News about Aggregate Demand and the Business Cycle*

Jang-Ting Guo†
University of California, Riverside

Anca-Ioana Sirbu‡
Western Washington University

Mark Weder§
The University of Adelaide

August 11, 2014

Abstract

We examine the plausibility of expectations-driven cyclical fluctuations in an otherwise standard one-sector real business cycle model with variable capital utilization and mild increasing returns-to-scale in production. Due to a dominating wealth effect, our model is able to generate qualitatively as well as quantitatively realistic aggregate fluctuations driven by news impulses to future consumption demand or government spending on goods and services. When the economy is subject to anticipated total factor productivity or investment-specific technology shocks, the relative strength of the intertemporal substitution effect needs to be enhanced for our model to exhibit positive macroeconomic co-movement and business cycle statistics that are consistent with the data.

Keywords: News Shocks; Aggregate Demand; Business Cycles.

JEL Classification: E32.

*We would like to thank an anonymous referee, Sergio Rebelo, Juin-Jen Chang, Yongsung Chang, Hung-Ju Chen, Shu-Hua Chen, Kim-Sau Chung, Alexandre Dmitriev, Chris Edmond, Nir Jaimovich, Andrew John, Ian King, Robert King, Charles Leung, Yiting Li, Teresa Lloyd-Braga, Chia-Hui Lu, Alexander Ludwig, Leonor Modesto, James Morley, Deokwoo Nam, Oscar Pavlov, Vincenzo Quadrini, Jyh-Yuan Shieh, Richard Suen, Yi-Chan Tsai, Hung-Jen Wang, Yin-Chi Wang, Yong Wang, Jacob Wong, Chong Kee Yip, and seminar participants at City University of Hong Kong, National Taiwan University, National Tsing Hua University (Taiwan), National Chengchi University (Taiwan), Deakin University, Soochow University (Taiwan), Melbourne Business School, Victoria University of Wellington, Universidade Católica Portuguesa, University of Tokyo, Chinese University of Hong Kong, National Taipei University (Taiwan), Workshop on Macroeconomic Dynamics, Midwest Macroeconomics Meeting, Annual Meetings of the Canadian Economic Association, North American Summer Meetings of the Econometric Society, Shanghai Macroeconomics Workshop, Annual Conference of the Society for Computational Economics, and Annual Meeting of the Canadian Macroeconomics Study Group for helpful discussions and comments. Part of this research was conducted while Guo was a visiting professor of economics at University of Adelaide, whose hospitality is greatly appreciated. Weder acknowledges generous financial support from the Australian Research Council. Of course, all remaining errors are our own.

†Corresponding Author. Department of Economics, 3133 Sproul Hall, University of California, Riverside, CA 92521, USA, 1-951-827-1588, Fax: 1-951-827-5685, E-mail: guojt@ucr.edu.

‡Department of Economics, Western Washington University, Bellingham, WA 98225, USA, 1-360-650-2134, Fax: 1-360-650-6315, E-mail: Anca-Ioana.Sirbu@wwu.edu.

§School of Economics, The University of Adelaide, 10 Pulteney Street, Adelaide SA 5005, Australia, 61-8-8303-4664, Fax: 61-8-8223-1460, E-mail: mark.weder@adelaide.edu.au.
1 Introduction

Since the work of Beaudry and Portier (2004, 2007), it is now well known that under the assumptions of perfectly competitive markets and constant returns-to-scale in production, a standard one-sector real business cycle (RBC) model is unable to exhibit qualitatively realistic expectations-driven cyclical fluctuations, i.e. simultaneous expansions of output, consumption, investment and hours worked in response to good news about future technological progress. Due to the dominating intertemporal income effect, forward-looking agents will raise their current consumption and leisure, which in turn lead to decreases in today’s output and investment. As a result, a news-driven prototypical one-sector RBC model fails to generate the positive co-movement among key macroeconomic aggregates observed in the data. Subsequent research resolves this “co-movement puzzle” by incorporating combinations among some of the following features into a RBC-type economy: a convex production possibility frontier, multiple production sectors, non-separable preferences, investment adjustment costs, knowledge capital, imperfect competition, countercyclical markups, sticky prices, and costly technology adoption, among others.\textsuperscript{1}

Parallel to the early development of the original real business cycle literature, almost all the existing studies have focused on news shocks to imminent productivity improvements (a supply disturbance). In this paper, our primary attention is turned to examine the theoretical as well as quantitative plausibility of expectations-driven business cycles (EDBC) within a one-sector RBC model subject to aggregate demand impulses.\textsuperscript{2} Specifically in our benchmark formulation, shocks to the marginal utility of consumption à la Baxter and King (1991) that may affect the household’s urge to consume are considered. As a result, this preference disturbance creates a wedge between the marginal rate of substitution between consumption and leisure versus the marginal product of labor.\textsuperscript{3} The main objective of this paper is striving for \textit{parsimonious departures} from a canonical one-sector RBC formulation, driven by expectational


\textsuperscript{2}See, for example, Beaudry and Lucke (2009) and Schmitt-Grohé and Uribe (2012) for empirical support that anticipated demand shocks play non-negligible roles in accounting for the U.S. business cycle. On the theoretical front, see Ramey (2011, section IV.B) for an analysis of expectational disturbances to government spending; and Beaudry and Portier (2007, section 4.4), Mertens and Ravn (2011) and Sirbu (2014) for studies on anticipated tax policy shocks.

\textsuperscript{3}The ratio between the marginal rate of substitution of consumption for leisure and the marginal product of labor is dubbed as the “labor wedge” in the literature. See Shimer (2009) for a recent review on the labor wedge.
shocks to future consumption demand, that is able to account for, not only qualitatively but also quantitatively, the postwar U.S. business cycle. In particular, variable capital utilization and positive productive externalities are incorporated into our analytical framework.

Our theoretic analysis shows that the necessary condition for consumption and investment to move in the same direction states that the equilibrium wage-hours locus is positively sloped and steeper than the labor supply curve. In a calibrated version of the model economy, the degree of aggregate returns-to-scale in production needed to satisfy the requisite condition for positive macroeconomic co-movement is found to be mild and empirically plausible vis-à-vis recent empirical findings of Laitner and Stolyarov (2004). Furthermore, in response to the favorable news about changes in future aggregate demand, a macroeconomic boom will occur in the economy as output, consumption, investment and labor hours all rise during the announcement period. Intuitively, an optimistic expectational impulse causes a leftward shift of the labor supply curve, which will raise the anticipated future real wage and hours worked. This in turn leads to an increase in current consumption, and in other key aggregates as well, because the household’s higher expected permanent income yields a dominating intertemporal wealth effect. We also generate simulated second moments from the benchmark specification, and compare them with the Hodrick-Prescott (H-P) filtered U.S. time series data. It turns out that our baseline model performs quite well at matching the main empirical regularities, i.e. the relative standard deviations to output and contemporaneous covariances, of U.S. cyclical fluctuations after 1954. In terms of sensitivity analysis, we find that through the above-mentioned mechanism, the results from our benchmark model continue to hold under alternative demand disturbances commonly studied in the real business cycle literature, specifically news impulses to multiplicative consumption shocks or government spending shocks.

To obtain further insights about our proposed mechanism for generating expectations-driven business cycles, anticipated innovations to total factor productivity or investment-specific technology shocks are also examined. Under the same parameterizations as those in the benchmark formulation, the model economy slides into a macroeconomic recession during the current period. In either environment, a positive news shifts the upward sloping equilibrium wage-hours locus to the left, which will then yield exactly the opposite outcome to that within demand-driven configurations since the expected future real wage and labor hours are now lower. In order to overturn this counterfactual result, the relative strength of the intertemporal substitution effect originating from agents’ expectations about an upcoming productivity improvement needs to be enhanced through the combinations of raising the
household’s intertemporal elasticity of substitution in consumption and incorporating convex adjustment costs into the law of motion for capital accumulation. Using empirically realistic calibrations associated with these two features, we show that qualitatively as well as quantitatively realistic EDBC may occur in our model economy subject to anticipated aggregate or investment-specific technology shocks.

The remainder of this paper is organized as follows. Section 2 describes the model and analyzes its equilibrium conditions. Section 3 analytically and quantitatively investigates the plausibility of expectations-driven business cycles under three formulations of news impulses to future aggregate demand. Section 4 examines our proposed mechanism when the model economy is subject to anticipated innovations to aggregate or investment-specific technology shocks. Section 5 concludes.

2 The Economy

Our economy is populated by a unit measure of identical infinitely-lived households, each endowed with one unit of time. The representative household maximizes a discounted stream of expected utilities over its lifetime

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{(c_t - \Delta_t)^{1-\sigma} - 1}{1 - \sigma} - A \frac{h_t^{1+\gamma}}{1+\gamma} \right], \quad 0 < \beta < 1, \quad \sigma > 0, \quad \gamma \geq 0 \quad \text{and} \quad A > 0,$$

where $E$ is the conditional expectations operator, $\beta$ is the discount factor, $c_t$ is consumption, $h_t$ is hours worked, $\gamma$ is the inverse of the (Frisch) labor supply elasticity, and $\sigma$ governs the degree of risk aversion or the intertemporal elasticity of substitution (IES) in consumption. As in Baxter and King (1991), $\Delta_t$ is a random shock to preferences that affects the household’s marginal utility of consumption. For example, an increase in $\Delta_t$ represents a positive disturbance to the economy’s aggregate demand as it raises agents’ urge to consume. We postulate that the unconditional mean of $\Delta_t$ (or its steady-state level denoted as $\Delta_{ss}$) is zero\(^4\), and that its innovation $\chi_t$ is specified as

\(^4\)It follows that the steady-state intertemporal elasticity of substitution in consumption is equal to $\frac{\sigma}{\gamma}$. Moreover, our quantitative results, reported in sections 3.2 and 3.3, are robust to the values of $\Delta_{ss}$ as long as it is smaller than the consumption counterpart $c_{ss}$.

\(^5\)If $\Delta_t$ is restricted to take on only positive values, then it can be interpreted as the time-varying minimum or subsistence consumption requirement that is taken as exogenous by all households. See, for example, Álvarez-Peláez and Díaz (2005).
$$\chi_t = \underbrace{\varepsilon_t}_{\text{unanticipated}} + \underbrace{\nu_{t-\tau}}_{\text{news}},$$

where $\varepsilon_t$ is a contemporaneous unanticipated impulse; and $\nu_{t-\tau}$ represents an anticipated component which was announced or observed $\tau$ periods beforehand and influences the forward-looking household’s current utility, hence a news shock. Both random errors are normally distributed with zero means and variances $\sigma_{\varepsilon}^2$ and $\sigma_{\nu}^2$. It is further assumed that each series is uncorrelated over time, and that there is no correlation between them.

The representative agent also faces the following resource constraint:

$$c_t + k_{t+1} - \Delta_t k_t = y_t, \quad k_0 > 0 \text{ given},$$

where $k_t$ is physical capital, $x_t$ is gross investment, and $\delta_t \in (0, 1)$ represents the time-varying capital depreciation rate which takes on the functional form

$$\delta_t = \frac{1}{\phi} u_t^\phi, \quad \phi > 1,$$

where $u_t$ is the rate of capital utilization that is endogenously determined by the household. The specification of $\phi > 1$ in (4) means that more intensive capital utilization accelerates its rate of depreciation. When $\phi \to \infty$, our model collapses to a standard RBC formulation with constant depreciation and utilization rates.

Output $y_t$ is produced by the Cobb-Douglas production function

$$y_t = Y_t^{\frac{1}{1+\eta}} (u_t k_t)^\alpha h_t^{1-\alpha}, \quad \eta \geq 0, \quad 0 < \alpha < 1,$$

where $Y_t$ stands for the economy’s aggregate output that is taken as given by each individual agent, and $\eta$ denotes the degree of productive externalities. In a symmetric equilibrium where $y_t = Y_t$, the social technology is given by

$$y_t = (u_t k_t)^{\alpha(1+\eta)} h_t^{(1-\alpha)(1+\eta)}.$$  

Notice that when $\eta = (>) 0$, equation (6) exhibits aggregate constant (increasing) returns-to-scale in utilized capital $u_t k_t$ and labor hours $h_t$.

The first-order conditions for the household’s dynamic optimization problem are

$$A (c_t - \Delta_t)^\sigma h_t^{\gamma} = (1 - \alpha) \frac{3\eta}{h_t},$$
\[
\delta_t = \frac{\alpha}{\phi} \frac{y_t}{k_t},
\]

\[
\frac{1}{(c_t - \Delta_t)^\sigma} = \beta E_t \left\{ \frac{1}{(c_{t+1} - \Delta_{t+1})^{\sigma}} \left( 1 - \delta_{t+1} + \frac{y_{t+1}}{k_{t+1}} \right) \right\},
\]

\[
\lim_{t \to \infty} \beta^t \frac{k_{t+1}}{(c_t - \Delta_t)^\sigma} = 0,
\]

where (7) equates the slope of household’s indifference curve to the marginal product of labor, (8) equates the marginal gain (additional output) and marginal loss (higher depreciation) of a change in the rate of capital utilization \( u_t \), (9) is the standard Euler equation for intertemporal consumption choices, and (10) is the transversality condition. Next, substituting (4) and (8) into (6) yields the following reduced-form social technology as a function of capital and labor inputs:

\[
y_t = \frac{\alpha(1+\eta)}{\phi - \alpha(1+\eta)} k_t^{\frac{\phi - \alpha(1+\eta)}{\phi - \alpha(1+\eta)}} h_t^{\frac{\alpha(1+\eta)}{\phi - \alpha(1+\eta)}},
\]

where \( 0 < \frac{\alpha(1+\eta)(\phi-1)}{\phi - \alpha(1+\eta)} < 1 \), i.e. diminishing marginal product of capital, in order to guarantee the existence of an interior steady state.\(^6\)

### 3 Expectations-Driven Business Cycles

This section examines whether the above one-sector RBC model is able to generate, not only qualitatively but also quantitatively, realistic cyclical fluctuations driven by news shocks to future aggregate demand. We first analytically derive the condition(s) under which the economy exhibits positive co-movement between consumption and investment. Under the assumption that this requisite condition is satisfied, we then undertake a quantitative investigation of the model’s dynamic responses and business cycle statistics within a calibrated version of our economy.

#### 3.1 Analytical Result

In our model economy, resolving the aforementioned “co-movement puzzle” amounts to looking for the condition(s) under which consumption \( c_t \), investment \( x_t \), and thus output \( y_t \) all move in the same direction. Hours worked \( h_t \) will co-move as well because capital is a pre-determined variable and there is no change in the current-period economic fundamentals. Per Beaudry and Portier’s (2004, Appendix A; 2007) temporary equilibrium approach, we use the

\(^6\)Since \( 0 < \alpha < 1 \), \( \eta \geq 0 \) and \( \phi > 1 \), the parametric restriction of \( 0 < \frac{\alpha(1+\eta)(\phi-1)}{\phi - \alpha(1+\eta)} < 1 \) implies that \( \phi - \alpha(1+\eta) > 0 \).
totally-differentiated version of equations (3) and (7), together with the aggregate production technology (11), to obtain the analytical expression of \( \frac{dc_t}{dx_t} \) as follows:

\[
\frac{dc_t}{dx_t} = \left\{ \frac{\sigma y_t}{\phi(1-\alpha)(1+\eta)} \frac{\phi(1-\alpha)(1+\eta)}{\phi-\alpha(1+\eta)} - 1 \right\}^{-1},
\]

which governs the sign of correlation between consumption and investment. Given \( \sigma > 0, \) \( 0 < \alpha < 1, \phi > 1, \eta \geq 0, \phi - \alpha(1 + \eta) > 0 \) (see footnote 6), and since \( \frac{1}{\phi-\alpha} > 0 \) represents the period-t marginal utility of consumption, \( \frac{dc_t}{dx_t} > 0 \) requires that

\[
\frac{\phi(1-\alpha)(1+\eta)}{\phi-\alpha(1+\eta)} - 1 - \gamma > 0 \quad \text{(13)}
\]

Hence, consumption and investment will move in the same direction only if

\[
\frac{\phi(1-\alpha)(1+\eta)}{\phi-\alpha(1+\eta)} - 1 > \gamma, \quad \text{(14)}
\]

which is independent of \( \sigma \) that governs the household’s intertemporal elasticity of substitution in consumption.\( ^{8} \)

To understand the above condition, we note that under the assumption of perfect competition in the labor market, agents’ intratemporal employment decision is governed by

\[
(1 - \alpha) \frac{y_t}{h_t} = w_t = A (c_t - \Delta_t)^{\gamma}, \quad \text{(15)}
\]

where \( w_t \) is the real wage rate. Notice that by plugging the social technology (11) into the logarithmic version of labor demand, we find that the slope of the equilibrium wage-hours locus is equal to \( \frac{\phi(1-\alpha)(1+\eta)}{\phi-\alpha(1+\eta)} - 1 \). In addition, taking logarithms on the second equality of (15) indicates that the slope of the household’s labor supply curve is \( \gamma \) (\( \geq 0 \)), and its position or intercept is affected by the level of “net consumption” \( (c_t - \Delta_t) \). It follows that the necessary condition for the economy to display positive co-movement between key macroeconomic

\footnote{The inequality in (14) is not a “if and only if” condition for \( \frac{dc_t}{dx_t} > 0 \) because a negative preference shock and/or \( \sigma < 1 \) could lead to \( \frac{\sigma}{\frac{y_t}{h_t}} > 1 \). However, when \( \Delta_t \) is restricted to be the positive subsistence level of consumption (see footnote 5) together with \( \sigma \geq 1, \frac{\sigma}{\frac{y_t}{h_t}} \) on the right-hand side of (13) must be smaller than one in that \( \sigma y_t \geq y_t > c_t > c_t - \Delta_t > 0 \). On the other hand, if (14) holds, then the left-hand side of (13) is larger than one. It follows that condition (14) is not only necessary, but also sufficient, for macroeconomic co-movement provided \( \Delta_t > 0 \) and \( \sigma \geq 1 \).}

\footnote{It is straightforward to show that under constant depreciation and utilization rates of capital (\( \phi \to \infty \)), the requisite condition for consumption and investment to co-move in our model economy becomes \( (1 - \alpha)(1 + \eta) - 1 > \gamma \).}
aggregates, as in (14), states that the equilibrium wage-hours locus is upward sloping and steeper than the labor supply curve. Wen (1998, p. 16) shows that (14) is also a necessary condition for our model with variable capital utilization to exhibit a continuum of stationary perfect-foresight equilibria.\(^9\) Therefore, as pointed out by Eusepi (2009), the requisite conditions for positive macroeconomic co-movement and equilibrium indeterminacy to occur within a one-sector RBC framework are tightly connected.

3.2 Dynamic Responses

Based on the preceding analytical result, this subsection quantitatively examines a calibrated version of our model in response to agents’ optimistic expectation about an upcoming change in consumption demand, while maintaining saddle-path stability and equilibrium uniqueness. As in Beaudry and Portier (2004), the stochastic process for exogenous preference disturbances fed into our numerical experiments is postulated as follows: the economy starts at its steady state in period zero. At period 1, households receive a signal that the taste shifter will permanently increase to 0.01 from period \(\tau = 4\) (denoted as \(\Delta_4\)) onwards, and this good news turns out to be materialized in period 4.

In the benchmark specification, we adopt the following quarterly parameterization that is commonly used in the real business cycle literature: \(\sigma = 1\) (i.e. the household utility is logarithmic in consumption), \(\alpha = 0.3\), \(\beta = 0.99\), \(\gamma = 0\) (i.e. perfectly elastic or indivisible labor supply \(\text{à la} \) Hansen [1985] and Rogerson [1988]), and the steady-state capital depreciation rate \(\delta_{ss} = 0.025\). The selected values of \(\beta\) and \(\delta_{ss}\) imply that \(\phi = 1.404\). In addition, the preference parameter \(A\) (= 2.6706) in (1) is chosen such that the household spends one third of its time endowment on working at the steady state. Given the baseline calibration of \(\alpha\), \(\gamma\) and \(\phi\), the threshold level of productive externalities that satisfies the necessary condition for positive co-movement between consumption and investment, as in (14), is \(\eta_{\text{min}} = 0.0945\).

Figure 1 presents the impulse response functions of our model economy to the above one-time positive innovation to future consumption expenditures under \(\eta = 0.1\) for the purpose of clear illustration. Notice that the resulting level of aggregate returns-to-scale in production \((= 1 + \eta)\) is empirically plausible \(\text{vis-à-vis}\) recent empirical findings of Laitner and Stolyarov (2004) who have reported a preferred range of 1.09 – 1.11 for the U.S. economy. As can been seen from Figure 1, an optimistic expectational shock yields a macroeconomic boom with

\(^9\)In an extended version of Wen’s (1998) indeterminate one-sector RBC model, Benhabib and Wen (2004) examine the quantitative business cycle driven by unanticipated disturbances to consumption demand and government spending (thus no news impulses), and sunspot shocks to agents’ animal spirits.
simultaneous expansions of output, consumption, investment and hours worked in period 1 after the announcement of good news is made. That is, our one-sector RBC model with mild increasing returns is able to generate qualitatively realistic business cycles driven solely by agents’ changing expectations about future consumption demand.

In order to understand the economic intuitions behind this result, it is useful to consider what will be the outcome that forward-looking agents, from their perspective at period 1, expect to occur in the period-4 labor market with a positively sloped equilibrium wage-hours locus which intersects the labor supply curve from below as depicted in Figure 2. Upon receiving the positive signal about future aggregate demand, the representative household anticipates that a higher $\Delta_4$ leads to an increase in consumption $c_4$. Due to the presence of sufficiently strong productive externalities ($\eta = 0.1$), the household’s “net consumption” $(c_4 - \Delta_4)$ will rise, thus a leftward shift of the labor supply curve ensues. Figure 2 shows that the resulting excess demand for labor moves the equilibrium from $E$ to $E'$, raising the expected real wage $w_4$ and hours worked $h_4$, which in turn increases the expected marginal product of capital $MPK_4$. It follows that how agents’ period-1 economic decisions react to these future changes depends on the relative strength of two opposing forces. On the one hand, the anticipation of a higher lifetime (labor) income results in an increase of consumption in $t = 1$ through a positive wealth effect. On the other hand, a higher expected rate of return on investment (i.e. $MPK_4$) induces households to reduce their consumption and invest more today through an intertemporal substitution effect. Our numerical simulations show that the income effect turns out to be stronger, hence current consumption $c_1$ rises in response to today’s good news. Since $\frac{dc_2}{dx_1} > 0$ under our parameterization where condition (14) is satisfied, investment together with output and labor hours will be higher as well at the announcement period $t = 1$.

3.3 Simulation Results

So far, we have shown that a slightly modified one-sector RBC model is able to generate qualitatively realistic co-movement of macroeconomic aggregates in response to an expectational impulse to future consumption demand. This subsection examines the corresponding statistical business cycle properties in comparison with those obtained from the H-P filtered cyclical

---

\(^{10}\)Although not shown here due to space limitation, our model also generates an aggregate period-1 expansion in response to a positive contemporaneous and unanticipated impulse to $\Delta$. In addition, this result continues to hold for contemporaneous shocks to other demand disturbances that we consider in section 3.4. These impulse response functions are available upon request.
components of the logarithmic U.S. quarterly time series for the period 1954:1 – 2009:2. We first derive the model’s unique interior steady state (a saddle point), and then take log-linear approximations to the equilibrium conditions in its neighborhood.\(^{11}\) In our numerical simulations, the calibrated values of \(\sigma, \alpha, \beta, \gamma, \delta_{ss}, \phi, \eta (= 0.1)\) and \(A\) remain unchanged as those in section 3.2.

With regard to identifying or measuring the stochastic process for the preference shock, we follow Baxter and King (1991) and obtain the time series of \(\Delta_t\) from the log-linearized version of the labor-supply portion in equation (15)\(^{12}\)

\[
\frac{\Delta_t}{c_{ss}} = \frac{1}{\sigma} \log A + \log c_t - \frac{1}{\sigma} \log w_t + \frac{\gamma}{\sigma} \log h_t, \quad (16)
\]

where \(c_{ss} (= 0.1977)\) denotes the model’s steady-state level of consumption. The resulting taste disturbance is found to be well described by the following first-order autoregressive regression with a linear time trend:

\[
\Delta_t = 1.5370 + 0.9685 \Delta_{t-1} + 0.00039 t + \chi_t, \quad (17)
\]

where numbers in parentheses are standard errors of the estimated parameters, and the standard deviation of innovations \(\sigma_\chi\) is equal to 0.001726. Notice that \(\Delta_t\) is highly persistent, with an autoregressive coefficient of \(\rho = 0.9685.\(^{13}\) In addition, the correlation coefficient between the H-P filtered cyclical components of U.S. output series and our measured preference shocks is 0.5221.

Since there is no direct evidence on the respective variabilities of the unanticipated and news components for the innovations to taste shocks (i.e. \(\sigma_\varepsilon\) and \(\sigma_v\) in equation 2), we use the Simulated Method of Moments to calibrate these parameters, as in Beaudry and Portier (2004) and Karnizova (2010). In particular, \(\sigma_\varepsilon\) is selected to minimize the squared error between output volatility of the data \(\sigma_y (= 1.6029\%)\) and that of model-generated time series

\(^{11}\)Since \(\Delta_{ss} = 0\), the proportionate deviations of the taste shock are computed relative the steady-state level of consumption \(c_{ss}\). The same procedure is implemented later in equation (16), and footnotes 13 and 19.

\(^{12}\)See the Appendix for detailed information on the U.S. time series data used in our quantitative analysis.

\(^{13}\)We obtain very similar point estimates of \(\rho\) and \(\sigma_\chi\) when the real wage is replaced with the marginal product of labor, i.e. \((1 - \alpha) \frac{y_t}{h_t}\), where \(\alpha = 0.3\), in the computation of \(\Delta_t\). In this case, equation (16) is changed to

\[
\frac{\Delta_t}{c_{ss}} = \frac{1}{\sigma} \log A + \frac{1}{\sigma} \log \left( 1 - \alpha \right) + \log c_t - \frac{1}{\sigma} \log y_t + \frac{(1 + \gamma)}{\sigma} \log h_t.
\]
averaged across simulations. Given the baseline parameterization described above, our model is simulated \( N = 1,000 \) times of length 220 periods. As a result,

\[
\sigma_\varepsilon = \arg \min \left( \sigma_y - \frac{1}{N} \sum_{i=1}^{N} \sigma_{y,i} \right)^2,
\]

where \( \sigma_{y,i} \) represents the standard deviation of output from the \( i \)-th simulation. Using equation (2), the volatility of the anticipated component for the random error to consumption demand can then be obtained by \( \sigma_\nu = \sqrt{\sigma_\chi^2 - \sigma_\varepsilon^2} \), where \( \sigma_\chi = 0.001726 \). This computational procedure yields that news impulses account for all the variance of preference innovations within the benchmark specification, \( i.e. \; \chi_t = \nu_{t-4} \).

Table 1 presents a set of H-P filtered second moments from our model economy driven by news shocks to future consumption demand, and compares them with the postwar U.S. data. The statistics reported in column 3 are sample means from the numerical simulations, which show that our baseline configuration does a reasonably good job in quantitatively mimicking the ranking of cyclical volatilities in investment, GDP, hours worked and consumption, as well as their contemporaneous correlations with output. Moreover, the benchmark model underpredicts the relative standard deviation of consumption and its correlation with GDP, and exaggerates the cross-correlation between output and labor hours.

### 3.4 Alternative Demand Shocks

This subsection shows that the above results from our benchmark specification will continue to hold under alternative demand disturbances commonly studied in business cycle models, specifically news impulses to multiplicative consumption shocks or government spending shocks. In this case, the representative household’s dynamic optimization problem is to maximize

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left[ \Lambda_t \frac{\epsilon_t^{1-\sigma} - 1}{1 - \sigma} - A \frac{k_t^{1+\gamma}}{1 + \gamma} \right], \quad 0 < \beta < 1, \quad \sigma > 0, \quad \gamma \geq 0 \quad \text{and} \quad A > 0,
\]

subject to the following aggregate resource constraint:

\[
c_t + k_{t+1} - (1 - \delta_t)k_t + g_t = y_t, \quad k_0 > 0 \quad \text{given},
\]

where \( y_t \) is total output as in (6), \( \Lambda_t \) is a multiplicative preference shock that affects agents’ marginal utility of consumption \( \ddot{a} \text{ la Bencivenga} (1992) \), and \( g_t \) represents public expenditures on goods and services that are financed by lump-sum taxes. Both disturbances (in logs) are
specified as stationary first-order autoregressive processes whose innovations are decomposed into their respective unanticipated and news components according to (2). We also postulate that the unconditional mean of $\Lambda_t$ is one, and that the steady-state ratio of government purchases to output is 0.2.

It is straightforward to derive that the requisite condition for consumption and gross investment to move in the same direction $\left(\frac{dc_t}{dx_t} > 0\right)$ within the aforementioned model is again given by (14). Under the same calibrated values of $\sigma$, $\alpha$, $\beta$, $\gamma$, $\delta_{ss}$, $\phi$ and $\eta (= 0.1)$ as those in our baseline parameterization, Figure 3 illustrates that GDP, consumption, investment and hours worked all rise in $t = 1$ when agents receive the positive signal of a one-percent permanent increase in the taste shifter or public spending to be realized at period 4. In either formulation, anticipated higher future demand causes the household’s period-4 labor supply curve to shift leftward as depicted in Figure 2, which in turn yields procyclical responses of key macroeconomic aggregates in the current period because of a dominating intertemporal income effect.

In terms of numerical simulations, we follow Bencivenga (1992) and set the persistence parameter for $\Lambda_t$ to be 0.144, and the standard deviation of its innovations to be 0.0744. Based on Christiano and Eichenbaum (1992), the autoregressive coefficient for $g_t$ is chosen to be 0.96, and the standard deviation of its innovations is set equal to 0.021. We then use the Simulated Method of Moments approach, as in (18), to calibrate $\sigma_x$ that produces the best fit between the observed and simulated output volatilities for each alternative demand disturbance. The last two columns of Table 1 report the simulation results. It turns out that our model driven by news about multiplicative consumption shocks exhibits very similar quantitative business-cycle properties to those in the benchmark specification. This implies that matching contemporaneous covariances in the data is not affected by the formulation of innovations to agents’ consumption expenditures under consideration. In addition, other than underpredicting the volatility of consumption series (absorbed by variations in public expenditures) and its contemporaneous correlation with output, our model subject to anticipated government spending shocks performs no worse than other demand-driven configurations in quantitatively mimicking the observed pattern of postwar U.S. business cycles.

\[\text{14} \text{The preference parameter } A \text{ is adjusted to maintain the steady-state labor hours as one third within each of the four variants considered in this section.} \]

\[\text{15} \text{These simulation results are quantitatively robust to the processes of } \Lambda_t \text{ and } g_t \text{ estimated from the U.S. time series data summarized in the Appendix.} \]
4 Total Factor Productivity and Investment-Specific Technology Shocks

To obtain further insights about our proposed mechanism for generating expectations-driven cyclical fluctuations, this section examines anticipated innovations to total factor productivity and investment-specific technology shocks. We first find that the model economy undergoes a counterfactual macroeconomic recession during the announcement period. However, after enhancing the relative strength of the intertemporal substitution effect associated with agents’ expectations about future productivity improvement, our model driven by news to aggregate or investment-specific technology shocks is able to generate qualitatively as well as quantitatively realistic business cycles.

In this environment, the representative household’s dynamic optimization problem is to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{c_t^{1-\sigma} - 1}{1 - \sigma} - A \frac{h_t^{1+\gamma}}{1 + \gamma} \right], \quad 0 < \beta < 1, \quad \sigma > 0, \quad \gamma \geq 0 \quad \text{and} \quad A > 0, \quad (21)$$

subject to the following aggregate resource constraint:

$$c_t + \frac{x_t}{z_t} = y_t = \theta_t (u_t k_t)^{\alpha (1+\eta)} h_t^{(1-\alpha)(1+\eta)}, \quad (22)$$

where $z_t$ is an investment-specific technological shock, and $\theta_t$ represents an impulse to the total factor productivity (TFP). Both disturbances are postulated to have unitary means, and follow (in logs) a stationary first-order autoregressive process whose innovations are decomposed into unanticipated and news components according to (2).

Once again, it can be shown that $c_t$ and $x_t$ will co-move within the aforementioned model only if the inequality in (14) is satisfied. Under the same parameterizations of $\sigma$ ($= 1$), $\alpha$, $\beta$, $\gamma$, $\delta_{ss}$, $\phi$ and $\eta$ as those in section 3, Figures 4a and 4b plot the impulse response functions of our model economy subject to a one-percent permanent increase in the aggregate or investment-specific technological improvement beginning at period 4.\textsuperscript{16} Although consumption and investment (as well as output and hours worked) are moving in the same direction as condition (14) would necessarily predict, a positive expectational shock to either formulation of technical progress generates a counterfactual recession in $t = 1$ whereby all key macroeconomic aggregates fall below their respective steady-state levels.

\textsuperscript{16}Figures 4a and 4b are separately presented because many points along these impulse response functions are quantitatively indistinguishable.
Intuitively, when households (standing at period 1) receive the good news about an increase in future TFP, they anticipate that a higher $\theta_4$ will shift the positively sloped equilibrium wage-hours locus leftward as depicted in Figure 5. The resulting excess supply of labor moves the market equilibrium from $E$ to $E'$, lowering the expected real wage $w_4$ and hours worked $h_4$, which in turn reduces the expected marginal product of capital $MPK_4$. It follows that the expectation of a lower permanent income results in a decrease of consumption in $t = 1$ through a negative wealth effect; whereas a reduction in the expected rate of return on investment induces agents to raise their current consumption through an intertemporal substitution effect. Figure 4a shows that the household’s period-1 consumption and investment expenditures are moving in the same, but not “correct”, direction because of a dominating wealth effect, i.e. EDBC does not emerge within this setting in spite of $\frac{d\bar{w}}{dx_t} > 0$.\(^{17}\)

Next, we shut down the TFP movements ($\theta_t = 1$, for all $t$) and substitute the first-order condition for the rate of capital utilization $u_t$, given by $\alpha \frac{w_t}{y_t} = \frac{u_t^{\phi-1}k_t}{z_t}$, into (6) to obtain the following reduced-form social technology as a function of investment-specific technical change $z_t$ together with capital and labor inputs:

$$y_t = \alpha \frac{\alpha(1+\eta)}{\phi-\alpha(1+\eta)} z_t^{\frac{\phi-\alpha(1+\eta)}{\phi-\alpha(1+\eta)}} k_t^{\frac{\phi(1+\eta)(1+\eta)}{\phi-\alpha(1+\eta)}} h_t^{\frac{\phi(1+\eta)(1+\eta)}{\phi-\alpha(1+\eta)}}.$$

(23)

As a result, agents’ anticipation of a higher $z_4$ will increase the marginal productivity of labor in $t = 4$. This shifts out the equilibrium wage-hours locus, which lowers the expected real wage $w_4$ and labor hours $h_4$ as shown in Figure 5. On the other hand, an improvement in the investment-specific technology produces two additional opposing forces at period 4: the substitution effect causes the household to invest more and consume less; and the income effect raises both consumption and investment. Since the substitution effect dominates, the resulting fall in the expected future consumption $c_4$ generates a downward shift of the period-4 labor supply curve, reinforcing the initial decreases of $w_4$ and $h_4$. Per the same intuition described above, Figure 4b shows that the favorable news of an upcoming investment-specific technological progress yields a macroeconomic recession during the announcement period.

In this case, the key for successfully generating qualitatively realistic expectations-driven business cycles in $t = 1$ is to boost the relative strength of the intertemporal substitution effect associated with news about future productivity improvement in the aggregate or investment-specific technology. This can be achieved through raising the household’s intertemporal elas-

\(^{17}\)See Guo, Sirbu and Suen (2012) for the same finding in a one-sector RBC model with fixed capital utilization and positive productive externalities coming from aggregate capital and labor inputs.
ticity of substitution in consumption \((= \frac{1}{2})\); and/or incorporating convex adjustment costs into the law of motion for capital accumulation given by

\[
k_{t+1} = (1 - \delta_t)k_t + x_t \left[ 1 - \psi \left( \frac{x_t}{x_{t-1}} - 1 \right)^2 \right], \quad \psi > 0 \quad \text{and} \quad x_{t-1} > 0 \quad \text{given}, \quad (24)
\]

where the elasticity parameter \(\psi\) governs the degree (or size) of adjustment costs for capital investment.

For the preference parameter \(\sigma\), most previous studies have adopted the range of one to three in their quantitative analyses. However, recent empirical research suggests that \(\sigma < 1\) thus the elasticity of intertemporal consumption substitution is higher than one. For example, Vissing-Jørgensen and Attanasio (2003) report the point estimates of IES to be 1.03 (with six instruments) and 1.44 (under one instrumental variable) for the group of all stock holders. Gruber (2006) finds that the IES is around 2 when endogenous tax rate variations are included in his cross-sectional estimation on U.S. total non-durable consumption expenditures, and that this result is in line with Mulligan’s (2002) earlier estimates based on time series data of total returns to capital. On the other hand, existing empirical estimates of the adjustment cost parameter \(\psi\) range in a wide spectrum: Christiano, Eichenbaum and Evans (2005) present the point estimate of 2.48 for their benchmark specification of a monetary model with nominal rigidities; Christiano, Ilut, Motto and Rostagno (2008) consider an estimated value of 15.1; and the point estimates reported in Schmitt-Gorhé and Uribe (2012) are 9.11 from Bayesian estimation, and 25.07 per the classical maximum likelihood method.

Drawing on these estimation results, it can be shown that raising either the intertemporal elasticity of substitution in consumption or the size of investment adjustment costs alone, while maintaining the respective empirical plausibilities of \(\sigma\) and \(\psi\), is unable to generate an aggregate expansion in \(t = 1\) since the expectations-driven wealth effect continues to dominate. We then choose \(\sigma = 0.5\), which corresponds to the highest possible value of IES that is regarded as realistic, and find that the minimum levels of \(\psi\) at which EDBC occurs at period 1 are 1.7 for the TFP and 4.2 for the investment-specific shocks.\(^{18}\) Notice that both values of \(\psi\) are at the lower end of its empirically plausible range. Figure 6 shows that due to a stronger intertemporal substitution effect within these parameterizations, the current-period output, consumption, investment and labor hours all rise after agents learn about the news of future

\(^{18}\)Alternatively, EDBC takes place when the adjustment-cost parameter \(\psi\) is set to be 1.3 \(\text{à la}~\text{Jaimovich and Rebelo} (2009)\), together with \(\sigma = 0.45\) for the TFP shock; or \(\sigma = 0.21\) for the investment-specific shock. The associated impulse response functions are available upon request.
technological progress.

With regard to numerical simulations, we set the persistence parameter for $\theta_t$ to be 0.979 and the standard deviation of its innovations to be 0.0072, as in King and Rebelo (1999). Based on Justiniano and Primiceri’s (2008) work, the autoregressive coefficient for $z_t$ is chosen to be 0.87, and the standard deviation of its innovations is set equal to 5.46. Next, we implement the SMM computational procedure described in section 3.3 to obtain $\sigma_x$ and $\sigma_v$ for each disturbance. Table 2 presents our simulation results. As in the three demand-driven variants discussed earlier, our model subject to anticipated aggregate or investment-specific technology shocks is able to generate quantitatively realistic business cycles in that they perform well at matching the relative variances and contemporaneous covariances observed in the postwar U.S. data.

Our final sensitivity assessment re-visits the aforementioned specification that allows for a non-unitary elasticity of intertemporal consumption substitution and investment adjustment costs, but driven by the three news shocks to future aggregate demand analyzed in section 3. In contrast to (14), we can no longer analytically derive the requisite condition under which the modified model exhibits EDBC for all feasible $\sigma$ and $\psi$. As a result, numerical examples are employed to quantitatively explore the macroeconomy’s co-movement properties. Using the same calibrations of $\alpha$, $\beta$, $\gamma$, $\delta_s$, $\phi$ and $\eta$ specified earlier, Table 3 presents the combinations of some empirically plausible values of $\sigma \geq 0.5$ together with their corresponding ranges of $\psi \geq 0$ that are able to produce qualitatively realistic expectations-driven cyclical fluctuations.\footnote{The results with the benchmark and multiplicative consumption shocks, as in columns 2 and 3 of Table 3, turn out to be identical. This can be understood from the following log-linearized versions (expressed in hat variables) for their respective marginal utility of consumption $\mu_t$: \[-\sigma \hat{c}_t + \sigma \frac{\Delta \hat{c}_{ss}}{c_{ss}} = \hat{\mu}_t,\] and \[-\sigma \hat{c}_t + \hat{\lambda}_t = \hat{\mu}_t.\] Notice that the only difference resides in the constant term in front of each disturbance. Since the magnitude of the driving uncertainties does not affect the economy’s qualitative business cycle properties, EDBC will take place under the same parameter constellations.}

Due to the existence of mild increasing returns-to-scale in production, a favorable signal about future aggregate demand induces the representative household to increase output in the current period through a positive wealth effect. On the other hand, a higher IES and/or the presence of investment adjustment costs will strengthen the intertemporal substitution effect when forward-looking agents invest more today. As discussed in sections 3.2 and 3.4, each parametric
configuration reported in Table 3 yields a stronger income effect across periods, which in turn leads to EDBC within these demand-driven business cycle models.

5 Conclusion

It is now well known that a standard one-sector real business cycle model fails to exhibit news-driven business cycles. This conundrum boils down to its inability to produce positive co-movement between output, consumption, investment and labor hours in response to agents’ changing expectations about future economic fundamentals. In this paper, we show that an otherwise prototypical one-sector real business cycle model, paired with variable capital utilization and mild increasing returns-to-scale in production, can successfully generate qualitatively as well as quantitatively realistic cyclical fluctuations driven by news shocks to future consumption expenditures or government spending on goods and services because of a dominating wealth effect. However, the known conundrum remains when our model economy is subject to expectational disturbances to total factor productivity or investment-specific technology shocks as a counterfactual macroeconomic recession takes place during the announcement period. In order for this environment to deliver aggregate co-movement and business cycle statistics that are consistent with the data, we find that the relative strength of the intertemporal substitution effect resulting from agents’ optimistic anticipation about future technological progress needs to be enhanced through raising the household’s intertemporal elasticity of substitution in consumption together with incorporating convex adjustment costs into the law of motion for capital accumulation.
6 Appendix

This appendix provides detailed information about the U.S. quarterly time series data used in our quantitative analysis. The time period covered is 1954:1 – 2009:2.

**Output:** Gross domestic product; NIPA Table 1.1.5 (line 1), in current dollars.

**Consumption:** Personal consumption expenditures on non-durable goods and services; NIPA Table 1.1.5 (line 5 + line 6), in current dollars.

**Investment:** Gross private investment expenditures + personal consumption expenditures on durable goods; NIPA Table 1.1.5 (line 8 + line 4), in current dollars.

**Government Spending:** Government consumption expenditures; NIPA Table 3.1 (line 16).

**Price Deflator:** The implicit GDP deflator; NIPA Table 1.1.9 (line 1).

**Population:** Civilian non-institutional population of ages 16 and older; Bureau of Labor Statistics CNP16OV.

**Average Hours Worked:** Average private nonfarm business hours, all persons; Valerie Ramey’s website http://weber.ucsd.edu/~vramey/research.html#data.

**Employment:** Total nonfarm employment, seasonally adjusted; Bureau of Labor Statistics PAYREMS.

**Wages:** Wage and salary disbursements by private industries; NIPA Tables 2.2A and 2.2B (line 2), in current dollars.

We use the series of GDP deflator and civilian population to obtain the real per-capita quantities of output, consumption, investment and government spending. Labor hours per capita are equal to total labor hours (the product of average hours worked and employment) divided by civilian population. We also use the series of GDP deflator and total labor hours to obtain the real wage per manhour.
References

[1] Álvarez-Peláez, María and Antonia Díaz (2005), “Minimum Consumption and Transi-

Cycles,” Institute for Empirical Macroeconomics at Federal Reserve Bank of Minneapolis
Discussion Paper 53.

Journal of Monetary Economics 51, 1183-1216.

Business Cycle Fluctuations in Neo-Classical Settings?” Journal of Economic Theory
135, 458-477.

Compete,” NBER Macroeconomics Annual 24, 413-455.


Theories and Aggregate Labor-Market Fluctuations,” American Economic Review 82,
430-450.

Rigidities and the Dynamic Effects of a Shock to Monetary Policy,” Journal of Political
Economy 113, 1-45.

[10] Christiano, Lawrence J., Cosmin Ilut, Roberto Motto and Massimo Rostagno (2008),
“Monetary Policy and Stock Market Boom-Bust Cycles,” European Central Bank Work-
ning Paper Series No. 955.

ment,” unpublished manuscript.


Economic Dynamics 14, 92-101.

Driven Business Cycles in Economies with Production Externalities: A Comment,” In-
ternational Journal of Economic Theory 8, 313-319.

Economics 16, 309-327.


Figure 1. Impulse Response Functions: Benchmark Model

Figure 2. Anticipated Labor Market Outcomes at Period 4
Figure 3. Impulse Response Functions: Alternative Demand Shocks
Figure 4a. Impulse Response Functions: Total Factor Productivity Shocks

Figure 4b. Impulse Response Functions: Investment-Specific Technology Shocks
Figure 5. Anticipated Labor Market Outcomes at Period 4

Figure 6. Impulse Response Functions under $\sigma = 0.5$: Total Factor Productivity and Investment-Specific Technology Shocks
Table 1. Business Cycle Statistics: Demand Shocks

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Relative Standard Deviations to Output</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_c/\sigma_y$</td>
<td>0.5442</td>
<td>0.3956</td>
<td>0.4638</td>
<td>0.0323</td>
</tr>
<tr>
<td>$\sigma_x/\sigma_y$</td>
<td>3.7931</td>
<td>3.8802</td>
<td>3.5924</td>
<td>3.8788</td>
</tr>
<tr>
<td>$\sigma_h/\sigma_y$</td>
<td>0.9905</td>
<td>0.9879</td>
<td>0.9931</td>
<td>0.9882</td>
</tr>
<tr>
<td><strong>Contemporaneous Correlations with Output</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{corr}(c, y)$</td>
<td>0.8005</td>
<td>0.6581</td>
<td>0.7456</td>
<td>0.3786</td>
</tr>
<tr>
<td>$\text{corr}(x, y)$</td>
<td>0.9383</td>
<td>0.9591</td>
<td>0.9484</td>
<td>0.9594</td>
</tr>
<tr>
<td>$\text{corr}(h, y)$</td>
<td>0.8852</td>
<td>0.9995</td>
<td>0.9999</td>
<td>0.9995</td>
</tr>
<tr>
<td>-----------------------</td>
<td>--------------------------</td>
<td>---------------------------------</td>
<td>--------------------------------------</td>
<td></td>
</tr>
<tr>
<td><strong>Relative Standard Deviations to Output</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_c/\sigma_y$</td>
<td>0.5442</td>
<td>0.7700</td>
<td>0.1898</td>
<td></td>
</tr>
<tr>
<td>$\sigma_x/\sigma_y$</td>
<td>3.7931</td>
<td>2.4599</td>
<td>4.6898</td>
<td></td>
</tr>
<tr>
<td>$\sigma_h/\sigma_y$</td>
<td>0.9905</td>
<td>0.6754</td>
<td>0.9425</td>
<td></td>
</tr>
<tr>
<td><strong>Contemporaneous Correlations with Output</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{corr}(c, y)$</td>
<td>0.8005</td>
<td>0.8980</td>
<td>0.6380</td>
<td></td>
</tr>
<tr>
<td>$\text{corr}(x, y)$</td>
<td>0.9383</td>
<td>0.8603</td>
<td>0.8899</td>
<td></td>
</tr>
<tr>
<td>$\text{corr}(h, y)$</td>
<td>0.8852</td>
<td>0.9676</td>
<td>0.9939</td>
<td></td>
</tr>
</tbody>
</table>
Table 3. Expectations-Driven Business Cycles: Demand Shocks

<table>
<thead>
<tr>
<th></th>
<th>Benchmark Consumption Shock</th>
<th>Multiplicative Consumption Shock</th>
<th>Government Spending Shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma = 0.5$</td>
<td>$0 \leq \psi \leq 0.02181$</td>
<td>$0 \leq \psi \leq 0.02181$</td>
<td>$0 \leq \psi \leq 0.02155$</td>
</tr>
<tr>
<td>$\sigma = 1$</td>
<td>$0 \leq \psi \leq 0.02133$</td>
<td>$0 \leq \psi \leq 0.02133$</td>
<td>$0 \leq \psi \leq 0.02123$</td>
</tr>
<tr>
<td>$\sigma = 2$</td>
<td>$0 \leq \psi \leq 0.02113$</td>
<td>$0 \leq \psi \leq 0.02113$</td>
<td>$0 \leq \psi \leq 0.02108$</td>
</tr>
<tr>
<td>$\sigma = 3$</td>
<td>$0 \leq \psi \leq 0.02107$</td>
<td>$0 \leq \psi \leq 0.02107$</td>
<td>$0 \leq \psi \leq 0.02104$</td>
</tr>
</tbody>
</table>