

On Indeterminacy and Growth under Progressive Taxation and Utility-Generating Government Spending*

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Abstract

We examine the theoretical interrelations between progressive income taxation and macroeconomic (in)stability in an otherwise standard one-sector AK model of endogenous growth with utility-generating government purchases of goods and services. In sharp contrast to traditional Keynesian-type stabilization policies, progressive taxation operates like an automatic destabilizer that generates equilibrium indeterminacy and belief-driven fluctuations in our endogenously growing macroeconomy. This instability result is obtained regardless of (i) the degree of the public-spending preference externality, and (ii) whether private and public consumption expenditures are substitutes, complements, or additively separable in the household's utility function.

Keywords: Progressive Income Taxation, Equilibrium (In)determinacy, Utility-Generating Government Spending, Endogenous Growth.

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1 Introduction

Starting with the important work of Jones and Manuelli (1990), King and Rebelo (1990) and Rebelo (1991), substantial progress has been made in exploring the aggregate effects of various fiscal policies within an endogenously growing macroeconomy. As it turns out, many existing theoretical studies consider a constant tax rate of income *and/or* useless government purchases of goods and services that do not contribute to utility or production.¹ Although these assumptions are commonly adopted for the sake of analytical simplicity, they are not consistent with those observed in the actual data. Motivated by this gap in the previous literature, we examine a parsimonious one-sector endogenous growth model with progressive/regressive taxation of income and utility-generating public expenditures. Specifically, this paper analytically investigates the interrelations between sustained economic growth, equilibrium (in)determinacy, and tax progressivity/regressivity governed by a single parameter. As a result, the current piece complements our earlier work, as in Chen and Guo (2013), which also analyzes the same research topic in a similar theoretical framework but with productive flow of government spending *à la* Barro (1990).

In this paper, we study the (de)stabilization effects of Guo and Lansing's (1998) nonlinear tax schedule in an otherwise prototypical one-sector AK model of endogenous growth with inelastic labor supply and utility-generating government purchases. Based on the empirical findings of Ni (1995), our analyses consider a constant-relative-risk-aversion (CRRA) Cobb-Douglas utility specification that postulates public spending as a positive preference externality. We focus on the model's unique balanced-growth equilibrium along which output, consumption, physical capital and government spending all grow at a common positive rate. As it turns out, our model economy exhibits equilibrium indeterminacy and endogenous belief-driven growth fluctuations under progressive taxation of income. Start from a particular balanced growth path, and suppose that agents become optimistic about the economy's future. Acting upon this expectation, the household will reduce consumption and raise investment today, which in turn yields another dynamic trajectory. When the tax progressivity is positive, we find that the equilibrium after-tax marginal product of capital is monotonically increasing along the convergent transitional path as the consumption-to-capital ratio rises.

¹See, for example, Barro and Sala-i-Martin (1992), Futagami, Morita and Shibata (1993), Pecorino (1993), Glomm and Ravikumar (1994, 1997), Cazzavillan (1996), Turnovsky (1996, 1997, 1999), Zhang (2000), Baier and Glomm (2001), Yamarik (2001), Palivos, Yip and Zhang (2003), Chen (2006), Greiner (2007), and Hu, Ohdoi and Shimomura (2008), among others.

Consequently, agents' initial optimistic anticipation is validated and the alternative path becomes a self-fulfilling equilibrium. On the contrary, the economy displays local determinacy and equilibrium uniqueness under regressive or flat income taxation.

The aforementioned findings demonstrate that in sharp contrast to conventional Keynesian-type stabilization policies, progressive taxation operates like an automatic destabilizer whereas regressive or flat taxation leads to saddle-path stability within our one-sector endogenous growth model. In addition, these (in)stability results do not depend on *any other* structural parameters, such as the degree of the public-spending preference externality. When this utility parameter approaches zero, our model collapses to one with wasteful government purchases, as analyzed in Chen and Guo (2016). Therefore, this paper shows the robustness of progressive income taxation destabilizing an endogenously growing macroeconomy that incorporates utility-generating public expenditures. We also find that whether private and public consumption goods are Edgeworth substitutes, complements, or additively separable in the household utility function does not affect the model's local stability properties.²

The remainder of this paper is organized as follows. Section 2 describes the model and analyzes its equilibrium conditions. Section 3 derives the economy's balanced growth equilibrium and examines its local stability properties. Section 4 concludes.

2 The Economy

We incorporate Guo and Lansing's (1998) progressive/regressive income tax schedule into a one-sector AK model of endogenous growth under perfect foresight and utility-generating government purchases of goods and services. The economy is populated by a unit measure of identical infinitely-lived households, each of which provides fixed labor supply and maximizes its discounted lifetime utility

$$\int_0^{\infty} \frac{\left(c_t^{\theta_1} g_t^{\theta_2}\right)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt, \quad \sigma > 0, \quad (1)$$

where c_t is private consumption, g_t is the flow of public expenditures that are determined outside an individual household's control, and $\rho > 0$ denotes the rate of time preference. Based on the empirical findings of Ni (1995), the instantaneous utility function (1) is postulated to (i) be increasing and strictly concave with respect to private consumption, thus $\theta_1 > 0$ and

²By contrast, Chen and Guo (2014) show that in the no-sustained-growth version of our model with variable labor supply, the degree of the public-spending preference externality as well as the utility complementarity/substitutability between private and public consumptions may influence the steady state's local dynamics.

$\theta_1(1 - \sigma) < 1$; (ii) be increasing in public consumption, thus $\theta_2 > 0$ indicating the presence of a positive preference externality; and (iii) exhibit the constant-relative-risk-aversion (CRRA) Cobb-Douglas formulation with linear homogeneity in “effective consumption” $c_t^{\theta_1} g_t^{\theta_2}$, thus $\theta_1 + \theta_2 = 1$ (see also Bean [1986], and Campbell and Mankiw [1990] for earlier work). In addition, when $\sigma < (>) 1$, the marginal utility of private consumption rises (falls) with respect to government spending, which in turn implies that c_t and g_t are Edgeworth complements (substitutes). When $\sigma = 1$, the household’s preference (1) displays additive separability between private and public consumption expenditures, hence the marginal utility of c_t is independent of g_t .

The budget constraint faced by the representative household is given by

$$c_t + \dot{k}_t + \delta k_t = (1 - \tau_t)y_t, \quad k_0 > 0 \text{ given}, \quad (2)$$

where k_t is the household’s capital stock, $\delta \in (0, 1)$ is the capital depreciation rate, y_t is GDP, and τ_t represents a proportional income tax rate. Output y_t is produced by a unit measure of identical competitive firms using the production function

$$y_t = Ak_t^\alpha \bar{k}_t^{1-\alpha}, \quad A > 0, \quad 0 < \alpha < 1, \quad (3)$$

where \bar{k}_t is the economy-wide average level of capital services that generate positive technological spillovers onto each firm’s individual productivity (Romer, 1986). In a symmetric equilibrium, all firms make the same decisions such that $k_t = \bar{k}_t$, which in turn yields the following social technology that allows for sustained economic growth:

$$y_t = Ak_t. \quad (4)$$

In terms of the income tax rate, we adopt the sustained-growth version of Guo and Lansing’s (1998, p.485, footnote 4) nonlinear tax structure and postulate τ_t as

$$\tau_t = 1 - \eta \left(\frac{y_t^*}{y_t} \right)^\phi, \quad \eta \in (0, 1), \quad \phi \in (\underline{\phi}, 1), \quad (5)$$

where y_t^* denotes a benchmark level of income that is taken as given by the representative household. In our model with persistent growth, y_t^* is set to be the per-capita output on the economy’s balanced growth path (BGP) whereby $\frac{\dot{y}_t^*}{y_t^*} = \theta > 0$ for all t . To guarantee the existence of a balanced growth path, the household’s taxable income y_t in equilibrium needs to grow at the same rate as the baseline level of output y_t^* .

Next, the marginal tax rate τ_{mt} , defined as the change in taxes paid by the household divided by the change in its taxable income, is given by

$$\tau_{mt} = \frac{\partial(\tau_t y_t)}{\partial y_t} = \tau_t + \eta \phi \left(\frac{y_t^*}{y_t} \right)^\phi. \quad (6)$$

Our subsequent analyses are restricted to an environment with $0 < \tau_t, \tau_{mt} < 1$ such that the government does not have access to lump-sum taxes or transfers; the government is not allowed to confiscate all productive resources; and households have incentive to provide capital services to firms' production process. Along the economy's balanced-growth equilibrium with $y_t = y_t^*$, the above considerations imply that $0 < \eta < 1$ and $\frac{\eta-1}{\eta} < \phi < 1$. On the other hand, the convexity of the household's budget set requires that the after-tax marginal product of capital $(1 - \tau_{mt})MPK_t$ must be strictly decreasing with respect to k_t , which in turn implies that $\phi > \frac{\alpha-1}{\alpha}$ on the balanced growth path. It follows that the lower bound on the parameter ϕ of the postulated tax policy rule (5) is determined by

$$\underline{\phi} = \max \left\{ \frac{\alpha - 1}{\alpha}, \frac{\eta - 1}{\eta} \right\}. \quad (7)$$

Given the aforementioned restrictions on η and ϕ , equation (6) shows that the marginal tax rate τ_{mt} is higher than the average tax rate τ_t when $\phi > 0$. In this case, the tax schedule is said to be "progressive". When $\phi = 0$, the average and marginal tax rates coincide at the value $1 - \eta$ and the tax schedule is said to be "flat". When $\phi < 0$, the tax schedule is "regressive".

We assume that agents take into account the way in which the tax schedule affects their earnings when they decide how much to consume and invest over their lifetimes. Therefore, it is the marginal tax rate of income τ_{mt} that governs the household's economic decisions. The first-order conditions for the representative agent with respect to the indicated variables and their associated transversality conditions (TVC) are

$$c_t : \quad \theta_1 c_t^{\theta_1(1-\sigma)-1} g_t^{\theta_2(1-\sigma)} = \lambda_t, \quad (8)$$

$$k_t : \quad \lambda_t \left[\underbrace{\eta(1-\phi) \left(\frac{y_t^*}{y_t} \right)^\phi}_{(1-\tau_{mt})} \underbrace{\alpha \frac{y_t}{k_t}}_{MPK_t} - \delta \right] = \rho \lambda_t - \dot{\lambda}_t, \quad (9)$$

$$\text{TVC} : \quad \lim_{t \rightarrow \infty} e^{-\rho t} \lambda_t k_t = 0, \quad (10)$$

where (8) equates the marginal utility of private consumption to its marginal cost λ_t , which is the Lagrange multiplier on the household's budget constraint (2) that also represents the shadow price of physical capital; (9) is the modified consumption Euler equation which takes into account the effect of public spending on the marginal benefit of private consumption; and (10) is the transversality condition.

The government sets the income tax rate τ_t according to (5), and balances its budget at each point in time. Hence, the instantaneous government budget constraint is given by

$$g_t = \tau_t y_t, \quad (11)$$

where government purchases of goods and services g_t in turn contributes to the household's utilities (1). With the government, the aggregate resource constraint for the economy is

$$c_t + \dot{k}_t + \delta k_t + g_t = y_t. \quad (12)$$

3 Balanced Growth Path and Macroeconomic (In)stability

We focus on the economy's balanced growth path along which output, private consumption, public spending, and physical capital exhibit a common, positive constant growth rate θ . To facilitate the subsequent dynamic analyses, we undertake the variable transformations whereby $x_t \equiv \frac{g_t}{k_t}$ and $z_t \equiv \frac{c_t}{k_t}$. Using these variable transformations, the model's equilibrium conditions (with $\frac{\dot{y}_t^*}{y_t^*} = \theta$ imposed) can be collapsed into the following autonomous dynamical system:

$$\dot{x}_t = -\phi(A - x_t)(\theta - A + \delta + x_t + z_t), \quad (13)$$

$$\frac{\dot{z}_t}{z_t} = \frac{\alpha(1 - \phi)(A - x_t) - \sigma(A - \delta - x_t - z_t) - \phi\theta_2(1 - \sigma)\left(\frac{A}{x_t} - 1\right)(\theta - A + \delta + x_t + z_t) - \delta - \rho}{1 - \theta_1(1 - \sigma)}. \quad (14)$$

A balanced-growth equilibrium is characterized by a pair of positive real numbers (x^*, z^*) that satisfy $\dot{x}_t = \dot{z}_t = 0$. It is straightforward to show that our model economy possesses a unique BGP with

$$x^* = A(1 - \eta) \quad (15)$$

and

$$z^* = \frac{A\eta[\sigma - \alpha(1 - \phi)] + (1 - \sigma)\delta + \rho}{\sigma}; \quad (16)$$

and that the common (positive) rate of economic growth is given by

$$\theta = \frac{\alpha A\eta(1 - \phi) - \delta - \rho}{\sigma}. \quad (17)$$

With regard to the BGP's local dynamics, we analytically derive the Jacobian matrix J of the dynamical system (13)-(14) evaluated at (x^*, z^*) , and find that its determinant and trace are

$$Det = -\frac{\alpha A\eta\phi(1 - \phi)z^*}{1 - \theta_1(1 - \sigma)} \geq 0 \quad \text{when } \phi \leq 0, \quad (18)$$

$$Tr = \frac{\sigma \left[\underbrace{1 - \eta(1 - \phi)}_{\tau_m^*} \right] z^* - \eta\phi [1 - \theta_1(1 - \sigma)] [A(1 - \eta) + z^*]}{(1 - \eta) [1 - \theta_1(1 - \sigma)]}, \quad (19)$$

where $\tau_m^* \in (0, 1)$ denotes the marginal tax rate on the economy's balanced-growth equilibrium path.

Proposition. The economy exhibits endogenous growth fluctuations driven by agents' self-fulfilling expectations or sunspots under progressive income taxation with $0 < \phi < 1$; whereas saddle-path stability and equilibrium uniqueness take place under regressive taxation with $\underline{\phi} < \phi < 0$, where $\underline{\phi}$ is given by (7).

Proof. The BGP's local stability property is determined by comparing the eigenvalues of J that have negative real parts versus the number of initial conditions in the dynamical system (13)-(14), which is zero because both x_t and z_t are non-predetermined jump variables in our model.³ Since $0 < \alpha, \eta < 1$, together with $A, z^* > 0$ and $\theta_1(1 - \sigma) < 1$ to ensure the preference concavity in private consumption, the Jacobian's determinant (18) is negative under progressive income taxation with $0 < \phi < 1$, indicating that the two eigenvalues are of opposite signs in their real parts. In this case, the economy's balanced-growth equilibrium exhibits local indeterminacy (*i.e.* a sink) and belief-driven aggregate fluctuations. When the tax schedule is regressive with $\underline{\phi} < \phi < 0$, the BGP displays saddle-path stability in that both eigenvalues of the Jacobian matrix J have positive real parts ($Det > 0$ and $Tr > 0$). ■

³As for the initial condition of consumption c_0 , the period-0 level of government spending g_0 (a flow variable) will be endogenously determined. Hence, both $x_0 = \frac{g_0}{k_0}$ and $z_0 = \frac{c_0}{k_0}$ are not predetermined.

The preceding Proposition shows that in the context of a one-sector AK model of endogenous growth with utility-generating government spending, progressive taxation operates like an automatic stabilizer whereas regressive taxation leads to equilibrium uniqueness. As it turns out, these (in)stability results do not depend on *any other* structural parameters, such as the degree of the public-spending preference externality $\theta_2 \in (0, 1)$. Notice that when $\theta_2 \rightarrow 0$, our model collapses to one with useless government purchases, as analyzed in Chen and Guo (2016). Therefore, this paper shows the robustness of progressive income taxation destabilizing an endogenous growth model that incorporates utility-generating government purchases of goods and services. In addition, our finding that progressive income taxation destabilizes an endogenously growing macroeconomy is independent of the parameter σ which governs whether private and public consumption goods are Edgeworth substitutes, complements, or additively separable in the household utility function (1). Nevertheless, as discussed below, its value will affect the model's phase diagram which in turn helps understand the above (in)determinacy results.

Figure 1 depicts our model's phase diagram when (i) c_t and g_t are Edgeworth substitutes or additively separable in the household's preference formulation ($\sigma \geq 1$), and (ii) the fiscal policy rule is progressive ($0 < \phi < 1$). Using (13) and (14), we find that the equilibrium loci $\dot{x}_t = 0$ and $\dot{z}_t = 0$ are both negatively-sloped, and that the associated downward-sloping stable arm (denoted as SS) is flatter than the $\dot{z}_t = 0$ locus, followed by $\dot{x}_t = 0$. Next, start from a particular BGP characterized by (x^*, z^*) , and suppose that agents become optimistic about the economy's future. Acting upon this belief, households will invest more and consume less today. This in turn will generate another dynamic trajectory $\{x'_t, z'_t\}$ that begins at (x'_0, z'_0) with $x'_0 > x^*$ and $z'_0 < z^*$. Figure 1 illustrates that for this alternative path to become a self-fulfilling equilibrium, the after-tax return on investment $(1 - \tau_{mt})MPK_t$ must be monotonically increasing along the transitional path SS as the consumption-to-capital ratio $z_t \equiv \frac{c_t}{k_t}$ rises. From (3), (5), (6) and (11), together with the chain rule, it can be shown that

$$\left. \frac{d[(1 - \tau_{mt})MPK_t]}{dz_t} \right|_{SS} = \underbrace{\left. \frac{d[(1 - \tau_{mt})MPK_t]}{dx_t} \right|_{SS}}_{= -\alpha(1-\phi) < 0} \underbrace{\left. \frac{dx_t}{dz_t} \right|_{SS}}_{\text{negative}} > 0. \quad (20)$$

As a result, agents' initial rosy expectation is validated.

When $0 < \sigma$, $\phi < 1$, c_t and g_t enter the household utility (1) as Edgeworth complements and the tax schedule (5) is progressive. In this case, the model's equilibrium dynamics can be discussed in three distinct parametric configurations: (i) $0 < \sigma < \tilde{\sigma}$, (ii) $\tilde{\sigma} < \sigma < \hat{\sigma}$,

and (iii) $\hat{\sigma} < \sigma < 1$, where $\tilde{\sigma} \equiv \frac{\eta\phi(1-\theta_1)}{1-\eta+\eta\phi(1-\theta_1)}$ and $\hat{\sigma} \equiv \frac{\eta\phi(1-\theta_1)+\alpha(1-\eta)(1-\phi)}{1-\eta+\eta\phi(1-\theta_1)} (> \tilde{\sigma})$. We find that the intuition for indeterminacy and sunspots to occur in the subcase (i) turns out to be identical to that under $\sigma \geq 1$, which is demonstrated by the phase diagram as in Figure 1. Moreover, Figures 2 and 3 plot the phase diagrams for subcases (ii) and (iii), respectively. When the household deviates from the original BGP characterized by (x^*, z^*) and decreases today's consumption due to its optimism about the economy's future, the resulting dynamic trajectory $\{x'_t, z'_t\}$ will begin at (x'_0, z'_0) with $x'_0 < x^*$ and $z'_0 < z^*$. It can be shown that when $x_t \equiv \frac{g_t}{k_t}$ rises along the convergent transitional path SS , the equilibrium after-tax marginal product of capital $(1 - \tau_{mt})MPK_t$ is monotonically increasing as well. As a result, agents' optimistic expectations are justified as a self-fulfilling equilibrium path.

When the tax schedule is regressive with $\underline{\phi} < \phi < 0$ and households decide to raise their investment expenditures today, the preceding mechanism that makes for multiple equilibria will generate divergent trajectories away from the original balanced growth path. This implies that given the initial capital stock k_0 , the period-0 levels of the household's consumption c_0 as well as the government's spending g_0 are uniquely determined such that the economy immediately jumps onto its original balanced-growth equilibrium (x^*, z^*) , and always stays there without any possibility of deviating transitional dynamics. It follows that equilibrium indeterminacy and endogenous growth fluctuations can never occur in this setting.

When the tax schedule is flat, average and marginal tax rates are equal whereby $\tau_t = \tau_{mt} = 1 - \eta$. Resolving our model with $\phi = 0$ leads to the following single differential equation in z_t that describes the equilibrium dynamics:

$$\frac{\dot{z}_t}{z_t} = \frac{[\sigma z_t + A\eta(\alpha - \sigma) - (1 - \sigma)\delta - \rho]}{1 - \theta_1(1 - \sigma)}, \quad (21)$$

which has a unique interior solution z^* that satisfies $\dot{z}_t = 0$. We then linearize (21) around the BGP and obtain the positive eigenvalue $\frac{z^*}{1 - \theta_1(1 - \sigma)} > 0$. This indicates that under flat income taxation, the balanced-growth equilibrium exhibits saddle-path stability and equilibrium uniqueness.

4 Conclusion

This paper examines the theoretical interrelations between progressive income taxation and macroeconomic (in)stability in an otherwise standard one-sector AK model of endogenous growth with fixed labor supply and utility-generating government spending. We show that the economy exhibits equilibrium indeterminacy and belief-driven growth fluctuations when

the tax progressivity is positive, and that the unique balanced-growth equilibrium displays saddle-path stability under regressive or flat taxation of income. It follows that in sharp contrast to a traditional automatic stabilizer, moving the fiscal policy toward progressive taxation may magnify the magnitude of aggregate fluctuations and thus destabilize an endogenously growing macroeconomy. We also find that these (in)stability results are independent of (i) the degree of the public-spending preference externality, and (ii) whether private and public consumption expenditures are substitutes, complements, or additively separable in the household's utility function. In terms of possible extensions, it would be worthwhile to explore alternative mechanisms for generating sustained economic growth (*e.g.* human capital accumulation) and/or an economy with multiple production sectors. We plan to pursue these research projects in the near future.

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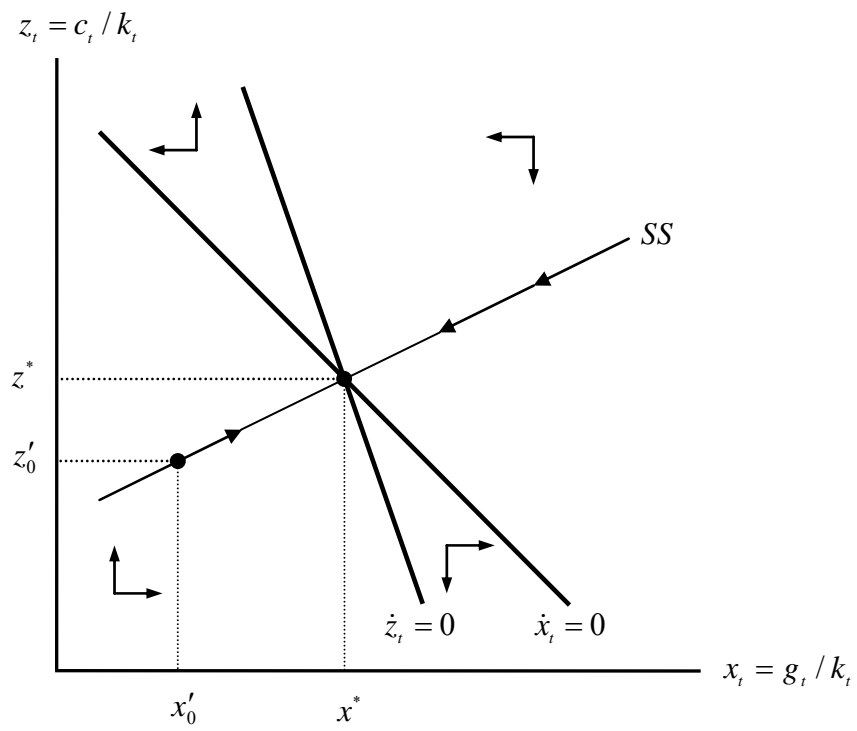


Figure 3. When $0 < \phi < 1$ and $\hat{\sigma} < \sigma < 1$: Indeterminacy