Abstract

We examine the effect of product market competition on the internal organization of a multi-divisional firm. The firm consists of two divisions which produce substitutes or complements or who use common resources (capital, labor, ...) in production. Information about the demand conditions of each division is held by self-interested division managers who communicate via cheap talk. The only available formal mechanism is the allocation of decision rights. The effect of an increase in competitive pressure on the optimal allocation of decision rights depends on how such an increase affects the demand functions that the divisions face. If it leads to a parallel, downward shift, it favors decentralization. If it leads to an anti-clockwise rotation, however, it tends to favor centralization. Our findings therefore caution against the view that competition unambiguously favors decentralization.

Keywords: coordination, competition, cheap talk, incomplete contracts

JEL classifications: D23, D83, L23

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1 Introduction

Firms are organized to compete successfully with their rivals. In spite of this simple fact, economists know little about how competition affects the internal organization of firms. Most economic explanations of the internal organization of firms assume that they exist in a bubble, shielded from all outside influences. Those explanations that do take into account that firms operate in markets tend to focus on single unit firms that consist of one headquarters and one operating division. Such single unit firms were pervasive in the United States in the 19th century and are still common in industries in which firms are of medium and small size. Since the First World War, however, many industries in the United States have become dominated by large scale, multidivisional firms (Chandler 1977). These multidivisional firms are the focus of this paper. How do such firms organize to compete with their rivals? How does the competitive environment affect their internal organization? Does more intense competition lead to decentralization, as is often argued, or might it have the opposite effect? In this paper we address these questions.

Alfred Sloan, the longtime Chairman and President of General Motors, is widely regarded as one of the inventors of the multidivisional firm. Before he became Chairman in 1923, General Motors was a loose collection of different businesses that operated independently and competed with each other fiercely in spite of being part of the same holding company. As soon as he became Chairman, Sloan planned to replace this organization with one in which a strong headquarters ensured that the activities of the different divisions were coordinated — either by making decisions itself or by encouraging collaboration between the divisions — but in which divisions also had enough independence to be able to adapt quickly to changes in the business environment. As he put it at the end of his autobiography:

"It has been the thesis of this book that good management rests on a reconciliation of centralization and decentralization. [...] From decentralization we get initiative, responsibility, development of personnel, decisions close to the facts, flexibility — in short, all the qualities necessary for an organization to adapt to new conditions. From co-ordination we get efficiencies and economies" (Sloan 1964, p.429).

This trade-off between the ability to adapt to new conditions and the ability to realize economies from coordination and cooperation is at the center of our view of multidivisional firms. The ability to respond quickly to new conditions tends to be more important in more competitive industries. This is why many commentators and economists argue that competition favors decentralization. This view is supported by anecdotal evidence that many firms do indeed respond to more intense
competition by decentralizing. Perhaps more convincing, a recent wave of empirical work (Acemoglu et al. (2007), Bloom and Van Reenen (2008), Guadalupe and Wulf (2008)) has documented a robust correlation between decentralized organizational structures and various proxies for competition.

The channels through which competition affects organizational structure, however, are only poorly understood. The above argument that more intense competition makes local information more valuable, is at best incomplete as it does not ask why these firms used to be centralized in the first place. While centralization may fail to make optimal use of local information, also decentralization results in inefficient decision-making. More intense competition is likely to affect both sides of the trade-off. In addition, the proxies used to measure the intensity of product market competition, such as the lerner index (Bloom and Van Reenen), or import tariffs (Guadalupe and Wulf (2008)), reflect lower profitability rather than the need to be fast, flexible and innovative. Other channels, such as the impact of reduced profitability on the agency problems may therefore explain the the observed correlations between competition and centralization. Changes in the intensity of competition further come in various forms: lower import tariffs, changing exchange rates, or still, reduced barriers to imitation or entry such as lower patent protection or decreasing brand value. Depending on the source of increased competition (for example, lower import tariffs versus increased imitation by competitors), the impact on organizational structure may well be different.

As important, what is easily overlooked, is that competition may not only increase the benefits from decentralization – the ability to respond quickly to new conditions – but also the costs – the economies that are wasted because the divisions fail to coordinate and cooperate efficiently. An increase in competitive pressure may, for instance, make it more important that divisions share their know-how and realize scale economies. In such a situation a firm may then respond to an increase in competitive pressure by centralizing authority rather than decentralizing it. Unilever Group, one of the world’s largest consumer goods companies, is an example of such a firm. Until 1999 Unilever Group was very decentralized and division managers had vast decision making authority. Since then, however, Unilever Group has put in place a series of reorganizations to centralize authority and limit the power of division managers. Recently, the Unilever Group Chief Executive Patrick Cescau explained these reorganizations:

“Historically, Unilever’s business had been built up around highly autonomous operating companies, with their own portfolio priorities and all the resources they needed – marketing, development, supply chain – to develop their business in whatever way they saw fit. This was a highly effective way of building a truly multinational business almost 50 years before the term was invented. But it
had become less suited to an increasingly globalised, competitive landscape, where battles were being fought and won with global scale and know-how, and top-down, strategically driven allocation of resources. In today’s world, a hundred different portfolio strategies run the risk of adding up to no strategy at all. It’s not efficient, it doesn’t leverage your best assets and it doesn’t build strong global positions.”

Both theoretical reasoning and anecdotal evidence therefore suggest that the effect of competition on the internal organization of multidivisional firms is in general ambiguous. The aim of this paper is to shed some light on when competition is likely to favor decentralization and when it is likely to favor centralization.

By explicitly modeling the trade-off between centralization and decentralization and imbed this trade-off in a model of product market competition, we aim to throw deeper insight into the interaction between competition and organization. We introduce multi-divisional firms in a classic model of competition between differentiated firms (building on Vives and Singh (1979, 1984).) In particular, we consider a firm consisting of two divisions who produce products that either use similar inputs (resulting in positive or negative cost externalities) and/or share the same customers (and hence are substitutes or complements of each other). Outside competition is modeled in the form of competitive fringes who produce products that are substitutes to one of these two divisions. Division managers are privately informed about the intercept of their (linear) demand curve, and may communicate this information to each other or head-quarters using cheap talk. But since divisional managers are biased towards the maximization of divisional profits, such communication is (endogenously) noisy. The only available formal mechanism is the allocation of decision rights. In particular, production decisions are either set by these division managers (decentralization) or by a head-quarters (centralization).

In this setting decentralization involves precisely the trade-off that Sloan described in the above quote. The benefit of decentralization is that it improves the firm’s ability to adapt to new conditions. This is the case since the division managers can adapt their decisions to the exact demand conditions that they observe while headquarters has to base its decisions on noisy information. The costs of decentralization are the economies that are wasted because the divisions fail to coordinate and cooperate. The division managers fail to coordinate their decisions because each division manager is uncertain about the demand conditions in the other market. As a result he

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1 This quote is taken from a presentation that Patrick Cescau gave at the Unilever Investor Seminar on 13 March 2007. The transcript is available at www.unilever.com. For more on the reorganizations at Unilever see, for instance, “Too Many Cooks: Despite Revamp, Unwieldy Unilever Falls Behind Rivals; Consumer-Products Giant Labors to Slim Operations; Two Chairmen in Charge; A Deodorant With 48 Formulas,” Wall Street Journal, January 3, 2005.
is unsure about what decision his counterpart is going to make which makes coordination difficult. The failure to coordinate is therefore primarily an informational problem. In addition to failing to coordinate their decisions, the division managers also fail to cooperate. What we mean by that is that neither division manager takes into account the (cost or demand) externalities that his decisions impose on the other division. Even if there were no informational problems, therefore, the division managers’ decisions would not maximize the firm’s profits.

The main question we are interested in is how the intensity of competition between the divisions and their competitive rivals affects this trade-off between centralization and decentralization. As we show, the effect of an increase in competitive pressure on the internal organization depends largely on whether it leads to a shift or a rotation of the inverse demand functions that the divisions face.

First, more competition in the form of increased price or cost pressures – where substitute products become cheaper or inputs become more expensive – always makes centralization more attractive. This change in competition results in a parallel downwards shift of the inverse demand curve, and its main effect is to reduce profit margins. While this does not affect the benefits of decentralization, it does reduce the benefits of centralization. Indeed, in our model, firms centralize decision-making to internalize demand or cost externalities between divisions. If the divisions produce substitutes, for example, it may be valuable to centralize pricing in order to reduce businessstealing or cannibalization. If profit margins are low to begin with, however, there is not much to be "stolen". It is then typically more valuable to decentralize pricing so divisions can make better use of their demand information. Indeed, getting pricing right is equally valuable on the margin regardless of whether profits are high or low. The same argument holds if division produce complements or if they compete in the same input market (e.g. when the firm has an internal labor markets or an internal capital market).

Second, more competition due to new entry or due to the re-positioning by existing competitors may actually result in centralization. Whereas increased price or cost pressures results in a downwards shift of the inverse demand curve, re-positioning by existing competitors results in a counter-clockwise rotation of the demand curve. This give rise to two opposing forces. On the one hand, as is the case with a parallel downward shift, this reduces profits margins and favors decentralization as argued above. On the other hand, the fact that a division faces a flatter demand exacerbates the coordination problem and incentive conflict between divisions and therefore increases the benefits of intervention by a central headquarter. As we show, the optimal response to a new entrant may therefore beto centralize pricing. Interestingly, a flatter demand curve also makes "pricing mistakes" more costly, supporting the view that more intense competition makes
information about demand more valuable. This does not result in more decentralization, however, as also "pricing mistakes" by the divisional managers, who ignore externalities, are punished more heavily. At the margins, these two effects tend to wash out. The argument that more competition makes information more valuable and, therefore, results in more decentralization is therefore flawed, as it only looks at one side of the trade-off.

Literature review: To be completed

2 The Model

A firm consists of two operating divisions, Divisions 1 and 2, and potentially a headquarters. Each operating division produces a single product. The profits of Division $j = 1, 2$ are given by

$$\pi_j = (p_j(\theta_j) - v_j) q_j,$$

where $p_j(\theta_j)$ and $q_j$ are the inverse demand function and the production level of product $j$ and $v_j$ the average production costs. The demand conditions in Market $j$ depend on the demand shock $\theta_j$ in a way that we will explain below. Headquarters does not produce anything and therefore does not generate any profits.

Information: Each division is run by one manager. Manager 1 is in charge of Division 1 and privately observes $\theta_1$, the demand shock in his market. He does not, however, observes $\theta_2$, the demand shock in the other market. Similarly, Manager 2 observes $\theta_2$ but does not know $\theta_1$. Headquarters observes neither $\theta_1$ nor $\theta_2$. It is common knowledge that $\theta_1$ and $\theta_2$ are uniformly distributed on $[-s, s]$ and that the draws of $\theta_1$ and $\theta_2$ are independent. We denote the variance of the demand shocks by $\sigma^2$.

Preferences: Manager 1 maximizes $\lambda \pi_1 + (1 - \lambda) \pi_2$ and Manager 2 maximizes $\lambda \pi_2 + (1 - \lambda) \pi_1$, where $\lambda \in [1/2, 1]$. The parameter $\lambda$ captures how biased each division manager is towards his own division. Headquarters cares about overall firm profits and thus maximizes $\pi_1 + \pi_2$. We follow our modelling approach in Alonso, Dessein, and Matouschek (2008) and assume that the own-division bias is exogenously given. It will become clear below that if the own-division bias $\lambda$ were endogenous and chosen by the firm, it would always set $\lambda = 1/2$. There would then be no incentive conflicts and all organizational structures would perform equally well. There are, however, many reasons outside of our model that explain why division managers are often biased towards their own divisions.2 The assumption that $\lambda > 1/2$ is therefore easy to defend. What is

2 See, for instance, Alonso, Dessein, and Matouschek (2008), Athey and Roberts (2001), and Rantakari (forthcoming). General Motors provides a historical example of a firm with a very large $\lambda$: "Under the incentive system in operation before 1918, a small number of division managers had contracts providing them with a stated share in the profits of their own divisions, irrespective of how much the corporation as a whole earned. Inevitably, this system exaggerated the self-interest of each division at the expense of the interests of the corporation itself. It was even possible for a division manager to act contrary to the interests of the corporation in his effort to maximize his own..."
more difficult to defend is that the own-division bias $\lambda$ is constant and does not change with the organizational structure of the firm or the extent of competition that the firm faces. We make this assumption nevertheless because it keeps the model clean and simple and still allows us to derive new insights into the effects of competition on the decentralization decision of firms. We discuss this issue further in Section 7.

Cost: The average production costs are given by

$$v_1 = c + \beta c q_1 + \gamma c q_2$$

and

$$v_2 = c + \beta c q_2 + \gamma c q_1$$

We have in mind that each product is produced using a leontief technology, where $v_1$ is the average price of all inputs needed for one unit of production. A positive interaction term $\gamma c > 0$ then reflects that divisions are competing with each other for (specialized) labor, internal capital, or other scarce company resources, driving up the "price" of these inputs. A more efficient global capital market, for example, is consistent with a lower $\gamma c$. A negative interaction term $\gamma c < 0$, on the other hand, points to scale economies, with higher production levels driving down the average costs of inputs.

Demand: The operating divisions face linear inverse demand functions

$$p_1 = a + \theta_1 - q_1 - \gamma^d q_2 - \rho q f_1$$

and

$$p_2 = a + \theta_2 - q_2 - \gamma^d q_1 - \rho q f_2,$$

where $a \in [1, \infty)$, $\gamma \in [-1, 1]$, and $\rho \in [0, 1]$. We interpret the intercept $a + \theta j$ as a measure of the strength of demand in Market $j = 1, 2$. A positive realization of $\theta j$ therefore indicates stronger than expected demand and a negative realization indicates weaker than expected demand. On average demand is equally strong in both markets. The parameter $\gamma$ captures the extent of differentiation between the goods produced by the two divisions. When $\gamma > 0$, the products are substitutes and when $\gamma < 0$, they are complements. When $\gamma = 0$, there are no demand externalities between products. Finally, Division $j = 1, 2$ faces competition from a competitive fringe that produces $q f_j$ units of a substitute product. The larger the parameter $\rho$, the more similar are goods produced by Division $j$ and its competitive fringe.3

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3 Maybe put in an illustration of this kind of demand structure (Hotelling or vertical differentiation).
The inverse demand functions of the two competitive fringes are given by

\[ p_{f1} = 1 - q_{f1} - \rho q_1 \]  
(3)
and

\[ p_{f2} = 1 - q_{f2} - \rho q_2. \]  
(4)

Entry/exit in the competitive fringe occurs until \( p_{f1} \) equals the marginal costs \( c_f \in [0,1] \). Substituting \( p_{f1} = c_f \) into (3), solving for \( q_{f1} \), and then substituting into (1) gives the residual inverse demand function for Market 1:

\[ p_1 = a - \rho (1 - c_f) + \theta_1 - (1 - \rho^2) q_1 - \gamma^d q_2. \]  
(5)

Similarly, the residual inverse demand function for Market 2 is given by

\[ p_2 = a - \rho (1 - c_f) + \theta_2 - (1 - \rho^2) q_2 - \gamma^d q_1. \]  
(6)

To simplify notation, we define the intercepts as \( \alpha_j \equiv a - \rho (1 - c_f) + \theta_j, \ j = 1,2 \), and the absolute value of the slope as \( \beta \equiv (1 - \rho^2) \). Further we denote the expected value of the intercept by \( \mu \equiv a - \rho (1 - c_f) \). We can then rewrite (5) and (6) as

\[ p_1 = \alpha_1 - \beta^d q_1 - \gamma^d q_2 \]

and

\[ p_2 = \alpha_2 - \beta^d q_2 - \gamma^d q_1. \]

In what follows we will abuse terminology somewhat and refer to these residual inverse demand functions simply as “inverse demand functions.”

**Competition:** There are two ways in which we capture an increase in the degree of product market competition that the firm faces. First, we say that the firm faces increased “cost pressure” if the competitive firms become more cost efficient, that is, if \( c_f \) is reduced. An increase in cost pressure reduces the expected intercept of the inverse demand functions \( \mu \equiv a - \rho (1 - c_f) \) but does not change the absolute value of the slope \( \beta \equiv (1 - \rho^2) \). Second, we say that the firm faces increased “imitation pressure” if the goods produced by the competitive firms become more similar to those produced by the two divisions, that is, if \( \rho \) is increased. An increase in imitation pressure reduces both the expected intercepts and the absolute value of the slope of the inverse demand functions. Thus, while an increase in cost pressure leads to a parallel, downward shift of the inverse demand functions, an increase in imitation pressure leads to an anti-clockwise rotation.
key insight of our paper is that the effect of an increase in product market competition on the decentralization decision of the firm depends crucially on whether it leads to a shift or a rotation of the inverse demand functions.

**Profits:** Total profits can then be rewritten as

\[
\pi_1 = p_1 q_1 - v_1 q_1 \\
= (\alpha_1 - c)q_1 - (\beta^c + \beta^d)q_1^2 - (\gamma^c + \gamma^d)q_1 q_2
\]

and

\[
\pi_2 = (\alpha_2 - c)q_2 - (\beta^c + \beta^d)q_2^2 - (\gamma^c + \gamma^d)q_1 q_2
\]

Abusing the terminology of Singh and Vives (1984), we define \( t \equiv (\gamma^c + \gamma^d)/(\beta^c + \beta^d) \) as the degree of product differentiation. In contrast to Singh and Vives, the product differentiation pertains here both to input markets (do the products use similar inputs) as to output markets (do they sell to similar consumers). We assume that

\[
\beta \equiv \beta^c + \beta^d > 0
\]

and

\[
t \leq \frac{E(\alpha - c) - s}{E(\alpha - c) + s}
\]

which ensures that in equilibrium both divisions engage in (bounded) production and thus avoids corner solutions.

**Decisions and Contracts:** The firm must either decide on the prices that the divisions charge or on their capacity levels. We follow the property rights literature (Grossman and Hart 1986, Hart and Moore 1990) in assuming that contracts are highly incomplete. In particular, the firm can only commit to an ex ante allocation of decision rights. The managers are unable to contract over the decisions themselves and over the communication protocol that is used to aggregate information. Once the decision rights have been allocated, they cannot be transferred before the decisions are made. We focus on two allocations of decision rights. Under Decentralization Manager \( j = 1, 2 \) has the right to set the capacities \( q_j \). Under Centralization the Headquarter Manager has the right to decide on the capacities.

We do not believe that in the real world firms are entirely unable to contract over capacities. Their ability to do so, however, must be at least somewhat limited. Otherwise there would be no need to decentralize pricing and capacity decisions, as many firms do (for evidence that many firms decentralize such decisions see Marin and Verdier 2008). The real world seems to be somewhere between the complete contracting benchmark that has been studied in the mechanism
design approach to organizational design and the incomplete contracting benchmark that we adopt here.\footnote{For a survey of the mechanism design approach to organizational design see Mookherjee (2006).} While neither benchmark is a perfect description of reality, they both provide useful insights into the organizational structure of firms.

**Communication:** Under Decentralization Manager 1 sends message $m_1 \in M_1$ to Manager 2 and, simultaneously, Manager 2 sends message $m_2 \in M_2$ to Manager 1. Under Centralization, Managers 1 and 2 send messages $m_1 \in M_1$ and $m_2 \in M_2$ to headquarters. We refer to communication between the division managers as “horizontal communication” and to that between the division managers and headquarters as “vertical communication.” Let $E[\theta_j | m_j]$ be the posterior of headquarters (under vertical communication) or division manager $i$ (under horizontal communication), where we assume that decision-makers are bayesian updaters.\footnote{Note that this implies that $E(E[\theta_j | m_j]) = E(\theta_j)$} We denote by

$$r_k^2 \sigma^2 = E \left[ (\theta_j - E[\theta_j | m_j])^2 \right], \, k = C, D$$

the residual variance, respectively under vertical communication ($r_C^2 \sigma^2$) and horizontal communication ($r_D^2 \sigma^2$). Whenever $r_C^2 \sigma^2 > 0$ or $r_D^2 \sigma^2 > 0$, some information is lost in the communication process. This loss in information could be simply due to physical communication constraints, as assumed and motivated in, for example, Aoki (1986), Dessein and Santos (2006) and Cremer, Garicano and Pratt (2007)). Alternatively, the loss in information could be due to strategic communication, as assumed in, for example, Crawford and Sobel (1982) and Alonso, Dessein and Matouschek (2006).

Rather than imposing a specific communication or monitoring technology, section 3 simply takes $r_C^2 \sigma^2 = r_D^2 \sigma^2 = r \sigma^2 > 0$ exogenously. Section 4 then endogenizes these information losses by assuming that division-managers communicate their information strategically, as in Crawford and Sobel (1982). Indeed, division managers have incentives to distort their (demand) information in order to influence decision-making by head-quarters or rival divisions. The incentive to distort information, however, are different for horizontal and vertical communication, and further depend on the degree of differentiation among the divisions, the level of competition, ... Nevertheless, we show that the results obtained with exogenous communication are generally robust.

**Timing:** The game is summarized in Figure 2. First, decision rights are allocated to maximize the total expected profits $E[\pi_1 + \pi_2]$. Under Decentralization each division manager has the right to decide on how much of his product to produce. Under Centralization the Headquarter Manager has the right to decide on production levels. Second, the division managers become informed about their local demand conditions, that is, they learn $\theta_1$ and $\theta_2$ respectively. Third, the division
managers communicate with the decision makers. Under Centralization they engage in vertical communication, sending messages $m_1$ and $m_2$ to headquarters, while under Decentralization they engage in horizontal communication, exchanging messages $m_1$ and $m_2$ with each other. Finally, the decision makers decide on production levels and payoffs are realized. Each decision maker chooses the decision that maximizes his or her payoff given the information that has been communicated.

We are almost ready to solve the model. Before we do so, though, we want to point out that we could easily change the model to allow for privately observed cost rather than demand shocks. In particular, the results would be identical if instead of observing the intercepts of the inverse demand functions, the division managers observed shocks to their variable production costs. The analysis can also be easily extended to Bertrand competition rather than Cournot competition. In fact, if $\gamma^d = 0$, that is there are only cost externalities between divisions, Bertrand and Cournot competition are equivalent.

## 3 Organizational Performance

We do first what the managers do last: examine their production decisions.

### 3.1 Centralized Decision-making

When decision rights are centralized, headquarters first receives messages $m_1$ and $m_2$ and then makes the decisions that maximize $E[\pi_1 + \pi_2 | m]$, where $m = (m_1, m_2)$. If headquarters decides on capacities, it sets

\[ q_1^C = E[\alpha_1 - c | m_1] - t q_2^C \]  

and

\[ q_2^C = E[\alpha_2 - c | m_2] - t q_1^C, \]

where $\alpha_1$, $\alpha_2$, and $\beta$ are the intercepts of the inverse demand functions and $t = \gamma/\beta$ is the degree of product differentiation. The capacity of each division depends on the expected demand conditions in its market and on the capacity of the other division. The first best capacities therefore need to be coordinated. To obtain closed form expressions, we solve (9) and (10) for $q_1^C$ and $q_2^C$. Doing so gives

\[ q_1^C = \frac{E[\alpha_1 - c | m_1] - t E[\alpha_2 - c | m_2]}{2 \beta (1 - t^2)} \]  

and

\[ q_2^C = \frac{E[\alpha_2 - c | m_2] - t E[\alpha_1 - c | m_1]}{2 \beta (1 - t^2)}. \]
Now that we have characterized decision making under Centralization, we can work out the expected profits under this organizational structure.

**Lemma 1:** Under Centralization, the firm’s expected profits are given by

$$\Pi^C (r^2_C) = \Pi^* - \frac{1}{2\beta (1 - t^2)} r^2_C \sigma^2,$$

where \(\Pi^*\) are the first best expected profits and

$$r^2_C \sigma^2 = E\left[ (\theta_j - E[\theta_j|m_j])^2 \right].$$

The lemma shows that the expected profits differ from the first best expected profits only because communication is noisy. If communication were perfect, the residual variance \(r^2_C \sigma^2\) would be equal to zero and the expected profits \(\Pi^C (r^2_C)\) would be equal to the first best expected profits \(\Pi^*\). The worse the quality of communication, the larger the residual variance and the lower the expected profits, as one would expect.

### 3.2 Decentralized Decision Making

When decision rights are decentralized, the division managers first exchange messages \(m_1\) and \(m_2\) and then make their decisions. The decision that Manager 1 makes maximizes \(E[\lambda \pi_1 + (1 - \lambda) \pi_2 | \theta_1, m]\) and the decision that Manager 2 makes maximizes \(E[(1 - \lambda) \pi_1 + \lambda \pi_2 | \theta_2, m]\), where \(\lambda\) is the own-division bias. If the divisions engage in Cournot Competition, the division managers set

$$q^D_1 = \frac{\alpha_1 - c}{2\beta} - \frac{t}{2\lambda} E[q^D_2 | m_2]$$

and

$$q^D_2 = \frac{\alpha_2 - c}{2\beta} - \frac{t}{2\lambda} E[q^D_1 | m_1],$$

where \(\alpha_1, \alpha_2,\) and \(\beta\) are the intercepts of the inverse demand functions and \(t = \gamma/\beta\) is the degree of product differentiation. A comparison between these expressions and the centralized capacity choices in (9) and (10) reveals a key difference between Centralization and Decentralization. Under Centralization headquarters’ imperfect knowledge of the demand conditions limits its ability to adapt the capacity of each division to the demand conditions in that market. It does not, however, limit its ability to coordinate capacities across divisions. In contrast, under Decentralization the division managers’ imperfect knowledge of the demand conditions in each others’ markets limits their ability to coordinate capacities across divisions. It does not, however, affect their ability to adapt their capacities to the demand conditions in their respective markets. To see this, note that headquarters’ capacity choice for Division 1 depends on the expected market conditions in Market
1 and on the actual capacity choice for Division 2, as can be seen in (9). In contrast, (13) shows that Manager 1’s capacity choice depends on the actual demand conditions in Market 1 and on the expected capacity choice for Division 2.

To obtain closed form expressions, we take expectations of (13) and (14), solve for the expected capacities, and then substitute back into (13) and (14). Doing so gives

\[ q_D^1 = \frac{\alpha_1 - c}{2\beta} - t \frac{2\lambda E[\alpha_2 - c | m_2] - tE[\alpha_1 - c | m_1]}{2\beta (4\lambda^2 - t^2)} \]  

and

\[ q_D^2 = \frac{\alpha_2 - c}{2\beta} - t \frac{2\lambda E[\alpha_1 - c | m_1] - tE[\alpha_2 - c | m_2]}{2\beta (4\lambda^2 - t^2)} \]  

We can now state the expected payoffs under Decentralization.

**Lemma 2:** Under Decentralization, expected profits are given by

\[
\Pi^D (\lambda, r_D^2) = \Pi^* - (2\lambda - 1)^2 t^2 (1 - t)(2\lambda - t)^2 E(\alpha - c)^2 + (4\lambda^2 + (4\lambda + 1)t^2) \sigma^2 \\
- t^2 4\lambda^2 (4\lambda - 1) - t^2 \frac{r_D^2 \sigma^2}{2\beta (4\lambda^2 - t^2)^2}
\]

where \( \Pi^* \) are the first best expected profits and \( r_D^2 \sigma^2 \) is the residual variance

\[ r_D^2 \sigma^2 = E[(\theta_j - E[\theta_j | m_j])^2] \]

The expected profits differ from the first best expected profits because decision making by the division managers is biased and because communication between them is imperfect. Note that the worse the quality of communication — that is, the larger the residual variance \( r_D^2 \sigma^2 \) — the lower the expected profits if the divisions engage in Cournot Competition.

### 4 Decomposing the Gains from Decentralization

In principle we could now turn to our main question and examine when it is optimal for the firm to decentralize. To do so, we would have to work out the gains from decentralization — by subtracting the expected profits of the centralized firm from those of the decentralized firm — and try to make sense of its sign. Doing so without direction, however, would bury most economic insights beneath tedious algebra. To avoid this fate, we use this section to decompose the gains from decentralization into three economically meaningful effects. This will allow us to make sense of the gains from decentralization and explain the firm’s decentralization decision in the next section.
Decentralization affects the firm’s expected profits through different channels. At least since Jensen and Meckling (1992) economists have known that decentralization involves a trade-off between the ability to adapt to new conditions and the costs of biased decisions making, what we called above the “failure to cooperate.” In the words of Jensen and Meckling: “Determining the optimal level of decentralization requires balancing the costs of bad decisions owing to poor information and those owing to inconsistent objectives” (Jensen and Meckling 1992, p.18). What has been somewhat overlooked is a third factor that is important when the firm consists of multiple divisions: decentralization then disperses decision rights among different managers which affects the firm’s ability to coordinate its decisions. This is what we call the coordination effect. In our setting the gains from decentralization can be decomposed into the gain in information, the loss of control, and the coordination effect. In particular, decentralization will be preferred over centralization whenever

\[ \Pi^D (\lambda, r^2_D) - \Pi^C (r^2_C) > 0 \]

which can be rewritten as

\[ \underbrace{\Pi^D (\lambda, r^2_D) - \Pi^D (1/2, r^2_D)}_{\text{loss of control}} + \underbrace{\Pi^D (1/2, r^2_D) - \Pi^D (1/2, 0)}_{\text{coordination loss}} + \underbrace{\Pi^C (0) - \Pi^D (r^2_C)}_{\text{gain in information}} \]

In the above expression, \( \Pi^D (r^2_D, 1/2) \) are the expected profit under Decentralization if both division managers are unbiased, but must rely on noisy communication about the other’s market demand condition and \( \Pi^D (0, 1/2) \) are the expected profits under Decentralization if both division managers are unbiased and perfectly informed about each others’ demand conditions. Since there is no difference between centralization and decentralization in the absence of incentive conflicts and information asymmetries, we have that \( \Pi^D (0, 1/2) = \Pi^C (0) \).

Next we discuss each effect and how it depends on the demand characteristics. We focus on the demand characteristics since ultimately we are interested in how changes in the intensity of competition affect the firm’s decentralization decision and since changes in the intensity of competition affect the firm only by changing demand for its product.

**Gain in Information** We measure the gain in information by the expected profits of the centralized firm if headquarters could directly observe the local demand conditions minus its expected profits if headquarters has to rely on noisy communication from the division managers. The gain in information is therefore given by

\[ \Pi^C (0) - \Pi^C (r^2_C) = \frac{r^2_C \sigma^2}{2 \beta (1 - t^2)} \geq 0. \]
Intuitively, the gain in information it is larger, the worse the quality of vertical communication. For a given quality of communication, the gain in information does not depend on the expected intercept of the inverse demand functions that the divisions face. A parallel shift of the inverse demand functions therefore does not have a direct effect on the gain in information. What does have a direct effect on the gain in information is a flattening of the inverse demand functions: for given \( r^2 \sigma^2 \), a reduction in \( \beta \) increases the gain information. Essentially, the flatter the inverse demand functions, the more costly is it for the firm to make pricing and capacity mistakes and thus the greater the value of information.

**Loss of Control** We measure the loss of control by the expected profits of the decentralized firm with fully aligned managers (\( \lambda = 1/2 \)) minus those of the decentralized firm with biased managers (\( \lambda > 1/2 \)), keeping the information structure fixed. This loss of control is given by

\[
\Pi^D (1/2, r_D^2) - \Pi^D (\lambda, r_D^2)
\]

\[
= \frac{(2\lambda - 1)^2 t^2 (1-t) (2\lambda - t)^2 E(\alpha - c)^2 + (4\lambda^2 + (4\lambda + 1)t^2) (\sigma^2 - r_D^2 \sigma^2)}{(1 - t^2)(4\lambda^2 - t^2)^2} \geq 0
\]

The flatter the inverse demand functions, that is, the smaller \( \beta \), the larger the control losses. In contrast, a reduction in the expected intercept \( \mu \) reduces the control losses. To understand this, note that a reduction in \( E(\alpha - c) \) reduces the price-cost margin and thus the profits of each division. It therefore also reduces the cost of the externalities that the divisions impose on each other when the firm is decentralized. In other words, it reduces the loss of control.

**Loss of Coordination** We measure the coordination effect by assuming that division managers are perfectly aligned (\( \lambda = 1/2 \)) and by comparing the expected profits of the decentralized firm if the division managers could observe the demand conditions in both markets – and thus could predict each others’ decisions – minus the actual expected profits under the actual information structure with \( r^2_D \sigma^2 > 0 \). This coordination effect is given by

\[
\Pi^D (1/2, 0) - \Pi^D (1/2, r_D^2) = \frac{t^2}{2\beta (1 - t^2)} r_D^2 \sigma^2 \geq 0
\]

The positive sign implies that the decentralized firm is adversely affected by the division managers’ inability to perfectly coordinate their capacity choices. Naturally, the worse the quality of horizontal communication, the larger the coordination effect. For a given quality of communication, the coordination effect does not depend on the expected intercept of the inverse demand functions. It does, however, depend on their slope. In particular, the flatter the inverse demand functions, the more important it is to coordinate capacities, other things equal.
5 The Internal Organization of the Firm

Having done all this work, we can now turn to our main questions: when does the firm decentralize and how does its decision depend on the intensity of competition? We answer the first question in the next section and the second one in Section 5.2.

5.1 The Decision to Decentralize

The firm decentralizes if the gains from decentralization \( \Pi^D (\lambda, r_D^2) - \Pi^C (r_C^2) \) are positive, that is if and only if the "gain in information" minus the "loss of coordination" is larger than the loss of control,

\[
\Pi^C (0) - \Pi^C (r_C^2) - [\Pi^D (1/2, 0) - \Pi^D (1/2, r_D^2)] \geq \Pi^D (1/2, r_D^2) - \Pi^D (\lambda, r_D^2)
\]

or still

\[
\frac{r_C^2 \sigma^2}{2 \beta (1 - t^2)} - \frac{r_D^2 t^2 \sigma^2}{2 \beta (1 - t^2)} \geq \left( \frac{(2\lambda - 1)^2 t^2 \sigma^2}{2 \beta} \right) \times \left( \frac{1}{(1 + t)(2\lambda + t)^2} \frac{E(\alpha - c)^2}{\sigma^2} + \frac{(4\lambda^2 + (4\lambda + 1)t^2)}{(1 - t^2)(4\lambda^2 - t^2)^2 (1 - r_D^2)} \right)
\]

If we assume \( r_C^2 \sigma^2 = r_D^2 \sigma^2 = r^2 \), the necessary and sufficient condition for decentralization to be preferred becomes

\[
r^2 \geq (2\lambda - 1)^2 t^2 \left( \frac{1}{(1 + t)(2\lambda + t)^2} \frac{E(\alpha - c)^2}{\sigma^2} + \frac{(4\lambda^2 + (4\lambda + 1)t^2)}{(1 - t^2)(4\lambda^2 - t^2)^2 (1 - r^2)} \right)
\]

One can verify that the RHS is increasing in the own-division bias of division managers (\( \lambda \)), the level of demand \( E(\alpha - c)^2 \) (and, hence, the expected profit margin \( 1/2)E(\alpha - c)^2 \) under centralization) and \( |t| \), the degree of (input or output) substitutability or complementarity. These changes therefore make centralization more attractive. On the other hand, decentralization is more likely to be optimal if either demand uncertainty is larger (\( \sigma^2 \) is larger) or communication is worse (\( r^2 \) is larger.

The following proposition characterizes our results:

PROPOSITION 1: Suppose \( r_C^2 = r_D^2 = r^2 > 0 \) and \( \lambda > 1/2 \), then decentralization is optimal if and only if

1) \( 0 < t \leq t_1 \) and \( \frac{\lambda}{E(\alpha - c)} > f_1(t) \), where \( t_1 \) and \( f_1(t) \) are defined in the proof and \( f_1(0) = 0 \) and \( f'_1(t) > 0 \) for any \( t \in [0, t_1] \). Furthermore, for any \( t \in [0, t_1] \), the function \( f_1(t) \) is increasing in \( \lambda \) and decreasing in \( r^2 \).
2) $t_2 < t \leq 0$ and $s/\mu > f_2(t)$, where $t_2$ and $f_2(t)$ are defined in the proof, $f_2(0) = 0$ and $f'_2(t) < 0$ for any $t \in [t_2, 0]$. Furthermore, for any $t \in [t_2, 0]$, the function $f_2(t)$ is increasing in $\lambda$ and decreasing in $r^2$.

The proposition is illustrated in Figures 1, where the curve most to the right are the boundaries of the permissible parameter space and the curve separating the Centralization and Decentralization regions are the functions $f_j(t)$, $j = 1, 2$, in the proposition. The figure makes clear how changes in the demand characteristics affect the firm’s decentralization decision: a reduction in the expected intercept of the inverse demand functions $\mu$ makes it more likely that the firm decentralizes and a reduction in $\beta$ makes it more likely that the firm centralizes. The proposition also shows that an increase in the own-division bias $\lambda$ or a improvement in communication (reduction in $r^2$) unambiguously favors Centralization. This is the case because an increase in $\lambda$ distorts decision making under Decentralization but not under Centralization, and because imperfect communication hurts centralization (information gain) more than decentralization (coordination loss).

5.2 The Effect of Competition on the Decision to Decentralize

The effect of an increase in competition on the firm’s decentralization decision depends crucially on how it affects the inverse demand functions that the firm faces. Suppose first that the increase in competition is caused by an increase in cost pressure, that is, by a reduction in the marginal costs $c_f$ of the firm’s competitors. As was discussed previously, an increase in cost pressure reduces
the expected intercept of the inverse demand functions \( \mu = a - \rho (1 - c_f) \) but does not affect the absolute value of the slope \( \beta = (1 - \rho^2) \). In other words, it leads to a parallel, downward shift of the inverse demand functions. We know from the previous section that such a shift favors Decentralization because it reduces the loss of control. This confirms the standard intuition that an increase in competition can induce firms to move decision rights down the hierarchy to managers who are closer to the customers and better informed about their needs and desires. We want to stress, however, that an increase in cost pressure does not favor Decentralization because it makes the division managers’ information more valuable, as is often argued. A parallel, downward shift of the inverse demand functions has no direct effect on the gain in information. If anything, such a shift weakens the information-based argument for Decentralization since it tends to improve the quality of vertical communication. In our setting an increase in cost pressure favors Decentralization because it mitigates the conflict between the division managers, not because it makes local information more valuable. The next proposition, which is illustrated in Figure 2, summarizes the effect of an increase in cost pressure on the firm’s decentralization decision.

PROPOSITION 2: Suppose that \( \tau_i^2 = \tau_D^2 > 0 \) and the division managers are biased towards their divisions. Then an increase in cost pressure (parallel downward shift of demand curve) makes it more likely that the firm decentralizes.

Suppose next that the increase in competition is caused by an increase in imitation pressure, that is, by the goods of the firm’s competitors becoming more similar to its own. As was discussed
previously, an increase in imitation pressure corresponds to an increase in \( \rho \) and thus lowers the expected intercept of the inverse demand functions \( \mu = a - \rho (1 - c_f) \) as well as the absolute value of the slope \( \beta^d = (1 - \rho^2) \). In other words, it leads to an anti-clockwise rotation of the inverse demand functions. Such a rotation has two opposing effects on the firm’s decentralization decision. On the one hand, the reduction in the expected intercepts favors Decentralization, as was just discussed. On the other hand, however, the flattening of the inverse demand functions favors Centralization, for reasons that were discussed in the previous section. The effect caused by the flattening of the inverse demand functions can be larger than that caused by the reduction in the expected intercept. As a result, an increase in competition can lead to Centralization. To see this, consider Figure 1 again. An increase in \( \rho \), increases both

\[
\begin{align*}
t &= \frac{\gamma^d + \gamma^c}{1 - \rho^2 + \beta^c} \\
1 - \frac{\gamma^d + \gamma^c}{t} + \beta^c &= \rho^2
\end{align*}
\]

and

\[
\frac{s}{E(\alpha - c)} = \frac{s}{a - c - \rho (1 - c_f)}.
\]

It therefore corresponds to a move in northeasterly direction in Figure 2. Simply eyeballing the figures already suggests that such a move can lead to a switch from Decentralization to Centralization.

To be more precise, consider Figure 3. This figure replicates Figure 1 but also shows all possible values of \( t \) and \( s/E(\alpha - c) \) for given \( s, \gamma, a, \) and \( c_f \). When \( \rho = 0 \), \( t = \gamma \) and \( s/\mu = s/a \). This point is indicated by point A in Figure 3. The largest permissible value of \( \rho \) is denoted by \( \overline{\rho} \) and solves the parameter restriction (7) with equality. It is therefore implicitly defined by

\[
\frac{\gamma^d + \gamma^c}{1 - \rho^2 + \beta^c} = \frac{a - c - \overline{\rho} (1 - c_f) - s}{a - c - \rho (1 - c_f) + s}.
\]

When \( \rho = \overline{\rho} \), \( t = (\gamma^d + \gamma^c)/(1 - \rho^2 + \beta^c) \) and \( s/\mu = s/(a - \overline{\rho} (1 - c_f)) \). This point is indicated by point B in Figure 3. The function \( g(t) \) that connects \( A \) and \( B \) shows the values of \( t \) and \( s/\mu \) for any \( \rho \) between \( \rho = 0 \) and \( \rho = \overline{\rho} \). We derive the function \( g(t) \) by solving (21) for \( \rho \) and then substituting for \( \rho \) in (22). This gives

\[
g(t) = \frac{s}{a - c - (1 - c_f) \sqrt{1 + \beta^c - (\gamma^d + \gamma^c)/t}} \text{ for } t \in \left[ \frac{\gamma^d + \gamma^c}{1 + \beta^c}, \frac{\gamma^c + \gamma^d}{1 - \overline{\rho}^2 + \beta^c} \right].
\]

We can now describe the effect of an increase in imitation pressure by examining the properties of \( g(t) \). The function has three key properties. First, \( g(t) \) is increasing in \( t \). Since \( t \) is itself
increasing in $\rho$, this implies that any increase in $\rho$ corresponds to a rightward move along $g(t)$. Second, at the lower bound $t = (\gamma^d + \gamma^c)/(1 + \beta^c)$, the slope of $g(t)$ goes to infinity. This implies that when $\rho$ is sufficiently small, the effect of an increase in $\rho$ on the expected intercept $E(\alpha)$ outweighs its effect on the slope $-\beta$. In other words, when imitation pressure is weak to begin with, an increase in imitation pressure favors Decentralization. Third, the slope of $g(t)$ is decreasing in $t$. This implies that the larger $\rho$, the larger the effect of an increase in $\rho$ on the slope $-\beta$ relative to its effect on the intercept $E(\alpha)$. An increase in imitation pressure therefore favors Centralization if imitation pressure is already strong before the increase. The next proposition follows from these properties of $g(t)$ and from the results in the previous proposition.

PROPOSITION 3: Assume $r_C^2 = r_D^2 = r^2 > 0$ and $\lambda > 1/2$. Then,

i. if $\frac{s}{E(a-c)}$ is sufficiently large, the firm decentralizes for any $\rho \in [0, \overline{\rho}]$.

ii. if $\frac{s}{E(a-c)}$ is sufficiently small and $\gamma^d + \gamma^c$ is sufficiently large, the firm centralizes for any $\rho \in [0, \overline{\rho}]$.

iii. if both $\frac{s}{E(a-c)}$ and $\gamma^d + \gamma^c$ are sufficiently small, the firm decentralizes when $\rho$ is small and centralizes when $\rho$ is large.

The key part of this proposition is Part iii. It shows that contrary to the standard intuition, an increase in competition can induce a firm to centralize. It is true, as is often argued, that an increase in competition can increase the value of information which, in turn, makes decentralization more attractive. It is also true, however, that an increase in competition can increase the need
for coordination and the conflict between divisions, which makes centralization more attractive. Moreover, the increase in the need for coordination and the conflict between the divisions can outweigh any increase in the value of information. This is why an increase in competition can lead to more centralized, rather than decentralized, firms.

6 Strategic Communication

In the previous section, we assumed that communication between division managers, and division managers and headquarters, was noisy for exogenous and technological reasons. This is the approach used in most of the team theory literature, and is consistent with communication technologies assumed and motivated, for example, in Aoki (1986), Dessein and Santos (2006) or, still, Cremer, Garicano and Pratt (2007). We further assumed that the level of noise in communication was independent of the organizational structure or the other parameters of the model.

In this section, we endogenize the quality of communication by taking an incentive perspective. In particular, we assume that managers distort their private information in order to influence decision-making in their favor. In particular, we assume that the private information is soft and non-contractible, and that managers communicate this information using cheap talk. We first show how the incentives to distort information, and hence the quality of communication, depends on the organizational structure (centralization vs decentralization), the cost or demand externalities between divisions and the level of competition. We then discuss how the fact that communication is endogenous affect our previous results with an exogenously specified quality of communication.

A crucial assumption of our communication game is that decision-makers make decisions in a "subgame-perfect" way, that is they have no commitment power. This lack of commitment further implies that the decision makers are not able to commit to paying transfers that depend on the information they receive or to make their decisions depend on such information in different ways. Communication therefore takes the form of an informal mechanism: cheap talk. For simplicity we assume that this informal communication occurs in one round of communication. In particular, under Decentralization Manager 1 sends message $m_1 \in M_1$ to Manager 2 and, simultaneously, Manager 2 sends message $m_2 \in M_2$ to Manager 1. Under Centralization, Managers 1 and 2 simultaneously send messages $m_1 \in M_1$ and $m_2 \in M_2$ to headquarters. It is well known in the literature on cheap talk games that repeated rounds of communication may expand the set of equilibrium outcomes even if only one player is informed. However, even for a simple cheap talk game such as the leading example in Crawford and Sobel (1982), it is still an open question as to what is the optimal communication protocol. Since it is our view that communication is an
informal mechanism which cannot be structured by the mechanism designer, it seems reasonable to focus on the simplest form of informal communication. In this sense, we take a similar approach as the property rights literature which assumes that players engage in ex post bargaining but limits the power of the mechanism designer to structure this bargaining game. As how to react

6.1 Vertical Communication

Before the managers make their capacity decisions, they communicate with each other. We now examine how they do so. We focus on vertical communication in this section and on horizontal communication in the next. To avoid confusion, note that we will think of the division managers as observing and communicating the intercepts of their inverse demand functions $\alpha_j$ rather than the demand shocks $\theta_j$, where $j = 1, 2$. This is innocuous since there is a one-to-one relationship between the demand shocks $\theta_j$ and the intercepts $\alpha_j = a - \rho (1 - c_f) + \theta_j$.

6.1.1 Incentives to Misrepresent Information

We start by examining the division managers’ incentives to misrepresent their information. Given the symmetry of the model, it is sufficient to consider only Manager 1’s incentives. Suppose then that Manager 1 can credibly misrepresent his information. In particular, suppose he can choose headquarters’ posterior beliefs about the demand conditions in his market. Formally, let $\nu \equiv E[\alpha_1 | m_1]$ be headquarters’ posterior belief about $\alpha_1$ after receiving message $m_1$ and suppose that Manager 1 can choose any $\nu$. The question we are interested in is what $\nu$ Manager 1 would choose. If headquarters sets capacities, Manager 1 would choose the posterior belief $\nu^*$ that solves

$$\max_\nu E[\lambda \pi_1 + (1 - \lambda) \pi_2 | \theta_1]$$

subject to the capacities being equal to $q_{C1}$ and $q_{C2}$ as defined in (11) and (12). In equilibrium the expected posterior of $\alpha_2$ is equal to the expected value of $\alpha_2$, that is, $E_{m_2}[\alpha_2 | m_2] = \mu$. Assuming that this relationship holds, the solution to (24) satisfies

$$\nu^* - \alpha_1 = tE(\alpha - c)\frac{2\lambda - 1}{2\lambda}.$$  

The right hand side of this expression is Manager 1’s “communication bias” and we denote it by

$$b_C \equiv tE(\alpha - c)\frac{2\lambda - 1}{2\lambda}.$$  

The communication bias is zero if $\lambda = 1/2$ or $\gamma = 0$. This implies that Manager 1 has no incentive to misrepresent his information if both division managers care only about overall firm profits or
the divisions produce independent products. If the division managers are biased towards their own divisions and the divisions produce substitutes, then Manager 1 has an incentive to overstate demand in his market. In this case the capacity in Market 1 that headquarters sets is too small from the perspective of Manager 1. To induce it to set a larger capacity, Manager 1 would like to convince headquarters that demand is stronger than it actually is. For similar reasons, Manager 1 understates demand if the division managers are biased towards their own divisions and the divisions produce complements.

The expression for the communication bias (26) also shows that an increase in competition can reduce the incentives of Manager 1 to misrepresent his information. If the increase in competition is caused by an increase in cost pressure — that is, by the firm’s competitors becoming more cost efficient — then it reduces the expected intercept of the inverse demand functions $E(\alpha)$. This in turn reduces the communication bias. If instead the increase in competition is caused by an increase in imitation pressure — that is, by the goods of the firms’ competitors becoming more similar to its own — then it reduces both the intercept $\mu$ and the absolute value of the slope $\beta$. While the reduction in $E(\alpha)$ reduces the communication bias, the reduction in $\beta$ increases it. Overall, an increase in imitation pressure reduces the communication bias at first and then increases it.

The fact that an increase in competition can reduce the division managers’ incentives to misrepresent their information is a first caution that the standard intuition about the effect of an increase on competition on the firm’s decentralization decision may not always be true. The standard intuition suggests that an increase in competition increases the value of local information which favors delegating decision rights to the managers who have this information. If the increase in competition reduces the division managers’ incentives to misrepresent their information, however, it will improve the quality of vertical communication, as we will see below. The improvement in the quality of vertical communication, in turn, implies that there is less reason to respond to an increase in competition by delegating the decision rights.

Finally, the expression for the communication bias (26) shows that it is independent of the demand realization. In other words, the amount by which Manager 1 wants to misrepresent demand does not depend on the true strength of demand in his market. We will see below that because of this feature vertical communication equilibria take a well-known form.

6.1.2 Communication Equilibria

Because the communication biases are constant and do not depend on the demand realization, the vertical communication equilibria are very similar to those of the leading uniform-constant bias
example in Crawford and Sobel (1982). Essentially, each division manager communicates with headquarters as if he had a constant bias which is equal to $b_C$ when headquarters sets capacities and which is equal to $-b_C$ when it sets prices. In this section we describe the communication equilibria. We do so informally since the uniform-constant bias example is well-known. The formal description of the communication equilibria is in the Appendix.

All Perfect Bayesian Equilibria of the communication game are equivalent to interval equilibria in which the state spaces are partitioned into intervals and the division managers only reveal which intervals their demand realizations belong to. Consider, for instance, Figure 1 and suppose that Manager 1 observes that the demand realization $\alpha_1$ in his market is equal to 0.6. He then simply tells headquarters that demand in his market is somewhere between $x$ and $y$. Similarly, if Manager 2 observes that his demand realization $\alpha_2$ is equal to 0.4, he tells headquarters that demand in Market 2 is somewhere between $x$ and $y$. Since the division managers only communicate the intervals that their demand realizations belong to, and not the realizations themselves, communication is noisy and information is lost.

The quality of vertical communication depends the size of the intervals. Roughly speaking, the smaller the intervals, the more information is communicated. The size of the intervals, in turn, depend on the communication bias. To see this, consider Manager 1 and let $a_{1,i}$ denote the value of $\alpha_1$ that lies just between interval $i-1$ and interval $i$, where $i = 1, 2, ...$. The length of interval $i$, $a_{1,i+1} - a_{1,i}$, is equal to the length of interval $i-1$, $a_{1,i} - a_{1,i-1}$, plus four times the communication bias:

$$a_{1,i+1} - a_{1,i} = a_{1,i} - a_{1,i-1} + 4b,$$

where $b = b_C$ if headquarters sets capacities and $b = -b_C$ if they set prices. When $b$ is positive, then the larger $\alpha_1$, the larger are the intervals. In other words, less information is communicated if Manager 1 reports a large demand realization than if he reports a small one. Intuitively, Manager 1 is less credible if he reports a high demand realization rather than a low one since headquarters knows that he has an incentive to exaggerate the strength of demand in his market. When $b$ is instead negative, then the larger $\alpha_1$, the smaller are the intervals. In this case less information is communicated when Manager 1 reports a low demand realization rather than a high one.

The quality of communication also depends on the number of intervals. Again roughly speaking, the more intervals, the more information is communicated. While there may be multiple equilibria with different numbers of intervals, we assume that the managers play the equilibrium with the largest number of intervals. This equilibrium ensures higher expected payoffs for all managers than any other equilibrium.
Formally, we measure the quality of communication by the residual variances $E \left[ (m_j - E[\theta_j | m_j])^2 \right]$, for $j = 1, 2$. The next proposition characterizes the quality of vertical communication.

**Lemma 1:** In the most efficient equilibrium in which the number of intervals is maximized, the residual variances are given by

$$r_C^2 \sigma^2 = E \left[ (m_j - E[\theta_j | m_j])^2 \right] = \frac{\sigma^2}{N^2} + \frac{1}{3} b_C^2 \left( N^2 - 1 \right), \text{ for } j = 1, 2,$$

where the number of intervals $N$ is given by

$$N = \text{int} \left( \frac{1}{2} \left( 1 + \sqrt{1 + \frac{4s}{|b_C|}} \right) \right).$$

Note that the quality of vertical communication does not depend on whether headquarters sets capacities or prices nor on whether the divisions’ products are substitutes or complements. This is so because the residual variances depend on the magnitude of the communication biases but not on their signs.

### 6.2 Horizontal Communication

We now turn to horizontal communication. We proceed as in the previous section: we first examine the division managers’ incentives to misrepresent their information, then describe the communication equilibria, and finally work out the expected profits.

#### 6.2.1 Incentive to Misrepresent Information

It is again sufficient to consider only Manager 1’s incentives to misrepresent his information. Suppose that Manager 1 can simply choose Manager 2’s posterior beliefs about the demand conditions in Market 1. Formally, let $\nu \equiv E[\alpha_1 | m_1]$ be Manager 2’s posterior belief about $\alpha_1$ after receiving message $m_1$ and suppose that Manager 1 can choose any $\nu$. The question we are interested in is what $\nu$ Manager 1 would choose.

If the divisions engage in Cournot Competition, Manager 1 would choose the belief $\nu^*$ that solves (24) subject to the quantities being equal to $q_1^D$ and $q_2^D$ as given in (15) and (16). Suppose first that $4\lambda(1 - \lambda) - t^2 < 0$. In this case, the objective function (24) is convex in $\nu$. This means that there is no limit to how much Manager 1 would like to misrepresent the strength of demand in his market. As a result, informative communication is not feasible. Suppose next that $4\lambda(1 - \lambda) - t^2 > 0$. The objective function (24) is then concave and the optimal belief $\nu^*$ that Manager 1 would like Manager 2 to have satisfies
\[ v^* - \alpha_1 = \frac{4\lambda(2\lambda - 1)}{4\lambda(1 - \lambda) - t^2} \left( \alpha_1 - c - \frac{t}{2\lambda} E(\alpha - c) \right). \]

The right hand side is Manager 1’s communication bias under Decentralization and we denote it by

\[ b_D \equiv \frac{4\lambda(2\lambda - 1)}{4\lambda(1 - \lambda) - t^2} \left( \alpha_1 - c - \frac{t}{2\lambda} E(\alpha - c) \right). \]

We will see below that informative communication is feasible provided that the communication bias is not too large. Note, however, that the communication bias depends on the demand realization: the larger \( \alpha_1 \), the larger the belief that Manager 1 wants Manager 2 to have. This is in contrast to the communication biases under Centralization which are independent of the demand realizations. Because of this difference horizontal communication equilibria no longer take the simple form that is familiar from the uniform-constant example.

To strengthen our intuition for when horizontal communication is feasible, it is useful to compare Manager 2’s capacity decision with the capacity decision that Manager 1 would like him to make if Manager 1 knew the demand conditions in Market 2 himself. For the purpose of this comparison, suppose that the firm’s products are substitutes; the comparison with complements is analogous.

We know from (16) that Manager 2’s capacity decision is given by

\[ q^D_2 = \frac{\alpha_2 - c}{2\beta} - \frac{2\lambda v - t E[\alpha_2 - c | m_2]}{2\beta (4\lambda^2 - t^2)}. \]

The capacity decision that Manager 1 would like Manager 2 to make is the capacity decision that Manager 1 would make himself if both decision rights were centralized in Division 1. When the division managers are strongly biased towards their own divisions, Manager 1 would always like Manager 2 to shut down production, independent of the strength of demand in Market 1. Formally, \( q^C_1 = 0 \) if \( 4\lambda (1 - \lambda) - t^2 < 0 \). Naturally, informative communication cannot take place if Manager 1 always wants to induce the same decision by Manager 2. When the division managers are less biased towards their own divisions, Manager 1 would like Division 2 to engage in some production. Formally,

\[ q^C_2 = \frac{-t\lambda (\alpha_1 - c) + 2 (1 - \lambda) (\alpha_2 - c)}{\beta(4\lambda(1 - \lambda) - t^2)} > 0 \]

if \( 4\lambda (1 - \lambda) - t^2 > 0 \) and \( -t\lambda (\alpha_1 - c) + 2 (1 - \lambda) (\alpha_2 - c) > 0 \). In general the division managers disagree on the the capacity choice for Market 2, that is, \( q^D_2 \neq q^C_2 \). They do, however, agree that the capacity of Market 2 should larger, the stronger the demand in Market 1. In other words, \( q^D_2 \) is decreasing in \( v = E[\alpha_1 | m_1] \) and \( q^C_2 \) is decreasing in \( \alpha_1 \). It is because of this commonality
in the division managers’ preferences that informative communication is feasible when the division managers are not too biased towards their own divisions.

6.2.2 Communication Equilibria

Having examined the incentives of the division managers to misrepresent their information, we can now describe the communication equilibria. We again relegate the formal analysis to the appendix.

Recall that informative communication between the division managers is not feasible if $4\lambda (1 - \lambda) - t^2 < 0$. In these cases the only communication equilibrium is the babbling equilibrium in which each division manager always sends the same message. The residual variance $r_D^2 \sigma^2$ is then equal to the variance $\sigma^2$.

Informative communication can be possible if $4\lambda (1 - \lambda) - t^2 > 0$. The communication equilibria are then very similar to the vertical communication equilibria that we described above. In particular, it is again the case that all Perfect Bayesian Equilibria are equivalent to interval equilibria in which the state spaces are partitioned into intervals and the division managers only reveal which intervals their demand realizations belong to. Moreover, the length of the intervals is again determined by (27) where now $b = b_D$. Finally, the expected payoffs of all managers are higher in the equilibrium with the largest number of intervals than in any other equilibrium. For this reason, we again focus on the equilibrium with the largest number of partitions.

In spite of all these similarities, there is one important difference: while the vertical communication bias $b_C$ is constant, the horizontal communication bias $b_D$ depends on the demand realization $\alpha_1$. Because of this difference, the expression for the residual variance $r_D^2 \sigma^2$ that characterizes the quality of horizontal communication is significantly more tedious than the expression for the residual variance $r_C^2 \sigma^2$ that characterizes the quality of vertical communication. For this reason we relegate the formal statement of $r_D^2 \sigma^2$ to the appendix.

6.3 Internal Organization of the Firm

Having endogenized $r_C^2$ and $r_D^2$ as a function of the organizational structure, the level of competition and the degree of demand or cost externalities between divisions, we now revisit the optimal organizational design of the firm. The following proposition shows that our results, obtained with exogenous communication, continue to hold with endogenous communication, provided that the divisions either product (input or output) substitutes, or, if they produce complements, $\lambda > 0.6$.

PROPOSITION 4: Assume $\lambda > 1/2$, then, decentralization is optimal if and only if

1) $0 < t \leq t_1$ and $\frac{s}{E(\alpha - c)} > f_1(t)$, where $t_1$ and $f_1(t)$ are defined in the proof and $f_1(0) = 0$ and
$f'_1(t) > 0$ for any $t \in [0, t_1]$. Furthermore, for any $t \in [0, t_1]$, the function $f_1(t)$ is increasing in $\lambda$.  
2) $t_2 < t \leq 0$, $\lambda > 0.6$ and $\frac{s}{E(\alpha-c)} > f_2(t)$, where $t_2$ and $f_2(t)$ are defined in the proof, $f_2(0) = 0$ and $f'_2(t) < 0$ for any $t \in [t_2, 0]$. Furthermore, for any $t \in [t_2, 0]$, the function $f_2(t)$ is increasing in $\lambda$.

As a corollary, with endogenous communication, the same comparative statics regarding the impact of competition on centralization/decentralization continue to hold as with exogenous communication. Only if $t < 0$ and $\lambda \in [1/2, 0.6]$, different results may obtain. (...).

7 Discussion

TBC

8 Appendix

TBC
REFERENCES


To be completed