Indeterminacy and Investment Adjustment Costs in an Endogenously Growing Small Open Economy*

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Abstract

This paper analytically examines the interrelations between macroeconomic (in)stability and investment adjustment costs in a one-sector endogenously growing small-open-economy representative agent model. We show that under costly capital accumulation, the economy exhibits indeterminacy and sunspots if and only if the equilibrium wage-hours locus slopes upwards and is steeper than the household's labor supply curve. By contrast, the economy without adjustment costs for capital investment always displays saddle-path stability and equilibrium uniqueness, regardless of the degree of increasing returns in aggregate production.

Keywords: Indeterminacy; Investment Adjustment Costs; Endogenous Growth; Small Open Economy.

JEL Classification: E22; E32; F41; O41.

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1 Introduction

It is now well known that dynamic general equilibrium macroeconomic models may possess an indeterminate steady state or balanced growth path that can be exploited to generate business cycle fluctuations driven by agents' self-fulfilling beliefs. In particular, Benhabib and Farmer (1994) show that one-sector representative agent models for a closed economy exhibit indeterminacy and sunspots under sufficiently strong increasing returns in aggregate production.[1] Subsequently, Kim (2003) finds that (both analytically and quantitatively) in the no-sustained-growth version of Benhabib and Farmer's model, the minimum level of returns-to-scale needed for equilibrium indeterminacy is *ceteris paribus* a monotonically increasing function of the size of investment adjustment costs.[2] It is straightforward to show that the same result also holds in the endogenous-growth specification of the Benhabib-Farmer economy. Intuitively, investment adjustment costs are observationally equivalent to a state-contingent tax that "leans against the wind" of intertemporal capital accumulation. As a result, when the representative agent becomes optimistic about the future of the economy and decides to invest more, the presence of investment adjustment costs will raise the required degree of aggregate increasing returns to fulfill the household's initial optimism.[3]

In this paper, we build upon Kim's (2003) analyses and examine the theoretical interrelations between equilibrium indeterminacy and investment adjustment costs in the endogenous-growth version of Benhabib and Farmer's (1994) one-sector representative agent model for a small open economy. This specification provides us a useful analytical benchmark as well as facilitates comparison with recent work that studies the condition(s) for indeterminacy and sunspots in closed-economy versus small-open-economy macroeconomic models without adjustment costs.[4] Under the assumption of perfect international capital markets, the domestic households are able to lend to and borrow from abroad freely. To guarantee positive equilibrium growth of consumption, the constant world real interest rate is postulated to be

[1] See Benhabib and Farmer (1999) for an excellent survey on this strand of the indeterminacy literature.

[2] Wen (1998) numerically verifies this positive relationship in a calibrated one-sector real business cycle model where capital adjustment costs are postulated as a quadratic function of the difference in gross investments between two consecutive time periods.

[3] A corollary of this finding is that adjustment costs for capital investment can be used to eliminate equilibrium multiplicity and select a locally unique equilibrium in the Benhabib-Farmer economy (Guo and Lansing, 2002, pp. $655 - 657$). The same result is obtained in different one-sector macroeconomic models for a closed economy (Georges, 1995) or in a two-sector representative agent model with sector-specific externalities for a small open economy (Herrendorf and Valentinyi, 2003).

[4] See, for example, Lahiri (2001), Weder (2001), and Meng and Velasco (2003, 2004), among others.

strictly higher than the representative agent's utility discount rate.[5] Moreover, without loss of generality, we consider a specific non-linear capital accumulation formulation that incorporates convex investment adjustment costs *a la* Lucas and Prescott (1971). The zero degree of joint homogeneity in capital stock and gross investment is imposed to ensure the economy's sustained growth.

We show that our model economy possesses a unique balanced growth path (BGP), and that its local dynamics depend on whether capital accumulation is costly or not. Specifically, in sharp contrast to Kim (2003) and other previous studies for closed economies, the level of increasing returns-to-scale needed for equilibrium indeterminacy is independent of the (positive) degree of investment adjustment costs in the small open economy version of Benhabib and Farmer's (1994) one-sector endogenous growth model. It follows that other things being equal, indeterminacy and sunspots are easier to obtain, in the sense that lower aggregate increasing returns are required, within a small open economy than its closed-economy counterpart. This result turns out to be qualitatively consistent with that of Lahiri (2001), Weder (2001), and Meng and Velasco (2003, 2004) in various two-sector dynamic general equilibrium macroeconomic models without adjustment costs for capital investment.[6]

We also find that under costly capital accumulation, our endogenously growing small open economy exhibits the same local stability properties as those in the original Benhabib-Farmer closed-economy model without investment adjustment costs. In particular, indeterminacy and belief-driven fluctuations arise *if and only if* the equilibrium wage-hours locus slopes upwards and is steeper than the labor supply curve. Intuitively, start from a particular equilibrium path, and suppose that households become optimistic about the future of the economy. Acting upon this belief, the representative agent will reduce consumption and increase investment today, thus substituting out of internationally traded bonds and into physical capital (a portfolio substitution effect). This in turn raises the relative shadow price of physical capital because of a higher demand. In addition, the labor supply curve shifts out because of lower consumption spending. If increasing returns in the firm's production technology are strong enough to make the equilibrium wage-hours locus intersect the labor supply curve from below, a posi-

[5] In the no-sustained-growth version of our one-sector small-open-economy model, international lending and borrowing lead to complete consumption smoothing, *i.e.* the level of equilibrium consumption is a fixed constant over time. As a result, indeterminacy and sunspots can never occur in this setting, regardless whether there exist investment adjustment costs or not (Weder, 2001, p. 346).

[6] Notice that the one-sector version of these authors' models exhibit degenerate macroeconomic dynamics, thus equilibrium indeterminacy cannot arise. It is the presence of an additional sector that leads to nondegenerate equilibrium dynamics and the possibility of multiple equiibria (Meng and Velasco, 2004, p. 509).

tive sunspot shock leads to simultaneous expansions in output, consumption and investment. Moreover, due to higher levels of employment and labor productivity, the marginal product of capital and its relative utility value both will rise, hence validating agents' initial optimistic expectations.

When the household's investment decision does not involve adjustment costs, we show that the BGP's shadow prices of physical capital and foreign debt are equal, and that the equilibrium level of labor hours remains as a constant over time. Therefore, when agents become optimistic and decide to raise their investment today, the mechanism described above that makes for indeterminate equilibria, *i.e.* movements of hours worked caused by the household's portfolio substitution, is completely shut down, regardless of the degree of aggregate increasing returns. It follows that our model economy *always* exhibits saddle-path stability and equilibrium uniqueness in this setting. As a corollary, the same stability/uniqueness result continues to hold if our analysis starts with fixed labor supply in the household's utility function, no matter whether capital accumulation is subject to investment adjustment costs or not.

The remainder of this paper is organized as follows. Section 2 describes our model economy and analyzes the equilibrium conditions. Section 3 examines the local dynamics of the economy's balanced growth path with or without investment adjustment costs. Section 4 concludes.

2 The Economy

We incorporate a small open economy with perfect capital mobility and convex investment adjustment costs into the endogenous-growth version of Benhabib and Farmer's (1994) one-sector representative agent model. To facilitate comparison with existing studies, we adopt the same preference and technology formulations as those in the Benhabib-Farmer economy. This will also allow our analysis to highlight the stability effects of costly capital accumulation in an endogenously growing small open economy.

2.1 Firms

There is a continuum of identical competitive firms, with the total number normalized to one. Each firm produces output y_t using the Cobb-Douglas production function as follows:

$$y_t = x_t k_t^{\alpha} h_t^{1-\alpha}, \ \ 0 < \alpha < 1, \tag{1}$$

where k_t and h_t are capital and labor inputs, respectively, and x_t represents productive externalities that are taken as given by the individual firm. We postulate that externalities take the form

$$x_t = K_t^{1-\alpha} H_t^{(1-\alpha)\eta}, \;\; \eta > 0, \tag{2}$$

where K_t and H_t denote the economy-wide levels of capital and labor services. In a symmetric equilibrium, all firms make the same decisions such that $k_t = K_t$ and $h_t = H_t$, for all t. As a result, (2) can be substituted into (1) to obtain the following aggregate production function that displays increasing returns-to-scale:

$$y_t = \; k_t \, h_t^{(1-\alpha)(1+\eta)}. \tag{3}$$

Notice that the economy exhibits sustained endogenous growth because the social technology (3) displays linearity in physical capital. Under the assumption that factor markets are perfectly competitive, the first-order conditions for the firm's profit maximization problem are given by

$$u_t = \alpha \frac{y_t}{k_t}, \tag{4}$$

$$w_t = (1-\alpha)\, \frac{y_t}{h_t}, \tag{5}$$

where u_t is the capital rental rate and w_t is the real wage.

2.2 Households

The economy is populated by a unit measure of identical infinitely-lived households, each has one unit of time endowment and maximizes a discounted stream of utilities over its lifetime

$$\int_0^\infty \left\{ \log c_t - A \frac{h_t^{1+\gamma}}{1+\gamma} \right\} e^{-\rho t} dt, \;\; A > 0, \tag{6}$$

where c_t is the individual household's consumption, $\gamma \geq 0$ denotes the inverse of the intertemporal elasticity of substitution in labor supply, and $\rho > 0$ is the discount rate. The budget constraint faced by the representative household is given by

$$c_t + i_t + \dot{b}_t = u_t k_t + w_t h_t + r b_t, \quad b_0 \;\text{ is given,} \tag{7}$$

where i_t is gross investment, and $r > 0$ is the exogenously-given constant world real interest rate on risk-free foreign bonds b_t. Under the assumption of perfect international capital markets, the domestic household is able to lend to and borrow from abroad freely.

As in Kim (2003, p. 400), the law of motion for capital stock is specified as

$$\frac{\dot{k}_t}{k_t} = \frac{\delta\left[\left(\frac{i_t}{\delta k_t}\right)^{1-\theta} - 1\right]}{1-\theta}, \quad k_0 > 0 \text{ given}, \quad \text{and} \quad 0 \leq \theta < 1, \tag{8}$$

where $\delta \in (0,1)$ is the capital depreciation rate. When $0 < \theta < 1$, this non-linear accumulation formulation exhibits decreasing returns to investment alone, which can be viewed as reflecting convex adjustment costs *a la* Lucas and Prescott (1971); and the parameter θ represents the degree (or size) of investment adjustment costs. When $\theta = 0$, (8) becomes the standard linear capital accumulation equation without adjustment costs.[7] Finally, the zero degree of joint homogeneity in i_t and k_t is needed to maintain the economy's sustained growth of output.[8]

The first-order conditions for the representative household with respect to the indicated variables and the associated transversality conditions (TVC) are

$$c_t : \quad \frac{1}{c_t} = \lambda_{at}, \tag{9}$$

$$h_t : \quad Ah_t^{\gamma} = \lambda_{at} w_t, \tag{10}$$

$$i_t : \quad \lambda_{kt}\left(\frac{i_t}{\delta k_t}\right)^{-\theta} = \lambda_{at}, \tag{11}$$

$$k_t : \quad \dot{\lambda}_{kt} = \rho\lambda_{kt} - \frac{\delta\lambda_{kt}}{1-\theta}\left[\theta\left(\frac{i_t}{\delta k_t}\right)^{1-\theta} - 1\right] - \lambda_{at}u_t, \tag{12}$$

$$b_t : \quad \dot{\lambda}_{at} = (\rho - r)\lambda_{at}, \tag{13}$$

$$\text{TVC}_1 : \quad \lim_{t\to\infty} e^{-\rho t}\lambda_{at} b_t = 0, \tag{14}$$

$$\text{TVC}_2 : \quad \lim_{t\to\infty} e^{-\rho t}\lambda_{kt} k_t = 0, \tag{15}$$

where λ_{at} and λ_{kt} are the shadow prices of wealth/asset and physical capital, respectively. Equation (9) states that the marginal benefit of consumption equals its marginal cost, which

[7] When $\theta = 1$, (8) corresponds to the loglinear law of motion for capital stock in discrete time whereby $k_{t+1} = k_t^{\delta} i_t^{1-\delta}$, $0 < \delta < 1$. In this case, Guo and Lansing (2003) show that the combination of logarithmic utility and Cobb-Douglas forms for production and capital accumulation generates exactly offsetting income and substitution effects of interest rate movements that govern the household's investment allocation. It follows that closed-form decision rules can be derived, thus the economy always exhibits saddle-path stability and equilibrium uniqueness.

[8] All the (in)determinacy results reported below remain unaffected under Hayashi's (1982) adjustment-cost formulation. In particular, we have examined the following investment expenditure function that includes capital installation costs and is consistent with balanced growth: $i_t(1 + \frac{\phi}{2}\frac{i_t}{k_t})$, where $\phi > 0$.

is the marginal utility of having an additional unit of internationally traded bonds. Moreover, equation (10) equates the slope of the representative household's indifference curve to the real wage, and equations (11) and (12) together govern the evolution of capital stock over time. Finally, equation (13) states that the marginal utility values of foreign debt holdings are equal to their respective marginal costs.

Combining equations (9) and (13) yields the following standard Keynes-Ramsey rule:

$$\frac{\dot{c}_t}{c_t} = r - \rho. \tag{16}$$

To guarantee positive equilibrium growth of consumption, we assume that the world interest rate is strictly higher than the household's utility discount rate, *i.e.* $r > \rho$.

3 Analysis of Dynamics

We focus on the economy's balanced growth path (BGP) along which labor hours are stationary, and output, consumption, physical capital and foreign bonds all exhibit a common, positive constant growth rate given by $r - \rho$. To facilitate the analysis of perfect-foresight dynamics under sustained economic growth, we make the following transformation of variables: $q_t \equiv \lambda_{kt}/\lambda_{at}$, $z_t \equiv c_t/k_t$ and $x_t \equiv b_t/c_t$. Notice that q_t corresponds to "Tobin's q" which defines the marginal utility value of physical capital measured in terms of the shadow price of foreign debt.

With these transformations, the model's equilibrium conditions can be expressed as an autonomous system of differential equations

$$\frac{\dot{q}_t}{q_t} = r - \frac{\delta\left(\theta q_t^{\frac{1-\theta}{\theta}} - 1\right)}{1-\theta} - \frac{\alpha}{q_t}\left(\frac{Az_t}{1-\alpha}\right)^{\frac{(1-\alpha)(1+\eta)}{(1-\alpha)(1+\eta)-(1+\gamma)}}, \tag{17}$$

$$\frac{\dot{z}_t}{z_t} = r - \rho - \frac{\delta\left(q_t^{\frac{1-\theta}{\theta}} - 1\right)}{1-\theta}, \tag{18}$$

$$\frac{\dot{x}_t}{x_t} = \rho + \frac{1}{z_t x_t}\left(\frac{Az_t}{1-\alpha}\right)^{\frac{(1-\alpha)(1+\eta)}{(1-\alpha)(1+\eta)-(1+\gamma)}} - \frac{\delta q_t^{\frac{1}{\theta}}}{z_t x_t} - \frac{1}{x_t}. \tag{19}$$

It is straightforward to show that our model economy possesses a unique balanced-growth equilibrium, characterized by $\{q^*, z^*, x^*\}$, that satisfies $\dot{q}_t = \dot{z}_t = \dot{x}_t = 0$.[9] The remaining endogenous variables on the economy's balanced growth path can then be derived accordingly.

In terms of the BGP's local stability properties, we note that the above dynamical system (17)-(19) is block recursive in that the evolutions of q_t and z_t do not depend on the foreign-bonds-to-consumption ratio x_t. As a result, whether the model exhibits equilibrium (in)determinacy will be completely determined by the two-by-two subsystem in q_t and z_t. We linearize equations (17) and (18) around (q^*, z^*), and then compute the resulting Jacobian matrix J. The trace and determinant of the Jacobian are

$$Tr = \rho > 0, \tag{20}$$

$$Det = \frac{\alpha\delta(1-\alpha)(1+\eta)(q^*)^{\frac{1-2\theta}{\theta}}}{\theta[1+\gamma-(1-\alpha)(1+\eta)]}\left(\frac{Az^*}{1-\alpha}\right)^{\frac{(1-\alpha)(1+\eta)}{(1-\alpha)(1+\eta)-(1+\gamma)}}. \tag{21}$$

Since q_t and z_t are both jump variables, the dynamical subsystem (17)-(18) does not have any initial condition. Therefore, the balanced growth path displays saddle-path stability and equilibrium uniqueness when both eigenvalues have positive real parts. Given the trace of the Jacobian J is positive,[10] the the BGP equilibrium is locally indeterminate (a sink) *if and only if* the two eigenvalues are of opposite sign, *i.e.* the Jacobian's determinant (21) is negative. In this case, the economy exhibits endogenous growth fluctuations driven by agents' self-fulfilling expectations or sunspots.

3.1 When $0 < \theta < 1$

In this specification, the non-linear capital accumulation equation (8) exhibits convex investment adjustment costs. It is straightforward to show that the BGP equilibrium is characterized by a pair of positive real numbers (q^*, z^*) given by

[9] By contrast, Benhabib and Farmer (1994) show that in a closed economy without investment adjustment costs, there may exist two balanced-growth equilibria whereby one is locally determinate and the other exhibits equilibrium indeterminacy.

[10] This implies that the case in which both eigenvalues have negative real parts is not possible.

$$q^{*}=\left[\frac{(1-\theta)(r-\rho)+\delta}{\delta}\right]^{\frac{\theta}{1-\theta}}, \tag{22}$$

$$z^{*}=\left(\frac{1-\alpha}{A}\right)\left[\frac{(1-\theta)r+\delta+\theta\rho}{\alpha}\right]^{\frac{(1-\alpha)(1+\eta)-(1+\gamma)}{(1-\alpha)(1+\eta)}}\left[\frac{(1-\theta)(r-\rho)+\delta}{\delta}\right]^{\frac{\theta[(1-\alpha)(1+\eta)-(1+\gamma)]}{(1-\theta)(1-\alpha)(1+\eta)}}. \tag{23}$$

Since q^*, z^*, A, $\eta > 0$, $0 < \alpha$, θ, $\delta < 1$, and $\gamma \geq 0$, the Jacobian's determinant (21) is negative if and only if

$$(1-\alpha)(1+\eta)-1>\gamma, \tag{24}$$

which is also the *necessary and sufficient* condition for our model economy to display equilibrium indeterminacy and belief-driven aggregate fluctuations.

To understand the above indeterminacy condition, we first substitute (9) into (10) and find that agents' equilibrium decision on hours worked is governed by

$$Ac_t h_t^{\gamma}=w_t. \tag{25}$$

Next, plugging the aggregate production function (3) into the logarithmic version of firms' labor demand condition (5) shows that the slope of the equilibrium wage-hours locus is equal to $(1-\alpha)(1+\eta)-1$. In addition, taking logarithms on both sides of (25) indicates that the slope of the household's labor supply curve is γ (≥ 0), and its position or intercept is affected by the level of consumption. Therefore, the condition that is needed to generate indeterminacy and sunspots, as in (24), states that the equilibrium wage-hours locus is positively sloped and steeper than the labor supply curve.

In terms of economic intuition, suppose that households become optimistic about the future of the economy and anticipate a higher return on investment. Acting upon this belief, agents will invest more and consume less today, thus substituting out of foreign bonds and into physical capital (a portfolio substitution effect). This in turn raises the relative shadow price of capital because of a higher demand. Using (25), lower spending of current consumption also shifts out the household's labor supply curve, which causes labor hours to rise and the real wage to fall. If the degree of external effects in firms' production function η is strong enough to yield a more-than-unity equilibrium labor elasticity of output in the social technology, namely

$(1-\alpha)(1+\eta) > 1$ in (3), increases in hours worked lead to a higher labor productivity. It follows that the labor demand curve will shift outward, reinforcing the initial employment effect generated by the reduction of consumption expenditures. Subsequently, higher labor hours raise the household's projected income stream, thereby increasing its ability to consume and shifting the labor supply curve to the left. When such a leftward shift makes the equilibrium wage-hours locus intersect the labor supply curve from below, *i.e.* $(1-\alpha)(1+\eta) - 1 > \gamma$, a positive sunspot shock produces simultaneous expansions in output, consumption, investment, hours worked and labor productivity. Moreover, due to higher levels of employment and labor productivity, the marginal product of capital and its relative shadow price both will rise, hence validating agents' initial optimistic expectations. On the contrary, if the strength of productive externalities η is not sufficiently high so that the equilibrium wage-hours locus is flatter than the labor supply curve, consumption will become countercyclical. As a result, agents' optimism cannot become self-fulfilling in equilibrium.

In sharp contrast to previous studies (*e.g.* Georges, 1995; Wen, 1998; Guo and Lansing, 2002; and Kim, 2003) for closed economies, we have shown that in the small open economy version of one-sector representative agent models with sustained economic growth, the level of increasing returns-to-scale needed for equilibrium indeterminacy $\eta_{\min}$ is independent of the degree of investment adjustment costs. As long as $\theta > 0$ and condition (24) is satisfied, the shadow-price wedge between physical capital and foreign bonds will generate the above-mentioned effects of portfolio substitution and labor adjustments in response to agents' optimistic expectations. This "independence" result implies that *ceteris paribus* indeterminacy and sunspots are more likely to arise, in the sense that lower productive externalities are required, within a small open economy $\left(\frac{\partial \eta_{\min}}{\partial \theta} = 0\right)$ than a closed economy $\left(\frac{\partial \eta_{\min}}{\partial \theta} > 0\right)$.[11] Notice that Lahiri (2001), Weder (2001), and Meng and Velasco (2003, 2004) obtain the same finding in various two-sector dynamic general equilibrium macroeconomic models without investment adjustment costs.[12]

On the other hand, (24) turns out to be identical to the necessary and sufficient condition that leads to multiple equilibria in the endogenous-growth formulation of Benhabib and

[11] For example, under Kim's (2003, p. 399) benchmark parameterization with $\alpha = 0.3$, $\gamma = 0$, $\rho = 0.05$ and $\delta = 0.1$, together $\theta = 0.05$, it is straightforward to show that $\eta_{\min} = 0.4286$ in our small-open-economy model, whereas $\eta_{\min} = 0.7383$ in the closed-economy counterpart.

[12] Due to perfect capital mobility, the small-open-economy models that these authors examine behave like a closed economy with linear preferences in consumption. It follows that there are no utility costs associated with constructing alternative equilibrium paths when agents become optimistic. This in turn enlarges the range of parameter values under which indeterminacy and sunspots may occur.

Farmer's (1994) model for a closed economy. Hence, the preceding (in)determinacy analysis shows that given identical preference and technology specifications, our one-sector endogenous growth model with investment adjustment costs for a small open economy exhibits exactly the same local (in)stability properties as those in the original Benhabib-Farmer closed-economy counterpart under the standard linear law of motion for capital accumulation.

3.2 When $\theta = 0$

In this specification, the capital accumulation equation (8) is linear and does not exhibit investment adjustment costs. Substituting $\theta = 0$ into (11) shows that the relative price (in utility terms) of physical capital to internationally traded bonds $q_t \equiv \lambda_{kt}/\lambda_{at} = 1$ for all t. As a result, the dynamical system (17)-(18) now becomes degenerate. Resolving our model with $\theta = 0$ and $q_t = 1$ leads to the following single differential equation in $x_t \equiv b_t/c_t$ that describes the equilibrium dynamics:

$$\frac{\dot{x}_t}{x_t} = \rho + \frac{1}{x_t}\left\{ \frac{A\left[(1-\alpha)\,r + \alpha\rho\right]}{\alpha\,(1-\alpha)} \left(\frac{r+\delta}{\alpha}\right)^{\frac{(1+\gamma)-(1-\alpha)(1+\eta)}{(1-\alpha)(1+\eta)}} - 1 \right\}, \tag{26}$$

which has a unique interior solution x^* that satisfies $\dot{x}_t = 0$ along the balanced growth path. We then linearize (26) around the BGP and find that its local stability properties are governed by $\rho > 0$. Consequently, the economy always displays saddle-path stability and equilibrium uniqueness because there is no initial condition associated with (26).

The intuition for the above determinacy result is straightforward. Combining equations (3), (4), (12) and (13), together with $\lambda_{kt} = \lambda_{at}$, yields a constant level of equilibrium labor hours over time

$$h_t = \left(\frac{r+\delta}{\alpha}\right)^{\frac{1}{(1-\alpha)(1+\eta)}}, \text{ for all } t. \tag{27}$$

Therefore, when households become optimistic and decide to increase their investment spending today, the mechanism described in the previous sub-section that makes for indeterminate equilibria, *i.e.* movements of hours worked induced by the representative agent's portfolio substitution, is completely shut down, regardless of the degree of productive externalities η. This implies that given the initial holding of foreign bonds b_0, the household's period-0 consumption c_0 will be uniquely determined such that the economy immediately jumps onto its balanced-growth equilibrium characterized by x^* that solves $\dot{x}_t = 0$ from (26), and always stays there without any possibility of deviating transitional dynamics. It follows that equilibrium indeterminacy and endogenous growth fluctuations can never occur in this setting. Notice that the same stability/uniqueness result will be obtained if our analysis starts with fixed labor supply in the household utility function (6), no matter whether capital accumulation is subject to investment adjustment costs or not.

4 Conclusion

We have analytically explored the theoretical relationship between macroeconomic (in)stability and investment adjustment costs within one-sector endogenous growth models for a small open economy. Under costly capital accumulation, the necessary and sufficient condition for our model economy to exhibit indeterminacy and sunspots is identical to that in Benhabib and Farmer's (1994) model for a closed economy without adjustment costs for capital investment. That is, the equilibrium wage-hours locus is positively sloped and steeper than the household's labor supply curve. This result turns out to be qualitatively consistent with previous studies which show that sunspot-driven business cycle fluctuations are more likely to occur in a small open economy than its closed-economy counterpart. On the other hand, our model economy without investment adjustment costs always displays saddle-path stability and equilibrium uniqueness, regardless of the degree of increasing returns-to-scale in aggregate production.

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