

# Ratifiability of Efficient Collusive Mechanisms in Second-Price Auctions with Participation Costs

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## Abstract

We investigate whether efficient collusive bidding mechanisms are affected by potential information leakage from bidders' decisions to participate in them within the independent private values setting. We apply the concept of *ratifiability* introduced by Cramton and Palfrey (1995) and show that when the seller uses a second-price auction with participation costs, the standard efficient cartel mechanisms such as preauction knockouts analyzed in the literature will *not* be ratified by cartel members. A high-value bidder benefits from vetoing the cartel mechanism since doing so sends a credible signal that she has high value, which in turn discourages other bidders from participating in the seller's auction.

*JEL Classifications:* C72, D44, D82

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# 1 Introduction

Auctions are important and widely employed trading mechanisms. The well-known advantages of using auctions diminish when bidders collude for which they have ample incentives. However, the bidders' problem of finding a way to profitably collude and share the generated collusive surplus is complicated by the possibility of each having private information.

Much of the theoretical literature on collusion in auctions treats it as a mechanism design problem with a special focus on feasibility of efficient collusive mechanisms when side-payments are possible.<sup>1</sup> Graham and Marshall (1987) analyze collusion in second-price sealed bid and English auctions with independent private values. For instance, in the context of a second-price auction with a reserve price, they show that a second-price preauction knockout run by an outside agent is an incentive-efficient and durable mechanism in the sense of Holmstrom and Myerson (1983). The knockout is also ex-ante budget balanced among bidders. Mailath and Zemsky (1991) consider second-price auctions with a reserve price and heterogeneous bidders, and show that for any subset of bidders there exists an incentive compatible and individually rational collusive mechanism that is ex post budget balanced and efficient. McAfee and McMillan (1992) study collusion in first-price sealed-bid auctions, also in the independent private values setting, and show that efficient collusion among all bidders with ex post budget balancing is possible and can be implemented with a first-price preauction knockout.<sup>2</sup>

All these papers follow the standard mechanism design approach in that when they consider (interim) individual rationality constraints they compare bidders' interim payoffs from the collusive mechanism with those from an equilibrium of the seller's auction, i.e., the *status quo*. Note, however, that the status quo here is a strategic interaction, and its outcome is affected by bidders' beliefs about others' values. This observation is important if bidders are making interim participation decisions for the collusive mechanism, as first pointed out by Cramton and Palfrey (1995; hereafter CP) in the general mechanism design problem. Others can make inferences about a bidder's type from her choice between the collusive mechanism and the status quo, which in turn may affect the outcome of the status quo game if it is

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<sup>1</sup>There is a recent literature on repeated auctions. See, for example, Aoyagi (2003), Blume and Heidhues (2002), and Skrzypacz and Hopenhayn (2004).

<sup>2</sup>Also see the recent work by Marshall and Marx (2003) on collusive mechanisms with a focus on those that do not rely on information from the auctioneer and are not all-inclusive.

played. CP refers to this possibility as the *information leakage* problem from participation decisions.<sup>3</sup> To address this problem they consider a two-stage process: Players simultaneously vote for or against the proposed mechanism. This vote occurs in the interim stage, i.e., when players have private information. If the mechanism is unanimously *ratified*, then it is implemented; otherwise the status quo game is played under revised beliefs where these satisfy consistency requirements in the spirit of rational expectations.<sup>4</sup>

In this paper we apply CP's approach to show that in the independent private values environment the standard efficient collusive mechanisms characterized in the literature are *not* ratifiable when the status quo is a second-price auction and there are participation costs, however small they may be.<sup>5</sup> In other words, in our setting the efficient collusive mechanisms will not be ratified by all types of all bidders. Bidders with high values will (credibly) signal that they have high values by vetoing the proposed cartel mechanism: A veto (believed to be by a high-valued bidder) discourages other bidders' participation in the auction, and as a result, the high-valued vetoer's payoff from the auction exceeds her payoff from the cartel mechanism. To be a little bit more specific, suppose that a veto of the proposed cartel mechanism by bidder  $i$  leads others to believe that her value is greater than  $v_N$ . Then, in the ensuing auction there is an equilibrium such that bidder  $i$ 's payoff in this equilibrium is higher than her payoff from the cartel *if and only if* her value is greater than  $v_N$ . In other words, a veto, believed to be by a bidder whose value is higher than  $v_N$ , is profitable only for these types, justifying others' beliefs about the vetoer. Hence, the cartel mechanism is not ratifiable. This is the case even though the cartel mechanism is individually rational in the standard sense, i.e., when bidders' interim payoffs from it are compared to those from the equilibrium of the auction with prior (or, *passive*) beliefs. Since entry fees and all kinds of participation costs are frequently observed in practice, our finding indicates that collusion may not be as easy to achieve as the previous literature has suggested. Moreover, the seller may use entry fees to destabilize potential collusive arrangements among bidders.

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<sup>3</sup>There is also the possibility of information leakage from the selection of the mechanism, i.e., the informed seller's problem; see, for example Myerson (1983), Maskin and Tirole (1990), and, in a bargaining context, Yilankaya (1999).

<sup>4</sup>The refinement is based on Grossman and Perry (1986).

<sup>5</sup>In the analysis below, "participation cost" can be replaced by "entry fee" without any change in the formal model or results. Accordingly, we will use both interpretations in our discussions.

Recently, Hendricks, Porter, and Tan (2004) studied the collusion problem in affiliated private value and common value environments, using the standard mechanism design approach. They show that, among other things, in some common value cases where the winner's curse effect is strong, no efficient collusive mechanism will be (interim) individually rational in the standard sense, i.e., with passive beliefs: bidders with high signals are better off if no one colludes. In this paper we show, in our independent private values setting, that the efficient collusive mechanism, which satisfies the standard interim individual rationality constraints, is not ratifiable because of the information leakage problem we discussed above. In addition to such factors as internal enforcement problem of a bidding ring, antitrust concern, and detection by the seller that are discussed in the literature, information leakage from bidders' participation decisions and the winner's curse are also potential sources of obstacles to collusion in auction markets.<sup>6</sup>

Our main result has a similar flavor to an example in CP (p.267-269) concerning collusion in a duoplistic setting, which we discuss next. Two firms have private information about their (constant) marginal costs that are drawn from a uniform distribution. The status quo is a (Bayesian) Cournot game, where the inverse demand function is affine. Using the standard mechanism design approach, one can find (see Cramton and Palfrey, 1990) the joint-monopoly mechanism that is incentive compatible, efficient (only the lowest-cost firm produces), and individually rational (in the standard sense). However, as CP shows this mechanism is not ratifiable. By vetoing the mechanism a firm sends a credible signal that it has relatively low marginal cost, which decreases the other firm's output in the ensuing Cournot game. The veto is a credible signal, since it is precisely these low cost types that benefit from the decrease in the other firm's output sufficiently that their profits in the Cournot game (with revised beliefs) end up being higher than their profits in the joint-monopoly mechanism.

In the next section we briefly describe the setup, discuss the concept of ratifiability and present our main finding. Section 3 contains discussions about possible extensions and some remarks. Proofs can be found in the Appendix.

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<sup>6</sup>Note that the ratifiability concept does not always strengthen participation constraints, see CP (p. 269) for an example. So, it may be possible that an efficient collusive mechanism in a common value setting is ratifiable even though it is not individually rational in the standard sense.

## 2 The Model and the Main Result

A single indivisible object is being sold by a seller who uses a sealed-bid second-price auction with no reserve price.<sup>7</sup> Consider a standard independent private values environment. There are  $n \geq 2$  risk-neutral (potential) bidders. Bidders' values for the object are independent draws from the same cumulative distribution function (cdf)  $F(\cdot)$  with continuous and strictly positive density  $f(\cdot)$  on its support  $[0, 1]$ . Bidder  $i$ 's value,  $v_i$ , is her private information. There is a bidder participation cost, or entry fee, common to all bidders, denoted by  $c \in (0, 1)$ . Bidders know their values before they decide whether to participate in the auction and must pay  $c$  in order to be able to submit a bid; they do not know others' participation decisions when they make theirs. All of the above is common knowledge.

### 2.1 The Status Quo: Non-cooperative Play

The status quo game we consider is the second-price auction. Let the feasible action set for any type of bidder be:  $\{No\} \cup [0, \infty)$ , where “No” denotes not participating; bidder  $i$  incurs the participation cost iff her action is different from “No”. Let  $b_i(\cdot)$  denote  $i$ 's strategy.

In the presence of a participation cost, it is *not* a weakly dominant strategy for a bidder to always bid her value in the seller's auction. However, if a bidder finds participating optimal, then she cannot do better than bidding her value.<sup>8</sup> Let  $G(y) = F(y)^{n-1}$  denote the cdf of the highest valuation of  $n - 1$  bidders. It is easy to see that there exists a unique (up to changes for a measure zero set of values) symmetric Bayesian-Nash equilibrium, where each bidder's strategy is (suppressing subscripts)

$$b^s(v) = \begin{cases} No & \text{if } v \leq v_0 \\ v & \text{if } v > v_0 \end{cases},$$

where  $v_0 G(v_0) = c$ .<sup>9</sup> The expected payoff of a type- $v$  bidder in this equilib-

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<sup>7</sup>We assume zero reserve price to keep the discussion and notation simple. The analysis and results hold when there is a binding reserve price.

<sup>8</sup>See, for example, Matthews (1995). Throughout the paper, we ignore equilibria in dominated strategies where bidders do not bid their values.

<sup>9</sup>There may be “reasonable” asymmetric equilibria as well. Tan and Yilankaya (2005) show that concavity (respectively, strict convexity) of  $F(\cdot)$  is a sufficient condition for uniqueness (respectively, multiplicity) of equilibrium within the class of *cutoff strategy*

rium is given by

$$U^s(v) = \begin{cases} 0 & \text{if } v \leq v_0 \\ vG(v_0) + \int_{v_0}^v (v-y)dG(y) - c & \text{if } v > v_0 \end{cases} ,$$

or, after using integration by parts

$$U^s(v) = \begin{cases} 0 & \text{if } v \leq v_0 \\ \int_{v_0}^v G(y)dy & \text{if } v > v_0 \end{cases} .$$

## 2.2 The Efficient All-Inclusive Cartel Mechanism

Suppose transfers among bidders are possible. Bidders may design a collusive bidding mechanism outside of the seller's auction. If the all-inclusive cartel mechanism is ex post efficient (i.e., the bidder with the highest value acquires the object if this value is greater than the participation cost), then it does not matter whether there is a participation cost or a reserve price in the seller's auction provided that their magnitudes are the same. Therefore, it follows from Mailath and Zemsky (1991) and McAfee and McMillan (1992) that there exists an incentive compatible, ex post budget balanced, and ex post efficient all-inclusive (symmetric) cartel mechanism  $m$  where the payoff of a type- $v$  bidder is given by

$$U^m(v) = \begin{cases} \pi & \text{if } v \leq c \\ \pi + \int_c^v G(y)dy & \text{if } v > c \end{cases} ,$$

where

$$\pi = \int_c^1 \left[ y - \frac{1-F(y)}{f(y)} - c \right] G(y) dF(y).$$

Since  $v_0 > c$  and  $\pi > 0$ , it follows that

$$U^m(v) > U^s(v) \tag{IR}$$

for all  $v \in [0, 1]$ . That is, the efficient cartel mechanism  $m$  is interim individually rational in the standard sense with respect to symmetric equilibrium payoffs in the seller's auction.

The efficient cartel mechanism can be implemented by a first-price or second-price preauction knockout in which all bidders compete for the right

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profiles (each bidder bids her value if it is greater than a cutoff, does not participate otherwise).

to be the sole bidder in the seller’s auction. In the case of a first-price knockout auction, the bidder with the highest bid wins and pays her bid, which will be shared equally among the losers.

### 2.3 Ratifying the Cartel Mechanism

The main objective of this paper is to study whether the efficient cartel mechanism described above can still be used by bidders if we allow the possibility of learning from their decisions to participate in it. The outside option faced by bidders when they are deciding to participate in the cartel, i.e., the status quo game, is a second-price auction with participation costs. The outcome of the auction, naturally, depends on bidders’ beliefs about others’ values, which in turn may be affected by their participation decisions in the cartel.

We consider a two-stage ratification game to analyze this problem, following CP. In the first stage, bidders simultaneously vote (interim) for or against  $m$ , the efficient cartel mechanism.<sup>10</sup> In the second stage, if the cartel mechanism is unanimously accepted then it is implemented; otherwise, bidders participate in the second-price auction knowing who had vetoed the cartel mechanism, and thus having updated beliefs about vetoers’ values.<sup>11,12</sup> The standard individual rationality conditions implicitly assume that beliefs following a possible veto are passive, i.e., the payoffs from the cartel mechanism are compared with the payoffs from an equilibrium of the auction with prior beliefs (values of all bidders, including the vetoer, are distributed on  $[0, 1]$  according to  $F(\cdot)$ ). As we remarked earlier,  $m$  is individually rational in this sense; see (IR). So, there is a sequential equilibrium of the ratification game, supported by passive beliefs after a possible veto, in which  $m$  is unanimously accepted in the first stage (see also Proposition 1 in CP).

As CP argued (in the general case) assuming passive beliefs following a veto may not be reasonable in the ratification game. Other bidders may draw inferences about the vetoer’s type. CP proposes a refinement of equilibria in the ratification game to address this issue. Applied to our problem, the

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<sup>10</sup>We follow CP, and much of the literature on collusion from a mechanism design perspective, in requiring that an “uninformed third party” proposes the cartel mechanism.

<sup>11</sup>Since our setup is symmetric, it is sufficient for our results that bidders only learn the number of vetoers, not their identities.

<sup>12</sup>Our main result still holds even if after a veto the remaining  $n - 1$  bidders manage to form an efficient partial cartel. (See the discussion in Section 3 for more on this.) We thank an anonymous referee for suggesting that we consider this possibility.

cartel mechanism is *ratifiable*, if, in addition, other bidders' beliefs about the vetoer's value satisfy certain consistency requirements.<sup>13</sup> In particular, if bidder  $i$  vetoes the mechanism, then the rest of the bidders try to rationalize  $i$ 's veto decision by identifying a set of types for  $i$  (the *veto set*, denoted by  $V_i$ ) that could have benefited from the veto. Let  $b$  be an equilibrium of the second-price auction with participation cost  $c$  where others believe that  $i$ 's type is in  $V_i$ .<sup>14</sup> For example, if  $V_i = [v_N, 1]$ , then  $b$  denotes an equilibrium of the auction where ratifiers' values are distributed on  $[0, 1]$  according to  $F(\cdot)$ , and the vetoer's value is distributed on  $[v_N, 1]$  according to  $F_N(v) \equiv \frac{F(v) - F(v_N)}{1 - F(v_N)}$ , which is derived from  $F(\cdot)$  using the Bayes' rule. Let  $U_i(v; b)$  be type- $v$  vetoer's payoff in  $b$ . The veto set  $V_i$  is *credible* if there is an equilibrium of the post-veto auction with updated beliefs,  $b$  which gives the types in (respectively, outside)  $V_i$  higher (respectively, lower) payoff than the cartel mechanism.<sup>15</sup> In other words, types in a credible veto set have an incentive, possibly weak, to veto the mechanism if doing so makes other bidders believe that they belong to this set, which in turn justifies others' beliefs. Finally, the cartel mechanism is *ratifiable* if either there is no credible veto set for any player or there exists one where all types in the veto set are actually indifferent between the cartel mechanism and the equilibrium in the seller's auction.

The formal definitions are as follows:

**Definition 1**  $V_i \subseteq [0, 1]$  is a credible veto set for  $i$  if there exists an equilibrium  $b$  in the post-veto auction (where other bidders' beliefs about  $i$ 's type  $v$  are Bayes update of their prior, conditioning on  $v \in V_i$ ) such that

- i)  $V_i \neq \emptyset$ ,
- ii)  $U_i(v; b) > U^m(v) \Rightarrow v \in V_i$ ,
- iii)  $U_i(v; b) < U^m(v) \Rightarrow v \notin V_i$ .

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<sup>13</sup>Since we will be looking for an equilibrium in which bidders unanimously ratify the cartel mechanism, we only need to be concerned about unilateral deviations.

<sup>14</sup>We restrict attention to equilibria where bidders use cutoff strategies (see Footnotes 8 and 9). Note that we are suppressing the dependence of  $b$  on  $V_i$  to keep the notation to a minimum.

<sup>15</sup>A credible veto set  $V_i$  is thus determined jointly with  $b$  that makes it credible. In particular, there may be multiple equilibria given  $V_i$ , and the vetoer may actually be worse off (compared to the cartel) in some of them. Credibility imposes a consistency condition on the  $V_i$  and  $b$  pair to rationalize a possible veto, which is not satisfied by these other equilibria.

**Definition 2** *The cartel mechanism is ratifiable against the seller’s auction if for all  $i$  either*

- i) there does not exist a credible veto set for  $i$ , or*
- ii) there exists a credible veto set  $V_i$  and a corresponding equilibrium  $b$  in the post-veto auction such that  $U_i(v; b) = U^m(v)$  for all  $v \in V_i$ .*

The above definitions are slightly different from CP’s, since we have already incorporated the fact that the cartel mechanism is incentive compatible and individually rational (with passive beliefs). Given these definitions, we now present the main result of our paper.

**Proposition 1** *The efficient cartel mechanism  $m$  is not ratifiable.*

The basic intuition behind the result is as follows. Low-value bidders have relatively more to gain by participating in the cartel mechanism than high-value bidders. By vetoing the cartel mechanism, a high-value bidder sends a signal that she has a high value, which would discourage other bidders from participating in the seller’s auction and benefit the vetoer. On the other hand, a low-value bidder is not able to gain from this effect of vetoing as much. This makes vetoing by high-value bidders credible.

To present the above intuition formally, and as the first step for proving the result, we will construct a credible veto set for any given bidder (Lemma 1). We will then show that there does not exist a credible veto set for any bidder where all types are indifferent between vetoing and ratifying (Lemma 2).

Suppose that when one of the bidders vetoes the cartel mechanism, others believe that her value exceeds a cutoff point  $v_N < 1$ .<sup>16</sup> We will show that there is an equilibrium of the auction with these revised beliefs such that the vetoer’s payoff in this equilibrium is larger (respectively, smaller) than her cartel payoff if her value is larger (respectively, smaller) than  $v_N$ . In other words, it is precisely the types in  $[v_N, 1]$  who would want to veto the cartel mechanism if a veto causes other bidders to believe that the vetoer’s type is in  $[v_N, 1]$ , so that  $V_i = [v_N, 1]$  is a credible veto set for any bidder  $i$ .<sup>17</sup>

Consider the second-price auction where the ratifiers’ values are distributed on  $[0, 1]$  according to  $F(\cdot)$  and the vetoer  $i$ ’s value is distributed on

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<sup>16</sup>Naturally,  $v_N$  depends on the participation cost  $c$ , as well as other exogenous variables, this dependence is suppressed in the notation. Also, “ $N$ ” is for saying “no” to the cartel mechanism (the vetoer), and “ $Y$ ” is for saying “yes” (ratifiers).

<sup>17</sup>Types in  $(v_N, 1]$  strictly prefer to veto, whereas  $v_N$  is indifferent.

$[v_N, 1]$  according to  $F_N(v)$ , defined earlier. The equilibrium we consider in this auction, denoted by  $b^*$ , is given by

$$\begin{aligned} b_i^*(v) &= v \text{ for all } v \in [v_N, 1] \\ b_j^*(v) &= \begin{cases} \text{No} & \text{if } v \leq v_Y \\ v & \text{if } v > v_Y \end{cases} \text{ for all } j \neq i. \end{aligned}$$

In this equilibrium, the ratifiers (who are symmetric) use a common cutoff  $v_Y$ , which is determined by the indifference to participation condition. For any given ratifier, the maximum of others' values is distributed on  $[v_N, 1]$  according to  $\hat{G}(y) \equiv F(y)^{n-2}F_N(y)$ . Let  $\tilde{v}_Y$  be the solution to

$$\int_{v_N}^{\tilde{v}_Y} (\tilde{v}_Y - y) d\hat{G}(y) = c,$$

where the left-hand side denotes the expected payoff of a  $\tilde{v}_Y$ -type ratifier whenever  $\tilde{v}_Y \leq 1$ . We have  $v_Y = \min\{1, \tilde{v}_Y\}$  and  $v_N < v_Y \leq 1$ . Notice that  $v_Y$  is a strictly increasing function of  $v_N$  until it reaches 1 for some value of  $v_N$  and stays there for higher values of  $v_N$ .

Since the best any player can do is to bid her valuation, if she chooses to participate in it, the expected payoff of *any* type- $v$  bidder  $i$  if she vetoes the cartel mechanism is (given the ratifiers' belief that her type is in  $[v_N, 1]$  and the equilibrium we are considering in the auction, namely  $b^*$ )

$$U_i(v; b^*) = \max\left\{ \int_0^{\max\{v, v_Y\}} (v - p(y)) dG(y) - c, 0 \right\},$$

where

$$p(y) = \begin{cases} 0 & \text{if } y \leq v_Y \\ y & \text{if } y > v_Y \end{cases}$$

is the price paid by the vetoer in case she wins the object.

Comparing this payoff with the one from the cartel mechanism leads to the following lemma.

**Lemma 1** *There exists a (unique)  $v_N \in (c, 1)$  such that  $U_i(v; b^*) \geq U^m(v)$  iff  $v \geq v_N$ , where the equality holds only when  $v = v_N$ .*

Lemma 1 shows that if after a veto of the cartel mechanism by bidder  $i$  others believe that the her value exceeds  $v_N$  (and the equilibrium  $b^*$  is

played), then  $i$  would benefit from such a veto iff her value exceeds  $v_N$ . Thus,  $V_i = [v_N, 1]$  is a credible veto set for player  $i$ . Furthermore, there does not exist any credible veto set for which *all* types in this set are indifferent between vetoing and ratifying. This is stated in the following lemma.

**Lemma 2** *For any given  $i$ , there does not exist a credible veto set  $V_i$  and a corresponding equilibrium  $b$  in the post-veto auction such that  $U_i(v; b) = U^m(v)$  for all  $v \in V_i$ .*

Proposition 1 then follows from Definition 2 and Lemmas 1 and 2.

The cutoff of the credible veto set  $v_N$  (Lemma 1) naturally depends on the distribution  $F(\cdot)$ , the number of bidders  $n$  and the participation cost  $c$ . To see the impact of a positive participation cost, consider two extreme cases. Given  $F(\cdot)$  and  $n$ , when  $c$  is large the veto set is such that  $v_Y = 1$ , i.e., the ratifiers do not participate in the auction following a veto. For instance, suppose  $n = 2$ ,  $F(v) = v$ , and  $c = 2/3$ . Then it can be easily computed that  $v_Y = 1$  and  $v_N = 0.686$ . In the other extreme, as  $c$  goes to 0, both  $v_N$  and  $v_Y$  approach the same limit  $v_N^*$  satisfying

$$v_N^* G(v_N^*) = \int_0^1 \left[ y - \frac{1 - F(y)}{f(y)} \right] G(y) dF(y) + \int_0^{v_N^*} G(y) dy,$$

where the left-hand side is the payoff of type- $v_N^*$  vetoer in the post-veto auction and the right-hand side is her payoff from the cartel mechanism (see Section 2.2), both when  $c = 0$ . Notice that  $v_N^* > 0$ . This implies that even when  $c = 0$  the efficient cartel mechanism is not ratifiable. However, we do not want to emphasize this point. The issue is, when  $c = 0$ , in the post-veto auction where the vetoer's type is known to be in  $[v_N^*, 1]$ , there is an equilibrium in weakly dominant strategies in which *all* types of ratifiers participate (rather than only those in  $[v_N^*, 1]$ ). If this is the post-veto equilibrium, then bidders clearly will not have an incentive to veto. Notice that, a positive  $c$ , however small, eliminates this issue, ratifiers with values less than  $v_N^*$  will never participate in the post-veto auction, and the cutoff they use will approach to  $v_N^*$  from above as  $c$  goes to zero.

### 3 Discussion

Since the seller's mechanism is a second-price auction, our analysis and main finding apply to the case of heterogeneous bidders. Our main result also carries over to partial cartels, however small. Suppose  $k$  bidders form a cartel,

where  $1 < k < n$ . Assume that the cartel uses a mechanism that is ex post efficient, incentive compatible and balances the budget (see Mailath and Zemsky, 1991), and that it sends a representative member to bid in the auction, if bid at all. From non-members' viewpoint, the cartel's (maximum) value has a distribution  $F^k(\cdot)$ . As before we assume that a unanimous agreement is required to ratify the cartel mechanism. To see whether this mechanism is ratifiable, suppose one of cartel members deviates and vetoes it. We can compute the equilibrium payoffs in the second-price auction where the ratifiers believe that the vetoer's type is in  $[v_N, 1]$  and the non-members think that the cartel is still in operation. We can then determine the cutoff  $v_N$  in the same way as we presented in Section 2, so that  $[v_N, 1]$  is a credible veto set, showing that the cartel mechanism is not ratifiable.<sup>18</sup>

We assumed that the cartel disintegrates after a veto by one of its members, following CP's original definitions. An alternative is to assume the formation of an efficient partial cartel by  $n - 1$  non-vetoers.<sup>19</sup> Even if this were the case, our main result would not change, i.e., the cartel mechanism we considered would still be not ratifiable. The reason is that, apart from participation cutoffs, the vetoer is concerned only about the highest valuation of the other bidders in the post-veto auction, and whether the other bidders collude efficiently (and send only the one with the highest-valuation to the auction) or bid non-cooperatively is immaterial in this respect. The magnitudes of participation cutoffs in the equilibrium of post-veto auction ( $v_N$  and  $v_Y$ ) may change if non-vetoers can collude, but this has no impact on our result, which is qualitative in nature, about the existence of a  $v_N \in (0, 1)$  for which  $[v_N, 1]$  is a credible veto set.

How would the seller respond to the presence of collusive bidding arrangements? The literature has mostly focused on the seller's response to possible collusion by appropriately choosing a reserve price. For instance, Graham and Marshall (1987) have shown that the seller's optimal reserve price with a bidding cartel is always greater than the one without a cartel. Our analysis suggests that an entry fee, independent of its magnitude, may be a useful tool in dealing with bidder collusion. Moreover, it is as good an instrument as a reserve price in terms of maximizing revenue in this setup.

There are several empirical studies that offer evidence of collusion in

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<sup>18</sup>Along the lines of Lemma 2, one can show that the second part of the ratifiability definition will not be satisfied either.

<sup>19</sup>This may not be warranted, since this partial cartel will also have the ratifiability problem, as suggested by our discussion in the previous paragraph.

many auction markets.<sup>20</sup> In these studies, the status quo game is typically a first-price sealed-bid auction. Even when bidders are ex-ante symmetric, to investigate ratifiability of collusive arrangements, one needs to consider asymmetric auctions induced by possible vetoes which are, generally, difficult to analyze.<sup>21</sup> We want to remark, however, that when there are two bidders and there are *no* participation costs, veto sets of the form  $[v_N, 1]$  that we considered in this paper will *not* be credible if the status quo is a first-price auction, since the vetoer will be hurt from being known as the “strong” bidder.<sup>22</sup> Sets of the form  $[0, v_N]$  will not be credible either, since the vetoer whose value is zero will receive a payoff of zero in every equilibrium of the post-veto auction, which is less than what she would get from the cartel mechanism.

Finally, it should be noted that, we have *only* shown that the well-known efficient cartel mechanism is not ratifiable in our setting.<sup>23</sup> We leave for future research the question of whether there are other (inefficient) collusive arrangements which are immune to problems associated with information leakage from members’ participation decisions. Comparing the performance of different auction formats in this regard also seems to be a worthwhile exercise.

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<sup>20</sup>Examples include highway construction contracts (Porter and Zona, 1993), school milk delivery (Pesendorfer, 2000, Porter and Zona, 1999), timber auctions (Baldwin, Marshall and Richard, 1997), and federal offshore oil and gas lease auctions (Hendricks, Porter, and Tan, 2004).

<sup>21</sup>As Marshall and Marx (2003) show, when the seller uses a first-price auction, there are other difficulties with the efficient collusive mechanism considered here if the cartel cannot enforce its recommended bids or shill bidding is possible. These problems do not arise in our setting where the seller uses a second-price auction.

<sup>22</sup>Consider the auction where the ratifier’s (the “weak” bidder) value is distributed by  $F(\cdot)$  on  $[0, 1]$ , and the vetoer’s (the “strong” bidder) by  $F_N(\cdot)$  on  $[v_N, 1]$ , derived from  $F(\cdot)$  using Bayes’ rule, with  $v_N > 0$ . It follows from Maskin and Riley (2000, p. 425) that the payoff of the vetoer is lower than what she would get if both bidders were weak, which in turn is lower than what she would get in the cartel mechanism (due to individual rationality with passive beliefs). Hence, no type in  $[v_N, 1]$  would like to veto the mechanism if a veto signals that the vetoer’s value is in  $[v_N, 1]$ .

<sup>23</sup>This nonratifiability result still holds if we consider mechanisms that send the highest-value bidder to the seller’s auction iff her value is larger than a certain cutoff that may be strictly higher than the participation cost.

## 4 Appendix

**Proof of Lemma 1.** In what follows we drop the subscript  $i$  to simplify notation. Note that we have

$$U(v; b^*) = \begin{cases} 0 & \text{if } v < \frac{c}{G(v_Y)} \\ vG(v_Y) - c & \text{if } \frac{c}{G(v_Y)} \leq v \leq v_Y \\ vG(v) - \int_{v_Y}^v ydG(y) - c & \text{if } v > v_Y \end{cases} .$$

We will first find a  $v_N$  for which  $U(v_N; b^*) = U^m(v_N)$ , and then check the inequalities.

Since  $c < v_N \leq v_Y$ , it suffices to show (making dependence of  $v_Y$  on  $v_N$  explicit)

$$v_N G(v_Y(v_N)) - c - \pi - \int_c^{v_N} G(y)dy = 0.$$

Let

$$\phi(v) = vG(v_Y(v)) - \int_c^v G(y)dy - c - \pi.$$

Notice that

$$\phi'(v) = G(v_Y(v)) + v g(v_Y(v)) v_Y'(v) - G(v) > 0$$

for  $v < 1$ , since  $v_Y(v) \geq v$ . Since  $\phi(v)$  is continuous,  $\phi(c) < 0$ , and  $\phi(1) > 0$ , a unique solution to  $\phi(v) = 0$  exists, and is our candidate for  $v_N$ . Next, we show

$$U(v; b^*) > (<) U^m(v) \text{ for } v > (<) v_N.$$

Fix  $c$ , and hence  $v_N$  and  $v_Y$ . Notice that,  $c \leq \frac{c}{G(v_Y)} < v_N \leq v_Y$ . The payoff difference  $U(v; b^*) - U^m(v)$  is continuous and given by

$$\begin{aligned} & -\pi & \text{if } v < c \\ & - \int_c^v G(y)dy - \pi & \text{if } c \leq v < \frac{c}{G(v_Y)} \\ & vG(v_Y) - \int_c^v G(y)dy - \pi - c & \text{if } \frac{c}{G(v_Y)} \leq v \leq v_Y \\ & vG(v) - \int_{v_Y}^v ydG(y) - \int_c^v G(y)dy - \pi - c & \text{if } v > v_Y \end{aligned} .$$

Also notice that  $U(v; b^*) - U^m(v) < 0$  for  $v < \frac{c}{G(v_Y)}$ . Let

$$\varphi(v) = vG(v_Y) - \int_c^v G(y)dy - \pi - c.$$

Note that  $\varphi(v_N) = \phi(v_N) = 0$ . Moreover,  $\varphi'(v) = G(v_Y) - G(v) > 0$  for  $v < v_Y$  and  $\varphi'(v_Y) = 0$ . It follows that  $\varphi(v) < 0$  for  $v < v_N$ ,  $\varphi(v_N) = 0$ , and  $\varphi(v) > 0$  for  $v > v_N$ , with  $\varphi(v)$  reaching its maximum, which is strictly positive, at  $v_Y$ . Finally, notice that

$$\frac{d}{dv}[U(v; b^*) - U^m(v)] = 0$$

for  $v > v_Y$ . Hence,

$$U(v; b^*) > (<)U^m(v) \text{ for } v > (<)v_N.$$

The claim follows. ■

**Proof of Lemma 2.** The proof is by contradiction. Suppose there exists a credible veto set  $V_i$  for bidder  $i$  and a corresponding equilibrium in the post-veto auction  $b$  such that

$$U_i(v; b) = U^m(v) \text{ for all } v \in V_i. \quad (1)$$

Note that it must be the case that  $U_i(v; b) > 0$  for all  $v \in V_i$ , because  $U^m(v) > 0$  for all  $v \in [0, 1]$ . Let  $a^*$  be the participation cutoff used by the vetoer in  $b$ . Since the vetoer's payoff in  $b$  would be zero if she were not participating in the post-veto auction, we have  $a^* = \inf V_i$ . Let the cutoffs used by others be  $a_1 < a_2 < \dots < a_k$ , where  $k \geq 1$ , and  $a_j \in [0, 1]$  is used by  $n_j$  bidders, with  $\sum_{j=1}^k n_j = n - 1$ .

If  $a^* < a_1$ , then for  $v \in (a^*, a_1)$ ,

$$U_i(v; b) - U^m(v) = v \prod_{j=1}^k F(a_j)^{n_j} - c - \int_c^v F(y)^{n-1} dy - \pi,$$

which is strictly increasing in  $v$ . So, types in  $(a^*, a_1)$  have a strict incentive to veto, and therefore must also belong to the veto set  $V_i$  (see condition ii) of Definition 1), which is a contradiction with (1).

If  $a^* = a_1$ , then for  $v \in (a^*, a_2)$ ,

$$U_i(v; b) - U^m(v) = \prod_{j=2}^k F(a_j)^{n_j} [F(a^*)^{n_1} a^* + \int_{a^*}^v F(y)^{n_1} dy] - c - \int_c^v F(y)^{n-1} dy - \pi,$$

which is again strictly increasing, as long as  $n_1 \neq n - 1$ , i.e., all the other bidders do not use the same cutoff as the vetoer. So, if  $n_1 \neq n - 1$ , types in

$(a^*, a_2)$  have a strict incentive to veto, leading to a contradiction as before. On the other hand, an equilibrium where all bidders use a common cutoff  $a^* < 1$  exists only if  $V_i = [0, 1]$ . But for that case we know that  $U_i(v; b) < U^m(v)$  (from (IR): the cartel mechanism is individually rational with passive beliefs), which is a contradiction with (1). If  $a^* = 1$ , i.e.,  $V_i = \{1\}$ , then in all  $b$  in which  $i$  participates, others will not participate, and we have

$$U_i(1; b) = 1 - c > U^m(1),$$

a contradiction.<sup>24</sup>

The remaining case, i.e.,  $a^* > a_1$ , is not possible either, since the bidders who use  $a_1$  are sure to lose in the auction, and thus have a negative payoff. The claim follows. ■

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<sup>24</sup>It cannot be the case that the vetoer mixes her participation decision in  $b$ . Indifference to participation implies that her overall payoff is zero as well, leading to a contradiction, i.e.,  $U_i(1; b) = 0 < U^m(1)$ .

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