 0.0019 estimated to match cash holdings). To match an average standard deviation of payouts of 0.1046 in the last period requires $\phi = 0.0133$.

As expected, Table 6 shows that the larger values of prudence $\phi$ generate larger cash holdings by stimulating the precautionary motive. Average cash holdings grow to 0.1071 in the first period (from 0.0890 using the benchmark calibration), and to 0.2183 in the last period (from 0.1793 using the benchmark calibration).

To understand the consequence of these changes, Figure 2 displays the tax and equity issuing cost schedule $T(D)$ for the benchmark calibrations of the first and last period, as well as the calibrations to match the payout volatility in the first and last periods. The benchmark calibrations show that
equity issuing costs have become less convex and taxes less progressive over time. In contrast, the
alternative calibrations to match the volatility of payouts suggest that equity costs have become
more convex and taxes more progressive over time. This is counterfactual. On the taxation side,
Piketty and Saez (2007) suggest that changes to the U.S. tax system have made the federal tax
system somewhat less progressive. On the equity issuing side, Chen and Ritter (2000) suggest
that initial public offerings on average have become less costly over time. While the model can
generate smooth payouts, the model feature used to generate the smoothness is not consistent with
documented changes in equity issuing costs and taxes over time. The smoothness of dividends
therefore remains a puzzle that our model cannot explain.

4.5.3 Introducing a Line of Credit

In the final model extension, we recognize that firms may have access to lines of credit to respond to
unexpected liquidity shocks. To evaluate the effect of credit lines on firms’ cash savings, our model
includes the main features of credit lines as described in Disatnik, Duchin, and Schmidt (2009),
Ham and Melnick (1987), and Sufi (2009). From these studies, we know that credit line loans may
not exceed a preset upper limit and that they carry an interest premium. Otherwise, the credit
line is flexible: the line is inexpensive to initiate, and the loan may be used at any time subject to
a covenant. The covenant generally specifies that the firm must be experiencing sufficiently high
cash flows to draw down a credit line. Hence, credit lines are substitutes for cash only for firms
with sufficiently high cash flows.

These features are modeled as follows. The firm may use its credit line after observing all shocks
on revenues $\epsilon_{zt}$ and on expenses $\epsilon_{ft}$. When doing so, the firm selects a loan size of $L_{t+1}$. The loan is
constrained to be non-negative and may not exceed a preset upper limit $\bar{L}$: $0 \leq L_{t+1} \leq \bar{L}$. Because
the firm may use its credit line to respond to the expense shock, the cash holding Equation (11)
becomes $M_{t+1} - L_{t+1} = S_t - (1 - \tau_C) f_t$.

For firms with sufficiently high cash flows, credit lines substitute for cash. To capture the
substitution and simplify the model, we define $G_{t+1} = M_{t+1} - L_{t+1}$. Either the firm accumulates
cash and does not use its credit line ($M_{t+1} = G_{t+1}$ and $L_{t+1} = 0$ whenever $G_{t+1} \geq 0$), or the firm
depletes its cash holdings and draws down its credit line \((M_{t+1} = 0\) and \(L_{t+1} = -G_{t+1}\) whenever \(G_{t+1} < 0\)). Furthermore, the preset upper limit implies \(G_{t+1} \geq -\bar{L}\). The interest rate on \(G_t\), \(\zeta\), is set to the real interest rate on cash \(\iota\) when \(G_t \geq 0\), or to the discount rate \(r\) plus an interest premium on the credit line loan \(p\) when \(G_t < 0\). Using this notation, Equation (11) further simplifies to:

\[
G_{t+1} = S_t - (1 - \tau_C) f_t, \tag{29}
\]

where the beginning-of-the-year cash saving of Equation (12) becomes

\[
S_t = (1 - \tau_C) (Y_t - \hat{F} - \delta K_t - \tau B_t + \zeta G_t) - \Delta K_{t+1} + \Delta B_{t+1} + G_t - \Omega^K_t - \Omega^B_t - D_t. \tag{30}
\]

The covenant requires that the firm maintains a high cash flow to use its credit line. In particular, the firm may use its credit line only when the expense shock is small enough \((f_t \leq \bar{f})\) and the revenue shock is large enough \((z_t \geq \bar{z})\). In these circumstances, it must nevertheless accumulate enough cash to cover the gap between the expense shock and the upper limit on the credit line: \(S_t \geq (1 - \tau_C) \bar{f} - \bar{L}\). The firm may not use the credit line when the expense shock is too large \((f_t > \bar{f})\) or when the revenue shock is too small \((z_t < \bar{z})\). In these circumstances, the firm may not use the line of credit as a substitute for cash so that all expense shocks must be met with cash: \(S_t \geq (1 - \tau_C) \sigma_F\).

Calibrating the credit line requires values for the premium \(p\), the preset limit \(\bar{L}\), as well as the cutoff levels of the expense shock \(\bar{f}\) and the revenue shock \(\bar{z}\). In 
\cite{Sufi2009}, the premium on the used credit line is on average 1.75 percent: \(p = 0.0175\). We set all other parameters to encourage the use of the credit line. We allow the preset limit on the credit line to respond to all expense shocks: \(\bar{L} = (1 - \tau_C) \sigma_F\). We do not make the use of the credit line contingent on the expense shock realization: \(\bar{f} \geq \sigma_F\). We specify only that the firm must be experiencing average or better revenue shocks \(\bar{z} \geq 0\) to use its line of credit.

Alternatively when the use of the credit line is contingent on the expense shock realization (i.e., when setting \(\bar{f} < \sigma_F\)), we note that the firm never uses its credit line. This occurs because the covenant blocks the firm’s access to the line of credit when expense shocks are large, so that the firm must respond to large expense shocks with cash only. The covenant therefore still constrains
the firm to accumulate enough cash at the beginning of the period to cover all expense shocks
\( (S_t \geq (1 - \tau C)\sigma_F) \). As a result, the firm always relies on cash and never uses its credit line, as in
the benchmark model without a line of credit. To make the line of credit relevant in the model, we
let \( \bar{f} \geq \sigma_F \) so that the covenant does not depend on the expense shock. As a result, the relevant
liquidity constraint (14) at the beginning of the period becomes
\[
S_t \geq [(1 - \tau C)\sigma_F - \bar{L}] 1(z_t \geq \bar{z}) + (1 - \tau C)\sigma_F [1 - 1(z_t \geq \bar{z})],
\]
where \( 1(z_t \geq \bar{z}) \) is an indicator function that takes a value of one when \( z_t \geq \bar{z} \), and takes a value
of zero otherwise.

Table 6 reports the average cash holdings with a credit line. The model with the credit line
still predicts a large increase in cash holdings: cash holdings grow from 7.10 percent of total assets
to 14.03 percent. The model however underpredicts the average cash holdings in both periods. In
addition, the model predicts an average used credit line of 0.23 percent of total assets in the first
period and 3.8 percent of total assets in the last period. Thus, summing the average cash and
credit line loans, the total liquidity is roughly 7.3 percent of total assets in the first period and
17.83 percent in the last period.

The model underpredicts cash holdings for two reasons. First, credit lines are substitute for
cash in meeting liquidity needs, which necessarily implies that cash holdings decrease with the
introduction of a line of credit. Second, the credit line greatly reduces the precautionary motive to
hold cash. To see this, note that the credit line greatly reduces liquidity in the first period, where
the precautionary motive was influential in the benchmark version of the model (from 8.90 percent
of total assets without the credit line to 7.3 percent with the credit line). The credit line, through
the effect of a higher interest rate paid by the firm, reduces only slightly liquidity as a percentage
of total assets in the last period, where the precautionary motive do not explain cash holdings in
the benchmark model (from 17.93 percent without the credit line to 17.83 percent with the credit
line).

The model extension predicts that credit line loans represent only a small fraction of total assets
(0.23 percent in 1971-1982 and 3.8 percent in 1995-2006). In the absence of extensive data, it is
difficult to judge whether these small numbers are reasonable. Using a sample of 300 firms over the 1996 to 2003 period, Sufi (2009) documents that the average used line of credit represents 5.7 percent of total assets but that the median used line of credit is zero. In this light, the model prediction of an average (and median) used line of credit of 3.8 percent of total assets in the 1995-2006 period appears reasonable.

Finally, we note that it is possible to match cash holdings in the model with a line of credit. For example, matching cash holdings of 8.90 percent of total assets in the first period can be achieved simply by increasing prudence from the benchmark calibrated value $\phi = 0.0019$ to $\phi = 0.0034$.

5 Conclusion

Cash holdings as a proportion of total assets of North American COMPUSTAT firms have roughly doubled from 1971 to 2006. Bates, Kahle, and Stulz (2009) attribute this increase to a rapid rise in cash flow volatility. The cash hoarding period provides an interesting setting in which to evaluate the different motives for corporate cash accumulation.

In particular, the literature highlights two important motives. The first is the precautionary motive (Leland, 1968). It arises because firms face various taxes, as well as adjustment and issuing costs, which may lead firms to act prudently and accumulate cash to self-insure against future adverse shocks. With precautionary self-insurance, a rise in volatility forces firms to hold more cash. The second is a liquidity motive (Miller and Orr, 1966). It arises because firms may find it more costly to alter their investment and other financial policies than to accumulate cash to meet their ongoing liquidity requirements. With liquidity requirements, a larger volatility increases the liquidity threshold and forces firms to hold more cash. The two motives are not mutually exclusive, and, in principle, both may explain how the increase in volatility generates higher cash holdings.

We find that the liquidity motive best explains how the large increase in the volatility generated the large increase in cash holdings. As for the precautionary motive, its importance has decreased to the point of generating no precautionary cash savings from 1995 to 2006. In other words, the increase in cash savings have not been generated by firms’ desire to self-insure against future
adverse shocks. This result is appealing, because it does not require that the increase in volatility be interpreted as a rise in undiversifiable risk against which firms must self-insure. It requires only that the increase in volatility impacts firms’ ongoing liquidity needs.

We re-examine our results in many other modeling environments. For example, when a firm cannot easily sell assets or issue more debt, the firm can rely more heavily on precautionary cash savings to self-insure against future adverse shocks. When a firm’s (unobservable) preference for prudence is calibrated based on the smoothness of its payouts, the model overpredicts the firm’s cash holdings. Conversely, when a firm has access to a line of credit in addition to cash to manage its liquidity needs, the model underpredicts cash holdings. Our qualitative conclusions remain unchanged across all model extensions: cash holdings nearly double from the 1971-1982 period to the 1995-2006 period despite the fact that the precautionary motive disappears in the latter period.

6 Appendix
6.1 Numerical Method

The model is solved numerically using finite element methods as described in Coleman’s (1990) algorithm. Accordingly, the policy functions \( K_{t+1}, M_{t+1}, B_{t+1} \), and co-states \( \lambda_t, V_t \) are approximated by piecewise linear interpolants of the state variables \( K_t, M_t, B_t \), and \( z_t \). The numerical integration involved in computing expectations is approximated with a Gauss-Hermite quadrature rule with two quadrature nodes.

This state space grid consists of 625 uniformly spaced points for the beginning-of-the-year state variables. The lowest and highest grid points for the endogenous state variables \( K_t, M_t, B_t \) are specified outside the endogenous choices of the firm. The lowest and highest grid points for the income shock \( z_t \) are specified three standard deviations away, at \( \exp\left(\frac{-3\sigma_z}{1-\rho_z}\right) \) and \( \exp\left(\frac{3\sigma_z}{1-\rho_z}\right) \).

The approximation coefficients of the piecewise linear interpolants are chosen by collocation, i.e., to satisfy the relevant system of equations at all grid points. The approximated policy interpolants are substituted in the equations, and the coefficients are chosen so that the residuals are set to zero at all grid points. The time-stepping algorithm is used to find these root coefficients. Given initial
coefficient values for all grid points, the time-stepping algorithm finds the optimal coefficients that minimize the residuals at one grid point, taking coefficients at other grid points as given. In turn, optimal coefficients for all grid points are determined. The iteration over coefficients stops when the maximum deviation of optimal coefficients from their previous values is lower than a specified tolerance level, e.g., 0.0001.

### 6.2 Estimation by Simulation

Our estimation procedure follows a moment matching procedure similar to Ingram and Lee (1991). We compute moments in the data and in the simulation as

\[
\hat{H}(x) = \frac{1}{F} \sum_{f=1}^{F} \left[ \frac{1}{T} \sum_{t=1}^{T} h(x_{f,t}) \right] \quad \text{and} \quad \hat{H}_s(\theta) = \frac{1}{F} \sum_{f=1}^{F} \left[ \frac{1}{T} \sum_{t=1}^{T} h(x_{s,f,t}(\theta)) \right],
\]

where \( \hat{H}(x) \) is an \( \tilde{m} \)-vector of statistics computed on the actual data matrix \( x \) and \( \hat{H}_s(\theta) \) is an \( \tilde{m} \)-vector of statistics computed on the simulated data for panel \( s \). The simulated statistics depend on the \( k \)-vector of parameters \( \theta \). We use these statistics to construct the \( m < \tilde{m} \) moments \( H(x) \) and \( H_s(\theta) \) on which our estimation is based. The estimator \( \hat{\theta} \) of \( \theta \) is the solution to

\[
\min_{\theta} \left[ H(x) - \frac{1}{S} \sum_{s=1}^{S} H_s(\theta) \right]^T W \left[ H(x) - \frac{1}{S} \sum_{s=1}^{S} H_s(\theta) \right] \tag{33}
\]

where \( W \) is a positive definite weighting matrix.

For the first period, we compute \( \tilde{m} = 9 \) statistics to form the \( m = 7 \) targeted moments and identify the \( k = 7 \) parameters of the first period (see Table 2). For the last period, we compute \( \tilde{m} = 8 \) statistics to form the \( m = 6 \) targeted moments to identify the \( k = 6 \) parameters of the last period (see Table 3). We construct simulated samples that have the same number of firm-year observations as the data. The actual data sample has 5,469 firms and 53,067 firm-year observations for the first period and 7,220 firms and 67,720 firm-year observations for the second sample. To replicate the data, the simulated samples have 53,070 firm-year observations (\( F = 5,307 \) firms and \( T = 10 \) years) for the first period and 67,720 firm-year observations (\( F = 6,772 \) firms and \( T = 10 \) years) for the last period. In practice, we simulate 50 years, but keep only the last 10 years. In both periods, we construct \( S = 5 \) simulated panels. We use weighting matrix \( W = [(1 + 1/S)\Omega^{-1}] \).
Our estimates of the covariance matrix is $\hat{\Sigma} = [B^T\hat{W}B]^{-1}$, where $\hat{W} = [(1 + 1/S)\hat{\Omega}]^{-1}$ and $B$ contains the gradient of the $m$ moments with respect to the $k$ parameters. We construct $\hat{\Omega}$ as $D'\hat{\Omega}_{\hat{m}}D$ where $D$ contains the gradient of the $m$ moments with respect to the $\tilde{m}$ statistics and $\hat{\Omega}_{\tilde{m}}$ is an heteroscedasticity-consistent covariance matrix of the $\tilde{m}$ statistics in the simulation.

### 6.3 Parameters Estimated from the Data

Table 1 presents the first set of parameter estimates for both periods. The capital share of revenues $\alpha$, the scale of revenues $\Gamma$, the persistence of the revenue shock $\rho_z$, and the volatility of its innovations $\sigma_z$ are estimated from the revenue Equation (3) and the autoregressive process (4). For each of the two time periods, the four parameters are estimated for each firm, and then averaged over all firms. Revenues $Y_t$ are measured as sales, and the beginning-of-the-period capital stock $K_t$ is measured as lagged property, plant, and equipment.\(^4\)

For the 1971-1982 period, the averages are $\alpha = 0.5785$, $\Gamma = 2.2731$, $\rho_z = 0.2452$, and $\sigma_z = 0.2104$. For the 1995-2006 period, the averages are $\alpha = 0.4233$, $\Gamma = 3.1799$, $\rho_z = 0.2062$ and $\sigma_z = 0.3093$. The share $\alpha$ of physical capital that explains revenues has decreased over time but the scale $\Gamma$ has increased. Note that the values for the capital share $\alpha$ are in line with the values used in Moyen (2004), Hennessy and Whited (2005, 2007), and Gamba and Triantis (2008). Over time, shocks to revenues became less persistent ($\rho_z$) and their innovations more volatile ($\sigma_z$). The parameters of the stochastic process indicate a significant increase in the unconditional variance of the revenue shock $\sigma^2_z/(1 - \rho^2_z)$ from 4.71 percent during the 1971-1982 period to 9.99 percent during the 1995-2006 period.

The corporate tax rate $\tau_C$ is set to the top marginal rate. The top marginal tax rate was 48 percent from 1971 to 1978 and 46 percent from 1979 to 1982. The top corporate marginal tax rate has been constant at 35 percent since 1993. As a result, the corporate tax rate is set to its twelve year average of $\tau_C = 0.4733$ for the first period and to $\tau_C = 0.35$ for the last period. The

\(^4\)As an alternative, the capital stock could be reconstructed from the accumulation Equation (7) using capital expenditures (CAPX) assuming an initial value for the capital stock and a value for the depreciation rate. We do not pursue this alternative approach for two reasons. First, it requires a value for the depreciation rate, which would prevent our estimation of the depreciation rate. Second, our results obtained by measuring the capital stock as lagged property, plant, and equipment yield parameter estimates similar to those obtained elsewhere in the literature.
personal tax rates are set to the average marginal tax rates reported in NBER’s TAXSIM. Over the
1971-1982 period, the marginal interest income tax rate averaged $\tau_r = 0.2761$ while the marginal
dividend tax rate averaged $\tau_D = 0.3948$. Over the 1995-2006 period, the marginal interest income
tax rate averaged $\tau_r = 0.2440$ while the marginal dividend tax rate averaged $\tau_D = 0.2325$.

The real interest rate $r$ is set to the average of the monthly annualized t-bill rate deflated by
the consumer price index. High inflation characterized much of the period from 1971 to 1982. As
a result, the real interest rate was quite low, at $r = 0.5848$ percent. As for the later period of 1995
to 2006, the real interest rate was higher, at $r = 1.6091$ percent. For the interest rate earned on
cash holdings $\iota$, we disentangle the two components of cash (CHE): short-term investments (IVST)
and cash (CH). In 1971-1982, firms held 30.66 percent of their cash in short-term investments
earning a rate of return $r$ and 69.34 percent in cash earning a zero nominal interest rate. Given
an average inflation rate of 7.93 percent, the interest rate on cash holdings is set to $\iota = 0.3066r +
0.6934(0 - 0.0793) = -5.32$ percent. In 1995-2006, firms held even less of their cash savings
in short-term investments but the inflation rate was much lower at 2.60 percent, so that $\iota =
0.1564r + 0.8436(0 - 0.026) = -1.94$ percent.

6.4 Parameters Estimated by Matching Moments

The last set of parameters cannot be estimated in isolation directly from the data. The estimation
procedure (described above) is based on the simulated method of moments. In spirit, the
estimation strategy targets a particular moment for each parameter. In practice, a change to one
parameter affects all simulated moments. The parameters to be estimated are the depreciation
rate $\delta$, the capital adjustment cost $\omega_K$, the long-run debt level $\bar{B}$, the debt deviation cost $\omega_B$, the
average expense level $\bar{F}$, the expense volatility $\sigma_F$, and the coefficient of absolute prudence $\phi$. The
estimation seeks to replicate the important features of the data, namely moments of investment,
debt, and cash policies.

Two moments of the capital policy are targeted to estimate the depreciation rate $\delta$ and the
adjustment cost parameter $\omega_K$. The estimate of the depreciation rate $\delta$ ensures that the average
of investment-to-total assets simulated from the model matches the average investment found in the
data. In the COMPUSTAT data, the ratio is computed as capital expenditures (CAPX) divided by total assets (AT). In the model simulated data, the ratio is computed as investment $I_t$ divided by total assets $A_t = V_t + (1 + (1 - \tau) r) B_t - B_{t+1}$. The estimate of the adjustment cost parameter $\omega_K$ ensures that the simulated standard deviation of investment $I_t/A_t$ normalized by the standard deviation of revenues $Y_t/A_t$ matches that of the data. We normalize by the standard deviation of revenues so that the capital adjustment cost $\omega_K$ can target the volatility of investment in reference to the volatility of the revenue shock $\sigma_r$.

For the long-run debt level $\bar{B}$ and the cost parameter $\omega_B$, we target two moments of the debt policy. Our estimate of $\bar{B}$ ensures that the simulated average leverage $B_t/A_t$ matches that of COMPUSTAT firms. Leverage is measured by the sum of long-term debt (DLTT) and debt in current liabilities (DLC) divided by total assets. Similarly to $\omega_K$, our estimate of $\omega_B$ ensures that the simulated standard deviation of debt relative to the standard deviation of revenues matches that of the actual data. Because costs are more relevant to long-term debt than to short-term debt, and because changes in debt are often related to changes in collateral, we focus on the standard deviation of long-term debt-to-capital stock. This standard deviation is then normalized by the standard deviation of revenues-to-capital stock.

The estimate of the average expense level $\bar{F}$ ensures that the average of operating income-to-total assets ratio $OI_t/A_t$ matches the data, where operating income $OI_t = Y_t - F_t$ is measured before depreciation (OIBDP). The estimate of $\sigma_F$ ensures that the standard deviation of net income-to-total assets $NI_t/A_t$ matches the data, where net income is measured as $NI_t = (1 - \tau_C) (Y_t - F_t - \delta K_t - rB_t + \iota M_t)$. We target net income because we want to allow for expenses similar to extraordinary items: expenses that may not be part of the regular operations of the firm but that can affect the firm’s financial health.

Finally, the convexity parameter $\phi$ is the coefficient of absolute prudence. We estimate $\phi$ to ensure that the average of cash holdings-to-total assets $M_{t+1}/A_t$ matches the data in the first time period of 1971-1982, where cash holdings are measured by cash and short-term investments (CHE).

Tables 2 and 3 present the results of the moment matching exercise. Table 2 shows the parameter
values and the target moments for the period covering 1971 to 1982, while Table 3 does so for the period covering 1995 to 2006.

In the data, the average investment-to-total assets is 7.97 percent in the first time period and 6.01 percent in the last time period. To hit these moments, the estimated depreciation rate $\delta$ is set to 0.1696 in the first time period and to 0.1656 in the last time period. The results indicate that the reduction in capital investment from 7.97 percent to 6.01 percent results mostly from the lower capital share $\alpha$.

In COMPUSTAT data, investment has an average standard deviation of 16.79 percent of the average standard deviation of revenues during the 1971-1982 years and a relative average standard deviation of 12.05 percent during the 1995-2006 years. To replicate these moments, the estimates of the capital adjustment cost $\omega_K$ are 0.9707 in the first time period and 0.5220 in the last time period. These estimates are of magnitudes similar to those obtained by Cooper and Haltiwanger (2006). All else equal, the lower capital adjustment cost in recent years stimulates the volatility of investments $I_t/A_t$. This higher volatility, however, is overwhelmed by the increased volatility of revenues $Y_t/A_t$. This denominator effect explains why the lower capital adjustment cost parameter replicates the lower ratio of the standard deviation of investment to the standard deviation of revenues in recent years.

The average leverage of COMPUSTAT firms has been fairly constant over time: 0.3052 during the 1971-1982 period and 0.2795 during the 1995-2006 period. To replicate these moments, the estimates of the long-run debt level (standardized by mean total assets) $\bar{B}/\bar{A}$ are 0.3049 in the first time period and to 0.2780 in the last time period.

The long-term debt-to-capital stock of COMPUSTAT firms has an average standard deviation of 16.78 percent of the average standard deviation of revenues-to-capital stock during the 1971-1982 years and a relative average standard deviation of 21.46 percent during the 1995-2006 years. To replicate these moments, the debt cost estimates $\omega_B$ are 0.0148 in the first time period and 0.0106

---

5The relative standard deviation computed in the data differs considerably from the relative standard deviation commonly shown using macroeconomic data. This simply results from different measurements. For example, we compute the standard deviation of the ratio of investment-to-total assets, while macroeconomists compute the standard deviation of the logarithm of investment detrended using the Hodrick-Prescott filter.
in the last time period.

In COMPUSTAT data, operating income has declined from an average of 13 percent of total assets during the 1971-1982 period to an average of 0.72 percent of total assets during the 1995-2006 period. A larger average expense is required to explain the reduction in the average operating income over time. The estimates of the average expense level (standardized by mean total assets) \( \bar{F} / \bar{A} \) are 0.0195 for the first period and 0.1614 for the last period.

In the data, the standard deviation of net income-to-total assets has greatly increased over time from an average of 0.0665 during the 1971-1982 years to an average of 0.1795 during the 1995-2006 years. The estimates of the volatility parameter (standardized by mean total assets) \( \sigma_{F/\bar{A}} \) are 0.1234 for the first period and 0.2743 for the last period.

7 References


Table 1
Parameter Estimates of the Calibration

The parameter estimates are based on North American data from COMPUSTAT for the sample periods 1971 to 1982 and 1995 to 2006. The COMPUSTAT samples include firm-year observations with positive values for total assets (COMPUSTAT Mnemonic AT), property, plant, and equipment (PPENT), and sales (SALE). The sample includes firms from all industries, except for utilities and financials, with at least five years of consecutive data. The data are winsorized to limit the influence of outliers at the 1% and 99% tails. For each of the two time periods, we estimate the four parameters of the revenue function per firm, and then average the estimates over all firms. The corporate tax rates are calibrated to the top marginal rate, while the personal tax rates are calibrated to the average marginal tax rates reported in NBER’s TAXSIM. The real interest rates are calibrated to the average of the monthly annualized t-bill rate deflated by the consumer price index. The interest rate earned on cash holdings is calibrated as the proportion of cash held in short-term investments (CHE) which earns the average of the monthly annualized t-bill rate deflated by the consumer price index, plus the proportion held in cash (CH) which earns a zero nominal interest rate deflated by the consumer price index.

\[
\begin{array}{lcccc}
\text{Parameters} & 1971-1982 Period & 1995-2006 Period \\
\hline
\text{Revenues} & & & \\
\alpha & 0.5785 & 0.4233 \\
\Gamma & 2.2731 & 3.1799 \\
\rho_z & 0.2452 & 0.2062 \\
\sigma_z & 0.2104 & 0.3093 \\
\text{Tax Rates} & & & \\
\tau_C & 0.4733 & 0.3500 \\
\tau_r & 0.2761 & 0.2440 \\
\tau_D & 0.3948 & 0.2325 \\
\text{Interest Rates ($\%$)} & & & \\
\tau & 0.5848 & 1.6091 \\
\nu & -5.3194 & -1.9417 \\
\end{array}
\]
Table 2
Matching Moments for the 1971-1982 Period

The observed moments are computed using a sample of North American data from COMPUSTAT for the sample period 1971 to 1982. The simulated moments are computed using 5 simulated panels of 5,307 firms over 10 years. $I$ denotes investment, $A$ total assets, $Y$ revenues, $B$ debt level, $K$ capital stock, $OI$ operating income, $NI$ net income, $M$ cash holdings, $D$ dividends, $\Delta B'$ debt issues, and primed variables refer to time $t+1$ values rather than time $t$ values. The model is solved using a finite-element method. The parameters are estimated using a just identified system of moment matching. The number in parenthesis are the standard deviations of the estimated parameters.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Targeted Moments</th>
<th>Simulated</th>
<th>Observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>0.1696 (0.0521)</td>
<td>Mean($I/A$)</td>
<td>0.0797</td>
<td>0.0797</td>
</tr>
<tr>
<td>$\omega_K$</td>
<td>0.9707 (0.0016)</td>
<td>SD($I/A$)/SD($Y/A$)</td>
<td>0.1679</td>
<td>0.1679</td>
</tr>
<tr>
<td>$\bar{B}/\bar{A}$</td>
<td>0.3049 (0.0194)</td>
<td>Mean($B'/A$)</td>
<td>0.3052</td>
<td>0.3052</td>
</tr>
<tr>
<td>$\omega_B$</td>
<td>0.0148 (0.0531)</td>
<td>SD($B'/K'/A$)/SD($Y/K'$)</td>
<td>0.1678</td>
<td>0.1678</td>
</tr>
<tr>
<td>$\bar{F}/\bar{A}$</td>
<td>0.0195 (0.0914)</td>
<td>Mean($OI/A$)</td>
<td>0.1300</td>
<td>0.1300</td>
</tr>
<tr>
<td>$\sigma_F/\bar{A}$</td>
<td>0.1234 (0.0201)</td>
<td>SD($NI/A$)</td>
<td>0.0665</td>
<td>0.0665</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.0019 (0.0202)</td>
<td>Mean($M'/A$)</td>
<td>0.0890</td>
<td>0.0890</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Other Moments</th>
<th>Simulated</th>
<th>Observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>SD($D/A$)</td>
<td>0.0405</td>
<td>0.0323</td>
</tr>
<tr>
<td>Corr($B'/A,Y/A$)</td>
<td>-0.1867</td>
<td>-0.3006</td>
</tr>
<tr>
<td>Corr($\Delta B'/A,Y/A$)</td>
<td>-0.1235</td>
<td>-0.2415</td>
</tr>
</tbody>
</table>
Table 3  
Matching Moments for the 1995-2006 Period

The observed moments are computed using a sample of North American data from COMPUSTAT for the sample period 1971 to 1982. The simulated moments are computed using 5 simulated panels of 6,772 firms over 10 years. $I$ denotes investment, $A$ total assets, $Y$ revenues, $B$ debt level, $K$ capital stock, $OI$ operating income, $NI$ net income, $M$ cash holdings, $D$ dividends, $\Delta B'$ debt issues, and primed variables refer to time $t+1$ values rather than time $t$ values. The model is solved using a finite-element method. The parameters are estimated using a just identified system of moment matching. The number in parenthesis are the standard deviations of the estimated parameters.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Targeted Moments</th>
<th>Simulated</th>
<th>Observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>0.1656</td>
<td>Mean($I/A$)</td>
<td>0.0601</td>
<td>0.0601</td>
</tr>
<tr>
<td></td>
<td>(0.1043)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega_K$</td>
<td>0.5220</td>
<td>SD($I/A$)/SD($Y/A$)</td>
<td>0.1205</td>
<td>0.1205</td>
</tr>
<tr>
<td></td>
<td>(0.3948)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{B}/\bar{A}$</td>
<td>0.2780</td>
<td>Mean($B'/A$)</td>
<td>0.2795</td>
<td>0.2795</td>
</tr>
<tr>
<td></td>
<td>(0.0201)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega_B$</td>
<td>0.0106</td>
<td>SD($B'/K'/SD(Y/K')$)</td>
<td>0.2146</td>
<td>0.2146</td>
</tr>
<tr>
<td></td>
<td>(0.0170)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{F}/\bar{A}$</td>
<td>0.1614</td>
<td>Mean($OI/A$)</td>
<td>0.0072</td>
<td>0.0072</td>
</tr>
<tr>
<td></td>
<td>(0.1216)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_F/\bar{A}$</td>
<td>0.2743</td>
<td>SD($NI/A$)</td>
<td>0.1795</td>
<td>0.1795</td>
</tr>
<tr>
<td></td>
<td>(0.1113)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Other Moments</th>
<th>Simulated</th>
<th>Observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>SD($D/A$)</td>
<td>0.1658</td>
<td>0.1046</td>
</tr>
<tr>
<td>Corr($B'/A,Y/A$)</td>
<td>-0.1025</td>
<td>-0.0411</td>
</tr>
<tr>
<td>Corr($\Delta B'/A,Y/A$)</td>
<td>-0.0482</td>
<td>-0.1877</td>
</tr>
</tbody>
</table>
Table 4
Cash Statistics

The observed moments are computed using a sample of North American data from COMPUSTAT for the sample periods 1971 to 1982 and 1995 to 2006. The simulated moments are computed using 5 simulated panels of 5,307 firms over 10 years for the first period calibration and 5 simulated panels of 6,772 firms over 10 years for the last period calibration. The model is solved using a finite-element method. $M$ denotes cash holdings, $A$ total assets, $Y$ revenues, and primed variables refer to time $t+1$ values rather than time $t$ values.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Simulated</td>
<td>Observed</td>
</tr>
<tr>
<td>Mean($M'/A$)</td>
<td>0.0890</td>
<td>0.0890</td>
</tr>
<tr>
<td>SD($M'/A$)</td>
<td>0.0626</td>
<td>0.0496</td>
</tr>
</tbody>
</table>

Table 5  
Sensitivity Analysis

The simulated moments are computed using 5 simulated panels of 5,307 firms over 10 years. For each parameter, we report the cash holdings obtained from changing the first period parameter value to its second period value, holding all other parameters constant.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1971-1982</td>
<td>0.0890</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1995-2006</td>
<td>0.1793</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Cash Policy Parameters**

- interest rate on cash (%): \(\iota\) -5.3194 -1.9417 0.1337
- interest rate on debt (%): \(r\) 0.5848 1.6091 0.1077
- corporate tax rate: \(\tau_C\) 0.4733 0.3500 0.0992
- interest income tax rate: \(\tau_r\) 0.2761 0.2440 0.0891
- dividend tax rate: \(\tau_D\) 0.3948 0.2325 0.0699
- average expense: \(\bar{F}/\bar{A}\) 0.0195 0.1614 0.0892
- expense shock volatility: \(\sigma_{F/A}\) 0.1234 0.2743 0.1741

**Debt Policy Parameters**

- debt adjustment cost: \(\omega_B\) 0.0148 0.0106 0.0881
- debt target: \(\bar{B}/\bar{A}\) 0.3049 0.2780 0.0893

**Capital Policy Parameters**

- depreciation rate: \(\delta\) 0.1696 0.1656 0.0920
- capital adjustment cost: \(\omega_K\) 0.9707 0.5220 0.0865
- revenue scale: \(\Gamma\) 2.2731 3.1799 0.1361
- capital share: \(\alpha\) 0.5785 0.4233 0.0654
- revenue shock persistence: \(\rho_z\) 0.2452 0.2062 0.0892
- revenue shock volatility: \(\sigma_z\) 0.2104 0.3093 0.0921
Table 6
Extensions

The simulated moments are computed using 5 simulated panels of 5,307 firms over 10 years for the first period calibration and 5 simulated panels of 6,772 firms over 10 years for the last period calibration. \( M \) denotes cash holdings and \( A \) total assets. Primed variables refer to time \( t+1 \) values rather than time \( t \) values. The observed and benchmark moments come from Table 4. For all three extensions, the moments are computed from versions of the model that hold all other parameters to their benchmark calibrated values.

<table>
<thead>
<tr>
<th></th>
<th>1971-1982 Period Mean(( M' / A ))</th>
<th>1995-2006 Period Mean(( M' / A ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed</td>
<td>0.0890</td>
<td>0.1709</td>
</tr>
<tr>
<td>Benchmark Model</td>
<td>0.0890</td>
<td>0.1793</td>
</tr>
<tr>
<td>Introducing Asymmetric Costs</td>
<td>0.0914</td>
<td>0.1793</td>
</tr>
<tr>
<td>Using ( \phi ) to Match Payout Volatility</td>
<td>0.1071</td>
<td>0.2183</td>
</tr>
<tr>
<td>Introducing a Credit Line</td>
<td>0.0710</td>
<td>0.1403</td>
</tr>
</tbody>
</table>
Figure 1: Average North American Firm Policies

- Cash M'/A
- Capital K'/A
- Debt B'/A
Figure 2: Tax and Equity Issuing Cost Schedule $T(D)$
Figure 3: Cash Holdings M'/A of Simulated Firms

1971-1982 Calibration
1995-2006 Calibration
Figure 4: Liquidity Constraint Multiplier $\lambda$

- 1971-1982 Calibration
- 1995-2006 Calibration
Figure 5: Cash Savings S/Mean(A) of Simulated Firms

- 1971-1982 Calibration
- 1995-2006 Calibration
Figure 6: Effect of Scale $\Gamma$ on Cash Savings $S/\text{Mean}(A)$
Figure 7: Effect of Capital Share $\alpha$ on Cash Savings $S/\text{Mean}(A)$