Credit Market Imperfections and Business Cycles

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Outline

- Introduction
- Imperfect Credit Markets: Some Examples
- Preview of Main Results
- Baseline Model
- Calibration
- Results
- Extension of Baseline Model
- Conclusion
Perfect credit market: The loan agreement is very simple. At the on-going interest rate, the agent can borrow any amount.

Examples of borrowing restrictions: e.g. borrowing limit, pledging of collaterals, requiring proof of income, minimum credit scores, credit ratings, etc.

Interest rate is neither the sole nor the most important factor in extending loans by the creditors.

Macro implication: in reaction to shocks, agents are less able to smooth their consumption.
Perfect credit market: The loan agreement is very simple. At the on-going interest rate, the agent can borrow any amount.

Macro implication: in reaction to shocks, agents have unlimited flexibility to smooth their consumption.
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- **Imperfect credit market**: The key concern is loan default by the borrowers. Therefore, creditors impose various restrictions in order to ensure the borrower has the ability and incentive to repay the loan in the future.

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Relation with Macro Literature

- Issue: standard real business cycle (RBC) models rely on productivity shocks to generate business cycles.
- But empirical evidence: the true productivity shocks are usually not large enough to generate quantitatively realistic business cycles (Rebelo 2005)
- Therefore, an amplification mechanism is needed. Typically the mechanism is some types of market frictions.
- Credit Market Imperfection: one example of such a mechanism
Collateral Value
How Imperfect Credit Markets Work? 2 Frameworks

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  - e.g. Kiyotaki & Moore (1997), Kocherlakota (2000)
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- e.g. Bernanke & Gertler (1989)
- premium on external financing, depends on net worth, which is pro-cyclical
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“Credit standards” : non-price lending terms specified in the typical bank loans or lines of credit: collateral, loan-to-valuation (LTV) ratio, loan limits, proof of incomes, credit scores, credit ratings, etc.


Federal Reserve Senior Loan Officer Opinion Survey on Bank Lending Practice: "Over the past three months, how have your bank’s credit standards for approving applications for C&I (commercial and industrial) loans or credit lines—other than those to be used to finance mergers and acquisitions—to large and middle-market firms and to small firms changed? Ans: 1) Tightened considerably 2) tightened somewhat 3) remained basically unchanged 4) eased somewhat 5) eased considerably."
Source: Federal Reserve Board (FRB)
Source: European Central Bank
Quantifying Credit Standards: Maximum LTVs

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Maximum LTV (Loan-to-Valuation) ratio: the highest mortgage loan that households can obtain from lenders as a % of the value of the property owned.

Source: Almeida, Campello and Liu (2006)
Djankov, Hart, McLiesh and Shleifer (2008):

- 88 developed and less developed countries at January 2006
- an average of 48% of the collateral value was dissipated in the debt enforcement procedures.
- Richest 8 countries: average 1.5 years to resolve debt enforcement, and transaction cost = 9% of collateral value
- Lower middle income countries: 2.8 years and 16%.

Altman, Brady, Resti and Sironi (2005):

- 1982-2001: average recovery rate after corporate debt defaults in the US was 37.2%, ranging from 23.4% in 1990 to 62% in 1987.
Output volatility can be amplified if a borrowing limit is imposed on the agents.
The increase in output volatility depends on the nature of the borrowing limit.
Exogenous borrowing limit does not amplify output volatility.
Endogenous borrowing limit can amplify output volatility, depending on the specification of the LTV.
- If the LTV ratio is fixed, the amplification is moderate, at most 5%.
- If the LTV ratio is endogenous (i.e. depending on aggregate output), output volatility can be amplified by up to 39% in the baseline model.

In the extended model that includes labor-leisure choice, endogenous LTV can still amplify output volatility. The amplification depends on income effect.
Baseline Model

Production Technology

- Agents: a group of identical agents
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- Production Technology: \( Y_t = K_t^{\alpha_1} L_t^{\alpha_2} N_t^{\alpha_3} Z_t \)
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- Factor markets are competitive
Baseline Model
Credit Market: basic features

- The agents can borrow an one-period loan $B_{t+1}$ at time $t$, at an interest rate $r_t$.
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Credit Market: basic features

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In general, this interest rate can be time-varying but for simplicity assume it to be constant in the baseline model.
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The gross interest rate $R$ is equal to $1 + r$.

To preclude Ponzi-schemes, the growth of borrowing is limited:
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- The agents can borrow an one-period loan $B_{t+1}$ at time $t$, at an interest rate $r_t$
- Assume a small open economy: $r_t$ is exogenously set by the rest of the world
- If $r_t$ is high enough, the agents can be creditors. But the focus here is the case when they are borrowers.
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- The gross interest rate $R$ is equal to $1 + r$.
- To preclude Ponzi-schemes, the growth of borrowing is limited:
  \[
  \lim_{j \to \infty} E_t \frac{B_{t+j}}{(1+r)^j} = 0
  \]
Problem of the Agent

\[
\max_{C_t, K_{t+1}, L_{t+1}, B_{t+1}} \quad E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma} \quad \text{subject to}
\]

\[
C_t + K_{t+1} - (1 - \delta) K_t + Q_t L_{t+1} + B_t R = K_t^{\alpha_1} L_t^{\alpha_2} Z_t + B_{t+1} + Q_t L_t
\]

where

\( C_t \): consumption

\( Q_t \): price of land in terms of consumption goods

\( \delta \in (0, 1) \): depreciation rate

\( \sigma > 0 \) is the constant coefficient of relative risk aversion.

\( \beta \in (0, 1) \) is the discount factor

where \( K_0, Z_0 \) and \( B_0 \) are given.

and Borrowing Constraints ...
Borrowing Constraint

- **Exogenous Borrowing Constraint**: the borrowing limit is exogenously given.
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- **Endogenous Borrowing Constraint**: the borrowing limit depends on the value of the collateral (land holding) pledged by the borrowers. The idea is to protect creditors against loan default. In the literature, the collateral can be evaluated at *current* price (e.g. Kocherlakota (2000)) or *future* price (e.g. Kiyotaki and Moore (1997)).
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  \[ B_{t+1} \leq \theta Q_t L_{t+1} \]
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\[ B_{t+1} \leq \theta Q_t L_{t+1} \]

or

\[ B_{t+1} \leq E_t (\theta Q_{t+1} L_{t+1}) \]
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where \( \theta \in [0, 1] \) is the LTV ratio
Trade-off between consumption and capital acquisition

\[ C_t^{-\sigma} = E_t \beta C_{t+1}^{-\sigma} \left( \alpha_1 K_{t+1}^{\alpha_1-1} Z_{t+1} + 1 - \delta \right) \] (All Cases)

Trade-off between consumption and land acquisition

- Exogenous Borrowing Constraint:
  \[ C_t^{-\sigma} Q_t = E_t \beta C_{t+1}^{-\sigma} \left( \alpha_2 K_{t+1}^{\alpha_1} Z_{t+1} + Q_{t+1} \right) \]

- Endogenous Borrowing Constraint (collateral evaluated at \( Q_t \))
  \[ C_t^{-\sigma} (1 - \theta) Q_t = E_t \beta C_{t+1}^{-\sigma} \left( \alpha_2 K_{t+1}^{\alpha_1} Z_{t+1} + Q_{t+1} - \theta Q_{t+1} R \right) \]

- Endogenous Borrowing Constraint (collateral evaluated at \( E_t Q_{t+1} \))
  \[ C_t^{-\sigma} (Q_t - E_t \theta Q_{t+1}) = E_t \beta C_{t+1}^{-\sigma} \left( \alpha_2 K_{t+1}^{\alpha_1} Z_{t+1} + Q_{t+1} - \theta Q_{t+1} R \right) \]

If \( \theta = 0 \), then these cases are the same \( \Rightarrow \) same dynamics
Baseline Model: Competitive Equilibrium

- Definition: a sequence of prices $\{Q_t\}_{t=0}^{\infty}$ and per capita allocations $\{C_t, K_{t+1}, L_{t+1}, B_{t+1}\}_{t=0}^{\infty}$, such that, given the initial allocations of capital goods, borrowing/ lending positions, the loan-to-valuation ratio $\theta$ and the world interest rate $r$, the following conditions hold:
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  (1) Each agent maximizes the expected lifetime utility subject to the budget constraint and one of the borrowing constraints.
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  1. Each agent maximizes the expected lifetime utility subject to the budget constraint and one of the borrowing constraints.
  2. All the following market-clearing conditions are satisfied in each period \( t \geq 0 \):

Goods market: the goods produced \( K_t^{\alpha_1} L_t^{\alpha_2} Z_t \) is equal to the sum of consumption \( C_t \) and investment \( K_{t+1}^\delta \).

Land market: the demand for land \( L_{t+1} \) is equal to the land supply, which is fixed at 1.
Baseline Model: Competitive Equilibrium

- Definition: a sequence of prices \( \{Q_t\}_{t=0}^\infty \) and per capita allocations \( \{C_t, K_{t+1}, L_{t+1}, B_{t+1}\}_{t=0}^\infty \), such that, given the initial allocations of capital goods, borrowing/lending positions, the loan-to-valuation ratio \( \theta \) and the world interest rate \( r \), the following conditions hold:
  
  (1) Each agent maximizes the expected lifetime utility subject to the budget constraint and one of the borrowing constraints.

  (2) All the following market-clearing conditions are satisfied in each period \( t \geq 0 \):

  - (a) goods market: \( K_t^{\alpha_1} L_t^{\alpha_2} Z_t = C_t + K_{t+1} - (1 - \delta) K_t \)

  - (b) land market: \( L_{t+1} = 1 \)
Baseline Model: Competitive Equilibrium

- Definition: a sequence of prices \( \{Q_t\}_{t=0}^{\infty} \) and per capita allocations \( \{C_t, K_{t+1}, L_{t+1}, B_{t+1}\}_{t=0}^{\infty} \), such that, given the initial allocations of capital goods, borrowing/ lending positions, the loan-to-valuation ratio \( \theta \) and the world interest rate \( r \), the following conditions hold:

  1. Each agent maximizes the expected lifetime utility subject to the budget constraint and one of the borrowing constraints.
  2. All the following market-clearing conditions are satisfied in each period \( t \geq 0 \):

     - (a) goods market: \( K_1^\alpha L_2^\alpha Z_t = C_t + K_{t+1} - (1 - \delta) K_t \)
     - (b) land market: \( L_{t+1} = 1 \)
Baseline Model: Competitive Equilibrium

- **Definition**: a sequence of prices \( \{Q_t\}_{t=0}^{\infty} \) and per capita allocations \( \{C_t, K_{t+1}, L_{t+1}, B_{t+1}\}_{t=0}^{\infty} \), such that, given the initial allocations of capital goods, borrowing/lending positions, the loan-to-valuation ratio \( \theta \) and the world interest rate \( r \), the following conditions hold:

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     (b) land market: \( L_{t+1} = 1 \)

- Goods market: the goods produced \( K_t^{\alpha_1} L_t^{\alpha_2} Z_t \) is equal to the sum of consumption \( C_t \) and investment \( K_{t+1} - (1 - \delta) K_t \).

- Land market: the demand for land \( L_{t+1} \) is equal to the land supply, which is fixed at 1.
### Benchmark Parameter Values

<table>
<thead>
<tr>
<th>Baseline Model</th>
<th>Description</th>
<th>Benchmark Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>discount factor</td>
<td>0.95</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>inverse of relative risk aversion</td>
<td>1</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>capital share of output</td>
<td>0.3</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>land share of output</td>
<td>0.1</td>
</tr>
<tr>
<td>$\delta$</td>
<td>depreciation rate</td>
<td>0.1</td>
</tr>
<tr>
<td>$\theta$</td>
<td>loan-to-valuation ratio (LTV)</td>
<td>1</td>
</tr>
<tr>
<td>$\bar{B}$</td>
<td>exogenous borrowing limit</td>
<td>1.2218</td>
</tr>
<tr>
<td>$r$</td>
<td>exogenous interest rate</td>
<td>0.0526</td>
</tr>
<tr>
<td>$\rho$</td>
<td>persistence of tech shock</td>
<td>0.9</td>
</tr>
<tr>
<td>$\sigma^2_\varepsilon$</td>
<td>variance of innovation to tech shock</td>
<td>0.005</td>
</tr>
</tbody>
</table>
Output Volatility: Baseline Model

Basic Models: SD of \( y(t) \) wrt Theta

- exo BC
- endo-BC, current landpx
- endo BC, exp landpx

<table>
<thead>
<tr>
<th>theta (collateral ratio)</th>
<th>SD of ( y(t) ) in %</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.94</td>
</tr>
<tr>
<td>0.2</td>
<td>1.96</td>
</tr>
<tr>
<td>0.4</td>
<td>1.98</td>
</tr>
<tr>
<td>0.6</td>
<td>2.00</td>
</tr>
<tr>
<td>0.8</td>
<td>2.02</td>
</tr>
<tr>
<td>1</td>
<td>2.04</td>
</tr>
<tr>
<td>1</td>
<td>2.06</td>
</tr>
<tr>
<td>1.2</td>
<td>2.08</td>
</tr>
<tr>
<td>1.4</td>
<td>2.10</td>
</tr>
</tbody>
</table>

W Leung (University of California, Riverside)  Credit Mkt Imperfection
### Volatilities: Baseline Model

<table>
<thead>
<tr>
<th>LTV $\theta$</th>
<th>LTV=0</th>
<th>LTV=1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case</td>
<td>1 2a 2b</td>
<td>1 2a 2b</td>
</tr>
<tr>
<td>$\sigma_{y_t}$</td>
<td>1.96 1.97 1.97</td>
<td>1.97 2.07 2.06</td>
</tr>
<tr>
<td>$\sigma_{c_t}$</td>
<td>1.39 1.39 1.39</td>
<td>1.57 1.54 1.54</td>
</tr>
<tr>
<td>$\sigma_{i_t}$</td>
<td>3.86 3.87 3.87</td>
<td>3.96 5.34 5.24</td>
</tr>
<tr>
<td>$\sigma_{b_t}$</td>
<td>n.a. 1.42 1.41</td>
<td>n.a. 0.89 0.83</td>
</tr>
<tr>
<td>$\sigma_{q_t}$</td>
<td>1.43 1.42 1.42</td>
<td>1.52 0.89 0.89</td>
</tr>
<tr>
<td>$\sigma_{c_t}/\sigma_{y_t}$</td>
<td>0.7 0.7 0.7</td>
<td>0.8 0.8 0.8</td>
</tr>
<tr>
<td>$\sigma_{i_t}/\sigma_{y_t}$</td>
<td>2.0 2.0 2.0</td>
<td>2.0 2.6 2.6</td>
</tr>
<tr>
<td>$\sigma_{b_t}/\sigma_{y_t}$</td>
<td>n.a. 0.7 0.7</td>
<td>n.a. 0.4 0.4</td>
</tr>
<tr>
<td>$\sigma_{q_t}/\sigma_{y_t}$</td>
<td>0.7 0.7 0.7</td>
<td>0.8 0.4 0.4</td>
</tr>
</tbody>
</table>

- **Case 1**: Exogenous Borrowing Constraint
- **Case 2a**: Endogenous Borrowing Constraint, collateral eval. at current px
- **Case 2b**: Endogenous Borrowing Constraint, collateral eval. at future px
Endogenous LTV

- Idea: the LTV ratio is endogenous, depending on current or expected aggregate output.
Endogenous LTV

- Idea: the LTV ratio is endogenous, depending on current or expected aggregate output.
- Literature: e.g. Dell’Ariccia and Marquez (2006)
Endogenous LTV

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- Literature: e.g. Dell’Ariccia and Marquez (2006)
- Individual agent treats LTV as given.
Endogenous LTV

- Idea: the LTV ratio is endogenous, depending on current or expected aggregate output.
- Literature: e.g. Dell’Ariccia and Marquez (2006)
- Individual agent treats LTV as given.
- Requirement for the functional form
  - LTV to be pro-cyclical to match stylized fact
  - LTV to be fluctuating between 0 and 1
  - LTV to be concave with respect to aggregate output
  - Interpretable parameters
  -tractability
  -parsimonious
Endogenous LTV

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  - interpretable parameters
  - tractability
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  - LTV to be concave with respect to aggregate output
  - Interpretable parameters
  - Tractability
  - Parsimonious
Endogenous LTV

\[ \theta_t = 1 - \exp(-\gamma Y_t) \quad \text{or} \]

\[ \theta_t = 1 - \exp(-\gamma E_t Y_t + 1) \]

where \( \gamma > 0 \) is a parameter governing the easiness of the LTV ratio.
Endogenous LTV

\[ \theta_t = 1 - \exp(-\gamma Y_t) \quad \text{or} \]
\[ \theta_t = 1 - \exp(-\gamma E_t Y_{t+1}) \]
Endogenous LTV

\[ \theta_t = 1 - \exp(-\gamma Y_t) \] or
\[ \theta_t = 1 - \exp(-\gamma E_t Y_{t+1}) \]

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Endogenous LTV

- \( \theta_t = 1 - \exp(-\gamma Y_t) \) or
- \( \theta_t = 1 - \exp(-\gamma E_t Y_{t+1}) \)

where \( \gamma > 0 \) is a parameter governing the easiness of the LTV ratio.
Output Volatility: Baseline Model

Basic Models: SD of $y(t)$ wrt $\theta$

- exo BC
- endo-BC, current landpx
- endo BC, exp landpx
- endo-LTV, current Y
- endo-LTV exp Y

$\theta$ (collateral ratio)

SD of $y(t)$ in %
### Volatilities: Baseline Model, LTV=0.75

<table>
<thead>
<tr>
<th>Case</th>
<th>1</th>
<th>2a</th>
<th>2b</th>
<th>3a</th>
<th>3b</th>
<th>3b vs. 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{y_t}$</td>
<td>1.97</td>
<td>2.05</td>
<td>2.05</td>
<td>2.79</td>
<td>2.72</td>
<td><strong>1.39</strong></td>
</tr>
<tr>
<td>$\sigma_{c_t}$</td>
<td>1.52</td>
<td>1.50</td>
<td>1.50</td>
<td>1.49</td>
<td>1.47</td>
<td>0.97</td>
</tr>
<tr>
<td>$\sigma_{i_t}$</td>
<td>3.93</td>
<td>4.96</td>
<td>4.93</td>
<td>21.90</td>
<td>21.35</td>
<td>5.43</td>
</tr>
<tr>
<td>$\sigma_{b_t}$</td>
<td>8.07</td>
<td>1.03</td>
<td>0.99</td>
<td>8.67</td>
<td>7.98</td>
<td>0.99</td>
</tr>
<tr>
<td>$\sigma_{q_t}$</td>
<td>1.50</td>
<td>1.03</td>
<td>1.03</td>
<td>8.16</td>
<td>7.50</td>
<td>5.01</td>
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<tr>
<td>$\sigma_{\theta_t}$</td>
<td>n.a.</td>
<td>n.a.</td>
<td>n.a.</td>
<td>1.27</td>
<td>1.22</td>
<td>n.a.</td>
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<tr>
<td>$\sigma_c / \sigma_{y}$</td>
<td>0.8</td>
<td>0.7</td>
<td>0.7</td>
<td>0.5</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>$\sigma_i / \sigma_{y}$</td>
<td>2.0</td>
<td>2.4</td>
<td>2.4</td>
<td>8.0</td>
<td>8.0</td>
<td></td>
</tr>
</tbody>
</table>

Case 1  Exogenous Borrowing Limit $\bar{B}$
Case 2a Exogenous Borrowing Limit, collateral evaluated at $Q_t$
Case 2b Exogenous Borrowing Limit, collateral evaluated at $E_t Q_{t+1}$
Case 3a Endogenous LTV, depending on $Y_t$
Case 3b Endogenous LTV, depending on $E_t Y_{t+1}$
Impulse Response Function

Impulse Response Function (LTV = 1.00)

% deviation from steady state

time

exo BC

endo BC, exp landpx

endo-BC, current landpx

endo-LTV, current Y

endo-LTV exp Y
Extension: Labor-Leisure Choice

- **Standard Preference:** \( U(C, N) = \frac{C^{1-\sigma}}{1-\sigma} - A \frac{N^{1+\chi}}{1+\chi} \)
Extension: Labor-Leisure Choice

- **Standard Preference:** \( U(C, N) = \frac{C^{1-\sigma}}{1-\sigma} - A \frac{N^{1+\chi}}{1+\chi} \)
- \( \chi \): the inverse of labor supply elasticity
Extension: Labor-Leisure Choice

- Standard Preference: \( U(C, N) = \frac{C^{1-\sigma}}{1-\sigma} - A \frac{N^{1+\chi}}{1+\chi} \)
- \( \chi \): the inverse of labor supply elasticity
- \( A > 0 \): parameter for calibration purpose.
Extension: Labor-Leisure Choice

- **Standard Preference:** \( U(C, N) = \frac{C^{1-\sigma}}{1-\sigma} - AN^{1+\chi} \)
- \( \chi \): the inverse of labor supply elasticity
- \( A > 0 \): parameter for calibration purpose.

- **GHH Preference:** \( U(C, N) = (1 - \nu)^{-1} \left[ \left( C - \frac{N\psi}{\psi} \right)^{1-\nu} - 1 \right] \)

Greenwood, Hercowitz and H"{u}fman (1988)

The marginal rate of substitution \( N\psi \) only depends on labor and not on consumption. In other words, the income effect brought by a change in wage rate is precluded.
Extension: Labor-Leisure Choice

- **Standard Preference:** \( U(C, N) = \frac{C^{1-\sigma}}{1-\sigma} - A \frac{N^{1+\chi}}{1+\chi} \)
  - \( \chi \): the inverse of labor supply elasticity
  - \( A > 0 \): parameter for calibration purpose.

- **GHH Preference:** \( U(C, N) = (1 - v)^{-1} \left[ \left( C - \frac{N^\psi}{\psi} \right)^{1-v} - 1 \right] \)
  - Greenwood, Hercowitz and Huffman (1988)
Extension: Labor-Leisure Choice

- **Standard Preference:** \( U(C, N) = \frac{C^{1-\sigma}}{1-\sigma} - AN^{1+\chi} \)

- \( \chi \): the inverse of labor supply elasticity

- \( A > 0 \): parameter for calibration purpose.

- **GHH Preference:** \( U(C, N) = (1 - v)^{-1} \left[ \left( C - \frac{N^\psi}{\psi} \right)^{1-v} - 1 \right] \)

- Greenwood, Hercowitz and Huffman (1988)

- The marginal rate of substitution \( N^{\psi-1} \) only depends on labor and not on consumption. In other words, the income effect brought by a change in wage rate is precluded.
## Calibration of the Labor-Leisure Choice Model

### Standard Labor Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Baseline Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_3$</td>
<td>labor share of output</td>
<td>0.6</td>
</tr>
<tr>
<td>$\chi$</td>
<td>inverse of labor supply elasticity</td>
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</tr>
<tr>
<td>$A$</td>
<td>disutility of labor</td>
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</table>

### GHH Labor Model\(^1\)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi$</td>
<td>parameter of utility function</td>
<td>1.4555</td>
</tr>
<tr>
<td>$\nu$</td>
<td>parameter of utility function</td>
<td>2</td>
</tr>
</tbody>
</table>

\(^1\)from Schmitt-Grohe and Uribe (2003)
Labor Models: SD of $y(t)$ wrt $\Theta$

- SD of $y(t)$ in %
- exo BC
- endo-BC, current landpx
- endo BC, exp landpx
- endo-LTV, current Y
- endo-LTV exp Y

 theta (collateral ratio)
## Volatilities: Labor Model

Standard Preference, LTV=0.7

<table>
<thead>
<tr>
<th>Case</th>
<th>1</th>
<th>2a</th>
<th>2b</th>
<th>3a</th>
<th>3b</th>
<th>3b vs. 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{y_t}$</td>
<td>2.03</td>
<td>2.14</td>
<td>2.14</td>
<td>2.28</td>
<td>2.30</td>
<td>1.13</td>
</tr>
<tr>
<td>$\sigma_{c_t}$</td>
<td>1.86</td>
<td>1.90</td>
<td>1.90</td>
<td>1.96</td>
<td>1.96</td>
<td>1.06</td>
</tr>
<tr>
<td>$\sigma_{i_t}$</td>
<td>4.02</td>
<td>5.56</td>
<td>5.52</td>
<td>7.55</td>
<td>8.32</td>
<td>2.07</td>
</tr>
<tr>
<td>$\sigma_{n_t}$</td>
<td>0.26</td>
<td>0.21</td>
<td>0.21</td>
<td>0.19</td>
<td>0.21</td>
<td>0.79</td>
</tr>
<tr>
<td>$\sigma_{b_t}$</td>
<td>n.a.</td>
<td>1.11</td>
<td>1.05</td>
<td>2.28</td>
<td>2.35</td>
<td>n.a.</td>
</tr>
<tr>
<td>$\sigma_{q_t}$</td>
<td>1.75</td>
<td>1.11</td>
<td>1.10</td>
<td>1.19</td>
<td>1.19</td>
<td>0.68</td>
</tr>
<tr>
<td>$\sigma_{\theta_t}$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>1.18</td>
<td>1.24</td>
<td>n.a.</td>
</tr>
</tbody>
</table>

**Case 1**  Exogenous Borrowing Limit $\bar{B}$

**Case 2a**  Endogenous Borrowing Limit, collateral evaluated at $Q_t$

**Case 2b**  Endogenous Borrowing Limit, collateral evaluated at $E_t Q_{t+1}$

**Case 3a**  Endogenous LTV, depending on $Y_t$

**Case 3b**  Endogenous LTV, depending on $E_t Y_{t+1}$
Correlation with Output: Labor Model
Standard Preference, LTV=0.7

<table>
<thead>
<tr>
<th>Case</th>
<th>1</th>
<th>2a</th>
<th>2b</th>
<th>3a</th>
<th>3b</th>
</tr>
</thead>
<tbody>
<tr>
<td>corr ( (c_t, y_t) )</td>
<td>0.96</td>
<td>0.99</td>
<td>0.98</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>corr ( (i_t, y_t) )</td>
<td>0.90</td>
<td>0.74</td>
<td>0.74</td>
<td>0.59</td>
<td>0.53</td>
</tr>
<tr>
<td>corr ( (n_t, y_t) )</td>
<td>0.45</td>
<td>0.65</td>
<td>0.64</td>
<td>0.85</td>
<td>0.84</td>
</tr>
<tr>
<td>corr ( (b_t, y_t) )</td>
<td>n.a.</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>0.99</td>
</tr>
<tr>
<td>corr ( (q_t, y_t) )</td>
<td>0.97</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>0.99</td>
</tr>
<tr>
<td>corr ( (\theta_t, y_t) )</td>
<td>n.a.</td>
<td>n.a.</td>
<td>n.a.</td>
<td>1.00</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Case 1  Exogenous Borrowing Limit \( \bar{B} \)
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Output Volatility: Labor Model

GHH Preference

Labor Model GHH: SD of y(t) wrt Theta

SD of y(t) in %

- exo BC
- endo BC, exp landpx
- endo-BC, current landpx
- endo-LTV, current Y
- endo-LTV exp Y

theta (collateral ratio)
Volatilities: Labor Model
GHH Preference, LTV=0.7

<table>
<thead>
<tr>
<th>Case</th>
<th>1</th>
<th>2a</th>
<th>2b</th>
<th>3a</th>
<th>3b</th>
<th>3b vs. 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{y_t}$</td>
<td>1.73</td>
<td>2.49</td>
<td>2.51</td>
<td>3.70</td>
<td>3.28</td>
<td><strong>1.89</strong></td>
</tr>
<tr>
<td>$\sigma_{c_t}$</td>
<td>1.48</td>
<td>1.95</td>
<td>1.96</td>
<td>2.69</td>
<td>2.44</td>
<td>1.64</td>
</tr>
<tr>
<td>$\sigma_{i_t}$</td>
<td>1.61</td>
<td>3.41</td>
<td>3.44</td>
<td>6.11</td>
<td>5.91</td>
<td>3.68</td>
</tr>
<tr>
<td>$\sigma_{n_t}$</td>
<td>1.03</td>
<td>1.49</td>
<td>1.50</td>
<td>2.21</td>
<td>1.97</td>
<td>1.90</td>
</tr>
<tr>
<td>$\sigma_{b_t}$</td>
<td>n.a.</td>
<td>1.49</td>
<td>1.53</td>
<td>3.75</td>
<td>2.96</td>
<td>n.a.</td>
</tr>
<tr>
<td>$\sigma_{q_t}$</td>
<td>1.66</td>
<td>1.52</td>
<td>1.53</td>
<td>2.13</td>
<td>1.91</td>
<td>1.15</td>
</tr>
<tr>
<td>$\sigma_{\theta_t}$</td>
<td>n.a.</td>
<td>n.a.</td>
<td>n.a.</td>
<td>1.71</td>
<td>1.10</td>
<td>n.a.</td>
</tr>
</tbody>
</table>

Case 1  Exogenous Borrowing Limit $\bar{B}$
Case 2a  Endogenous Borrowing Limit, collateral evaluated at $Q_t$
Case 2b  Endogenous Borrowing Limit, collateral evaluated at $E_t Q_{t+1}$
Case 3a  Endogenous LTV, depending on $Y_t$
Case 3b  Endogenous LTV, depending on $E_t Y_{t+1}$
Credit market restrictions can amplify the impact of productivity shocks.
Conclusion

- Credit market restrictions can amplify the impact of productivity shocks.
- In the baseline model with a fixed LTV, the amplification of output volatility is moderate, approximately 5%.

In the baseline model with endogenous LTV, output volatility can be 39% larger than the baseline case. The increase in output volatility is mainly driven by a drastic increase in the volatility of capital investment.

The relationship between the endogenous LTV ratio and output volatility is typically non-monotonic. When the steady-state value of the LTV is calibrated to approximately 0.75, the output volatility reaches its maximum.

In the model with labor-leisure choice, the endogenous LTV still amplifies output volatility. The amplification depends on the magnitudes of income effects.
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Appendix 1: Steady State of Baseline Model

- $K = (\beta \alpha_1)^{\frac{1}{1-\alpha_1}} [1 - \beta (1 - \delta)]^{-\frac{1}{1-\alpha_1}}$
- $Y = K^{\alpha_1}$
- $Q = \frac{\alpha_2 K^{\alpha_1}}{r}$
- $B = \theta Q$
- $C = K^{\alpha_1} - (1 - \delta) K - Br$
- $\theta = 1 - \exp(-\gamma Y)$

Note: For models with labor-leisure choice, closed-form solutions for the steady state are not available.
Log-Linearized System: Baseline Model
Case: Endo-BC, expected landpx, endo-LTV on current output

\[
\sigma c_t = E_t [\sigma c_{t+1} + (1 - \alpha_1) [1 - \beta (1 - \delta)] k_{t+1} - [1 - \beta (1 - \delta)] z_{t+1}]
\]

\[
\sigma (1 - \theta) c_t - q_t = E_t [\sigma (1 - \theta) c_{t+1} - \beta \alpha_1 r k_{t+1} - \beta r z_{t+1} - \beta q_{t+1}]
\]

\[
[\alpha_1 + s_k (1 - \delta)] k_t + z_t - s_c c_t - s_b R b_t = s_k k_{t+1} - s_b b_{t+1}
\]

\[
b_{t+1} - E_t q_{t+1} = 0
\]

\[
\tilde{\theta}_t = \left( \frac{1-\theta}{\theta} \right) Y (\alpha_1 k_t + z_t)
\]

\[
\rho z_t = z_{t+1} - \tilde{\epsilon}_{t+1}
\]

, where \( s_k = K / Y \), \( s_c = C / Y \) and \( s_b = B / Y \)

, where \( x_t \) or \( \tilde{x}_t \) is % deviation from steady state, \( x = c, k, b, q, z, \theta \)