

# Progressive Taxation, Endogenous Growth, and Macroeconomic (In)stability\*

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## Abstract

In the context of a standard one-sector  $AK$  model of endogenous growth, we show that the economy exhibits equilibrium indeterminacy and belief-driven aggregate fluctuations under progressive taxation of income. When the tax schedule is regressive or flat, the economy's balanced growth path displays saddle-path stability and equilibrium uniqueness. These results imply that in sharp contrast to a conventional automatic stabilizer, progressive income taxation may destabilize an endogenously growing macroeconomy by generating cyclical fluctuations driven by agents' self-fulfilling expectations or sunspots.

*Keywords:* Progressive Income Taxation, Endogenous Growth, Equilibrium (In)determinacy.

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# 1 Introduction

Following the resurgent interest in long-run economic growth that began in the mid-1980's, there has been an extensive literature analyzing the macroeconomic impacts of income taxation within various formulations of endogenously growing dynamic general equilibrium models.<sup>1</sup> As it turns out, the vast majority of previous theoretical studies postulate a constant tax rate on the households' taxable income.<sup>2</sup> While this assumption is commonly adopted for the sake of analytical simplicity, it is not consistent with the U.S. federal individual income tax schedule that is characterized by several "tax brackets" (branches of income) taxed at progressively higher rates. Motivated by this gap in the existing literature, we examine the (in)stability effects of Guo and Lansing's (1998) nonlinear taxation structure, whereby the associated tax progressivity is governed by a single parameter, in the simplest one-sector  $AK$  model of endogenous growth: a representative-agent economy with fixed labor supply and useless government spending that does not contribute to utility or production.<sup>3</sup> Not only does our work provide valuable theoretical insights, it also yields important implications about the (de)stabilization role of tax policies in a macroeconomy with sustained economic growth.

In sharp contrast to traditional Keynesian-type stabilization policies, progressive income taxation does not operate as an automatic stabilizer in our model because it leads to equilibrium indeterminacy and endogenous belief-driven growth fluctuations.<sup>4</sup> Start from a particular balanced growth path, and suppose that agents become optimistic about the economy's future. Acting upon this expectation, the representative household will reduce consumption and raise investment today, which in turn generates another dynamic trajectory. When the tax progressivity is positive, we show that the equilibrium after-tax marginal product of capital is monotonically increasing along the downward-sloping transitional path as the consumption-to-capital ratio rises. As a consequence, agents' initial optimistic anticipation is validated and the alternative path becomes a self-fulfilling equilibrium. On the contrary, our model exhibits

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<sup>1</sup>Early examples in this area include Barro (1990), King and Rebelo (1990), Rebelo (1991), and Barro and Sala-i-Martin (1992), among many others.

<sup>2</sup>Noted exceptions are Yamarik (2001), Li and Sarte (2004), and Greiner (2006), among others. However, none of these papers focus on the (de)stabilization effects of a nonlinear tax policy rule.

<sup>3</sup>Useless or wasteful government spending is a simplifying assumption that is commonly adopted in the endogenous growth literature as the benchmark theoretical specification. Subsequent research may consider useful public expenditures that are productive (*e.g.* infrastructure) and/or utility-generating (*e.g.* national health care and public education).

<sup>4</sup>By contrast, Guo and Lansing (1998) find that sufficiently progressive taxation can stabilize a one-sector real business cycle model, which possesses an indeterminate steady state under *laissez-faire* and no sustained endogenous growth, against cyclical fluctuations driven by agents' animal spirits.

saddle-path stability and equilibrium uniqueness under regressive<sup>5</sup> or flat income taxation. These findings altogether imply that in the context of a basic one-sector  $AK$  model of endogenous growth, changing the tax schedule from being flat or regressive to progressive will magnify the magnitude of aggregate fluctuations and thus destabilize the macroeconomy.

In our previous work, Chen and Guo (2013) analytically explore the interrelations between nonlinear income taxation and equilibrium (in)determinacy in a one-sector endogenous growth model as well. Although their main theoretical finding about the potential destabilization role of progressive taxation is qualitatively identical, there are important differences between the two studies. First, sustained endogenous growth in the current piece is driven by Romer's (1986) learning-by-doing technological spillovers from physical capital, whereas the presence of aggregate constant returns-to-scale in production within Chen and Guo's (2013) model is caused by useful public expenditures contributing to firms' productivity. Second, the local (in)stability effects of our postulated fiscal policy rule analyzed in this paper is solely governed by the sign of the tax progressivity parameter; however, the relative strength of the demand-side versus the supply-side impact from government purchases of goods and services also affects the equilibrium dynamics of Chen and Guo's (2013) macroeconomy. As a result, our analysis shows the robustness of progressive taxation operating as an automatic destabilizer within a one-sector endogenous growth model without productive flow of public spending.

The remainder of this paper is organized as follows. Section 2 describes our model and derives the economy's unique balanced-growth equilibrium path. Section 3 examines the theoretical interrelations between tax progressivity and macroeconomic (in)stability. Section 4 concludes.

## 2 The Economy

We incorporate a progressive/regressive income tax schedule *à la* Guo and Lansing (1998) into the one-sector  $AK$  model of endogenous growth under perfect foresight. The economy is populated by a unit measure of identical infinitely-lived households. Each household provides fixed labor supply and maximizes its discounted lifetime utility

$$U = \int_0^{\infty} \frac{c_t^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt, \quad \sigma \geq 1, \quad (1)$$

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<sup>5</sup>By contrast, Schmitt-Grohé and Uribe (1997) show that a prototypical one-sector real business cycle model, coupled with no sustained endogenous growth and a balanced-budget rule that qualitatively resembles regressive income taxation, may exhibit indeterminacy and sunspots.

where  $c_t$  is consumption,  $\rho > 0$  denotes the subjective rate of time preference, and  $\sigma$  represents the inverse of the intertemporal elasticity of substitution in consumption. Based on the empirical evidence for this preference parameter in the mainstream macroeconomics literature, our analysis is restricted to the cases with  $\sigma \geq 1$ . The budget constraint faced by the representative household is

$$c_t + \dot{k}_t + \delta k_t = (1 - \tau_t)y_t, \quad k_0 > 0 \text{ given}, \quad (2)$$

where  $k_t$  is the household's capital stock,  $\delta \in (0, 1)$  is the capital depreciation rate,  $y_t$  is GDP, and  $\tau_t$  represents a proportional income tax rate. Output  $y_t$  is produced by a unit measure of identical competitive firms with the Cobb-Douglas production function

$$y_t = Ak_t^\alpha \bar{k}_t^{1-\alpha}, \quad A > 0, \quad 0 < \alpha < 1, \quad (3)$$

where  $\bar{k}_t$  is the economy-wide average level of capital services that generate positive technological spillovers onto each individual firm's productivity (Romer, 1986). In a symmetric equilibrium, all firms make the same decisions such that  $k_t = \bar{k}_t$ , which in turn yields the following social technology that allows for sustained economic growth:

$$y_t = Ak_t. \quad (4)$$

With regard to the income tax rate, we adopt the sustained-growth version of Guo and Lansing's (1998, p.485, footnote 4) nonlinear tax structure and postulate  $\tau_t$  as

$$\tau_t = 1 - \eta \left( \frac{y_t^*}{y_t} \right)^\phi, \quad \eta \in (0, 1), \quad \phi \in (\underline{\phi}, 1), \quad (5)$$

where  $y_t^*$  denotes a benchmark level of income that is taken as given by the representative household. In our model with endogenous growth,  $y_t^*$  is set equal to the level of per capita output on the economy's balanced growth path (BGP) whereby  $\frac{\dot{y}_t^*}{y_t^*} = \theta > 0$  for all  $t$ .<sup>6</sup> In addition, the marginal tax rate  $\tau_{mt}$ , defined as the change in taxes paid by the household divided by the change in its taxable income, is given by

$$\tau_{mt} = \frac{\partial(\tau_t y_t)}{\partial y_t} = \tau_t + \eta \phi \left( \frac{y_t^*}{y_t} \right)^\phi. \quad (6)$$

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<sup>6</sup>To guarantee the existence of a balanced growth path in our subsequent analyses, the household's taxable income  $y_t$  in equilibrium needs to grow at the same rate as the baseline level of income  $y_t^*$ . Moreover, the constant (positive) growth rate  $\theta$  for  $y_t^*$  will be endogenously determined through the model's equilibrium conditions (see equation 16).

Our analyses are restricted to an environment with  $0 < \tau_t, \tau_{mt} < 1$  such that (i) the government does not have access to lump-sum taxes or transfers, (ii) the government cannot confiscate all productive resources, and (iii) households have incentive to provide factor services to firm's production process. Along the economy's balanced-growth equilibrium path where  $y_t = y_t^*$ , these considerations imply that  $0 < \eta < 1$  and  $\frac{\eta-1}{\eta} < \phi < 1$ . On the other hand, the convexity of the household's budget set requires that the after-tax marginal product of capital  $(1 - \tau_{mt})MPK_t$  must be strictly decreasing with respect to  $k_t$ , which in turn implies that  $\phi > \frac{\alpha-1}{\alpha}$  on the balanced growth path. It follows that the lower bound on the parameter  $\phi$  of the tax schedule (5) is determined by

$$\underline{\phi} = \max \left\{ \frac{\alpha - 1}{\alpha}, \frac{\eta - 1}{\eta} \right\}. \quad (7)$$

Given the postulated restrictions on  $\eta$  and  $\phi$ , equation (6) shows that the marginal tax rate  $\tau_{mt}$  is higher than the average tax rate  $\tau_t$  when  $\phi > 0$ . In this case, the tax schedule is said to be "progressive". When  $\phi = 0$ , the average and marginal tax rates coincide at the value  $1 - \eta$  and the tax schedule is said to be "flat". When  $\phi < 0$ , the tax schedule is "regressive".

In making decisions about how much to consume and invest over their lifetimes, agents take into account the effect in which the tax schedule influences their net earnings. As a result, it is the marginal tax rate of income  $\tau_{mt}$  that will govern the household's economic decisions. The first-order conditions for the representative agent with respect to the indicated variables and the associated transversality conditions (TVC) are

$$c_t : \quad c_t^{-\sigma} = \lambda_t, \quad (8)$$

$$k_t : \quad \lambda_t \left[ \underbrace{\eta(1 - \phi) \left( \frac{y_t^*}{y_t} \right)^\phi}_{(1 - \tau_{mt})} \underbrace{\alpha \frac{y_t}{k_t}}_{MPK_t} - \delta \right] = \rho \lambda_t - \dot{\lambda}_t, \quad (9)$$

$$\text{TVC} : \quad \lim_{t \rightarrow \infty} e^{-\rho t} \lambda_t k_t = 0, \quad (10)$$

where (8) states that the marginal utility of consumption is equal to the Lagrange multiplier  $\lambda_t$  on the household's budget constraint (2). Equation (9) is the Keynes-Ramsey condition that describes how the stock of physical capital evolves over time, and (10) is the transversality condition. Finally, the government sets the income tax rate  $\tau_t$  according to (5), and balances

its budget at each point in time. Hence, the instantaneous government budget constraint is given by

$$g_t = \tau_t y_t, \quad (11)$$

where  $g_t$  is public spending on goods and services that does not contribute to the household's utility or firms' production.<sup>7</sup>

We focus on the economy's balanced growth path along which output, consumption and physical capital exhibit a common, positive constant growth rate  $\theta$ . To facilitate the subsequent dynamic analyses, we adopt the following variable transformations:  $x_t \equiv \frac{g_t}{k_t}$  and  $z_t \equiv \frac{c_t}{k_t}$ . Per these variable transformations, the model's equilibrium conditions (with  $\frac{y_t^*}{y_t^*} = \theta$  imposed) can be collapsed into the following autonomous dynamical system:

$$\dot{x}_t = -\phi(A - x_t)(\theta - A + \delta + x_t + z_t), \quad (12)$$

$$\frac{\dot{z}_t}{z_t} = \frac{1}{\sigma} [\alpha(1 - \phi)(A - x_t) - \delta - \rho] - A + \delta + x_t + z_t. \quad (13)$$

A balanced-growth equilibrium is characterized by a pair of positive real numbers  $(x^*, z^*)$  that satisfy  $\dot{x}_t = \dot{z}_t = 0$ . It is straightforward to show that our model economy possesses a unique BGP with

$$x^* = A(1 - \eta) \quad (14)$$

and

$$z^* = \frac{A\eta[\sigma - \alpha(1 - \phi)] + (1 - \sigma)\delta + \rho}{\sigma}; \quad (15)$$

and that the common (positive) rate of economic growth is given by

$$\theta = \frac{\alpha A \eta (1 - \phi) - \delta - \rho}{\sigma}. \quad (16)$$

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<sup>7</sup>Substituting the social technology (4) into the government's budget constraint (11), together with the restriction of  $0 < \tau_t < 1$  described earlier, leads to  $0 < \frac{g_t}{k_t} < A$ , for all  $t$ . This in turn imposes a lower bound on the initial value of capital stock  $k_0 > \frac{g_0}{A}$ , where  $g_0$  is endogenously determined (see footnote 8).

### 3 Macroeconomic (In)stability

In terms of the BGP's local dynamics, we analytically derive the Jacobian matrix  $J$  of the dynamical system (12)-(13) evaluated at  $(x^*, z^*)$ , and find that its determinant and trace are

$$Det = -\frac{\alpha A \eta \phi (1 - \phi) z^*}{\sigma}, \quad (17)$$

$$Tr = -A \eta \phi + z^*. \quad (18)$$

**Proposition.** The economy's balanced-growth equilibrium exhibits local indeterminacy (*i.e.* a sink) and belief-driven aggregate fluctuations under progressive income taxation with  $0 < \phi < 1$ ; whereas saddle-path stability and equilibrium uniqueness take place under regressive taxation with  $\underline{\phi} < \phi < 0$ , where  $\underline{\phi}$  is given by (7).

*Proof.* The BGP's local stability property is determined by comparing the eigenvalues of  $J$  that have negative real parts with the number of initial conditions in the dynamical system (12)-(13), which is zero because  $x_t$  and  $z_t$  are both non-predetermined jump variables in our model.<sup>8</sup> Since  $\sigma \geq 1$ ,  $0 < \alpha$ ,  $\eta < 1$  and  $A, z^* > 0$ , the Jacobian's determinant (17) is negative when  $0 < \phi < 1$ , indicating that the two eigenvalues are of opposite signs in their real parts. In this case, the economy exhibits endogenous growth fluctuations driven by agents' self-fulfilling expectations or sunspots. When  $\underline{\phi} < \phi < 0$ , the BGP displays local determinacy in that both eigenvalues of the Jacobian matrix  $J$  have positive real parts ( $Det > 0$  and  $Tr > 0$ ). ■

To help understand the above (in)determinacy results, we use (12) and (13) to construct the model's phase diagram under progressive taxation as depicted in Figure 1. It is straightforward to show that the equilibrium loci  $\dot{x}_t = 0$  and  $\dot{z}_t = 0$  are downward sloping, and that the associated negatively-sloped stable arm (denoted as  $SS$ ) is flatter than the  $\dot{z}_t = 0$  locus, followed by  $\dot{x}_t = 0$ . Next, start from a particular balanced growth path characterized by  $(x^*, z^*)$ , and suppose that agents become optimistic about the future of the economy. Acting upon this belief, households will invest more and consume less today, which in turn lead to another dynamic trajectory  $\{x'_t, z'_t\}$  that begins at  $(x'_0, z'_0)$  with  $x'_0 > x^*$  and  $z'_0 < z^*$ . Figure 1 shows that for this alternative path to become a self-fulfilling equilibrium, the after-tax return on investment  $(1 - \tau_{mt})MPK_t$  must be monotonically increasing along the transitional path

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<sup>8</sup>As for the initial condition of consumption  $c_0$ , the period-0 level of government spending  $g_0$  (a flow variable) will be endogenously determined. Using the model's equilibrium conditions, it can be shown that  $x_t = A[1 - \eta e^{\phi(\theta + \delta - A + x_t + z_t)}]$ , thus both  $x_0 = \frac{g_0}{k_0}$  and  $z_0 = \frac{c_0}{k_0}$  are not predetermined.

$SS$  as the consumption-to-capital ratio  $z_t \equiv \frac{c_t}{k_t}$  rises. From (3), (5), (6) and (11), together with the chain rule, it can be shown that

$$\frac{d[(1 - \tau_{mt})MPK_t]}{dz_t} \Big|_{SS} = \underbrace{\frac{d[(1 - \tau_{mt})MPK_t]}{dx_t}}_{= -\alpha(1-\phi) < 0} \underbrace{\frac{dx_t}{dz_t} \Big|_{SS}}_{\text{negative}} > 0. \quad (19)$$

As a consequence, agents' initial rosy anticipation is validated.

Under regressive income taxation, when households become optimistic and decide to raise their investment expenditures today, the preceding mechanism that makes for multiple equilibria, *i.e.* an increase in the equilibrium after-tax marginal product of capital, will generate divergent trajectories away from the original balanced growth path. This implies that given the initial capital stock  $k_0$ , the period-0 levels of the household's consumption  $c_0$  as well as the government's spending  $g_0$  are uniquely determined such that the economy immediately jumps onto its balanced-growth equilibrium  $(x^*, z^*)$ , and always stays there without any possibility of deviating transitional dynamics. Figure 2 demonstrates this case with the corresponding phase diagram. It follows that equilibrium indeterminacy and endogenous growth fluctuations can never occur in this setting.

When the tax schedule is flat with  $\tau_t = \tau_{mt} = 1 - \eta$ , we substitute  $\phi = 0$  into (12) and find that the ratio of public expenditures to physical capital  $x_t$  remains unchanged over time. This implies that the dynamical system (12)-(13) now becomes degenerate. Resolving our model with  $\phi = 0$  leads to the following single differential equation in  $z_t$  that describes the equilibrium dynamics:

$$\frac{\dot{z}_t}{z_t} = \frac{\alpha A \eta - \delta - \rho}{\sigma} - A \eta + z_t + \delta, \quad (20)$$

which has a unique interior solution  $z^*$  that satisfies  $\dot{z}_t = 0$  along the balanced-growth equilibrium path. We then linearize (20) around the BGP and find that its local stability property is governed by the positive eigenvalue  $z^* > 0$ , as is shown in the phase diagram of Figure 3. Consequently, our endogenously growing economy exhibits saddle-path stability and equilibrium uniqueness under flat income taxation since there is no initial condition associated with (20).

Finally, it is worth pointing out that our main quantitative finding, whereby progressive taxation may destabilize an endogenously growing macroeconomy, is obtained in a highly stylized framework with two important caveats. First, although the fiscal policy rule under examination can be progressive, households are postulated to be homogeneous. It follows that



our analysis can not account for the redistributive effects on income inequality from changes in the tax progressivity. Second, the government's budget is always balanced, which does not stand up for scrutiny in light of the recent global financial crisis. Nevertheless, in spite of its analytical simplicity, our one-sector endogenous model is able to generate sharply contrasting results *vis-à-vis* a conventional viewpoint. In addition, maintaining the assumptions of representative agent and balanced budget highlights the destabilization role of progressive income taxation, which in turn allows the comparison of our findings with those from previous studies to be focused and transparent.

## 4 Conclusion

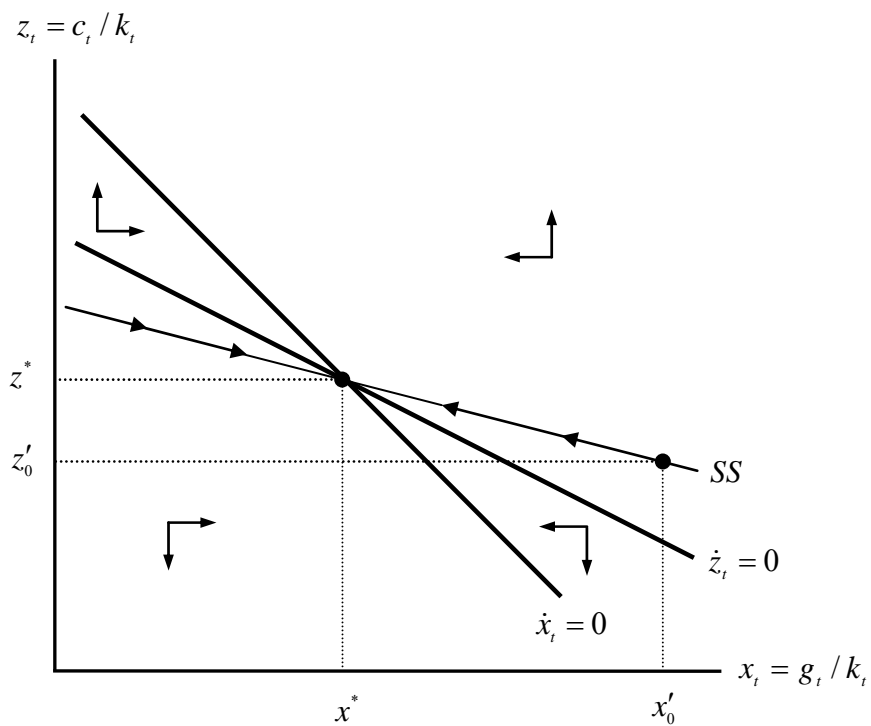
This paper examines the theoretical interrelations between progressive taxation of income and equilibrium (in)determinacy in a one-sector  $AK$  model of endogenous growth with fixed labor supply and useless public spending. We find that the economy exhibits an indeterminate balanced-growth equilibrium path when the tax progressivity is positive, and that saddle-path stability and equilibrium uniqueness take place under regressive or flat taxation of income. These results imply that in sharp contrast to a conventional automatic stabilizer, moving the government's fiscal policy rule toward progressive taxation may destabilize an endogenously growing macroeconomy with aggregate fluctuations driven by agents' self-fulfilling expectations or sunspots. In terms of possible extensions, it would be worthwhile to explore alternative mechanisms for generating sustained economic growth (*e.g.* human capital accumulation), and/or an economy with heterogeneous agents, non-balanced budget or multiple production sectors.<sup>9</sup> We plan to pursue these research projects in the near future.

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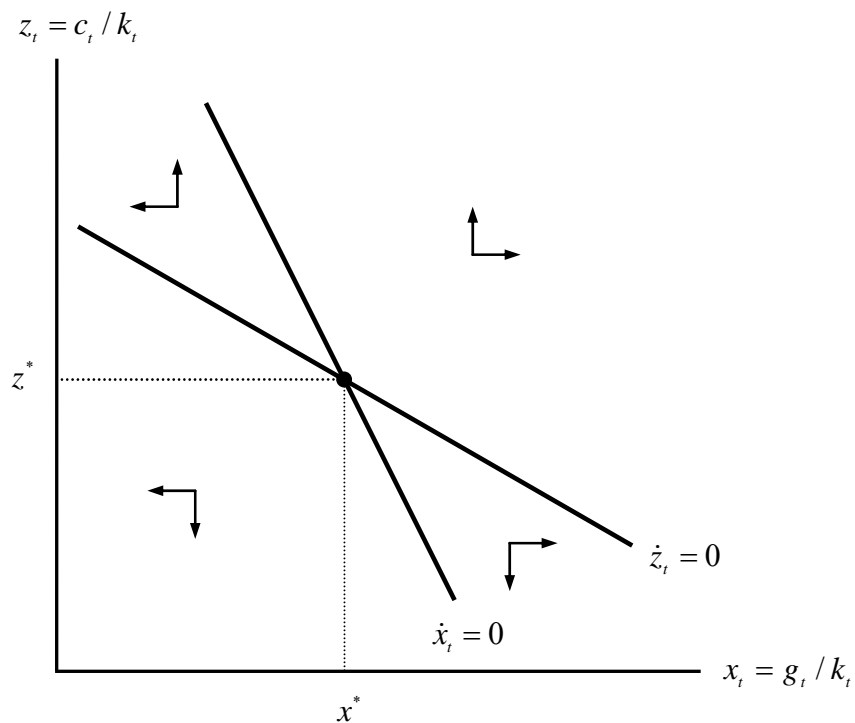
<sup>9</sup>Chen and Guo (2015) show that the results reported in this paper remain qualitatively unchanged when variable labor supply is incorporated into a one-sector endogenous growth model.

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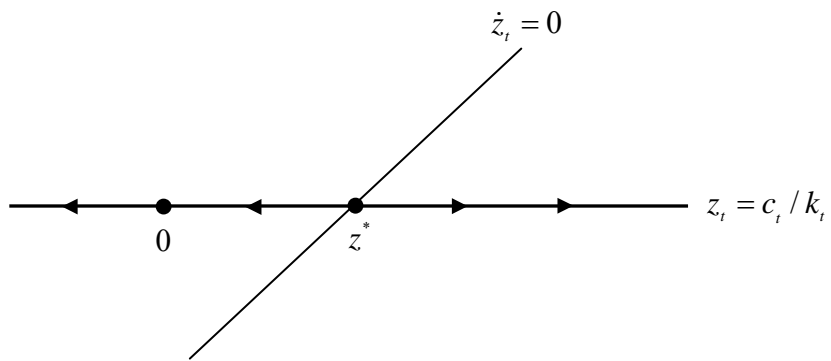
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**Figure 1. Progressive Taxation with  $0 < \phi < 1$ : Indeterminacy**



**Figure 2. Regressive Taxation with  $\underline{\phi} < \phi < 0$ : Determinacy**



**Figure 3. Flat Taxation with  $\phi = 0$ : Determinacy**