

1. Old and new growth theories: a unifying structure?

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1.1. INTRODUCTION

1.1.1. Some Background

Long-run growth was first introduced by Solow (1956) and Swan (1956) into the traditional neoclassical macroeconomic model by considering a growing population coupled with a more efficient labour force. The direct consequence of this approach is that the long-run growth rate in these models is ultimately tied to demographic factors, such as the growth rate of population, the structure of the labour force and its productivity growth, all of which were typically taken to be exogenously determined. Hence, the only policies that could contribute to long-run growth were those that would increase the growth of population or the efficiency of the labour force. Conventional macroeconomic policy had no influence on long-run growth performance.

Since then, growth theory has evolved into a voluminous literature in two distinct phases. The Solow–Swan model was the inspiration for a first generation of growth models during the 1960s, which, being associated with exogenous sources of long-run growth, are now sometimes referred to as *exogenous growth models*. Research interest in these models tapered off around 1970, as economists turned their attention to other issues, perceived as being of more immediate significance, such as inflation, unemployment, and oil shocks, and the design of macroeconomic policies to deal with them.¹ Beginning with the seminal work of Romer (1986), there has been a resurgence of interest in economic growth theory giving rise to a second generation of growth models. This revival of activity has been motivated by several issues. These include: (1) an attempt to explain aspects of the data not addressed by the neoclassical model; (2) a more satisfactory explanation of international differences in economic growth rates; (3) a more central role for

the accumulation of knowledge; and (4) a larger role for the instruments of macroeconomic policy in explaining the long-run growth process (see Romer, 1994). These new models seek to explain the growth rate as an endogenous equilibrium outcome of the behavior of rational optimizing agents, reflecting the structural characteristics of the economy, such as technology and preferences, as well as macroeconomic policy. For this reason they have become known as *endogenous growth models*.

The new growth theory is far-ranging. It has been analysed in both closed economies, as well as in open economies. In fact, one of the characteristics of the new growth theory is that it has more of an international orientation (see e.g. Grossman and Helpman, 1991). This may reflect the increased importance of the international aspects in macroeconomics in general. In comparison with the first generation of growth models, the newer literature places a greater emphasis on empirical issues and the reconciliation of the theory with the empirical evidence. In this respect a widely debated issue concerns the so-called convergence hypothesis. The question here is whether or not countries have a tendency to converge to a common per capita level of income or growth rate.

But new growth theory is also associated with important theoretical advances as well, and one can identify two main strands of theoretical literature, emphasizing different sources of economic growth. One class of models, closest to the neoclassical growth model, stresses the *accumulation of private capital* as the fundamental source of economic growth. This differs in a fundamental way from the neoclassical growth model in that it does not require exogenous elements, such as a growing population, to generate an equilibrium of ongoing growth. Rather, the equilibrium growth rate is an internally generated outcome.

In the simplest such model, in which the only factor of production is capital, the constant returns to scale condition implies that the production function must be linear in physical capital, being of the functional form $Y = AK$. For obvious reasons, this technology has become known as the 'AK model'. As a matter of historical record, explanation of growth as an endogenous process in a one-sector model is not new. In fact it dates back to Harrod (1939) and Domar (1946). The equilibrium growth rate characterizing the AK model is essentially of the Harrod–Domar type, the only difference being that consumption (or savings) behavior is derived as part of an intertemporal optimization, rather than being posited directly. These one-sector models assume (often only implicitly) a broad interpretation for capital, taking it to include both human, as well as nonhuman, capital (see Rebelo, 1991). A direct extension to this basic model are two-sector investment based growth models, originally due to Lucas (1988), that

disaggregate private capital into human and nonhuman capital (see also Mulligan and Sala-i-Martin, 1993; and Bond et al., 1996).

A second class of models emphasizes the *endogenous development of knowledge*, or research and development, as the engine of growth. The basic contribution here is that of Romer (1990), who constructs a two-sector model in which new knowledge produced in one sector is used as an input in the production of final output. Indeed, the rigorous treatment of the accumulation of knowledge (technology) is viewed by many as being one of the key distinctions between the old and the new growth theory. Early efforts to introduce technology into the original Solow–Swan model assumed that the rate of technological change was exogenous, and the extent to which this was the case, the model could generate exogenous long-run per capita growth. Subsequent authors such as Uzawa (1965) and Shell (1967) endogenized the rate of technological change, though at a purely macroeconomic level without providing any underlying microeconomic foundations. The microeconomic foundations for understanding the evolution of technology were gradually developed later and one of the important contributions of the new growth theory is to incorporate these developments into aggregate growth models in a more rigorous way.

More recently, the knowledge sector has been extended in various directions by a number of authors (see e.g. Aghion and Howitt, 1992, and more recently, Eicher, 1996, among others). A related class of models deals with innovation and the diffusion of knowledge across countries (see Barro and Sala-i-Martin, 1995, Ch. 8).

Despite its attractive features – chiefly the role it gives to macro policy as a determinant of long-run growth – the endogenous growth model is characterized by two aspects, both of which have been the source of empirical and theoretical criticism. First, both classes of models are often characterized by having ‘scale effects’, meaning that variations in the size or scale of the economy, as measured by say population, affect the size of the long-run growth rate. For example, the Romer (1990) model of research and development implies that a doubling of the population devoted to research will double the growth rate. Whether the AK model is associated with scale effects depends upon whether there are production externalities that are linked to the size of the economy (see Barro and Sala-i-Martin, 1995).

However, empirical evidence does not support the presence of scale effects. For example, OECD data suggest that variations in the level of research employment have exerted no influence on the long-run growth rates of the OECD economies, in contrast to the predictions of the Romer (1990) model (see Jones, 1995b). In addition, the systematic empirical analysis of Backus et al. (1992) finds no conclusive evidence of a relation between US GDP growth and measures of scale. Moreover, Easterly and Rebelo (1993)

and Stokey and Rebelo (1995) find at best weak evidence for the effects of tax rates on the long-run rate of growth, although Kneller et al. (1999) argue that these results are biased because of the incomplete specification of the government budget constraint.

The second limitation of recent endogenous growth models is the requirement that to generate an equilibrium of ongoing growth all production functions must in general exhibit constant returns to scale in the factors of production that are being accumulated endogenously. This is a strong condition, one that imposes a strict knife-edge restriction on the production structure, and has been extensively criticized (see Solow, 1994).

These considerations have stimulated the development of non-scale growth models (see Jones, 1995a, 1995b; Segerstrom, 1998; and Young, 1998). The advantage of such models is that they are consistent with balanced growth under quite general production structures. Indeed, if the knife-edge restriction that generates traditional endogenous growth models is not imposed, then any stable balanced growth equilibrium is characterized by the absence of scale effects. From this standpoint, non-scale growth equilibria should be viewed as being the norm, rather than the exception. In this case the long-run equilibrium growth rate is determined by technological parameters, in conjunction with the exogenously given growth rate of labour, and is independent of macro policy instruments.

In light of the empirical evidence, and the theoretical restrictions associated with endogenous growth models, the generality of the production structures compatible with balanced-growth paths in the non-scale growth model enhances the importance of the latter. These models are in many aspects a hybrid of endogenous and neoclassical models.² Technology can still be endogenous and the outcome of agents' optimizing behavior, as in Romer's (1990) work, yet the long-run growth rate is determined very much as in the Solow–Swan model, which in fact is an early example of a non-scale model. Thus, one is beginning to see a merger between the old and the new growth theory.

The non-scale model offers advantages on another issue, namely the speed of convergence. This issue is important for the reason that this speed is the crucial determinant of the relevance of the steady state relative to the transitional path. Influential empirical work by Barro and Sala-i-Martin (1992b, 1995), Sala-i-Martin (1994), Mankiw et al. (1992) and others established 2–3 per cent as a benchmark estimate of the convergence rate.³ But both the one-sector Ramsey growth model and the two-sector Lucas (1988) endogenous growth model generate speeds of convergence that greatly exceed this empirical benchmark, being approximately 7 per cent in the neoclassical model and 10 per cent in the Lucas model. Eicher and Turnovsky, (1999b, 2001), show how the two-sector non-scale model can

replicate the empirical estimates of the rate of convergence with relative ease.⁴

Interestingly, the speed of convergence offers another connection between the new and the old growth theory. After Solow (1956), Swan (1956), Uzawa (1961) and others established the formal dynamic structure of the neoclassical growth model, other authors including R. Sato (1963), K. Sato (1966), Conlisk (1966), and Atkinson (1969) addressed the question of the speed of adjustment toward the steady state. Already early on, this was recognized as being important, for the reason that this speed is the crucial determinant of the relevance of the steady state relative to the transitional path. The range of estimates of convergence speeds obtained in this early literature was extensive, being sensitive to the specific characteristics of the model, such as the returns to scale, the structure of the technology, the rate of technological progress, and number of sectors.

1.1.2. Scope of this Chapter

It is beyond the scope of this chapter to present an exhaustive discussion of growth theory. For that the reader should refer to Grossman and Helpman (1991), Barro and Sala-i-Martin (1995), and Aghion and Howitt (1998) who provide comprehensive treatments of the subject, albeit from different perspectives. Rather, the purpose of this chapter is to exposit the investment-based growth models, focusing on the common structures between the old and the new growth theories. We shall restrict ourselves to closed economies, although the international aspects of traded and growth are increasingly important.⁵

We begin our discussion in Section 1.2 by expositing a small canonical model of a growing economy, which is sufficiently general to encompass alternative models. Sections 1.3 and 1.4 then consider two alternative versions of the AK growth model as special cases. Such models have been used particularly to analyse the effects of fiscal policy on growth performance.

Section 1.3 begins with the simplest Romer (1986) model with fixed labour supply and then modifies this model to the case where labour is supplied elastically. It emphasizes how going from one assumption to the other dramatically changes the determination of the equilibrium growth rate and the impact of fiscal policy. Section 1.4 discusses the Barro (1990) model where government expenditure is productive and analyses optimal fiscal policy in that setting.

These initial models have the characteristic that the economy is always on its balanced growth path, and hence abstract from transitional dynamics. This is clearly a limitation and Section 1.5 discusses one straightforward way that

transitional dynamics may be introduced. This is through the introduction of government capital, so that in contrast to the Barro model, government expenditure impinges on production as a stock, rather than as a flow. But transitional dynamics can also arise in other ways, and these are discussed in the next two sections.

Section 1.6 discusses the Lucas (1988) model where the production technology is augmented to two sectors, one producing final output and physical capital, the other producing human capital, showing the nature of the dynamics this introduces. Other authors to analyse the two-sector model include Mulligan and Sala-i-Martin (1993), Devereux and Love (1994), and Bond et al. (1996). Section 1.7 considers the non-scale growth model. This is characterized by a higher order dynamic system than that of the corresponding endogenous growth model, so that even the simplest one-sector model is associated with transitional dynamics.

1.2. A CANONICAL MODEL OF A GROWING ECONOMY

We begin by describing the generic structure of an economy that consumes and produces a single aggregate commodity. There are N identical individuals, each of whom has an infinite planning horizon and possesses perfect foresight. Each agent is endowed with a unit of time that can be allocated either to leisure, l , or to labour, $(1-l)$. Labour is fully employed so that total labour supply, equal to population, N , grows exponentially at the steady rate $\dot{N} = nM$. Individual domestic output, Y_i , of the traded commodity is determined by the individual's private capital stock, K_i , his labour supply, $(1-l)$, and the aggregate capital stock $K = NK_i$.⁶ In order to accommodate growth under more general assumptions with respect to returns to scale, we assume that the output of the individual producer is determined by the Cobb–Douglas production function:⁷

$$Y_i = \alpha(1-l)^{1-\sigma} K_i^\sigma K^\eta \quad 0 < \sigma < 1, \quad \eta \gtrless 0. \quad (1.1a)$$

This formulation is akin to the earliest endogenous growth model of Romer (1986). The spillover received by an individual from the aggregate stock of capital can be motivated in various ways. One is to interpret K as knowledge capital, as Romer suggested. Another, is to assume N specific inputs (subscripted by i) with aggregate K representing an intra-industry spillover of knowledge.⁸

Each private factor of production has positive, but diminishing, marginal physical productivity. To assure the existence of a competitive equilibrium

the production function exhibits constant returns to scale in the two private factors (Romer, 1986). In contrast to the standard neoclassical growth model, we do not insist that the production function exhibits constant returns to scale, and indeed total returns to scale are $1+\eta$, and are increasing or decreasing, according to whether the spillover from aggregate capital is positive or negative.

As we will show in subsequent sections, the production function is sufficiently general to encompass a variety of models. For example, we will show that the model is consistent with long-run stable growth, provided returns to scale are appropriately constrained. This contrasts with models of endogenous growth and externalities in which exogenous population growth can be shown to lead to explosive growth rates (see Romer, 1990). We should also point out that the standard AK model emerges when $\sigma + \eta = 1$, $n = 0$, and the neoclassical model corresponds to $\eta = 0$.

Aggregate consumption in the economy is denoted by C , so that the per capita consumption of the individual agent at time t is $C/N = C_i$, yielding the agent utility over an infinite time horizon represented by the intertemporal isoelastic utility function:

$$\Omega \equiv \int_0^{\infty} (1/\gamma) (C_i l^\theta)^\gamma e^{-\rho t} dt; \quad -\infty < \gamma < 1; \quad \theta > 0, \quad 1 > \gamma(1+\theta), \quad 1 > \gamma\theta, \quad (1.1b)$$

where $1/(1-\gamma)$ equals the intertemporal elasticity of substitution, and θ measures the substitutability between consumption and leisure in utility.⁹ The remaining constraints on the coefficients in (1.1b) are required to ensure that the utility function is concave in the quantities C and l . We shall assume that income from current production is taxed at the rate τ_y , while consumption is taxed at the rate τ_c . We shall illustrate the contrasting implications of different models by analysing the purely distortionary aspects of taxation and assume that revenues from all taxes are rebated to the agent as lump sum transfers T_i . Agents accumulate physical capital, which for simplicity we assume does not depreciate, so that the individual's net rate of capital accumulation is given by his instantaneous budget constraint:

$$\dot{K}_i = (1-\tau_y)Y_i - (1+\tau_c)C_i + T_i - nK_i. \quad (1.1c)$$

The agent's decisions are to choose his consumption, C_i , leisure, l , and rate of capital accumulation, \dot{K}_i , to maximize the intertemporal utility function, (1.1b), subject to the production function, (1.1a), and accumulation equation, (1.1c). The optimality conditions with respect to C_i and l are respectively:

$$C_i^{\gamma-1} l^{\theta\gamma} = \lambda(1 + \tau_c) \quad (1.2a)$$

$$\theta C_i^{\gamma} l^{\theta\gamma-1} = \frac{\lambda(1 - \tau_y)(1 - \sigma)Y_i}{(1 - l)}, \quad (1.2b)$$

where λ is the shadow value of capital (wealth). Equation (1.2a) equates the marginal utility of consumption to the tax-adjusted shadow value of capital, while (1.2b) equates the marginal utility of leisure to its opportunity cost, the after-tax marginal physical product of labour (real wage), valued at the shadow value of wealth. Optimizing with respect to K_i implies the arbitrage relationship:

$$\rho - \frac{\dot{\lambda}}{\lambda} = \frac{(1 - \tau_y)\sigma Y}{K_i} - n. \quad (1.2c)$$

Equation (1.2c) is the standard Keynes–Ramsey consumption rule, equating the marginal return on consumption to the growth-adjusted after-tax rate of return on holding capital. Finally, in order to ensure that the agent's intertemporal budget constraint is met, the following transversality condition must be imposed:

$$\lim_{t \rightarrow \infty} \lambda K_i e^{-\rho t} = 0. \quad (1.2d)$$

The government in this canonical economy plays a limited role. It taxes income and consumption, and then rebates the tax revenues. In aggregate, these decisions are subject to the balanced budget condition:

$$\tau_y Y + \tau_c C = T. \quad (1.3)$$

Aggregating (1.1c) over the N individuals, and imposing (1.3) leads to the aggregate goods market clearing condition, expressed as the growth rate:

$$\frac{\dot{K}}{K} = \frac{Y}{K} - \frac{C}{K}. \quad (1.4)$$

The model can thus be summarized by the 3 optimality conditions (1.2a)–(1.2c), together with the goods market clearing condition, (4). Many models assume that labour is supplied inelastically, in which case the optimality condition, (1.2b), ceases to be operative.

1.3. THE ENDOGENOUS GROWTH MODEL

The investment-based endogenous growth model has been the subject of intensive research since 1986, with much of the focus being on the role of fiscal policy on growth and welfare.¹⁰ As we shall demonstrate, the endogeneity (or otherwise) of labour is an important determinant of the equilibrium growth rate, and the effects of policy.

The key feature of the endogenous growth model is that it can generate ongoing growth in the absence of population growth; i.e. if $n = 0$. For this to occur, the production function, (1.1a), must have constant returns to scale in the accumulating factors, individual and aggregate capital, that is:

$$\sigma + \eta = 1. \quad (1.5)$$

Substituting this into (1.1a), this implies individual and aggregate production functions of the form:

$$Y_i = \alpha[(1-l)K]^\eta K_i^{1-\eta}; \quad Y = \alpha[(1-l)N]^\eta K. \quad (1.6)$$

The individual production function is thus constant returns to scale in private capital, K_i , and in labour, measured in terms of ‘efficiency units’ $(1-l)K$. Summing over agents, the aggregate production function is thus linear in the endogenously accumulating capital stock. Note that as long as $\eta \neq 0$ so that there is an aggregate externality, the average (and marginal) productivity of capital depends upon the size of the population. Increasing the population, holding other technological characteristics constant, increases the productivity of capital and the equilibrium growth rate. The economy is thus said to have a ‘scale effect’ (see Jones, 1995a). Such scale effects run counter to the empirical evidence and have been a source of criticism of the AK growth models (see Backus et al., 1992). These scale effects can be eliminated from the AK model if either (1) there are no externalities ($\eta = 0$), or (2) if the individual production function (1.1a) is modified to:

$$Y_i = \alpha(1-l)^{1-\sigma} K_i^\sigma (K/N)^\eta$$

so that the externality depends upon the average, rather than the aggregate capital stock (see Mulligan and Sala-i-Martin, 1993). Henceforth throughout this section, we shall normalize the size of the population at $N = 1$ and thereby eliminate the issue of scale effects.

1.3.1. Inelastic Labour Supply

We begin with the widely discussed case where labour supply is inelastic, i.e. $l = \bar{l}$. With population normalized, the individual and aggregate production functions are of the pure AK form:

$$Y_i = AK_i; Y = AK, \quad (1.7)$$

where $A \equiv \alpha\sigma(1-\bar{l})^{1-\sigma}$ is a fixed constant. With the labour supply fixed, both the marginal and average productivity of capital is constant. The specification of the technology, consistent with ongoing growth, is a strong knife-edge condition, one for which the endogenous growth model has been criticized (see Solow, 1994).¹¹

The solution proceeds by trial and error, by postulating a solution of the form $C = \mu K$, where μ is a constant to be determined. Differentiating (1.2a) with respect to time, and combining with (1.4) and (1.7) with fixed employment (and normalized population), leads to the following explicit solutions for the equilibrium growth rate and consumption/capital ratio:

$$\frac{\dot{C}}{C} = \frac{\dot{K}}{K} \equiv \psi = \frac{A(1-\tau_y) - \rho}{1-\gamma} \quad (1.8a)$$

$$\frac{C}{K} \equiv \mu = \frac{\rho + A(1-\gamma) - A(1-\tau_y)}{1-\lambda}. \quad (1.8b)$$

These two expressions specify the ratio of consumption/capital, and the rate of growth of capital (and therefore consumption) along the optimal path, in terms of all exogenous parameters. Noting that $Y = AK$, the right-hand side of (1.8a) can be written as $(Y/K)(1-C/Y)$. Thus (1.8a) asserts that the equilibrium rate of growth of capital equals the product of the savings rate, $(1-C/Y)$, together with the output/capital ratio, precisely as in Harrod (1939). The difference is – and it is significant – that the savings rate is endogenously determined through intertemporal optimization, rather than being assumed exogenously.

In addition, the transversality condition (1.2d) must hold. Using (1.8a), (1.8b), and the fact that $\dot{\lambda}/\lambda = (\gamma-1)\psi$, this implies the following constraint on the equilibrium growth rate in the economy:

$$\rho - \gamma\psi > 0; \text{ i.e. } \rho > \gamma A(1-\tau_y). \quad (1.8c)$$

Condition (1.8c) is automatically satisfied for $\gamma \leq 0$ (thereby including the logarithmic utility function); otherwise it imposes a constraint on the tax

rate.¹² In any event, the condition (1.8c) for a viable solution is a weak one that any reasonable economy must surely satisfy.

The following general characteristics of this equilibrium can be observed.

1. Consumption, capital, and output are always on their steady-state growth paths, growing at the rate ψ . This is driven by the difference between the after-tax rate of return on capital and the rate of time preference.
2. With all taxes being fully rebated and labour supply fixed, the consumption tax is completely neutral. It has no effect on any aspect of the economic performance and acts like a pure lump-sum tax.
3. An increase in income tax leads to less investment and growth, and more consumption.

A key issue concerns the effects of tax changes, on the level of welfare of the representative agent, when consumption follows its equilibrium growth path. This is given by the expression:

$$\Omega = \int_0^{\infty} \frac{1}{\gamma} [C(0)e^{\psi t}]^{\gamma} e^{-\rho t} dt = \frac{[\mu]^{\gamma} K_0^{\gamma}}{\gamma(\rho - \gamma\psi)}. \quad (1.9)$$

Thus, the overall intertemporal welfare effect of any policy change depend upon its effects on (1) the initial consumption level, and (2) the growth rate of consumption. On impact, the higher tax raises current consumption, which is welfare improving in the short run. But at the same time, it reduces the growth rate, and this is welfare deteriorating. Evaluating (1.9) we can show that the overall effect on intertemporal welfare is:

$$\frac{d\Omega}{d\tau_y} \frac{1}{\gamma\Omega} = - \frac{\tau_y A^2}{(1-\gamma)c(\rho - \gamma\psi)} \leq 0.$$

Starting from a zero tax, the imposition of a tax has no effect on welfare; the benefits to current consumption are exactly offset by the discounted adverse effects on the growth rate. But with a positive initial tax, the adverse growth effects dominate and there is a net welfare loss.

1.3.2. Elastic Labour Supply

The endogenous growth model we have been discussing includes two interdependent critical knife-edge restrictions: (1) inelastic labour supply, and (2) fixed productivity of capital. The structure of the equilibrium changes

significantly when the labour supply is generalized. This introduces two key changes. The first is that the production function is modified to (1.6), so that the productivity of capital now depends positively upon the fraction of time devoted to labour. Second, the fixed endowment of a unit of time leads to the requirement that the steady-state allocation of time between labour and leisure must be constant.

This latter condition provides a link between the long-run rate of growth of consumption and the rate of growth of output. This can be seen by dividing the optimality condition (1.2a) by (1.2b). On the left hand side we see that with the allocation of time remaining finite, the marginal rate of substitution between consumption and leisure grows with consumption. On the right-hand side we see that, given the fixed labour supply, the wage rate grows with output. For these conditions to remain compatible over time, the equilibrium consumption/output ratio must therefore remain bounded, being given by:

$$\frac{C_t}{Y_t} = \frac{C}{Y} = \left(\frac{1-\tau_y}{1+\tau_c} \right) \left(\frac{l}{1-l} \right) \frac{1-\sigma}{\theta}. \quad (1.10)$$

To obtain the macroeconomic equilibrium we take the time derivatives of: (1) the optimality condition for consumption, (1.2a), (2) the equilibrium consumption-output ratio, (1.10), and, (3) the production function, (1.6). Combining the resulting equations with (1.6) and (1.2c), the macroeconomic equilibrium can be expressed by the differential equation in l :

$$\frac{dl(t)}{dt} = \frac{G(l)}{F(l)}, \quad (1.11)$$

where

$$F(l) \equiv [1-\gamma(1+\theta)] \frac{1}{l} + \frac{(1-\gamma)(1-\eta)}{1-l} > 0$$

$$G(l) = \left\{ (1-\tau_y)\sigma - (1-\gamma) \left[1 - \left(\frac{1-\tau_y}{1+\tau_c} \right) \left(\frac{l}{1-l} \right) \frac{1-\sigma}{\theta} \right] \right\} \frac{Y}{K} - \rho.$$

1.3.2.1. Balanced-growth equilibrium

Steady-state equilibrium is attained when the fraction of time allocated to leisure (or equivalently to labour supply) ceases to adjust. Setting $\dot{l} = 0$, steady state is characterized by the following balanced-growth conditions:

$$\frac{\dot{C}}{C} = \frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = -\frac{1}{1-\gamma} \frac{\dot{\lambda}}{\lambda} \equiv \tilde{\psi}.$$

Substituting for (Y/K) from (1.6) into (1.2c) yields:

$$\mathbf{RR:} \quad \tilde{\psi} = \frac{1}{1-\gamma} \left[(1-\tau_y) (\alpha\sigma(1-\tilde{l})^{1-\sigma} - \rho) \right]. \quad (1.12a)$$

Equation (1.12a) describes the trade-off locus, RR , between the equilibrium growth rate, $\tilde{\psi}$, and leisure, \tilde{l} , that ensures the equality between the after-tax rate of return on capital and the return on consumption. This curve can be shown to be negatively sloped and concave with respect to the origin (see Figure 1.1). This is because a higher fraction of time devoted to leisure will reduce the productivity of, and return, to capital. For rates of return to remain in equilibrium, the rate of return on consumption must fall correspondingly, and this requires the growth rate of the marginal utility of capital to rise, that is, the balanced-growth rate of the economy must decline.

Similarly, substituting (C/Y) from (1.10) and (Y/K) into (1.4) yields

$$\mathbf{PP:} \quad \tilde{\psi} = \left[1 - \left(\frac{1-\tau_w}{1-\tau_c} \right) \frac{1-\sigma}{\theta} \left(\frac{\tilde{l}}{1-\tilde{l}} \right) \right] \alpha(1-\tilde{l})^{1-\sigma}. \quad (1.12b)$$

The locus, PP describes the trade-off between the equilibrium growth rate and leisure that ensures product market equilibrium is maintained. This, too, is negatively sloped and concave. Intuitively, a higher fraction of time devoted to leisure reduces the productivity of capital. It also increases the consumption/output ratio, thus having a negative effect on the growth rate of capital in the economy. As l increases, employment declines, the marginal product of labour increases, and thus the trade-off along the PP locus becomes steeper.¹³

1.3.2.2. Equilibrium dynamics

We return to the dynamic equation (1.11) and consider its stability in the neighborhood of the balanced-growth equilibrium defined by (1.12a, 1.12b). In steady-state equilibrium, $G(\tilde{l}) = 0$, and the linearized dynamics about that point are represented by:

$$\frac{dl(t)}{dt} = \frac{G'(\tilde{l})}{F(\tilde{l})} (l(t) - \tilde{l}). \quad (1.13)$$

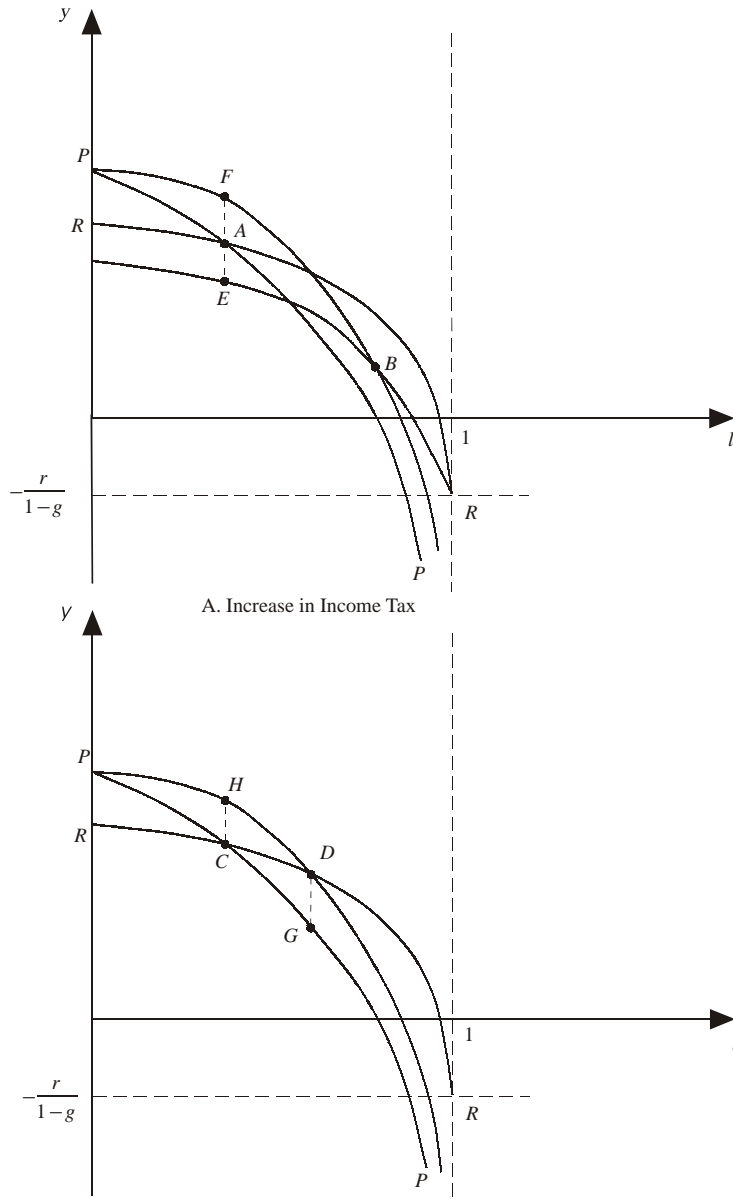


Figure 1.1 – Tax changes with endogenous labour

The existence of a balanced-growth equilibrium suffices to ensure that $G'(\tilde{l}) > 0$, in which case (1.13) is an unstable differential equation. The only solution consistent with a balanced-growth equilibrium is for $l = \tilde{l}$ at all points of time so that the economy is always on the balanced-growth path (1.12a)–(1.12b).¹⁴

1.3.2.3. Fiscal shocks

The model can be conveniently employed to analyse the effects of various kinds of fiscal disturbances. The qualitative effects of simple changes in the fiscal instruments on the macroeconomic equilibrium are readily obtained by considering shifts in the *RR* and *PP* curves. Others, involving composite changes require more formal analysis.

One important consequence of endogenizing the labour supply is that because of the work–leisure choice, the consumption tax, τ_c is no longer neutral. Indeed it has the same qualitative effect as an increase in the income tax rate, τ_y , decreasing the fraction of time devoted to work and reducing the equilibrium growth rate. These responses are illustrated in Figure 1.1. Part A illustrates the case of an increase in the income tax, τ_y , with revenues rebated in lump-sum fashion. To the extent that it is levied on capital income, it leads to a downward rotation in the *RR* locus, which given the fraction of time devoted to leisure, leads to an immediate reduction in the growth rate, as measured by the move from *A* to *E*. This in turn reduces the return on consumption, causing a substitution of leisure for labour, and reducing the rate of return on capital to that of consumption.¹⁵ This decrease in employment reduces output, leading to a further reduction in the growth rate. To the extent that it is levied on wage income, it leads to an upward rotation in the *PP* locus. Given leisure, this leads to an immediate reduction in the consumption/output ratio, and a corresponding rise in the growth rate of output, from *A* to *F*. This raises the return on consumption, causing agents to increase consumption and leisure over work. This causes a reduction in output and the growth rate, leading to a reduction in the return to capital and in consumption. The ultimate shift in equilibrium is thus represented by a move from *A* to *B*, being a combination of the moves *EB* and *FB*, with both components of the tax increase reinforcing their common effects on leisure and the growth rate. An increase in the consumption tax leads only to an upward rotation in the *PP* locus, as illustrated in Part B and is therefore equivalent to a wage tax.¹⁶

1.4. PRODUCTIVE GOVERNMENT EXPENDITURE

1.4.1. The Barro Model

Thus far we have focused on the taxation side of the government budget. But most tax revenues are used to finance government expenditures that provide some benefits to the economy. We shall focus on government expenditure that enhances the productive capacity of the economy, identifying such expenditures as being on some form of infrastructure. This model was developed by Barro (1990), and like Barro, we shall make the simplifying assumption that the benefits are derived from the *flow* of productive government expenditures. In Section 1.5 we shall briefly discuss the more plausible case where it is the accumulated *stock* of government expenditure that is relevant.

We abstract from labour (setting $l = 0$) and assume that the production function of the representative firm is now specified by:

$$Y_i = \alpha G_s^\eta K_i^{1-\eta} \equiv \alpha \left(\frac{G_s}{K_i} \right)^\eta K_i, \quad 0 < \eta < 1, \quad (1.14)$$

where G_s denotes the flow of productive services enjoyed by the individual firm. Thus productive government expenditure has the property of positive, but diminishing, marginal physical product, while enhancing the productivity of private capital. As in Section 1.3, we assume that the population growth $n = 0$.

We shall assume that the services derived from aggregate expenditure, G , are:

$$G_s = G \left(\frac{K_i}{K} \right)^{1-\varepsilon}, \quad (1.15)$$

where K denotes the aggregate capital stock. Most public services are characterized by some degree of congestion and (1.15) provides one convenient formulation that builds on the public goods literature (see Edwards, 1990). The parameter ε can be interpreted as describing the degree of relative congestion associated with the public good and the following special cases merit comment. If $\varepsilon = 1$, the level of services derived by the individual from the government expenditure is fixed at G , independent of both the firm's own usage of capital and aggregate usage. The good G is a non-rival, non-excludable public good that is available equally to each individual; there is no congestion. Since few, if any such public good exist, it is probably best viewed as a benchmark. At the other extreme, if $\varepsilon = 0$, then

only if G increases in direct proportion to the aggregate capital stock, K , does the level of the public service available to the individual firm remain fixed. We shall refer to this case as being one of *proportional* (relative) congestion. In that case, the public good is like a private good, in that since $K = NK_i$, the individual receives his proportionate share of services, $G_s = G/N$.

In order to sustain an equilibrium of ongoing growth, the level of government expenditure must be tied to the scale of the economy. This can be achieved most conveniently by assuming that the government sets its level of expenditure as a share of aggregate output, $Y = NY_i$:

$$G = gY . \quad (1.16)$$

In an environment of growth this is a reasonable assumption. Government expenditure thus increases with the size of the economy, with an expansionary government expenditure being denoted by an increase in g .

Summing (14) over the N identical agents and substituting (1.15) and (1.16), we obtain

$$G = (\alpha g N^{\eta \epsilon})^{\frac{1}{1-\eta}} K; \quad Y = (\alpha g^{\eta} N^{\eta \epsilon})^{\frac{1}{1-\eta}} K . \quad (1.17)$$

Aggregate output thus has the fixed AK technology, where the productivity of capital depends (positively) upon the productive government input. Notice that provided $\eta \epsilon > 0$, the productivity of capital depends upon the size (scale) of the economy, as parameterized by the fixed population, N . This is because the size of the externality generated by government expenditure increases with the size of the economy, playing an analogous role to aggregate capital in the Romer (1986) model. As in that model, this scale effect disappears if $\epsilon = 0$, so that there is proportional congestion and each agent receives his own individual share of government services, G/N .

We now re-solve the representative individual's optimization problem. In so doing, he is assumed to take aggregate government spending, G , and the aggregate stock of capital, K , as given, insofar as these impact on the productivity of his capital stock. Performing the optimization, the optimality conditions (1.2a) and (1.2b) remain unchanged. The optimality condition with respect to capital is now modified to:

$$\rho - \frac{\dot{\lambda}}{\lambda} = \frac{(1 - \tau_y)(1 - \eta \epsilon) Y_i}{K_i} . \quad (1.2c')$$

The difference is that the private marginal physical product of capital is now proportional to $(1 - \eta \epsilon)$, depending both upon the degree of congestion

and the productivity of government expenditure. The less congestion (the larger ε) the smaller the benefits of government expenditure are tied to the usage of private capital, thus lowering the return.

The other modification is to the government budget constraint, (1.4) which becomes:

$$\tau_y Y + \tau_c C = G. \quad (1.3')$$

1.4.2. Optimal Fiscal Policy

It is clear from Section 1.4.1 that growth and economic performance are heavily influenced by fiscal policy in this model. This naturally leads to the important question of the optimal tax structure. To address this issue it is convenient to consider, as a benchmark, the first-best optimum of the central planner, who controls resources directly, against which the decentralized economy can be assessed. The central planner is assumed to internalize the equilibrium relationship $NK_i = K$, as well as the expenditure rule (16). The optimality conditions are now modified to:

$$C_i^{\gamma-1} = \lambda \quad (1.2a')$$

$$\rho - \frac{\dot{\lambda}}{\lambda} = \frac{(1-g)Y_i}{K_i}. \quad (1.2c'')$$

The key difference is that the social return to capital nets out the fraction of output appropriated by the government.

It is straightforward to show that the decentralized economy will mimic the first-best equilibrium of the centrally planned economy if and only if

$$(1-g) = (1-\tau_y)(1-\varepsilon\eta). \quad (1.18a)$$

Intuitively, government expenditure, by being tied to the stock of capital in the economy, induces spillovers into the domestic capital market, generating distortions that require a tax on capital income in order to ensure that the net private return on capital equals its social return.

To understand (1.18) better, it is useful to observe that the welfare-maximizing share of government expenditure, is (Barro, 1990):

$$\hat{g} = \eta. \quad (1.19)$$

Substituting (1.19) into (1.18) and simplifying, the optimal income tax can be expressed in the form:

$$\hat{\tau}_y = \frac{g - \eta\varepsilon}{1 - \eta\varepsilon} = \frac{g - \hat{g}}{1 - \eta\varepsilon} + \frac{\hat{g}(1 - \varepsilon)}{(1 - \eta\varepsilon)}. \quad (1.18a')$$

In order to finance its expenditures, (1.3'), the government must, in conjunction with $\hat{\tau}_y$, set a corresponding consumption tax $\hat{\tau}_c$:

$$\hat{\tau}_c = \frac{\eta\varepsilon(1 - g)}{(1 - \eta\varepsilon)(C/Y)}. \quad (1.18b)$$

Equation (1.18a') emphasizes how the optimal tax on capital income corrects for two distortions. The first is due to the deviation in government expenditure from the optimum the second is caused by congestion. Comparing (1.18a') and (1.18b) we see that there is a tradeoff between the income and the consumption tax in achieving these objectives, and that this depends upon the degree of congestion. This tradeoff is seen most directly if g is set optimally in accordance with (1.19). In this case, if there is no congestion, government expenditure should be fully financed by a consumption tax alone; capital income should remain untaxed. As congestion increases (ε declines), the optimal consumption tax should be reduced and the income tax increased until with proportional congestion, government expenditure should be financed entirely by an income tax.

It is useful to compare the present optimal tax on capital with the well-known Chamley (1986) proposition which requires that asymptotically the optimal tax on capital should converge to zero. The Chamley analysis did not consider any externalities from government expenditure. Setting $\eta = 0$, we still find that the optimal tax on capital is equal to the share of output claimed by the government ($\bar{\tau} = g$). The difference is that by specifying government expenditure as a fraction of output, its level is not exogenous, but instead is proportional to the size of the growing capital stock. The decision to accumulate capital stock by the private sector leads to an increase in the supply of public goods in the future. If the private sector treats government spending as independent of its investment decision (when in fact it is not), a tax on capital is necessary to internalize the externality and thereby correct the distortion. Thus, in general, the Chamley rule of not taxing capital in the long-run will be nonoptimal, although it will emerge in the special case where $g = \eta\varepsilon$, in which case there is no spillover from government expenditure to the capital market.

We conclude by noting two further points. The equilibrium growth rate in the Barro model is:

$$\tilde{\psi} = \frac{(1-g)(\alpha g^\eta)^{\frac{1}{1-\eta}} - \rho}{1-\gamma}.$$

It is straightforward to show that the equilibrium growth rate is maximized at the optimal government expenditure/output ratio. Maximizing the productivity of government expenditure net of resource costs maximizes both the growth rate and utility. The coincidence of the growth-maximizing size of government and its welfare-maximizing size is a strong result, but it is not robust. For example, it ceases to hold if the installation of new capital involves adjustment costs of the type where the productive government investment (plausibly) reduces the costs of adjustment, along with enhancing the productivity of private capital (see Turnovsky, 1996b). It also ceases to apply if the government production good enters the private production function as a stock, rather than as a flow (see Futagami et al., 1993). Third, it ceases to apply to the economy being considered here if that economy is subject to stochastic productivity. This is because growth maximization in general entails too much risk for a risk-averse representative agent (see Turnovsky, 1999a).

The final point is that it is possible to augment this model to include endogenous labour. The qualitative nature of the solution is analogous to that of Section 1.3.2. A detailed analysis of fiscal policy in this case is provided by Turnovsky (2000).

1.5. PUBLIC AND PRIVATE CAPITAL

The models discussed thus far all have the characteristic that consumption and output (capital) are on their balanced growth paths; there are no transitional dynamics. Instead, the economy adjusts infinitely fast to any exogenous shock, thus contradicting the empirical evidence pertaining to speeds of convergence. As noted, the point of this literature is that the economy adjusts relatively slowly, with the rate of 2 per cent per annum being a benchmark estimate. This implies that the economy is mostly off its balanced growth path, on some transitional path, which only gradually converges to steady state. It is therefore important to modify the model to allow for such transitional dynamics, and this can be achieved in several ways, all of which assign a central role to a second state variable to the dynamics.

In this section, we consider one important modification to the basic model that accomplishes this objective: the introduction of public in addition to private capital. As we will show in Section 1.6 the two-sector endogenous

growth model of Lucas (1988), in which the two capital goods are physical capital and human capital, is an alternative way. In both these cases, the stable adjustment path is a one-dimensional locus, implying that all variables converge to their respective long-run equilibrium at the same constant rate. But one further way transitional dynamics can be introduced is through a modification of the technology to allow it to have non-scale growth, along the lines initially proposed by Jones (1995a, 1995b). It is shown that incorporating a more general production technology increases the dimensionality of the transitional path, thus allowing for more flexible dynamic transitional behavior patterns. This model will be discussed further in Section 1.7.

Most models analysing productive government expenditure treat the current *flow* of government expenditure as the source of contribution to productive capacity. While the flow specification has the virtue of tractability, it is open to the criticism that insofar as productive government expenditures are intended to represent public infrastructure, such as roads and education, it is the accumulated *stock*, rather than the current flow, that is the more appropriate argument in the production function.

Despite this, relatively few authors have adopted the alternative approach of modeling productive government expenditure as a stock. Arrow and Kurz (1970) were the first authors to model government expenditure as a form of investment in the Ramsey model. More recently, Baxter and King (1993) study the macroeconomic implications of increases in the stocks of public goods. They derive the transitional dynamic response of output, investment, consumption, employment, and interest rates to such policies by calibrating a real business cycle model. Fisher and Turnovsky (1998) address similar issues from a more analytical perspective.

The literature introducing both private and public capital into growth models is sparse. Futagami et. al. (1993), Glomm and Ravikumar (1994), and Turnovsky (1997a) do so in a closed economy, and an extension to an open economy is developed by Turnovsky (1997b). Private capital in the Glomm–Ravikumar model fully depreciates each period, rather than being subject to at most gradual (or possibly zero) depreciation. This enables the dynamics of the system to be represented by a single state variable alone, so that the system behaves much more like the Barro model in which government expenditure is introduced as a flow. In particular, under constant returns to scale in the reproducible factors, there are no transitional dynamics and the economy is always on a balanced growth path.

We now outline the main features of such a model, in the simpler case where labour supply is fixed. The key modification to the model is the production function, which is now of the form:

$$Y = \alpha K_g^\eta K^{1-\eta} \quad \alpha > 0; \quad 0 < \eta < 1, \quad (1.20)$$

where K denotes the representative firm's capital stock of private capital and K_g denotes public (government) capital. The production function is analogous to that introduced in Section 1.4, except that the government input is introduced as a stock, rather than as a flow. Equation (1.20) embodies the assumption that the public capital enhances the productivity of private capital, though at a diminishing rate. Public capital is not subject to congestion and neither form of capital is subject to depreciation.¹⁷ The model abstracts from labour, so that as in the basic AK model of Sections 1.3 and 1.4, private capital should be interpreted broadly to include human, as well as physical, capital.

The other main modification is that government expenditure leads to the accumulation of public capital, and assuming that the government sets its investment as a proportion, g , of output:

$$\dot{K}_g = G = gY \quad 0 < g < 1. \quad (1.21a)$$

In setting policy, the government is subject to its budget constraint:

$$G = \tau_y Y + \tau_c C + T, \quad (1.21b)$$

where T denotes lump-sum taxes. Summing the private and public budget constraint yields the economy-wide resource constraint:

$$Y = C + \dot{K} + \dot{K}_g. \quad (1.22)$$

To derive the macroeconomic equilibrium, we need to express it in terms of stationary variables. For this purpose it is convenient to express it in terms of quantities relative to the growing stock of private capital, namely, $z \equiv K_g / K$; $c \equiv C / K$. Taking the time derivatives of these quantities and combining with (1.21) and (1.22), together with the optimality conditions for the private agent, the equilibrium dynamics can be represented by:

$$\frac{\dot{z}}{z} = g\alpha z^{\eta-1} - (1-g)\alpha z^\eta + c \quad (1.23a)$$

$$\frac{\dot{c}}{c} = \frac{(1-\tau_y)\alpha(1-\eta)z^\eta - \rho}{1-\gamma} - (1-g)\alpha z^\eta + c. \quad (1.23b)$$

The first of these equations describes the differential growth rate between public and private capital and is obtained from the relationship $\dot{z}/z = \dot{K}_g/K_g - \dot{K}/K$. The second equation determines the differential growth rate between consumption and private capital. It is obtained from the relationship $\dot{c}/c = \dot{C}/C - \dot{K}/K$, where \dot{C}/C is obtained, in turn, from combining the time derivative of (1.2a') with (1.21a) and (1.22).

1.5.1. Steady-State

Steady-state equilibrium is determined by

$$\tilde{c} = (1-g)\alpha(\tilde{z})^\eta - g\alpha(\tilde{z})^{\eta-1} \quad (1.24a)$$

$$\frac{(1-\tau_y)\alpha(1-\eta)(\tilde{z})^\eta - \rho}{1-\gamma} - (1-g)\alpha(\tilde{z})^\eta + \tilde{c} = 0, \quad (1.24b)$$

which jointly determine the equilibrium values of \tilde{z} and \tilde{c} . However, equations (24a) and (24b) define a pair of nonlinear equations in \tilde{z} and \tilde{c} , and may or may not be consistent with a well-defined steady state in which $\tilde{c} > 0, \tilde{z} > 0$ (see Turnovsky, 1997a). Having determined these quantities, the corresponding (common) equilibrium growth rate of consumption and the two capital stocks may be expressed in the following equivalent forms:

$$\tilde{\Psi} \equiv \frac{\tilde{C}}{C} = \frac{\tilde{K}}{K} = \frac{\tilde{K}_g}{K_g} = g\alpha\tilde{z}^{\eta-1} = (1-g)\alpha\tilde{z}^\eta + \tilde{c}. \quad (1.25)$$

The long-run effects of fiscal policies on the relative capital stock, \tilde{z} , consumption ratio, \tilde{c} , and the growth rate, $\tilde{\Psi}$, are obtained by considering (1.24a), (1.24b), and (1.25). We shall discuss the effects of changes in (1) the income tax rate and (2) the share of government expenditure, assuming that the government budget constraint is met through an appropriate adjustment in lump-sum taxes. The equilibrium is independent of the consumption tax, τ_c , which therefore operates as a lump-sum tax, and may also serve as the balancing item in the government budget.

Omitting details, the following responses to fiscal policy can be established. An increase in the income tax rate, τ_y , (with tax revenues rebated) reduces the net rate of return to private capital, thereby inducing investors to switch from saving to consumption, thus raising the consumption/capital ratio and decreasing the growth rate of private capital. This negative effect on the return to private capital favors its decumulation and leads to a long-run increase in the ratio of public to private capital. By

contrast, an increase in the share of output claimed by the government, financed by a lump-sum tax, raises the equilibrium growth rate of capital unambiguously.¹⁸ The case considered by Futagami et al. (1993), in which government expenditure is determined by tax revenues, corresponds to $g = \tau_y$ and hence $dg = d\tau_y$ and is a hybrid of these two cases. It is straightforward to verify that while the increase in g raises the growth rate, the corresponding increase in τ does the opposite, rendering a net effect that depends upon $(\eta - g)$, precisely as in the Barro model, discussed in Section 1.4.1. It then follows that, setting $g = \eta$ is growth-maximizing.

1.5.2. Transitional Dynamics: Tax Cuts

The transitional dynamics for z and c are obtained by linearizing (1.23) about (1.24) in the usual manner. And having derived these relationships, one can further derive the linearized dynamics for the growth rates themselves. The formal solutions are provided by Turnovsky (1997a). Figure 1.2 illustrates the transitional dynamics of private and public capital in response to a fiscal expansion taking the form of a permanent cut in the income tax rate, financed by a lump-sum tax.

Suppose that the economy is initially in steady-state equilibrium at the point P , and that a *permanent* tax cut is introduced. The immediate effect of the lower tax is to raise the net return to private capital, inducing agents to reduce their level of consumption and to increase their rate of accumulation of private capital. This increase in the growth of private capital causes the ratio of public to private capital, z , to begin to decline. As z declines, the average productivity of private capital, αz^η falls, causing its growth rate to decrease. The transitional adjustment in the growth rate of private capital (ψ_k) is illustrated by the initial jump from P to A , on the new stable arm $X'X'$, followed by the continuous decline AQ , to the new steady state at Q . With the growth of public capital being tied through aggregate output to the capital stocks in accordance with (1.21a), the growth rate of public capital does not respond instantaneously to the lower tax rate, τ_y . The stable arm YY remains fixed. Instead, as z declines, the average productivity of public capital $\alpha z^{\eta-1}$ rises, causing the growth rate of public capital (ψ_g) to rise gradually over time along the path PQ . During the transition, the growth rates of the two capital stocks approach their common long-run equilibrium growth rate from different directions.

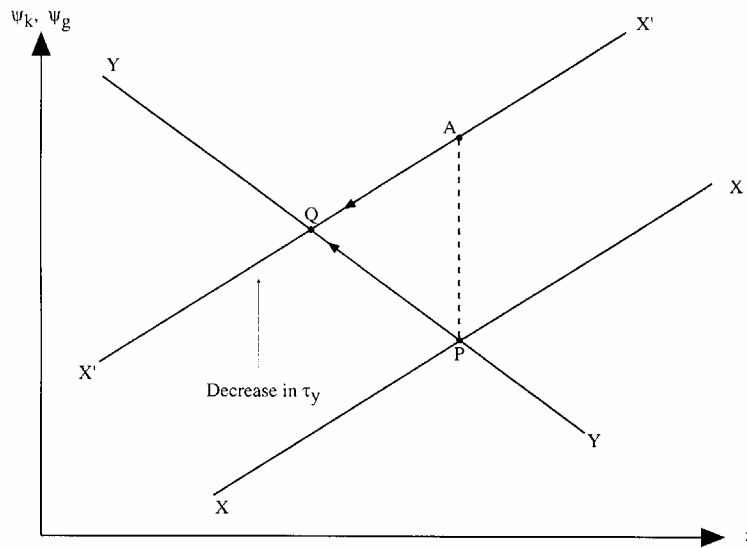


Figure 1.2 – Transitional dynamics of capital: tax cut

1.5.3. Optimal Fiscal Policy

Futagami et al. (1993) analyse the optimal (welfare-maximizing) size of government (optimal income tax) in this economy and show that it is smaller than the growth-maximizing size. This is in contrast to Barro (1990), who, introducing government expenditure as a flow in the production function, finds that the welfare-maximizing and growth-maximizing shares of government expenditure coincide. The difference is accounted for by the fact that when government expenditure influences production as a flow, maximizing the marginal product of government expenditure net of its resource cost maximizes the growth rate of capital. But it also maximizes the social return to public expenditure, thereby maximizing overall intertemporal welfare. By contrast, when government expenditure affects output as a stock, public capital needs to be accumulated to attain the growth maximizing level. This involves forgoing consumption, leading to welfare losses relative to the social optimum. Intertemporal welfare is raised by reducing the growth rate, thereby enabling the agent to enjoy more consumption.

Turnovsky (1997a) analyses optimal tax policy in this economy and shows that the first-best tax policy comprises two components, one fixed, the other time-varying. This is because there are two objectives to attain. The fixed

component ensures that the steady-state of the first-best optimum is replicated, and this has the general characteristics of the optimal tax structure of Section 1.4.2. But because the internal dynamics of the decentralized economy do not in general coincide with those of the first-best optimum, the time-varying component of the optimal tax is necessary to introduce the appropriate correction along the transitional path.

1.6. TWO-SECTOR MODEL OF ENDOGENOUS GROWTH

Two-sector models of economic growth were pioneered by Uzawa (1961), Takayama (1963), and others. In this early literature, the two sectors corresponded to the production of the consumption good and the production of the investment good, respectively. The key results in that early literature focused on the uniqueness and stability of equilibrium, which was shown to depend critically upon the capital intensity of the investment good sector relative to the consumption sector.

In a seminal paper, Lucas (1988) introduced the two-sector endogenous growth model. The model includes two capital goods, physical capital and human capital. The former is produced along with consumption goods in the output sector, using as inputs both human and physical capital. Human capital is produced in the education sector using both physical and human capital. The agent's decisions at any instant of time are: (1) how much to consume, (2) how to allocate his physical and human capital across the two sectors, and (3) at what rates to accumulate total physical and human capital over time. Having two capital goods, this model is characterized by transitional dynamics. However, because two-sector endogenous growth models initially proved to be intractable, much of the analysis was restricted to balanced-growth paths (Lucas 1988; Devereux and Love 1994), or to analyzing transitional dynamics using numerical simulation methods (Mulligan and Sala-i-Martin 1993; Pecorino 1993; Devereux and Love 1994). One important exception to this is Bond et al. (1996) which, by using the methods of the two-sector trade model, provides an effective analysis of the dynamic structure of the two-sector endogenous growth model.¹⁹

1.6.1. The Two-Sector Economy

There is a single, infinitely lived representative agent who accumulates two types of capital for rental at the competitively determined rental rate. The first is physical capital, K , and the second is human capital, H . Neither of

these capital goods is subject to depreciation and their accumulation is free of adjustment costs. For expositional simplicity, there is no government.

These two forms of capital are used by the agent to produce a final output good, X , taken to be the numeraire, by means of a linearly homogeneous production function:

$$X = aK_X^\alpha H_X^{1-\alpha} \quad 0 < \alpha < 1, \quad (1.26a)$$

where, K_X and H_X denote the allocations of the two capital goods to the production of final output. Goods may be either consumed, C , or added to the capital stock, which therefore evolves as follows:

$$\dot{K} = aK_X^\alpha H_X^{1-\alpha} - C \quad 0 < \alpha < 1. \quad (1.26b)$$

The agent also adds to human capital, using an analogous production function:

$$\dot{H} = Y = bK_Y^\delta H_Y^{1-\delta} \quad 0 < \delta < 1. \quad (1.26c)$$

The key feature of the production setup is that the two production functions are linearly homogeneous in the two reproducible factors, K and H . This is critical for an equilibrium with steady growth to exist. This specification is more general than the assumption frequently adopted that physical capital is not an input into the production of human capital (i.e. $\delta = 0$) in (26c). Other authors, however, have introduced externalities analogous to those introduced into the one-sector model (see Lucas, 1988), Mulligan and Sala-i-Martin (1993). As Bond et al. note, their approach can be easily extended to include such externalities. Both forms of capital are costlessly and instantaneously mobile across the two sectors, with the sectoral allocations being constrained by:

$$K_X + K_Y = K \quad (1.26d)$$

$$H_X + H_Y = H. \quad (1.26e)$$

The agent chooses the rate of consumption, C , capital allocations K_X , K_Y , H_X and H_Y , and rates of capital accumulation, \dot{K} and \dot{H} , to maximize the intertemporal isoelastic utility function subject to the constraints (1.26a)–(1.26e), and the initial stocks of assets, H_0 and H_0 . From the optimality conditions we immediately obtain:

$$\frac{\dot{C}}{C} = \frac{r_K(q) - \rho}{1 - \gamma} \equiv \psi(t), \quad (1.27)$$

where q denotes the relative price of human to physical capital. While (1.27) is analogous to previous expressions for the growth rate, for example, (1.8a), in general, q evolves during the transition, thereby rendering the return on capital, r_K , and the growth rate, $\psi(t)$ time-varying.

As Bond et al. (1996) show, the macroeconomic dynamics can be represented by three differential equations in: (1) $k \equiv K/H$, the relative stocks of physical to human capital; (2) $c \equiv C/H$, the ratio of consumption to human capital, and (3) q . As we will see, the steady-state equilibrium will have the characteristic that $\dot{k} = \dot{h} = \dot{c} = 0$, so that consumption and the two types of capital will all grow at an equilibrium rate determined by (1.27), while the relative price of the two types of capital will be constant.

The derivation of the macroeconomic equilibrium proceeds in two stages. The first stage determines the static allocation of existing resources. We express the sectoral capital intensities and marginal physical products of capital in terms of the relative price of nontraded to traded goods, and also express the absolute levels of the allocation of capital in terms of the gradually evolving aggregate stocks, K and H . The second stage then determines the dynamics. As is characteristic of two-factor, two-sector growth models, the dynamics of the system decouple, with the price dynamics evolving independently of the quantity dynamics.

1.6.2. Transitional Dynamics

Working through the model one can show that the linearized transitional dynamics is represented by a matrix equation of the following form:

$$\begin{pmatrix} \dot{q} \\ \dot{c} \\ \dot{k} \end{pmatrix} = \begin{pmatrix} a_{11} & 0 & 0 \\ a_{21} & 0 & a_{23} \\ a_{31} & -1 & a_{33} \end{pmatrix} \begin{pmatrix} q(t) - \tilde{q} \\ c(t) - \tilde{c} \\ k(t) - \tilde{k} \end{pmatrix} \quad (1.28)$$

where the elements of the matrix have the following sign patterns:

$$\text{sgn}(a_{11}) = -\text{sgn}(a_{23}) = -\text{sgn}(a_{33}) = \text{sgn}(\delta - \alpha); \quad a_{31} < 0 \quad (1.29)$$

The characteristic equation to (28) can be written as follows:

$$(\mu - \alpha_{11})(\mu^2 - a_{33}\mu + a_{23}) = 0, \quad (1.30)$$

where the factorization of (1.30) reflects the decoupling of the price and the quantity dynamics in (1.28).

As is familiar from two-sector models of international trade, the dynamics depends crucially upon the relative sectoral capital intensity, which in the present context is measured by $\alpha - \delta$. Thus $\alpha > \delta$ corresponds to the output sector being relatively more intensive in physical capital than is the human capital sector, and the common assumption, $\delta = 0$, is an extreme example of this. We now show that irrespective of the relative sectoral intensity condition, (1.30) always describes a stable saddle path. Two cases must be considered.

First, suppose $\delta > \alpha$. Then (1.29) yields $a_{11} > 0, a_{23} < 0, a_{33} < 0$. Equation (1.30) then implies that the eigenvalue $\mu = a_{11} > 0$. The remaining two roots are solutions to the quadratic factor in (1.30) and, for the sign pattern noted, one of the roots is positive and the other is negative. The root $\mu = [a_{33} - (a_{33}^2 - 4a_{23})^{1/2}]/2 < 0$ is thus the unique stable root. Second, if $\alpha > \delta$, then $a_{11} < 0, a_{23} > 0, a_{33} > 0$, so that the eigenvalue $\mu = a_{11} < 0$. Moreover, the two roots to the quadratic factor in (1.30) are now both positive, implying that $\mu = a_{11} < 0$ is the unique stable root. These two alternative cases of capital intensity have important differences for the dynamics that we shall now discuss.

(i) $\delta > \alpha$: *Human Capital Sector Relatively More Intensive in Physical Capital*

In this case the only solution for q consistent with the transversality condition is for q to remain constant at its steady-state value, \tilde{q} . The stable solution to (1.28) is thus:

$$q(t) = \tilde{q} \quad (1.31a)$$

$$k(t) - \tilde{k} = (k_0 - \tilde{k})e^{\mu t} \quad (1.31b)$$

$$c(t) - \tilde{c} = \frac{a_{23}}{\mu} (k(t) - \tilde{k}), \quad (1.31c)$$

where $\mu = [a_{33} - (a_{33}^2 - 4a_{23})^{1/2}]/2 < 0$. Note that the consumption/human capital ratio moves directly with the physical to human capital ratio; the stable saddlepath in $c - k$ space is positively sloped. Intuitively, an increase in k results in a decrease in the relative supply of final output when the human capital sector is more intensive in physical capital, so that c must fall to keep the two factors accumulating at the same rate. The constancy in the relative price, q , translates to the constancy in the sectoral physical capital to human capital ratios.

It is convenient to focus on the transitional dynamics of the growth rates. With the relative price q constant, the consumption growth rate is fixed at the common steady-state growth rate:

$$\frac{\dot{C}}{C} \equiv \Psi_c(t) = \frac{r_k(\tilde{q}) - \rho}{1 - \gamma} \equiv \tilde{\Psi}. \quad (1.32)$$

By evaluating the expressions for the growth rate of physical capital, $\dot{K}/K \equiv \Psi_K$, and human capital, $\dot{H}/H \equiv \Psi_H$, and comparing with (1.32) one can establish the following ordering. Assuming that the economy begins from an initial point at which $k_0 < \tilde{k}$, we see from (1.31a)–(1.31c) that the growth rates can be ranked as follows along the transitional paths:

$$\Psi_K(t) > \Psi_c(t) \equiv \tilde{\Psi} > \Psi_H(t). \quad (1.33)$$

The growth rates of physical and human capital approach their common equilibrium growth rate from opposite sides.

(ii) $\alpha > \delta$: *Physical Capital Sector Relatively More Intensive in Physical Capital*

In this case the stable eigenvalue $\mu = a_{11} < 0$, and the stable adjustment path is:

$$k(t) - \tilde{k} = (k_0 - \tilde{k})e^{a_{11}t} \quad (1.34a)$$

$$q(t) - \tilde{q} = \left[\frac{a_{11}(a_{11} - a_{33}) - a_{23}a_{32}}{a_{11}a_{31} - a_{21}} \right] [k(t) - \tilde{k}] \quad (1.34b)$$

$$c(t) - \tilde{c} = \left[\frac{a_{21}(a_{11} - a_{33}) - a_{23}a_{31}}{a_{11}a_{31} - a_{21}} \right] [k(t) - \tilde{k}]. \quad (1.34c)$$

The important point is that the relative price, q , is no longer constant, but follows a stable adjustment path. Evaluating the term in brackets in (1.34b), one can establish that the relative price of human to physical capital, q , is an increasing function of the relative stock of physical to human capital, k . The slope of the stable $c - k$ locus includes two conflicting influences: the direct effect as in case (i) and the relative price effect.

One important difference between cases (i) and (ii) is that with the relative price q following a transitional path, the growth rate of consumption now also has the same characteristic:

$$\frac{\dot{C}}{C} \equiv \psi_c = \frac{r'_k(q - \bar{q})}{1 - \gamma} + \tilde{\psi}. \quad (1.35)$$

For $\alpha > \delta$, we have $r'_k < 0$, so that the growth rate of consumption is a decreasing function of the physical capital to human capital ratio. The adjustment of the relative price has an additional impact on the transitional dynamics of the two types of capital (see Bond et al., 1996).

1.6.3. Fiscal Policy in Two-Sector Model

Bond et al. (1996) show that when one introduces distortionary taxes, the dichotomy in the dynamics introduced by the sectoral intensity condition $(\delta - \alpha)$ is somewhat broken. It now becomes possible for the economy to be unstable, but it is also possible for it to have ‘too many’ stable roots, and for the economy to have a continuum of equilibria, the latter property often being associated with increasing returns and externalities (Benhabib and Perli, 1994). Other authors have also examined fiscal policy in the two-sector Lucas model. Pecorino (1993) has conducted a simulation study of commodities taxes. More recently, Ortigueira (1998) has studied the effects of labour and capital income taxes on the transitional dynamics and determined a measure of inefficiency derived from the taxation of capital income. His analysis is also based primarily on numerical simulations.

1.7. NON-SCALE GROWTH MODEL

As noted, the endogenous growth model has been criticized on both empirical and theoretical grounds. We therefore now turn to an alternative model, the non-scale model, which has been proposed partly in response to these criticisms. The increased flexibility of the production function is associated with higher order dynamics in comparison to the corresponding AK growth model. Thus, in cases where the AK model is always on its balanced growth path, the corresponding non-scale model will follow a first-order adjustment path. The non-scale counterpart to the two-sector Lucas model, which as we have seen follows a one-dimensional stable locus, now follows a two-dimensional path, and so on.

We start our examination of the dynamics by examining the simplest one-sector non-scale model, with inelastic labour supply. The main virtue of this is pedagogic, in that analytical results are easily obtained. The two-sector non-scale model, briefly discussed in Section 1.7.3, is significantly more complex, requiring that we characterize the transition paths numerically.

Our objective is to analyse the dynamics of the aggregate economy about a balanced growth path. Along such an equilibrium path, aggregate output and the aggregate capital stock are assumed to grow at the same constant rate, so that the aggregate output/capital ratio remains unchanged. Summing the individual production functions (1a) over the N agents, the aggregate production function with inelastic labour supply is:

$$Y = \alpha(1-\bar{l})^{1-\sigma} K^{\eta+\sigma} N^{1-\sigma} \equiv AK^{\sigma_K} N^{\sigma_N}, \quad (1.36)$$

where $A \equiv \alpha(1-\bar{l})^{1-\sigma}$, $\sigma_N \equiv 1-\sigma$ = share of labour in aggregate output, $\sigma_K \equiv \eta+\sigma$ = share of capital in aggregate output. Thus $\sigma_K + \sigma_N \equiv 1+\eta$ measures total returns to scale of the social aggregate production function. Taking percentage changes of (1.36) and imposing the long-run condition of a constant Y/K ratio, the long-run equilibrium growth of capital and output, g , is:

$$g \equiv \frac{\sigma_N}{1-\sigma_K} n > 0. \quad (1.37)$$

Equation (1.37) exhibits the key feature of the non-scale growth model, namely, that the long-run equilibrium rate is proportional to the population growth rate by a factor that reflects the productivity of labour and capital in the aggregate production function. Under constant returns to scale, $g = n$, the rate of population growth, as in the standard neoclassical growth model, to which the present one-sector model reduces. Otherwise g exceeds n or is less than n , that is, there is positive or negative per capita growth, according to whether returns to scale are increasing or decreasing, $n \gtrless 0$. In any event, g is independent of any macro policy instrument, particularly the tax rate. In contrast to the basic AK model of Section 1.3.1 where the growth of output decreases with the tax rate, in the non-scale model, the response to taxes occurs through the gradual adjustment in k . We shall show below that as long as the dynamics of the system are stable, $\sigma_K < 1$, in which case the long-run equilibrium growth is $g > 0$, as indicated.

The implication that long-run growth cannot be sustained in the absence of population growth has itself been criticized. Accordingly, an alternative class of non-scale growth models, that eliminates the effect of country size, but still permits growth in the absence of population growth, has more recently emerged. These models permit at least a limited role for government policy to influence the long-run growth rate, through taxes and subsidies to research and development.²⁰

1.7.1. Equilibrium Dynamics

To analyse the transitional dynamics of the economy about its long-run stationary growth path, it is convenient to express the system in terms of the following stationary variables:

$$c \equiv \frac{C}{\frac{\sigma_N}{N^{1-\sigma_K}}}; \quad k \equiv \frac{K}{\frac{\sigma_N}{N^{1-\sigma_K}}}.$$

With constant social returns to scale ($\sigma_N + \sigma_K = 1$) these reduce to standard per capita quantities; i.e. $c = C/N = C_i$, etc. Otherwise they represent 'scale-adjusted' per capita quantities.

We begin by determining the consumption dynamics. To do so we take the time derivative of (1.2a) and combine with (1.4a) to find that the individual's consumption grows at the constant rate:

$$\frac{\dot{C}_i}{C_i} = \frac{(1 - \tau_y) \sigma A K_i^{\sigma-1} K^n - \rho - n}{1 - \gamma} \equiv \psi_i. \quad (1.38)$$

With all individuals being identical, the growth rate of aggregate consumption is $\psi = \psi_i + n$, so that:

$$\frac{\dot{C}}{C} = \frac{(1 - \tau_y) \sigma A K^{\sigma-1} N^{\sigma_N} - \rho - \gamma n}{1 - \gamma} \equiv \psi. \quad (1.39)$$

Differentiating c and using (1.46), the growth rate of the scale-adjusted per capita consumption is:

$$\dot{c} = c \left[\frac{(1 - \tau_y) \sigma A k^{\sigma-1}}{1 - \gamma} - \frac{\rho}{1 - \gamma} - \left(\frac{\gamma}{1 - \gamma} + \frac{\sigma_N}{1 - \sigma_K} \right) n \right]. \quad (1.40a)$$

To derive the dynamics of the scale adjusted aggregate capital stock we take the time derivative of k and combine with the production market equilibrium condition (4) to yield:

$$\dot{k} = k \left[A k^{\sigma-1} - \frac{c}{k} - \left(\frac{\sigma_N}{1 - \sigma_K} \right) n \right]. \quad (1.40b)$$

The steady-state values of the transformed variables, k and \tilde{c} , are given by:

$$\tilde{k} = \left\{ \frac{1}{A\sigma(1-\tau_y)} \left[\rho + \left(\gamma + \frac{(1-\tau_y)\sigma_N}{1-\sigma_K} \right) n \right] \right\}^{\frac{1}{\sigma_K-1}} \quad (41a)$$

$$\tilde{c} = \frac{1}{\sigma(1-\tau_y)} \left\{ \rho + \left[\gamma + (1-\gamma - (1-\tau_y)\sigma) \left(\frac{\sigma_N}{1-\sigma_K} \right) \right] n \right\} \tilde{k} \quad (41b)$$

and we see that the tax rates impinge on the level of activity through k, \tilde{c} . Note also, that (1.41a), (1.41b) reduce to the standard expressions for the neoclassical model, in the absence of externalities, when the aggregate production function has constant returns to scale.

Linearizing (1.40a) and (1.40b) around steady state, it is immediately apparent that $\sigma_K < 1$ is a necessary and sufficient condition for saddlepoint stability, in which case (1.37) ensures that the equilibrium growth rate of output, g is positive. The stability condition asserts $\eta < \sigma_N = 1 - \sigma$, so that the share of external spillover generated by private capital accumulation, and hence the overall social increasing returns to scale, cannot exceed the exogenously growing factor's share (labour) in production. Thus stability is consistent with decreasing, constant, or even increasing returns to scale, provided that the latter are not too excessive.

The model we have presented is the direct analogue to the endogenous growth model presented in Section 1.3.1. In contrast to that model, which was always on its balanced growth path, the present model involves transitional dynamics, the linearized approximation to which is described by a first order path. Thus whereas in the AK model, a tax increase leads to an instantaneous jump in the price level, in the present model it leads to a gradual decline in the capital stock, with no impact on the long-run growth rate.

1.7.2. Optimal Fiscal Policy

Optimal fiscal policy can be easily characterized in this model. The corresponding first-best equilibrium in the centrally planned economy, in which the central planner internalizes the externalities, is described by:

$$\dot{c} = c \left[\frac{(1-\tau_y)\sigma A k^{\sigma_K-1}}{1-\gamma} - \frac{\rho}{1-\gamma} - \left(\frac{\gamma}{1-\gamma} + \frac{\sigma_N}{1-\sigma_K} \right) n \right] \quad (1.40a')$$

in conjunction with (1.40b). Comparing (1.40a) and (1.40a') we immediately see that the tax rate:

$$\hat{\tau}_y = -\eta/\sigma < 0 \quad (1.42)$$

will ensure that the decentralized economy replicates the centrally planned economy. The reason that the optimum calls for a subsidy is because aggregate capital generates a positive externality that individual agents fail to take into account. Furthermore, the simple fixed rule (1.42) enables the decentralized economy to mimic the entire optimal dynamic path. Eicher and Turnovsky (2000) have extended this model to include alternative specifications of congestion, including (1.15), and examined optimal policy in this more generalized model.

1.7.3. Two-Sector Non-Scale Model

The non-scale model was originally introduced by Jones (1995a, 1995b) in the context of Romer's (1990) two-sector model of technological change. As formulated by Romer, his model had an acute scale effect in the sense that it implied that doubling the population and the number of people engaged in research would double the equilibrium growth rate, a proposition that was clearly counterfactual. Jones introduced decreasing returns to knowledge and showed how this leads to an equilibrium in which the long-run growth rates in the two sectors, – final output and technology – were independent of the size of the economy. Instead, they were functions of the technological elasticities of production function of the knowledge-producing sector, together with the population growth rate, and as such, were a direct generalization of (1.43), the equilibrium one-sector growth rate.

Jones' analysis was based on the specific assumption of constant returns to scale in the final output sector in capital and labour efficiency units. Eicher and Turnovsky (1999a) characterize the equilibrium growth rates under more general conditions with respect to returns to scale in the two sectors. One important conclusion they show is that essentially the only production function consistent with a balanced growth path under arbitrary returns to scale is the Cobb–Douglas. Thus, for example, the CES production function generalized to allow for non-constant returns to scale, is inconsistent with a balanced growth path. That is why we have imposed the Cobb–Douglas production function throughout.²¹

Eicher and Turnovsky (1999b, 2001) characterize the transitional dynamics of the two-sector non-scale model in detail, relying extensively on numerical simulation methods. They show how the stable adjustment path, which is a one-dimensional locus in the corresponding endogenous growth model, now becomes a two-dimensional locus in the two-sector non-scale model. This is important for issues pertaining to the speed of convergence. The basic one-sector neoclassical model and the two-sector endogenous

growth model both imply speeds of convergence that are generally viewed as implausibly high, around 10 per cent per annum.

The two-sector non-scale models generally imply much slower rates of convergence. In a preliminary examination of this issue, Jones (1995a) obtained excessively slow rates of convergence, mainly because he assumed that the sectoral allocation of each factor remains constant during the transition. Dinopoulos and Thompson (1998) analyze the transition dynamics of the alternative non-scale model numerically to find that the rate of convergence is approximately equal to the rate of population growth, but they do not highlight separate transition paths of output, capital, and technology. Eicher and Turnovsky (1999b) show how the empirical findings regarding the differential rates of convergence of output and technology can be reconciled by introducing a two-sector *non-scale* model of capital accumulation that incorporates endogenous technological change and population growth. Their key contribution is to show how the stable transition path in the two-sector non-scale growth model is characterized by a two-dimensional stable saddlepath, which permits the growth rates and the convergence speeds *to vary both across time and variables*. The presence of a two-dimensional stable manifold introduces important flexibility to the convergence characteristics, by allowing capital, output, and technology to converge at different time-varying rates toward possibly different long-run equilibrium growth rates. These properties are consistent with Bernard and Jones (1996a, 1996b) who show that different sectors exhibit distinctly different convergence time profiles, suggesting that the process of convergence is more complex than indicated by changes in any single aggregate measure. Furthermore, they show that reasonable asymptotic convergence speeds are achieved with a wide variety of parameter values.

1.8. CONCLUDING COMMENTS

In this chapter we have discussed some of the recent theoretical developments in growth theory, tying them to the earlier growth theories. We have done this by setting out a basic generic model and have shown how it may yield two of the key models that have played a prominent role in the recent literature on economic growth theory, the endogenous growth model and the non-scale growth model. We focused initially on the former, emphasizing how the simplest such model leads to an equilibrium in which the economy is always on its balanced growth path. We have also shown how the endogeneity or otherwise of the labour supply is crucial in determining the equilibrium growth rate and the responsiveness of the equilibrium growth rate to macroeconomic policy.

But transitional dynamics are an important aspect of the growth process and indeed, much of the recent discussion of convergence concerns the speed with which the economy approaches its balanced growth path. We have discussed alternative ways that such transitional dynamics may be introduced. Within the endogenous growth framework this occurs naturally through the introduction of a second capital stock and two such examples have been considered. The first is the introduction of government capital in the one sector model, and the second is the two-sector production model, pioneered by Lucas (1988), in which the two capital goods relate to physical and human capital.

As we have noted, the endogenous growth model has been the source of criticism, leading to the development of the non-scale model. This too is characterized by transitional dynamics, which are more flexible than those of the corresponding endogenous growth model. But the fact that the long-run growth rate is independent of policy in such models does not mean that policy is unimportant. On the contrary, since such models are typically associated with slow convergence speeds (of the order of 2–3 per cent) policy can influence the accumulation of capital for extended periods of time, leading to significant long-run level effects.

We observed at the outset that our discussion is necessarily limited and restricted. We have focused entirely on models where the growth occurs through capital accumulation, emphasizing particularly the role of the technological conditions. We have not addressed issues pertaining to innovation and technology transfer and the related issue of the indigenous development of technology for which the knowledge-based models of Romer (1990), Grossman and Helpman (1991), Eicher (1996), Aghion and Howitt (1998) are particularly relevant. Moreover, our analysis has been restricted to the real aspects, and one can also introduce monetary aspects, including inflation and monetary policy. Some work along these lines has been carried out by Palivos and Yip (1995) and others. Other issues that are important but are not addressed here include the role of fertility and education in the growth process, as well as the relationship between growth, the distribution of income, and income inequality.

As one sorts through the growth literature over the last half century, one sees striking parallels between the old and the new. The structures have many similarities; it is the methods of analysis that have changed, and presumably become more sophisticated. The AK technology of the basic one-sector Barro–Rebelo endogenous growth model is identical to that of the Harrod–Domar model. Furthermore, the equilibrium rate of growth in the AK model can be expressed as the product of the savings rate and the output/capital ratio, and is identical to Harrod’s warranted growth rate. Moreover, the rigidities that were associated with the Harrod–Domar technology, and led to

the development of the Solow–Swan neoclassical model, have their parallels in the more recent literature. The scale effects identified with the constant returns to scale in innovation in the Romer model led to the more flexible specification of technology in the Jones non-scale model. But this model is just an extension of the Solow–Swan neoclassical model to allow for deviations from returns to scale. Imposing the assumption of constant returns to scale gets us right back to the neoclassical framework. What is new here is the endogeneity of technology and the more serious effort devoted to trying to reconcile the theoretical models with the empirical facts, although that too was important in the early work.

NOTES

1. A comprehensive treatment of the early growth theory through 1970 is provided by Burmeister and Dobell (1970).
2. Jones (1995a) referred to such models as ‘semi-endogenous’ growth models.
3. Subsequent studies suggest that the convergence rates are more variable and sensitive to time periods and the set of countries than originally suggested and a wider range of estimates have been obtained. For example, Islam (1995) estimates the rate of convergence to be 4.7 per cent for non-oil countries and 9.7 per cent for OECD economies. Temple (1998) estimates the rate of convergence for OECD countries to be between 1.5 per cent and 3.6 per cent and for non-oil countries to be between 0.3 per cent and 6.7 per cent. Evans (1997) obtains estimates of the convergence rate of around 6 per cent per annum. Caselli et al. (1996) obtain an even higher rate of convergence of around 10 per cent.
4. This contrasts with Ortigueira and Santos (1997) who achieve a similar reduction in the rate of convergence in the two-sector endogenous growth model, by introducing appropriate adjustment costs to investment.
5. Turnovsky (2002) provides a parallel discussion to that presented here, describing recent developments of growth theory in an open economy.
6. Since all agents are identical, all aggregate quantities are simply multiples of the individual quantities, $X = NX$. Note that since all agents allocate the same share of time to work, there is no need to introduce the agent’s subscript to l .
7. When production functions exhibit non-constant returns to scale in all factors, the existence of a balanced growth equilibrium requires the production function to be Cobb–Douglas, as assumed in (1.1a) (see Eicher and Turnovsky, 1999a).
8. A negative exponent can be interpreted as reflecting congestion, along the lines of Barro and Sala-i-Martin (1992a).
9. This form of utility function is consistent with the existence of a balanced growth path (see Ladrón-de-Guevara et al., 1997). The specification in (1.1b) is the case of pure leisure; they also consider the case where utility derived from leisure depends upon its interaction with human capital.
10. See, for example, Barro, (1990), Jones and Manuelli (1990), King and Rebelo (1990), Rebelo (1991), Jones et al. (1993), Ireland (1994), Turnovsky (1996a, 2000), Ortigueira (1998).

11. Note that the technology (1.7) is identical to that of the original Harrod–Domar model, to which the AK model is a modern counterpart. It was Harrod, himself, who originally referred to the ‘knife-edge’ characteristics of his model.
12. The empirical evidence on the intertemporal elasticity of substitution overwhelmingly suggests it is less than unity, i.e. $\gamma < 0$, thus supporting the transversality condition (1.8c).
13. Being nonlinear, (1.12a) and (1.12b) may, or may not, intersect, so that a balanced growth equilibrium may, or may not, exist. A sufficient condition for a unique steady-state balanced-growth path is that $(1 - \tau_y) \sigma / (1 - \gamma) - 1 < [\min(1 - \tau_y) \sigma / (1 - \tau_c) \theta, \rho / \alpha]$. And whether this condition is met depends upon tax rates and other parameters.
14. For more general production structures it is possible that (1.13) is stable, giving rise to potential problems of indeterminate equilibria. This may quite plausibly occur for production functions more general than the Cobb–Douglas. In a model with both physical and nonhuman capital Benhabib and Perli (1994), Ladrón-de-Guevara et al. (1997) show how the steady-state equilibrium may become indeterminate. Other authors have emphasized the existence of externalities as sources of indeterminacies of equilibrium (see Benhabib and Farmer, 1994).
15. The rate of return on consumption equals $\rho - \dot{\lambda} / \lambda$, which varies positively with the growth rate, ψ .
16. The contrast in the effects of tax rates between models having fixed and endogenous labour supply is much more dramatic in an open economy. For example, the effect of an increase in the income tax rate, which reduces growth when labour supply is fixed, is fully borne by labour supply, when labour supply is endogenous, leaving the equilibrium growth rate unaffected (see Turnovsky, 1999b).
17. Turnovsky (1997a) extends the model to allow for congestion, along the lines of the analysis of Section 4.1. Glomm and Ravikumar (1994) make the opposite sign regarding depreciation, namely that capital lasts a single period. This assumption leads to the economy always being on its balanced growth path.
18. This result does not mean that the government can increase the growth rate indefinitely by increasing g indefinitely. There are constraints due to the fact that $g < 1$ and that the transversality conditions must be met.
19. Caballé and Santos (1993) provide an earlier analytical discussion of the transitional dynamics in the two-sector model.
20. Examples of the second form of non-scale model include Young (1998), Aghion and Howitt (1998), Dinopoulos and Thompson (1998), Howitt (1999), and Jones (1999).
21. The existence of a balanced growth equilibrium is consistent with an arbitrary production function if and only if it has constant returns to scale. Eicher and Turnovsky also show that a balanced growth path may arise under a nonconstant returns to scale production function provided it is of a specific homogeneously separable form.

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