Empirical Macroeconometric Modelling*

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Abstract

The paper describes an approach to macroeconometric modelling which has roots back to Trygve Haavelmo’s classic contribution to econometrics (Haavelmo (1944)). Building onto a long tradition in the field, we suggest an approach to macroeconometric modelling which is based on fundamental statistical concepts like the joint distribution function of all observable variables for the whole sample period. Users of macroeconomic models often demand a detailed description of the economy, and in order to accommodate that demand, realistic macroeconomic models invariably become too large to be specified simultaneously. The suggested methodology therefore relies on valid conditioning and marginalisation of this function in order to arrive at tractable subsystems, which can be analysed with statistical methods. The objective is to obtain data congruent sub-models that represents partial structure, which can be combined into a model for the entire economy. There is however a Catch 22: a general theory for the procedure will contain criteria and conditions which are formulated for the full system. Two case studies - the modelling of the household sector and of wages and prices in the Norges Bank RIMINI model - highlight these issues.

Keywords: macroeconometric modelling, large vs small models, role of statistics, Haavelmo distribution, sequential conditioning and marginalisation, dynamic modelling, aggregate consumption, wages and prices, testing of rival models

JEL classification: B23, C50, C51, C52, C53, E21, E31, E37

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1 Introduction: Small vs. large models

Macroeconometric modelling aims at explaining the empirical behaviour of an actual economic system. Such models will be systems of inter-linked equations estimated from time-series data using statistical or econometric techniques.

A conceptual starting point is the idea of a general stochastic process that has generated all data we observe for the economy, and that this process can be summarised in terms of the joint probability distribution of random observable variables in a stochastic equation system, see Section 4. For a modern economy the complexity of such a system, and the corresponding joint probability distribution, is evident. Nevertheless, it is always possible to take a highly aggregated approach in order to represent the behaviour of a few 'headline' variables (e.g., inflation, GDP growth, unemployment) in a small scale model. If small enough, the estimation of such econometric models can be based on formally established statistical theory as with low dimensional VARs, where the statistical theory has recently been extended to cointegrated variables.

However, it takes surprisingly little in terms of user-instigated detailing of model features - e.g., more than one production sector, separate modelling of consumption and investment, - to render simultaneous modelling of all equations impossible in practice. Hence, models that are used for analysing the impact of the governmental budget on the economy are typically very large systems of equations. Even in the cases where the model user in the outset targets only one variable, as with the recently contrived inflation targeting, policy choices are made against the backdrop of a broader analysis of the effects of the interest rate on the economy (the nominal and real exchange rates, output growth, employment and unemployment, housing prices, credit growth and financial stability). Thus, it has been a long standing task of model builders to establish good practice and develop operational procedures for model building which secures that the end product of piecewise modelling is tenable and useful. Important contributions in the literature include Klein (1983), Klein et al. (1999), Christ (1966), Fair (1984, 1994) and the surveys in Bodkin et al. (1991) and Wallis (1994).

In Bårdsen et al. (2004a) we supplement the existing literature by suggesting the following operational procedure:1

1. By relevant choices of variables we define and analyse subsectors of the economy (by marginalisation).

2. By distinguishing between exogenous and endogenous variables we construct (by conditioning) relevant partial models, which we will call models of type A.

3. Finally, we need to combine these submodels in order to obtain a model B for the entire economy.

Our thesis is that, given that Model A is a part of Model B, it is possible to learn about Model B from Model A. The alternative to this thesis amounts to a

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1See Jansen (2002), reply to Søren Johansen (Johansen (2002)).
kind of creationism\(^2\), i.e., unless of course macroeconometrics should be restricted to aggregate models.

Examples of properties that can be discovered using our procedure includes cointegration in Model B. This follows from a corollary of the theory of cointegrated systems: any nonzero linear combination of cointegrating vectors is also a cointegrating vector. In the simplest case, if there are two cointegrating vectors in Model B, there always exists a linear combination of those cointegrating vectors that “nets out” one of the variables. Cointegration analysis of the subset of variables (i.e., Model A) excluding that variable will result in a cointegrating vector corresponding to that linear combination. Thus, despite being a property of Model B, cointegration analysis of the subsystem (Model A) identifies one cointegration vector. Whether that identification is economically meaningful or not remains in general an open issue, and any such claim must be substantiated in each separate case. Several examples are readily at hand: In Section 5 below we discuss the identification of a consumption function as a cointegrating relationship, and link that discussion to the concept of partial structure. In Section 6 the identification of cointegrating relationships corresponding to price and wage setting are discussed in detail.

Other important properties of the full model that can be tested from subsystems include the existence of a natural rate of unemployment, see Bårdsen et al. (2004a, Ch. 6) and the relevance of forward looking terms in wage and price setting, see Bårdsen et al. (2004b).

Nevertheless, as pointed out by Johansen (2002), there is a Catch 22 to the above procedure: a general theory for the three steps will contain criteria and conditions which are formulated for the full system. However, sophisticated piecewise modelling can be seen as a sort of gradualism - seeking to establish submodels that represents partial structure: i.e., partial models that are invariant to extensions of the sample period, to changes elsewhere in the economy (e.g. due to regime shifts) and remains the same for extensions of the information set. However, gradualism also implies a readiness to revise a submodel. Revisions are sometimes triggered by forecast failure, but perhaps less often than believed in academic circles, see the final paragraph of Section 4.2. More mundane reasons include data revisions and data extensions which allow more precise and improved model specification. The dialogue between model builders and model users often results in revisions too. For example, experienced model users are usually able to pinpoint unfortunate and unintended implications of a single equation’s (or submodel) specification on the properties of the full model.

Obviously, gradualism does not preclude thorough testing of a submodel. On the contrary, the first two steps in the operational procedure above do not require that we know the full model, and testing those conditions have some intuitive appeal since real life provides ”new evidence” through the arrival of new data and by ”natural experiments” through regime shifts like e.g. changes in government or the financial deregulation in many European economies in the recent past. For the last of the three steps, we could in principle think of the full model as the ultimate extension of the information set, and so establishing structure or partial structure represents

\(^2\)Theory that attributes the origin of matter and species to a special creation (or act of God), as opposed to the evolutionary theory of Darwin.
a way to meet with the Søren Johansen’s observation. In practice, we know that the full model is not attainable. What we do then is to graft the sector model in simplified approximations of Model B, and test the relevant exogeneity assumptions of the partial model within that frame. To the extent that the likelihood function of the simplified Model B is adequately representing or approximating the likelihood function of the full Model B, there is no serious problem left. It is also possible to corroborate the entire procedure, since it is true that Model A can be tested and improved gradually on new information, which is a way of gaining knowledge that parallels modern Darwinism in the natural sciences. We develop these views further in Section 7.

A practical reason for focusing on submodels is that the modellers may have good reasons to study some parts of the economy more carefully than other parts. For a central bank that targets inflation, there is a strong case for getting the model of the inflationary process right. This calls for careful modelling of the wage and price formation conditional on institutional arrangements for the wage bargain, the development in financial markets and the evolving real economy in order to answer a number of important questions: Is there a natural rate (of unemployment) that anchors unemployment as well as inflation? What is the importance of expectations for inflation and how should it be modelled? What is the role of money in the inflationary process?

We find that in order to answer such questions - and to probe the competing hypotheses regarding supply side economics - a detailed modelling, drawing on information specific to the economy under study - is necessary. Taking account of the simultaneity is to a large extent a matter of estimation efficiency. If there is a trade off between such efficiency and the issue of the getting the economic mechanisms right, the practitioners of macroeconometric modelling should give priority to the latter.

**Organization of this paper**

In the following Section 2 we discuss briefly the roles of statistics and economic theory in macroeconomic modelling. The rest of this paper is organised as follows: In Section 3 we outline an empirical macroeconometric model for the Norwegian economy, RIMINI, and show the working of the model by describing two monetary transmission channels within that model. The main point of this is to demonstrate the complexity and interdependencies in a realistic macroeconometric model. It transpires that such a model is too big and too complex to be modelled, or let alone estimated, simultaneously. Thus, there is a need to deal with subsectors of the economy - i.e. we try to make sense out of bits and pieces rather than handling a complete model. The modelling of subsystems implies making simplifications of the joint distribution of all observable variables in the model through sequential conditioning and marginalisations, as discussed in Section 4.

The methodological approach adopted in the RIMINI project is then illustrated by means of two case studies. First, the strategy of sequential simplification is illustrated for the household sector in Section 5. The empirical consumption function which we arrive at, has been the main "work-horse" in RIMINI for more than a decade. Thus, it is of particular interest to compare it with rival models in the literature, as we do in Section 5.2. Section 6 focuses on the modelling of wages and prices. This is an exercise that includes all ingredients regarded as important
for establishing an econometrically relevant submodel. The credentials of that submodel can be seen as indirect evidence for the validity of the assumptions the larger model must rely on. Being RIMINI “writ small”, we also regard it as a working laboratory for various modelling experiments that are cumbersome, time-consuming and in some cases impossible to carry out with the fullblown RIMINI model. In a final Section 7 we return to the issue of when it is legitimate to combine submodels of this kind into a model for the entire economy.

2 The roles of statistics and economic theory in macroeconometrics

Macroeconometrics draws upon and combines two academic disciplines - economics and statistics. There is hardly any doubt that statisticians have had a decisive influence on quantitative economics in general and on modern macroeconometric modelling in particular.

2.1 The influx of statistics into economics

The history of macroeconomic modelling starts with the Dutch economist Jan Tinbergen who built and estimated the first macroeconometric models in the mid-1930s (Tinbergen (1937)). Tinbergen showed how one could build a system of equations into an econometric model of the business cycle, using economic theory to derive behaviourally motivated dynamic equations and statistical methods (of that time) to test them against data. However, there seems to be universal agreement that statistics enters the discipline of economics and econometrics with the contributions of the Norwegian economist Trygve Haavelmo in his treatise "The Probability Approach in Econometrics", (Haavelmo (1944)), see Royal Swedish Academy of Science (1990), Klein (1988), Morgan (1990), or Hendry and Morgan (1995). Haavelmo was inspired by some of the greatest statisticians of that time. As Morgan (1990, p 242) points out, he was converted to the usefulness of probability ideas by Jerzy Neyman and he was also influenced by Abraham Wald whom Haavelmo credited as the source of his understanding of statistical theory.

For our purpose it is central to note that Haavelmo recognised and explained in the context of an economic model, that the joint distribution of all observable variables for the whole sample period provides the most general framework for statistical inference, see Hendry et al. (1989). This applies to specification (op.cit., pp. 48-49), as well as identification, estimation and hypothesis testing:

...all come down to one and the same thing, namely to study the properties of the joint probability distribution of random (observable) variables in a stochastic equation system... (Haavelmo (1944), p. 85)]

Haavelmo’s probabilistic revolution changed econometrics. His thoughts were immediately adopted by Jacob Marschak - a Russian-born scientist who had studied statistics with Slutsky - as the research agenda for the Cowles Commission for the period 1943-1947, in reconsidering Tinbergen’s work on business cycles cited above. Marschak was joined by a group of statisticians, mathematicians and economists,
including Haavelmo himself. Their work was to set the standards for modern econometrics and found its way into the textbooks of econometrics from Tintner (1952) and Klein (1953) onwards.

The work of the Cowles Commission also laid the foundations for the development of macroeconomic models and model building grew into a large industry in the US in the next three decades, see Bodkin et al. (1991) and Wallis (1994). These models were mainly designed for short (and medium) term forecasting, i.e. modelling business cycles. The first model, Klein (1950), was made with the explicit aim of implementing Haavelmo’s ideas into Tinbergen’s modelling framework for the US economy. Like Tinbergen’s model, it was a small model and Klein put much weight on the modelling of simultaneous equations. Later models became extremely large systems in which more than 1000 equations were used to describe the behaviour of a modern industrial economy. In such models, less care could be taken about each econometric specification, and simultaneity could not be treated in a satisfactory way. The forecasting purpose of these models meant that they were evaluated on their performance. When the models failed to forecast the effects on the industrial economies of the oil price shocks in 1973 and in 1979, the macroeconomic modelling industry lost much of its position, particularly in the US.

In the 1980’s, macroeconometric models took advantage of the methodological and conceptual advances in time series econometrics. Box and Jenkins (1970) had provided and made popular a purely statistical tool for modelling and forecasting univariate time series. The second influx of statistical methodology into econometrics has its roots in the study of the non-stationary nature of economic data series. Clive Granger - with his background in statistics - has in a series of influential papers shown the importance of an econometric equation being balanced. A stationary variable cannot be explained by a non-stationary variable and vice versa, see e.g. Granger (1990). Moreover, the concept of cointegration (see Granger (1981), Engle and Granger (1987, 1991b)), - that a linear combination of two or more non-stationary variables can be stationary - has proven useful and important in macroeconometric modelling. Within the framework of a general vector autoregressive model (VAR), the statistician Søren Johansen has provided (see Johansen (1988, 1991, 1995b)) the most widely used tools for testing for cointegration in a multivariate setting, drawing on the analytical framework of canonical correlation and multivariate reduced rank regression in Anderson (1951).

Also, there has been an increased attention attached to the role of evaluation in modern econometrics, see Granger (1990, 1999). The so-called LSE methodology emphasizes the importance of testing and evaluating econometric models, see Hendry (1993a, 1995a), Mizon (1995), and Ericsson (2004). Interestingly, Hendry et al. (1989) claim that many aspects of the Haavelmo research agenda were ignored for a long time. For instance, the joint distribution function for observable variables was recognised by the Cowles Commission as central to solving problems of statistical inference, but the ideas did not influence empirical modelling strategies for decades. By contrast, many developments in econometrics after 1980 are in line with this and other aspects of Haavelmo’s research programme. This is also true for the role of economic theory in econometrics:

Theoretical models are necessary tools in our attempts to understand and “explain” events in real life (Haavelmo (1944), p. 1)
But whatever “explanations” we prefer, it is not to be forgotten that they are all our own artificial inventions in a search for an understanding of real life; they are not hidden truth to be “discovered” (ibid., p. 3).

With this starting point you would not expect that the facts or the observations would agree with any precise statement that is derived from a theoretical model. Economic theories must then be formulated as probabilistic statements and Haavelmo viewed probability theory as indispensable in formalizing the notion of models being approximations to reality.

2.2 The role of economic theory in macroeconometrics

The Cowles Commission research agenda focused on Simultaneous Equation Models (SEMs) and put much weight on the issue of identification. In dealing with these issues economic theory plays an important part. The prominent representative of this tradition, Lawrence Klein, writes in a very readable survey of the interaction between statistics and economics in the context of macroeconometric modelling - Klein (1988) - that the model building approach can be contrasted to pure statistical analysis, which is empirical and not so closely related to received economic theory as is model building.

Still, it is on this score the traditional macroeconomic model building has come under attack, see Favero (2001). Whereas the LSE methodology largely ascribes the failure of those early macroeconomic models to missing dynamics or model misspecification (omitted energy price effects), other critiques like Robert Lucas and Christopher Sims have claimed that the cause is rather that they had a weak theoretical basis. The Lucas critique (see e.g. Lucas (1976)) claims that the failure of conditional models is caused by regime shifts, as a result of policy changes and shifts in expectations. The critique carries over to SEMs if expectations are non-modelled. On the other hand, Sims (1980) argued that SEMs embodied ”incredible” identifying restrictions: The restrictions needed to claim exogeneity for certain variables would not be valid in an environment where agents optimize intertemporally.

Sims instead advocated the use of a low order vector autoregression to analyse economic time series. This approach appeared to have the advantage that it did not depend on an ”arbitrary” division between exogenous and endogenous variables and also did not require ”incredible” identifying restrictions. Instead Sims introduced identifying restrictions on the error structure of the model, and this approach has been criticized for being equally arbitrary. Later developments have led to structural VAR models in which cointegration defines long-run relationships between non-stationary variables and where exogenous variables are reintroduced, see Pesaran and Smith (1998) for a survey in which they reanalyse an early model by King et al. (1991).3

Ever since the Keynes-Tinbergen controversy, see Morgan (1990) and Hendry and Morgan (1995), the role of theory in model specification has represented a major controversy in econometrics, cf. Granger (1990, 1999) for recent surveys. At one end of the theory-empiricism line we have theory-driven models that take the

3Jacobson et al. (2001) uses a structural VAR with emphasis on the common trends to analyse the effect of monetary policy under an inflation targeting regime in a small open economy.
received theory for granted, and do not test it. Prominent examples are the general equilibrium models, dubbed real business cycle models, that have gained a dominating position in academia, see e.g. Kydland and Prescott (1991). There is also a new breed of macroeconometric models which assume intertemporally optimizing agents endowed with rational forward-looking expectations, leading to a set of Euler equations, see Poloz et al. (1994), Willman et al. (2000), Hunt et al. (2000) and Nilsson (2002) for models from the central banks of Canada, Finland, New Zealand and Sweden, respectively. Also dynamic stochastic general equilibrium models, as e.g., in Smets and Wouters (2002), belong to this model class. At another extreme we have data based VAR models which initially were statistical devices that made only minimal use of economic theory. As noted above, in the less extreme case of structural VARs, theory restrictions can be imposed as testable cointegrating relationships in levels or they can be imposed on the error structure of the model.

The approach we are advocating has much in common with the LSE methodology referred to above, and it focuses on evaluation as recommended in Granger (1999). It represents a compromise between data based (purely statistical) models and economic theory: On the one hand learning from the process of trying to take serious account of the data, whilst on the other hand avoiding to make strong theoretical assumptions - needed to make theories “complete” - which may not make much sense empirically, i.e. that are not supported by the data. Moreover, there are common sense arguments in favour of not adopting a theory-driven model as a basis for policy decisions, which indeed affect reality, until it has gained convincing empirical support, see Granger (1992).

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As is clear from the discussion above, econometric methodology lacks a consensus, and thus the approach to econometric modelling we are advocating is controversial. Heckman (1992) questions the success, but not the importance, of the probabilistic revolution of Haavelmo. Also, Keuzenkamp and Magnus (1995) offer a critique of the Neyman-Pearson paradigm for hypothesis testing and they claim that econometrics has exerted little influence on the beliefs of economists over the past fifty years, see also Summers (1991). For sceptical accounts of the LSE methodology, see Hansen (1996) and Faust and Whiteman (1995, 1997), to which Hendry (1997) replies.
The transmission mechanism in RIMINI

In modern economies, the transmission mechanism can be seen as a complex system where different groups of agents interact through markets which are often strongly interlinked, and an attractive feature of a macroeconomic model is that it represents the different linkages in a consistent framework. As an example, we take a closer look at the transmission mechanism of the medium term macroeconomic model, RIMINI.\(^5\)

By Norwegian standards RIMINI is an aggregated macroeconometric model.\(^6\) The core consists of some 30 important stochastic equations, and there are about 100 non-trivial exogenous variables which must be projected by the forecaster. Such projections involve judgements and they are best made manually based on information from a wide set of sources. The model should be run repeatedly to check for consistency between the exogenous assumptions and the results before one arrives at a baseline forecast. In this way the model serves as a tool taking account of international business cycle development, government policy and market information, e.g. forward market interest rates.

RIMINI is a fairly closed model in the sense that the most important variables for the Norwegian economy are determined by the model, while the model conditions upon “outside” variables like foreign prices and output and domestic policy variables like interest rates and tax rates. The model distinguishes between five production sectors. The oil and shipping sectors are not modelled econometrically, nor is the sector for agriculture, forestry and fishing. The two main sectors for which there exist complete sub-models are manufacturing and construction (traded goods) and services and retail trade (non-traded goods). There are reasons to expect important differences in for instance the responses to changes in interest rates and exchange rates between traded and non-traded goods.

In RIMINI there are two main channels through which monetary policy instruments affect employment, output and prices - the interest rate channel and the exchange rate channel. For the first channel - the effect of the interest rate - Figure 1 shows the role of the household sector in RIMINI (first dotted box from the top) and also the main interaction between the demand side (second dotted box) and the supply side (bottom dotted box). The main point here is to illustrate the complexity and interdependencies that are typical of macroeconometric systems.

Assuming fixed exchange rates, an increase in the central bank interest rate for loans to the banks (the signal rate) immediately affects the money market interest rate. The money market rate in turn feeds into the deposit and lending rates of commercial and savings banks with a lag. Aggregate demand is affected through several mechanisms: There is a negative effect on housing prices (for a given stock of housing capital), which causes real household wealth to decline, thus suppressing total consumer expenditure. Also, there are negative direct and indirect effects on real investment in the traded and non-traded sectors and on housing investment.

CPI inflation is reduced after a lag, mainly through the effects from changes in

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\(^5\)RIMINI has been used by the Central Bank of Norway for more than a decade to make forecasts for the Norwegian economy four to eight quarters ahead as part of the Inflation report of the Bank, see Olsen and Wulfsberg (2001)

\(^6\)See Bjerkholt (1998) for an account of the Norwegian modelling tradition.
aggregate demand on aggregate output and employment, but also from changes in unit labour costs. Notably, productivity first decreases and then increases - due to temporary labour hoarding - to create a cyclical pattern in the effects of the change in the interest rate.

An appreciation of the krone has a more direct effect on CPI inflation compared to the interest rate. As illustrated by the first dotted box in Figure 2, it mainly works through reduced import prices with a lagged response which entails a complete pass-through to import and export prices after about two years. The model specification is consistent with a constant markup on unit labour costs the long-run. A currency appreciation has a negative effect on the demand for traded goods. The direct effects are not of a large magnitude, because there are small relative price elasticities in the export equations and secondly because export prices (in local currency) adjust with a lag and tend to restore the relative prices. However, there are also important feedback mechanisms as the decrease in the price level caused by the appreciation feeds back into aggregate demand from domestic sectors.
Interest rate channels in RIMINI

Money market rates (3-month Euro-NOK) → Bank rates - deposit rate - lending rate

Disposable income and savings ratio for the household sector

Housing prices (resale) → Loans to households

Net financial wealth Total household wealth

Private consumption Housing investment Business sector fixed investment

Employment Output level Inventories “Output gap“

Productivity Unemployment

Sector wages Wage inflation Sector prices CPI inflation

Figure 1: Interest rate channels in RIMINI. Effects on CPI inflation assuming constant exchange rates
Figure 2: Exchange rate channels in RIMINI. Effects on CPI inflation assuming constant interest rates
4 Identifying partial structure in submodels

Model builders often face demands from model users that are incompatible with a 3-5 equations closed form model. Hence, modellers often find themselves dealing with submodels for the different sectors of the economy. Thus it is often useful to think in terms of a simplification of the joint distribution of all observable variables in the model through sequential factorization, conditioning and marginalisations.

4.1 The theory of reduction

Consider the joint distribution of \( x_t = (x_{1t}, x_{2t}, \ldots, x_{nt})' \) \( t = 1, \ldots, T \), and let \( x_T^T = \{x_t\}_{t=1}^T \). Sequential factorisation means that we factorize the joint density function \( D_x(x_T^T | x_0; \lambda_x) \) into

\[
D_x(x_T^T | x_0; \lambda_x) = D_x(x_1 | x_0; \lambda_x) \prod_{t=2}^T D_x(x_t | x_{t-1}^T, x_0; \lambda_x)
\]

which is what Spanos (1989) named the Haavelmo distribution. It explains the present \( x_t \) as a function of the past \( x_{t-1}^T \), initial conditions \( x_0 \), and a time invariant parameter vector \( \lambda_x \). This is - by assumption - as close as we can get to representing what Hendry (1995a) calls the data generating process (DGP), which requires the error terms, \( \varepsilon_t = x_t - E(x_t | x_{t-1}^T, x_0; \lambda_x) \), to be an innovation process. The following approach has been called “the theory of reduction” as it seeks to explain the origin of empirical models in terms of reduction operations conducted implicitly on the DGP to induce the relevant empirical model (see Hendry and Richard (1982, 1983)).

The second step in data reduction is further conditioning and simplification. We consider the partitioning \( x_t = (y_t', z_t') \) and factorize the joint density function into a conditional density function for \( y_t | z_t \) and a marginal density function for \( z_t \):

\[
D_x(x_t | x_{t-1}^T, x_0; \lambda_x) = D_{y|z}(y_t | z_t, x_{t-1}^T, x_0; \lambda_{y|z}) \cdot D_z(z_t | x_{t-1}^T, x_0; \lambda_z)
\]

In practice we then simplify by using approximations by \( k \)th order Markov processes and develop models for

\[
D_x(x_t | x_{t-1}^T, x_0; \lambda_x) \approx D_x(x_t | x_{t-k}^T; \theta_x)
\]

\[
D_{y|z}(y_t | z_t, x_{t-1}^T, x_0; \lambda_{y|z}) \approx D_{y|z}(y_t | z_t, x_{t-k}^T; \theta_{y|z})
\]

for \( t > k \). The validity of this reduction requires that the residuals remain innovation processes.

A general linear dynamic class of models with a finite number of lags which is commonly used to model the \( n \)-dimensional process \( x_t \) is the \( k \)th order VAR with Gaussian error, \( i.e. \)

\[
x_t = \mu + \sum_{i=1}^k \Pi_i x_{t-i} + \varepsilon_t
\]

where \( \varepsilon_t \) is normally and identically distributed, \( Niid (0, \Lambda_\varepsilon) \). A VAR model is also the starting point for analysing the cointegrating relationships that may be
identified in the $x_t$-vector, see Johansen (1988, 1991, 1995b). Economic theory helps in determining which information sets to study and in interpreting the outcome of the analysis. In the following we assume for simplicity that the elements of $x_t$ are non-stationary $I(1)$-variables that become stationary after being differenced once. Then, if there is cointegration, it is shown in Engle and Granger (1987) that the VAR system always has a Vector Equilibrium Correcting Model (VEqCM) representation, which can be written in differences and levels (disregarding the possible presence of deterministic variables like trends) in the following way:

\[
\Delta x_t = \sum_{i=1}^{k-1} A_i \Delta x_{t-i} + \alpha (\beta' x_{t-1}) + \varepsilon_t
\]

where $\alpha$ and $\beta$ are $n \times r$ matrices of rank $r$ ($r < n$) and $(\beta' x_{t-1})$ comprises $r$ cointegrating $I(0)$ relationships. Cointegrated processes are seen to define a long-run equilibrium trajectory and departures from this induce "equilibrium correction" which moves the economy back towards its steady state path. These models are useful as they often lend themselves to an economic interpretation of model properties and their long-run (steady state) properties may be given an interpretation as long-run equilibria between economic variables that are derived from economic theory. Theoretical consistency, i.e., that the model contains identifiable structures that are interpretable in the light of economic theory, is but one criterion for a satisfactory representation of the economy.

4.2 Congruence

If one considers all the reduction operations involved in the process of going from the hypothetical DGP to an empirical model it is evident that any empirical econometric model is unlikely to coincide with the DGP. An econometric model may however possess certain desirable properties, which will render it as a valid representation of the DGP. According to the LSE methodology, see Mizon (1995) and Hendry (1995a), such a model should satisfy the following six criteria:

1. The model contains identifiable structures that are interpretable in the light of economic theory.
2. The residuals must be innovations in order for the model to be a valid simplification of the DGP.
3. The model must be data admissible on accurate observations.
4. The conditioning variables must be (at least) weakly exogenous for the parameters of interest in the model.
5. The parameters must be constant over time and remain invariant to certain classes of interventions (depending on the purpose for which the model is to be used).
6. The model should be able to encompass rival models. A model $M_i$ encompasses other models ($M_j$, $j \neq i$) if it can explain the results obtained by the other models.

Models that satisfy the first five criteria are said to be congruent, whereas an encompassing congruent model satisfy all six. Below, we comment on each of the requirements.

Economic theory (item 1) is a main guidance in the formulation of econometric models. Clear interpretation also helps communication of ideas and results among researchers and it structures the debate about economic issues. However, since economic theories are necessarily abstract and build on simplifying assumptions, a direct translation of a theoretical relationship to an econometric model will generally not lead to a satisfactory model. Notwithstanding their structural interpretation, such models will lack structural properties, see Nymoen (2002).

There is an important distinction between seeing theory as representing the correct specification (leaving parameter estimation to the econometrician), and viewing theory as a guideline in the specification of a model which also accommodates institutional features, attempts to accommodate heterogeneity among agents, addresses the temporal aspects for the data set and so on, see e.g., Granger (1999). Likewise, there is a huge methodological difference between a procedure of sequential simplification whilst controlling for innovation errors as in Section 4.1 above and the practice of adopting an axiom of a priori correct specification which by assumption implies white noise errors.

Innovation residuals (item 2) mean that residuals cannot be predicted from the model’s own information set. Hence it is relative to that set. This is a property that follows logically from the reduction process and it is a necessary requirement for the empirical model to be one that is derived from the DGP. If the errors do not have this property, e.g. if they are not homoscedastic white noise, some regularity in the data has not yet been included in the specification.

The requirement that the model must be data admissible (item 3) entails that the model must not produce predictions that are not logically possible. For example, if the data to be explained are proportions, the model should force all outcomes into the zero to one range.

Criterion 4 (weak exogeneity) holds if the parameters of interest are functions of $\theta_{y|z}$, see (4), which vary independently of $\theta_x$, see equation (3) and Engle et al. (1983) for a formal definition. This property relates to estimation efficiency: weak exogeneity of the conditioning variables $z_t$ is required for estimation of the conditional model for $y_t$ without loss of information relative to estimation of the joint model for $y_t$ and $z_t$. In order to make conditional forecasts from the conditional model without loss of information, strong exogeneity is required. This is defined as the joint occurrence of weak exogeneity and Granger noncausality, which is absence of feedback from $y_t$ to $z_t$, that is $x_{t-1}^1$ in the marginal density function for $z_t$, $D_z(z_t | x_{t-1}^1, x_0^1, \lambda_z)$ in equation (2), does not include lagged values of $y_t$.

Item 5 in the list is spelt out in greater detail in Hendry (1995a), pp.33-34, where he gives a formal and concise definition. He defines structure as the set of basic permanent features of the economic mechanism. A vector of parameters defines a structure if it is invariant and directly characterizes the relations under analysis.
i.e. it is not derived from more basic parameters. A parameter can be structural only if it is

- constant and so is invariant to an extension of the sample period;
- unaltered by changes elsewhere in the economy and so is invariant to regime shifts, etc. and
- remains the same for extensions of the information set and so is invariant to adding more variables to the analysis

This invariance property is of particular importance for a progressive research programme: ideally, empirical modelling is a cumulative process where models continuously become overtaken by new and more useful ones. By useful we understand models that possess structural properties (items 1 - 5 above), in particular models that are relatively invariant to changes elsewhere in the economy, i.e., they contain autonomous parameters, see Frisch (1938), Haavelmo (1944), Johansen (1977), and Aldrich (1989). Models with a high degree of autonomy represent structure: They remain invariant to changes in economic policies and other shocks to the economic system, as implied by the definition above.

However, structure is partial in two respects: First, autonomy is a relative concept, since an econometric model cannot be invariant to every imaginable shock. Second, all parameters of an econometric model are unlikely to be equally invariant. Parameters with the highest degree of autonomy represent partial structure, see Hendry (1993b, 1995b). Examples are elements of the $\beta$-vector in a cointegrating equation, which are often found to represent partial structure, as documented by Ericsson and Irons (1994). Finally, even though submodels are unlikely to contain partial structure to the same degree, it seems plausible that very aggregated models are less autonomous than the submodels, simply because the submodels can build on a richer information set.

Data congruence, i.e. ability to characterize the data remains an essential quality of useful econometric models, see Granger (1999) and Hendry (2002). In line with this, our research strategy is to check any hypothesized general model which is chosen as the starting point of a specification search for data congruence, and to decide on a final model after a general-to-spesific (Gets) specification search. Due to recent advances in the theory and practice of data based model building, we know that by using Gets algorithms a researcher stands a good chance of finding a close approximation to the data generating process, see Hoover and Perez (1999), and Hendry and Krolzig (2000), and that the danger of over-fitting is in fact surprisingly low.7

7Naturally, with a very liberal specification strategy, overfitting will result from Gets modelling, but with “normal” requirements of levels of significance, robustness to sample splits etc, the chance of overfitting is small. Thus the documented performance of Gets’ modelling now refutes the view that the axiom of correct specification must be invoked in applied econometrics, Leamer (1983). The real problem of empirical modelling may instead be to keep or discover an economically important variable that has yet to manifest itself strongly in the data, see Hendry and Krolzig (2001). Almost by implication, there is little evidence that Gets leads to models that are prone to forecast failure, see Clements and Hendry (2002).
A congruent model is not necessarily a true model. Hendry (1995a, Ch. 4) shows that an innovation is relative to its information set but may be predictable from other information. Hence, a sequence of congruent models could be developed and each of them encompassing all previous models. So satisfying all six criteria provides a recipe for a progressive research strategy. Congruency and its absence, can be tested against available information, and hence, unlike truth, it is an operational concept in an empirical science, see Bontemps and Mizon (2003).

Finally, it should be noted that a strategy that puts a lot of emphasis on forecast behaviour, without a careful evaluation of the causes of forecast failure ex post, runs a risk of discarding models that actually contain important elements of structure. Hence, for example Doornik and Hendry (1997) and Clements and Hendry (1999, Ch. 3) show that the main source of forecast failure is deterministic shifts in means (e.g., the equilibrium savings rate), and not shifts in such coefficients (e.g., the propensity to consume) that are of primary concern in policy analysis. Structural breaks are a main concern in econometric modelling, but like any hypothesis of theory, the only way to judge the quality of a hypothesized break is by confrontation with the evidence in the data. Moreover, given that an encompassing approach is followed, a forecast failure is not only destructive but represent a potential for improvement, since respecification follows in its wake, see Section 5.2 below.

5 An example: Modelling the household sector

The complete Haavelmo distribution function - e.g. the joint distribution (1) of all variables of the macro model - is not tractable and hence not an operational starting point for empirical econometric analysis. In practice, we have to split the system into subsystems of variables and to analyse each of them separately. Joint modelling is considered only within subsystems. But by doing so one risks to ignore possible influences across the subsystems. This would translate into invalid conditioning (weak exogeneity is not fulfilled) and invalid marginalisation (by omitting relevant explanatory variables from the analysis), which are known to imply inefficient statistical estimation and inference. The practical implementation of these principles are shown in an example drawn from the modelling of the household sector of the RIMINI model, see Section 3 above.

The process of sequential decomposition into conditional and marginal models is done repeatedly within the subsystems of RIMINI. In the household sector subsystem, total consumer expenditure, \( ch_t \), is modelled as a function of real household disposable income, \( yh_t \), and real household wealth, \( wh_t \). (Here and in the rest of the paper small letters denote logs of variables). Total wealth consists of the real value of the stock of housing capital plus net financial wealth. The volume of the residential housing stock is denoted \( H_t \) and the real housing price is \( (PH)_t/P_t \), where \( P_t \) is the general national accounts price deflator for total consumption expenditure. The sum of net real financial assets is equal to the difference between real gross financial assets and real loans \( (M_t - L_t) \), yielding

\[
wh_t = \ln WH_t = \ln [(PH_t/P_t)H_{t-1} + M_t - L_t].
\]

The joint distribution function for this subsystem can be written as (1) with \( x_t = (ch_t, yh_t, wh_t) \) The conditional submodel for total real consumer expenditure,
$\textit{ch}_t$ (Brodin and Nymoen (1992) - B&N hereafter) is

$$D_{c|y,w}(\textit{ch}_t \mid y_{ht}, w_{ht}; \lambda_c),$$

relying on the corresponding conditional density function, (4), to be a valid representation of the DGP. RIMINI contains submodels for $y_{ht}$ and for all individual components in $w_{ht}$. For example, the conditional submodel for simultaneous determination of housing prices, $p_{ht}$ and real household loans, $l_t$, is

$$D_{w|y}(p_{ht}, l_t \mid RL_t, y_{ht}, h_{t-1}; \lambda_w),$$

where $RL_t$ denotes the interest rate on loans, and conditional submodels for the net addition to housing capital stock $\Delta h_t$, and the price on new housing capital, $ph_{nt}$

$$D_{\Delta h|\cdot}(\Delta h_t \mid p_{ht}, ph_{nt}, RL_t, y_{ht}, h_{t-1}; \lambda_{\Delta h})$$

$$D_{phn|\cdot}(ph_{nt} \mid p_{ht}, pj_t, h_{t-1}; \lambda_{phn})$$

where $pj_t$ is the deflator of gross investments in dwellings.

5.1 The aggregate consumption function

The model for aggregate consumption in B&N satisfies the criteria we listed in Section 4.2. They provide a model in which cointegration analysis establishes that the linear relationship

$$ch_t = \text{constant} + 0.56y_{ht} + 0.27w_{ht},$$

is a cointegrating relationship and that the cointegration rank is one. Hence, while the individual variables in (6) are assumed to be non-stationary and integrated, the linear combination of the three variables is stationary with a constant mean showing the discrepancy between consumption and its long-run equilibrium level $0.56y_{ht} + 0.27w_{ht}$. Moreover, income and wealth are weakly exogenous for the cointegration parameters. Hence, the equilibrium correction model for $\Delta ch_t$ satisfies the requirements of valid conditioning. Finally, there is evidence of invariance in the cointegration parameters. Estimation of the marginal models for income and wealth shows evidence of structural breaks. The joint occurrence of a stable conditional model (the consumption function) and unstable marginal models for the conditional variables is evidence of within sample invariance of the coefficients of the conditional model and hence super exogenous conditional variables (income and wealth). The result of invariance is corroborated by Jansen and Teräsvirta (1996), using an alternative method based on smooth transition models.

The empirical consumption function in B&N has proven to be relatively stable for more than a decade, in particular this applies to cointegration part of the equation. Thus, it is of particular interest to compare it with rival models in the literature.
5.2 Rival models

Financial deregulation in the mid-1980s led to a strong rise in aggregate consumption relative to income in several European countries. The preexisting empirical macroeconometric consumption functions in Norway, which typically explained aggregate consumption by income, all broke down, i.e. they failed in forecasting, and failed to explain the data ex post.

As stated in Eitrheim et al. (2002), one view of the forecast failure of consumption functions is that the failure provided direct evidence in favour of the rivaling rational expectations, permanent income hypothesis. In response to financial deregulation, consumers revised their expected permanent income upward, thus creating a break in the conditional relationship between consumption and income. The breakdown has also been interpreted as a confirmation of the relevance of the Lucas critique, in that it was a shock to a non-modelled expectation process that caused the structural break in the existing consumption functions.

In Eitrheim et al. (2002) we compare the merits of the two competing models: the empirical consumption function (CF), conditioning on income in the long-run, and an Euler equation derived from a model for expectation formation. We find that while the conditional consumption function (CF) encompasses the Euler equation (EE) on a sample from 1968.2 to 1984.4, both models fail to forecast the annual consumption growth in the next years. In the paper we derive the theoretical properties of forecasts based on the two models. Assuming that the EE is the true model and that the consumption function is a mis-specified model, we show that both sets of forecasts are immune to a break (i.e. shift in the equilibrium savings rate) that occurs after the forecast have been made. Moreover, failure in “before break” CF-forecasts is only (logically) possible if the consumption function is the true model within the sample. Hence, the observed forecast failure of the CF is corroborating evidence in favour of the conditional consumption function for the period before the break occurred.

However, a re-specified consumption function - B&N of the previous section - that introduced wealth as a new variable was successful in accounting for the breakdown ex post, while retaining parameter constancy in the years of financial consolidation that followed after the initial plunge in the savings rate. The re-specified model was able to adequately account for the observed high variability in the savings rate, whereas the earlier models failed to do so.

B&N noted the implication that the re-specification explained why the Lucas critique lacked power in this case: First, while the observed breakdown of conditional consumption functions in 1984 - 1985 is consistent with the Lucas critique, that interpretation is refuted by the finding of a conditional model with constant parameters. Second, the invariance result shows that an Euler equation type model (derived from e.g., the stochastic permanent income model) cannot be an encompassing model. Even if the Euler approach is supported by empirically constant parameters, such a finding cannot explain why a conditional model is also stable. Third, finding that invariance holds, at least as an empirical approximation, yields an important basis for the use of the dynamic consumption function in forecasting and policy analysis, the main practical usages of empirical consumption functions.

In Eitrheim et al. (2002) we extend the data set by nine years of quarterly observations, i.e. the sample is from 1968.3 to 1998.4. There are major revisions in
the national accounts in that period. We also extended the wealth measure to include non-liquid financial assets. Still we find that the main results of B&N are confirmed. There is empirical support for one and only one cointegrating vector between $ch_t$, $yh_t$ and $wh_t$, and valid conditioning in the consumption function is reconfirmed on the new data. In fact, full information maximum likelihood estimation of a four equation system explaining (the change in) $ch_t$, $yh_t$, $wh_t$ and $(p_h - p_t)$ yields the same empirical results as estimation based on the conditional model. These findings thus corroborate the validity of the conditional model of B&N.

6 Testing a sub-model within a completing model. An example

Our approach to macromodelling is to combine sector models into a model for the macro economy. Only in that way is it possible to capture all relevant economic aspects of the sector. It is also important to build a model where the experts are able to recognise ”their” sector. In this chapter we use the model for wages and prices in Bårdsen et al. (2003) in order to highlight all ingredients which we regard as important for establishing an econometrically relevant submodel. First, economic theory is used to derive theoretical equilibrium conditions for nominal wages and prices within a model for wage bargaining and price setting under incomplete competition.\footnote{In Bårdsen et al. (2004a) we interpret and evaluate the last 40 years’ of international research on the modelling of wages and prices. The Norwegian “main course” model of inflation for the small open economy serves as a benchmark and the dynamic model of incomplete competition of this section lies in the other end of the evolutionary line. The latter is evaluated against alternatives as diverse as the Phillips curve, the Nickell-Layard type of wage curves, the New Keynesian Phillips curve and monetary inflation models on data for the Euro area, UK and Norway.} The theory analysis also yields an information set $I_t$ of relevant explanatory variables which we use to model these long-run properties of the system by means of multivariate cointegration techniques, assuming that the explanatory variables are weakly exogenous to the long-run parameters of interest.

A core model for wage and price growth is established by imposing those steady state equations on (vector) ADL (autoregressive distributed lag) equations which are modelled general to specific. Tests of overidentifying restrictions plays an important part in that process. The core model is grafted into a somewhat larger model, obtained by supplementing the core model with marginal models for it’s important explanatory variables. The embedding model allows us to test for valid conditioning in the wage price submodel. We interprete the extended model as RIMINI ”writ small” and if the embedding model captures the relevant [feed-back] properties of RIMINI, non-rejection of these tests can be taken as indirect evidence for the validity of the wage price model specification as an integral part of RIMINI.

6.1 A theoretical model for wages and prices

The core model for wages and prices considered has conflicting real wage claims for trade unions and firms in a small open economy, see Kolsrud and Nymoen (1998). The real wage claims on the part of the trade unions are in the first place affected by consumer prices ($p_t$) and indirect taxes ($\tau_2_t$), whereas the claims of firms are
assumed to be determined by producer prices \((pp_t)\), productivity \((pr_t)\) and payroll taxes \((\tau_1)\). Moreover, the unemployment rate \((u_t)\) is taken to represent the tightness of the labour market and is assumed to affect both parties. In an economy with imperfect competition firms set producer prices as a markup over marginal costs, that is wages corrected for productivity and payroll taxes. We are focusing on nominal wages and the consumer prices defined as a weighted sum of producer prices and import prices \((pi_t)\) with weights \((1-\zeta)\) and \(\zeta\), corrected for the effect \((\eta)\) of indirect taxes \((\tau_3)\). Based on these assumptions we can derive target equations in levels for the two parties - the nominal wage claim \((w_t^*)\) of workers in equation (7) and the corresponding claim in terms of prices \((p_t^*)\) of firms in equation (8):

\[
(7) \quad w_t^* = (1 + \zeta d_12)p_t + \delta_13 pr_t - \zeta d_12 pi_t - \delta_15 u_t - \delta_16 \tau_1 t - \delta_17 \tau 2 t - \eta d_12 \tau 3 t, \\
(8) \quad p_t^* = (1 - \zeta) (w_t - pr_t + \tau_1 t) + \zeta pi_t + \eta \tau 3 t, 
\]

where \(\{\delta_{ij}\}\) are structural coefficients derived from the wage bargaining, and \(d_{12} = \delta_{12}/(1 - \zeta)\). (7)–(8) can also be written in terms of two conflicting real wage claims, one for workers and one for firms. If they are set equal, we arrive at a static equilibrium of real wages. In terms of economic content the model is incomplete since nothing has been said about the development of targeted and actual real wages. Although firms and unions have separate views about what the real wage level should be, they can only influence real wages through nominal adjustment of wages and prices. In this way conflicting views about the desired real wage level become an important source of price and wage adjustments.

This conflict is embedded in a model that captures a number of other relevant causes of inflation. In particular we allow wage growth \(\Delta w_t\) to interact with current and past price inflation, changes in unemployment, changes in tax-rates, and previous deviations from the desired wage level. In the dynamic price setting, inflation \(\Delta p_t\) interacts with wage growth and productivity gains and with changes in the payroll tax-rate, as well as with corrections from an earlier period’s deviation from the equilibrium price. Moreover, in the short-run, the marginal cost curve is upward sloping, and hence any increase in output above the trend in potential output exerts a (lagged) positive pressure on prices, measured by the output gap, \(gap_t\), defined below.

The theory points out the relevant information set for the empirical work, i.e. which variables we need to examine in order to empirically establishing long-run cointegrating relationships for wages and prices, and in fact it also suggests a VEQUCM model for wage and price growth in accordance with these equilibrium relationships.

6.2 The core model

6.2.1 Modelling the steady state

In order to model the long-run relationships of this wage price submodel we carry out a cointegration analysis of a congruent 5th order VAR in the variables that the theory of Section 6.1 suggests we should include. Compared to the theoretical model
the income tax rate $\tau_2$ is omitted from the empirical model, since it is insignificant in the model. In addition to the variables in the wage-claims part of the system, we include $\text{gap}_{t-1}$—the lagged output gap measured as deviations from output trend obtained by the Hodrick-Prescott filter. The other non-modelled variables contain first the length of the working day $\Delta h_t$, which captures wage compensation for reductions in the length of the working day—see Nymoen (1989). Second, incomes policies and direct price controls have been in operation on several occasions in the sample period and they are represented by the intervention variables $Wdum$ and $Pdum$, and one impulse dummy $i70q2$. Finally, $i70q1$ is a VAT dummy. The resulting unrestricted conditional sub-system, where all main variables enter with three lags, is estimated over the period 1966(4)–1996(4).

The analysis yields support for two cointegrating equations which are interpretable in the light of the above theory. These long-run equations are then simplified by imposing a sequence of data admissible restrictions. We start from the two general claims equations equation (7) and (8). The omission of the income tax-rate $\tau_2$ from the system implies that $\delta_{17} = 0$. Panel 2 in Table 1 reports the statistical long-run relationship in the form of the theoretical equations in Panel 1. The remaining panels report a sequence of valid simplifications of Panel 2. Panel 3 shows a simplification where $\delta_{12} = 0$ (and hence $d_{12} = 0$), corresponding to full wage indexation to consumer prices. Panel 4 allows productivity to be fully reflected in wages ($\delta_{13} = 1$). Finally, if there are no effects from producer prices, but the full payroll tax-incidence is borne by the firms, so $\delta_{16} = 0$, the two target equations can be formulated as:

\begin{align*}
(10) & \quad \quad w^* = p + pr - \delta_{15}u, \\
(11) & \quad \quad p^* = (1 - \zeta)(w - pr + \tau1) + \zeta pi + \eta \tau3, \\
\end{align*}

with estimation results in Panel 4.

Interestingly, an alternative that was rejected is defined by $\delta_{17} = 0$ and $\delta_{16} = \delta_{12} = 1$, which amounts to an equation where wage-costs depend on the real exchange rate ($p_t - \tilde{p}_t$)

\begin{align*}
(9) & \quad \quad w^*_t + \tau1_t - p_t = \delta_{13}pr_t - \delta_{15}u_t + \frac{\zeta}{1 - \zeta}(p_t - \tilde{p}_t) - \frac{\eta}{1 - \zeta} \tau3_t.
\end{align*}
Table 1: Testing claim hypotheses.

Panel 1: The theoretically identified claims equations with nonlinear cross equation restrictions

\[ w = p + \delta_{13}pr - \delta_{15}u - \delta_{16}\tau + d_{12}\zeta \left( p - pi - \frac{\eta}{\tau} \right) \]

\[ p = (1 - \zeta)(w + \tau - pr) + \zeta pi + \eta\tau \]

Panel 2: Nonlinear cross equation restrictions

\[ w = p + 0.85pr - 0.08u + 1.60\tau - 0.03(p - pi + 2.66\tau) \]

\[ p = 0.64(w + \tau - pr) + 0.36pi + 0.95\tau \]

\( \chi^2(4) = 7.49[0.11] \)

Panel 3: No effect from producer prices and full effect of indirect taxation

\[ w = p + 0.84pr - 0.08u + 1.51\tau \]

\[ p = 0.63(w + \tau - pr) + 0.37pi + \tau \]

\( \chi^2(6) = 7.59[0.27], \chi^2(2) = 0.1[0.95] \)

Panel 4: Full effect of productivity and no effect of payroll-tax

\[ w = p + pr - 0.09u \]

\[ p = 0.62(w + \tau - pr) + 0.38pi + \tau \]

\( \chi^2(8) = 10.48[0.23], \chi^2(2) = 2.89[0.24] \)

Diagnostic tests for the unrestricted conditional subsystem

\( AR 1 \sim 5 F(20,150) = 1.25[0.22] \)

\( Normality \chi^2(4) = 1.05[0.90] \)

\( Heteroscedasticity F(66,183) = 0.49[0.99] \)

The sample is 1966(4) to 1996(4), 121 observations.

The numbers in [..] are p-values.

The last results are very close to the results for Norway in Bårdsen et al. (1998) for a sample ending in 1993(1), which is evidence of invariance to a sample extension of 15 new observations. Figure 3 records the stability over the period 1978(3)-1996(4) of the coefficient estimates of Panel 4 in Table 1 (\( \beta \) in the graphs) with \( \pm 2 \) standard errors (\( \pm 2se \) in the graphs), together with the tests of constant cointegrating vectors over the sample. The estimated wage responsiveness to the rate of unemployment is approximately 0.1, which is close to the finding of Johansen (1995a) on manufacturing wages. This estimated elasticity is numerically large enough to represent a channel for economic policy on inflation.
Elasticity of unemployment in wage equation

Elasticity of import price index in price equation

Test of overidentifying restrictions on the cointegrating vectors

Figure 3: Identified cointegration vectors. Recursively estimated parameters and the $\chi^2(8)$ test of parameter constancy of Table 1, Panel 4.

On the basis of Table 1 we therefore conclude that the steady-state solution of our system can be represented as

$$w = p + pr - 0.1u$$

$$p = 0.6(w + \tau 1 - pr) + 0.4p\dot{i} + \tau 3.$$  

6.2.2 The dynamic wage-price model.

Our next step is to impose these steady state equations on the dynamic equations for growth in wages and prices, which are modelled general to specific. Again, the general model is formulated as a vector autoregressive distributed lag model of order 3. The end result is the following wage price model:

$$\Delta w_t = \Delta p_t - 0.4 \times 0.36\Delta p\dot{i}_t - \Delta \tau 1_{t-2} - 0.36\Delta \tau 3_{t-2} - 0.3 \Delta h_t$$

$$-0.08[w_{t-2} - p_{t-2} - pr_{t-1} + 0.1u_{t-2}] + \text{ dummies}$$

$$\hat{\sigma}_{\Delta w} = 1.01\%$$

$$\Delta p_t = 0.13(\Delta w_t + \Delta \tau 1_{t-2}) + 0.06gap_{t-1} + 0.4 \times 0.07\Delta p\dot{i}_t - 0.07\Delta \tau 3_{t-2}$$

$$-0.08[p_{t-3} - 0.6(w_{t-1} - pr_{t-1} + \tau 1_{t-1}) - 0.4p\dot{i}_{t-1} + \tau 3_{t-3}] + \text{ dummies}$$

$$\hat{\sigma}_{\Delta p} = 0.41\%$$
The coefficient estimate of $\Delta \pi_t$ is restricted to be 40 per cent of the coefficient estimate of $\Delta \tau_{3t-2}$ in both equations and this is denoted by \textit{e.g.} $0.4 \times 0.36$ in the wage growth equation. The two equations in (12) show that the equilibrium correcting terms are significant with equal coefficients. (Standard errors are given in parentheses.) These terms act as \textit{attractors} on wages and prices, see Engle and Granger (1991a). The long-run wage attractor is given by prices adjusted for productivity and an effect from unemployment, whereas the price attractor is made up of the indirect tax rate and a weighted sum of import prices and wages corrected for productivity and payroll taxes. Recursive estimates of the long-run coefficients of unemployment in the wage equation and of import prices in the price equation signal stability over time, and tests reported in Bårdsen et al. (2003) show that the parameters of the cointegrating vectors are constant over the sample. In the wage equation we have short-run homogeneity in consumer prices and in the price equation we find significant effects of wage growth and excess demand.

Table 2: Diagnostic tests of the model in (8).

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<table>
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<tr>
<td>The sample is 1966(4) to 1996(4), 121 observations.</td>
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<tr>
<td>$\hat{\sigma}_{\Delta w}$ = 1.01%</td>
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<tr>
<td>$\hat{\sigma}_{\Delta p}$ = 0.41%</td>
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<tr>
<td>Correlation of residuals = $-0.5$</td>
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<tr>
<td>Overidentification $\chi^2(9)$ = 9.92[0.60]</td>
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<tr>
<td>$AR\ 1-5\ F(20,200)$ = 1.20[0.26]</td>
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<tr>
<td>Normality $\chi^2(4)$ = 4.14[0.39]</td>
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<tr>
<td>Heteroscedasticity $F(66,257)$ = 0.81[0.84]</td>
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References: Overidentification test (Anderson and Rubin (1949, 1950), Koopmans et al. (1950), Sargan (1988)), AR-test (Godfrey (1978) and Doornik (1996)), Normality test (Doornik and Hansen (1994)), and Heteroscedasticity test (White (1980) and Doornik (1996)).

The numbers in [..] are p-values.

The quality of the model is corroborated by displaying constant parameters, as shown in Figure 4, and non-systematic residuals, documented by the diagnostics of Table 2. Notably, the model is encompassing the system at every sample size—as shown in the lower left panel of Figure 4.
6.3 Outline of an embedding larger model

Equipped with this core model of the determination of wages and prices, we can go on to outline the model we alluded to as “RIMINI writ small” in the introduction to Section 6. In the context of this larger embedding model the explanatory variables of the wage price model fall into two groups: On the one hand feedback variables, that are endogenised in the larger model and on the other hand, non-modelled variables (tax-rates, world prices), and policy instruments, which remain determined outside the system.

The feedback variables include unemployment, output gap, productivity, and import prices. The term feedback is derived from a causal link from lagged wages and prices onto these variables (Granger causality). However, these variables are not only functions of lagged wages and prices. Empirically the feedback variables may depend on both the non-modelled explanatory variables and on the policy variables. In the wage-price core model the feedback variables are treated as weakly exogenous variables. This is a testable property that we address after presenting the feedback relationships in Section 6.4.

Central banks do set interest rates, but presumably not in the same way under the different monetary policy regimes found in our sample. Thus finding an
empirically constant reaction function from inflation forecasts to interest rates is not what we would anticipate and indeed we find that the short-run interest rate can be treated as strongly exogenous, defined as joint occurrence of weak exogeneity and the absence of feedback from the wage and price variables to the interest rate, see Engle et al. (1983). The important monetary feedback variable turns out to be the exchange rate, which depend both on inflation and foreign variables.

Regime shifts may induce non-constancies in the parameters of the wage-price model. If that were the case, the usefulness of the model for policy analysis is reduced, as it then falls prey to the Lucas-critique. However, invariance can be tested within the sample and we return to this in Section 6.5, after introducing marginal models for the feedback variables below.

6.4 Marginal models for the feedback variables

As we have seen, the core model for wages and prices is developed conditional upon the rate of unemployment $u_t$, average labour productivity $pr_t$, import prices $pi_t$, and GDP mainland output $y_t$ (excluding oil and gas production). For forecasting purposes we add to the model relationships for $u_t$, $pr_t$, $y_t$ and nominal exchange rate $v_t$, which multiplied by foreign prices, $pf_t$, yields $pi_t$. All of these variables could be affected by interest rates and are therefore potential channels for monetary instruments to influence inflation. Also, none of these variables is likely to be strongly exogenous. For example, import prices depend by definition on the nominal exchange rate and the relationship below links the exchange rate to the lagged real exchange rate, which in turn depend on the domestic price level. Bårdsen et al. (2003) give details of the additional relationships, but their qualitative properties can be summarized as

$$
\Delta v_t = f \left( ppp_{t-1}, oil_t, \Delta RS_t \right)
$$

$$
\Delta y_t = g \left( EqCM_y_t, \Delta y_{t-1}, \Delta cr_{t-1} \right)
$$

$$
\Delta u_t = h \left( \Delta y_t, \Delta u_{t-1}, u_{t-1}, stu_{t-1}, \Delta (w - p)_{t-1}, amun_t \right)
$$

$$
\Delta pr_t = k \left( \Delta 3pr_{t-1}, \Delta u_{t-1} \right)
$$

where $ppp$ is purchasing power parity (see Akram (2000) for documentation of the exchange rate equation), $oil$ is the USD price of North Sea oil, $RS$ is the money market interest rate, $EqCM_y$ is an equilibrium correction term for an aggregate demand relationship, and $cr$ is a function of credit demand—see Bårdsen and Klovland (2000). Furthermore, $stu$ denotes non-linear effects in unemployment adjustment, while $amun$ is an indicator picking up the effect of labour market programmes.
6.5 Testing exogeneity and invariance

Weak and super exogeneity refer to different aspects of “exogeneity”, namely the question of “valid conditioning” in the context of estimation and policy analysis respectively—see Engle et al. (1983). In the light of the results reported above, it is important to assess the possible endogeneity of output, productivity, unemployment, and exchange rates. First, the cointegrating vectors have been estimated conditional on output, productivity, unemployment, and exchange rates, and efficient estimation requires that these variables are weakly exogenous for the cointegration vectors (see e.g. Johansen (1992)). Second, policy analysis involves as a necessary condition that the wage and price equations are invariant to the interventions occurring in the marginal models of output, productivity, unemployment, and exchange rates; together with weak exogeneity (if that holds) invariance implies super exogeneity.

Following Johansen (1992) weak exogeneity of the conditioning variables with respect to the cointegration parameters requires that the equilibrium-correction terms for wages and prices do not enter the marginal models. Table 3 shows the results of testing weak exogeneity of exchange rates, output growth, unemployment and productivity within the marginal system.

Table 3: Testing weak exogeneity

| EqCM –terms in core model: |  
|-----------------------------|-----------------|
| ∆νₜ | F(2, 91) = 0.16 [0.85] |
| ∆yₜ | F(2, 89) = 0.02 [0.98] |
| ∆uₜ | F(2, 102) = 0.63 [0.53] |
| ∆prₜ | F(2, 111) = 1.59 [0.21] |

The weak exogeneity assumptions of exchange rates, output growth, unemployment and productivity for the long-run parameters appear to be tenable. Looking at the detailed results, only the coefficient of $EqCM_w(t)$ in the productivity equation obtained a $t$-value of that came even close to significance ($-1.7$), all the other coefficients had $t$-values equal to one or smaller than one in absolute value.

To test for parameter invariance, we need the interventions occurring in the parameterizations of the marginal models for $Z_t = (ν_t, y_t, u_t, pr_t)$. Consider therefore the following stacked form of the (estimated) single-equation marginal models referred to in Section 6.4:

$$
\Delta Z_t = A(L) \left( \begin{array}{c} \Delta Z \\ \Delta Y \end{array} \right)_{t-1} + B \cdot EqCM(Z_{t-1}) + C \cdot X_t + D \cdot INT_t + \varepsilon_{zt}.
$$

The matrix $B$ contains the coefficients of the equilibrium correction terms (if any) in the marginal models (with the loadings along the diagonal). The matrix $C$ contains the coefficients of the maintained exogenous variables $X_t$ in the marginal models for $Z_t$. Intervention variables affecting the mean of the variables under investigation—significant dummies and non-linear terms$^{10}$—are collected in the $INT_t$.

$^{10}$The marginal equations for $ν_t$, $y_t$, and $pr_t$ are endowed with composite dummies necessary
matrix, with coefficients $D$. By definition, the elements in $\mathbf{INT}_t$ are included because they pick up linear as well as non-linear features of $Z_t$ that are left unexplained by the information set underlying the wage-price model.\(^{11}\)

To test for parameter invariance in the wage-price model, we test for the significance of the intervention variables of $\mathbf{INT}_t$ relevant in each equation.\(^{12}\) The results for adding the set of intervention variables to the wage-price model (12) are reported below in table 4.

\[ \Delta w_t = \cdots + 0.0054 \ STU_{t-1} - 0.011 \ \Delta co \times STU_{t-3} + 0.055 \ \Delta yf \times STU_{t-3}, \]
\[ - 0.047 \ \Delta oil_t \times OILST_t - 0.0079 \ \Delta oil_{t-2} \times OILST_t + 0.018 \ \Delta oil_{t-1} \times OILST_t \]
\[ - 0.00075 \ Vdum_t + 0.0011 \ PRdum_t \]
\[ \Delta p_t = \cdots + 0.02 \ \Delta oil_t \times OILST_t - 0.0031 \ \Delta oil_{t-2} \times OILST_t \]
\[ - 0.0075 \ \Delta oil_{t-1} \times OILST_t + 0.00075 \ Vdum_t \]
\[ - 0.0016 \ Ydum_t - 0.0025 \ PRdum_t \]

| Testing the invariance with respect to all interventions: $\chi^2(14) = 28.8[0.01]$ |
| Testing the invariance, not including exchange rate interventions: $\chi^2(6) = 5.3.[0.51]$ |

We cannot reject the significance of two of the oil-price terms in either equation. Therefore the parameters of the growth rate in import prices $\Delta p_i t$ cannot be treated as invariant to changes in oil prices. This finding suggests that the impact of changing oil-prices on domestic inflation is not fully captured by our simple assumption of full and immediate pass-through of oil-price induced exchange rate movements, through $\Delta p_i t = \Delta v_t + \Delta p^*_f t$ in the model. This means that valid policy simulations of the impact of changing oil prices on inflation cannot be conducted with this model.

Considering the other intervention variables, they are individually and jointly insignificant, with a joint test statistic of $\chi^2(6) = 5.3.[0.5]$, providing ample evidence of parameter invariance. Moreover, the finding of weak exogeneity of the conditioning variables is further corroborated by the fact that full information maximum

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\(^{11}\)The idea to first let the marginal models include non-linear terms in order to obtain stability and second to use them as a convenient alternative against which to test invariance in the conditional model, was first proposed by Jansen and Teräsvirta (1996).

\(^{12}\)There is no marginal model for the impact of import prices $\Delta p_i t$. Instead, we have assumed full and immediate pass-trough of the exchange rate, imposing $\Delta p_i t = \Delta v_t + \Delta p^*_f t$ on the model. We therefore use the intervention variables of $\Delta v_t$ to test for invariance of the parameters of $\Delta p_i t$. 

---
likelihood estimates of the full system of eleven equations explaining (the change in) all model variables yields virtually the same empirical results for the wage and price equations as reported in (12) for the conditional model.

7 Is modelling sub-systems and combining them to a global model a viable procedure?

The traditional approach to building large scale macroeconometric models has been to estimate one equation (or submodel) at a time and collect the results in the simultaneous setting. Most often this has been done without testing for the adequacy of that procedure. The approach could however be defended from the estimation point of view. By adopting limited information maximum likelihood methods, one could estimate the parameters of one equation, while leaving the parameters of other equations unrestricted, see Anderson and Rubin (1949) and Koopmans and Hood (1953). It has however also been argued that the limited information methods were more robust against mis-specified equations elsewhere in the system in cases where one had better theories or more reliable information about a subset of variables than about the rest (cf Christ (1966), p. 539). Historically, there is little doubt that limited information methods - like limited information maximum likelihood (LIML) - were adopted out of practical considerations, to avoid the computational burden of full information methods - like full information maximum likelihood (FIML). The problem of sorting out the properties of the system that obtained when the bits and pieces were put together, remained unsolved.

That said, it is no doubt true that we run into unchartered territory when we - after constructing relevant submodels by marginalisation and conditioning - combine the small models of subsectors to a large macroeconometric model. As we alluded to in the introduction, it is pointed out by Johansen (2002) that a general theory for the validity of the three steps will invariably contain criteria and conditions which are formulated for the full system. The question thus is: Given that the full model is too large to be modelled simultaneously, is there a way out?

One solution might be to stay with very aggregated models that are small

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13 These include the core model, equations for all feedback variables of Section 6.4 and five auxiliary equations needed to close the model for forecasting. See Bårdsen et al. (2003) for further details.

14 In Bårdsen et al. (2003) this model, which we has alluded to as "RIMINI writ small", is used to forecast inflation and simulate effects of monetary policy. Moreover in a companion paper - Bårdsen et al. (2002a) - we evaluate the core model in (12) against two variants of the Phillips curve - the standard open economy Phillips curve and a New Keynesian expectations augmented Phillips curve model. The New Keynesian Phillips curve is rejected as a valid representation of the inflationary process in Norway, see also Bårdsen et al. (2002b, 2004b). The standard Phillips curve appears to be well specified according to the specification tests, but it is encompassed by the core model in (12). Further, the core model in (12) both forecasts annual inflation better and has significantly smaller forecast error uncertainty than the Phillips curve model.

15 Interestingly, the papers that introduced the limited information methods are also introducing the first tests of overidentifying restrictions in econometrics.

16 Johansen (2002) has pointed out that LIML does not work with cointegrated systems, where relaxing cross equation restrictions (implied by cointegration) changes the properties of the system.
enough to be analysed as a complete system. Such an approach will necessarily leave out a number of economic mechanisms which we have found to be important and relevant in order to describe the economy adequately.

Our general approach can be seen as one of gradualism - seeking to establish structure (or partial structure) in the submodels. In Section 4.2 we gave a formal definition of partial structure as partial models that are i) invariant to extensions of the sample period, ii) invariant to changes elsewhere in the economy (e.g. due to regime shifts) and iii) remain the same for extensions of the information set.

The first two of these necessary conditions do not require that we know the full model. The most common cause for them to be broken is that there are important omitted explanatory variables. This is detectable within the frame of the submodel once the correlation structure between included and excluded variables change.

For the last of these conditions we can, at least in principle, think of the full model as the ultimate extension of the information set, and so establishing structure or partial structure represents a way to break free of Søren Johansen’s Catch 22. In practice, however, we know that the full model is not attainable. Nevertheless, we note that the conditional consumption function of Section 5.2 is constant when the sample is extended with nine years of additional quarterly observations; it remains unaltered through the period of financial deregulation and it also sustains the experiment of simultaneous modelling of private consumption, household disposable income, household wealth and real housing prices. We have thus found corroborating inductive evidence for the conditional consumption function to represent partial structure. The simultaneous model in this case is hardly an ideal substitute for a better model of the supply side effects that operate through the labour market, nonetheless it offers a safeguard against really big mistakes of the type that causation “goes the other way”, e.g. income is in fact equilibrium correcting not consumption.

When modelling wages and prices in Section 6 we go one step further. In that case we are in fact entertaining two models: One detailed and carefully modelled core model for wage and price determination (Model A), where we condition on a number of explanatory variables and a second model (Model B*) which in fact is an aggregated model for the entire economy. Model B* embeds Model A - it consists of Model A supplemented with marginal models for that model’s explanatory variables. We have called Model B* “RIMINI writ small”, as it is a serious attempt to capture the properties of the full RIMINI model. There is little doubt that the modelling of demand in Model B* suffers from crude aggregation, but this simplification may not be important for the core model of wages and prices. So we carry on to test the validity of the assumptions made in Model A - e.g. valid conditioning (weak exogeneity) as well as invariance (which together with weak exogeneity defines super exogeneity) - on criteria and conditions formulated within Model B*. To the extent the simultaneous likelihood function of Model B* is adequately representing or approximating the likelihood function of the full model (Model B of Section 1), there is no serious ”Catch 22“ left.

There may be an interesting difference in focus between statisticians and macroeconomic modellers. A statistician may be concerned about the estimation perspective, i.e. the lack of efficiency by analysing a sequence of submodels instead of a full model, whereas a macroeconomic modeller primarily wants to avoid misspecified relationships. The latter is a due to pragmatic real-world considerations as
macroeconomic models are used as a basis for policy making. From that point of view it is important to model the net coefficients of all relevant explanatory variables by also conditioning on all relevant and applicable knowledge about institutional conditions in the society under study. Relying on more aggregated specifications where gross coefficients pick up the combined effects of the included explanatory variables and correlated omitted variables may lead to misleading policy recommendations. Our conjecture is that such biases are more harmful for policy makers than the simultaneity bias one may incur by combining submodels. Whether this holds true or not is an interesting issue which it is tempting to explore by means of Monte Carlo simulations on particular model specifications.

That said, it is of particular importance to get the long-run properties of the submodel right. We know that once a cointegrating equation is found, it is invariant to extensions of the information set. On the other hand, this is a property that needs to be established in each case. We do not know what we do not know. One line of investigation that may shed light on this is associated with the notion of separation in cointegrated systems as described in Granger and Haldrup (1997). Their idea is to decompose each variable into a persistent (long-memory) component and a transitory (short-memory) component. Within the framework of a vector equilibrium correcting model like (5), the authors consider two subsystems, where the variables of one subsystem do not enter the cointegrating equations of the other subsystem (cointegration separation). Still, there may be short term effects of the variables in one subsystem on the variables in the other and the cointegrating equations of one system may also affect the short term development of the variables in the other. Absence of both types of interaction is called complete separation whilst if only one of these is present it is referred to as partial separation. These concepts are of course closely related to strong and weak exogeneity of the variables in one subsystem with respect to the parameters of the other. Both partially and completely separated submodels are testable hypotheses, which ought to be tested as part of the cointegration analysis. Hecq et al. (2002) extend the results of Granger and Haldrup (1997). The conclusion of Hecq et al. (2002) is however that testing of separation requires that the full system is known, which is in line with Søren Johansen’s observation above.
References


