On Money, Social Status and Endogenous Growth*

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June 26, 2009

Abstract

Motivated by the substantial increase of nominal money supply in the U.S. economy since late 2008, this paper examines the equilibrium growth effect of money/inflation within a standard one-sector AK model of endogenous growth with wealth-enhanced preferences for social status and the most generalized cash-in-advance constraint. We show that the sign for the correlation between money and output growth depends crucially on (i) the liquidity-constrained ratio of consumption to investment, and (ii) how the shadow price of physical capital responds to a change in the monetary growth rate. This money-growth correlation, as well as the growth effect of social status, turns out to be closely related to the local stability properties of the economy's balanced growth path(s).

Keywords: Money, Endogenous Growth, Cash-in-Advance Constraint, Social Status, Indeterminacy.

JEL Classification: E52, O42.

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*We would like to thank Shu-Hua Chen and Richard Suen for helpful discussions and comments. Part of this research was conducted while Guo was a visiting professor of economics at the National Taiwan University, whose hospitality is greatly appreciated. The financial support provided by the National Taiwan University's Public Economic Research Center and Program for Globalization Studies at the Institute for Advanced Studies in Humanities (grant number: 95R0064-AH03-03) is also gratefully acknowledged.

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1 Introduction

Since the economic recession officially started in December 2007, the Federal Reserve has undertaken some aggressive policy actions to contend with the cyclical contraction and financial crisis. First, the Federal Funds rate was cut several times from 4.75% in September 2007 to the $0\% - 0.25\%$ target range that was announced in December 2008. Second, realizing that the U.S. economy did not respond fast enough to these interest rate reductions, the Fed started to considerably raise the size of its balanced sheet, through purchasing Treasury Bills, government bonds and agency mortgage debt, in late 2008. As a result, the quantity of monetary base almost doubled between September 2008 and January 2009. This monetary expansion in turn revives interest in theoretical analysis on the macroeconomic effects of a change in the growth of nominal money supply within dynamic general equilibrium models.[1] Such academic research is worthwhile not only for its topical relevance, but also for its important implications for understanding the design and implementation of monetary policies.

In this paper, we address the above research question, systematically and comprehensively, in a one-sector AK model of monetary endogenous growth with two salient features. First, in addition to consumption goods, the representative household's non-separable constant-relative-risk-aversion (CRRA) preference formulation includes wealth-enhanced social status represented by its physical-capital ownership relative to the economy's aggregate level.[2] Based on the empirical evidence in the mainstream macroeconomics literature, we also postulate that the intertemporal elasticity of substitution in consumption is not strictly greater than unity. Second, money is introduced to the model by the most generalized cash-in-advance (CIA) or liquidity constraint. Specifically, positive fractions (including 100%) of both consumption and investment expenditures must be financed by the household's real money holdings. Our analysis is focused on the economy's balanced growth path (BGP) along which GDP, consumption, physical capital and real balances all grow at a common positive rate.

We find that the output-growth effect of money/inflation depends crucially on (i) whether the ratio between the CIA-constrained proportion of consumption expenditures and that of gross investment is higher or lower than a critical value; and (ii) how the shadow price of

[1] Early work in this area includes Tobin (1965), Sidrauski (1967), Dornbusch and Frenkel (1973), Brock (1974), Stockman (1981), Abel (1985), Wang and Yip (1992), among many others.

[2] There has been a growing literature that examines the macroeconomic effects of agents' wealth-induced preferences for social status within neoclassical models of capital accumulation, economic growth, and asset pricing. See, for example, Zou (1994, 1995, 1998), Bakshi and Chen (1996), Corneo and Jeanne (1997, 2001*a*, 2001*b*), Chang, Hsieh and Lai (2000), Gong and Zou (2001), Chang and Tsai (2003), Clemens (2004), Chang, Tsai and Lai (2004), Fisher and Hof (2005), and Chen and Guo (2009), among many others.

physical capital responds to an increase in the monetary growth rate, which is governed by the relative strength of two opposing forces dubbed as the portfolio substitution effect (out of real balances and into capital) and the intertemporal substitution effect (from consumption to investment, strengthened by agents' status-seeking motive). Moreover, this money-growth correlation result turns out to be closely associated with the local stability properties of the economy's balanced growth path(s).

Our main results from three different model specifications are summarized as follows. When the household utility is separable and logarithmic in consumption and relative wealth, the economy's unique balanced-growth equilibrium exhibits saddle-path stability and dominating portfolio substitution effect. In addition, the sign for the growth effect of money/inflation is theoretically ambiguous. If consumption is sufficiently more liquidity-constrained relative to investment, it will be less stringent for the household to accumulate physical capital in response to an injection of nominal money supply. As a result, the BGP's consumption-to-capital ratio may fall, which in turn generates a faster output growth through stimulated capital accumulation.

Next, we examine two possibilities for the specification in which the intertemporal elasticity of consumption substitution is strictly smaller than one. In the first case, the BGP's utility value of capital and consumption-to-capital ratio are moving in the same direction. As in the logarithmic formulation, the economy exhibits a unique balanced-growth equilibrium that is locally determinate. However, since the liquidity-constrained ratio of consumption to investment is not sufficiently high, an increase in the monetary growth rate raises the household's relative cash-in-advance cost for capital accumulation. This leads to a negative relationship between money/inflation and the rate of economic growth.

In the other case with the BGP's shadow price of capital and consumption-to-capital ratio moving in the opposite direction, the economy exhibits two balanced-growth equilibria when the intertemporal elasticity of substitution in consumption falls below a threshold level that is strictly lower than one. Since the portfolio substitution effect dominates in the high-growth BGP equilibrium, which turns out to be a saddle path, the growth effect of money/inflation is negative. On the contrary, due to a stronger intertemporal substitution effect, the low-growth equilibrium path is locally indeterminate (a sink), and displays a positive correlation between GDP growth and money/inflation.

Finally, we find that the equilibrium growth effect of social status is closely linked with the BGP's local stability properties as well. In particular, under saddle-path stability, a higher degree for "the spirit of capitalism" provides strong incentives for the representative household to accumulate more physical capital that raises its social status and thus utilities. This will decrease the equilibrium consumption-to-capital ratio, which in turn enhances the BGP's output growth. On the contrary, an indeterminate balanced-growth equilibrium path displays a negative growth effect of social status because of the dominating intertemporal substitution effect.

This paper is related to recent work of Suen and Yip (2005) and Chen and Guo (2008*a*, 2008*b*) who also study the macroeconomic effects of money/inflation in a one-sector AK model of monetary endogenous growth. However, wealth-induced preferences for social status are not considered in these studies. In addition, indeterminacy and endogenous growth fluctuations occur in their models only when the intertemporal elasticity of consumption substitution is strictly higher than one – a parameterization that is not consistent with exiting empirical evidence. Another piece of relevant research is Chen and Guo (2009) who incorporate relative wealth into the household's separable CRRA utility function, and restrict their analysis to Stockman's (1981) formulation whereby the entire consumption and investment purchases are subject to the CIA constraint. Moreover, their model economy always exhibits equilibrium uniqueness, saddle-path stability and a negative growth effect of money/inflation.

The remainder of this paper is organized as follows. Section 2 examines an AK model of endogenous growth with fixed labor supply, wealth-enhanced preferences for social status and the most generalized cash-in-advance constraint. Section 3 analyzes the existence and number of the economy's balanced growth path(s). Section 4 investigates their local stability properties as well as the associated growth effects of money/inflation and social status. Section 5 concludes.

2 The Economy

We incorporate a constant-relative-risk-aversion (CRRA) preference formulation that exhibits wealth-enhanced social status *a la* Clemens (2004) and the most generalized cash-in-advance (CIA) constraint into a prototypical one-sector AK model of endogenous growth. The economy is populated by a unit measure of identical infinitely-lived households. Each household provides fixed labor supply and maximizes its lifetime utility

$$U = \int_0^\infty \frac{\left[c_t\left(\frac{k_t}{K_t}\right)^\beta\right]^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt, \quad \sigma \geqslant 1, \quad \text{and} \quad \beta > 0, \tag{1}$$

where c_t and k_t are the individual household's consumption and capital stock, respectively, $\rho \in (0,1)$ denotes the time discount rate, and σ is the inverse of the intertemporal elasticity of substitution in consumption. Based on the empirical evidence for this preference parameter in the mainstream macroeconomics literature, we restrict our analysis to the specifications in which $\sigma \geqslant 1$. In addition to consumption goods, the representative agent derives utilities from the wealth-based social status represented by its physical-capital ownership k_t relative to the economy-wide level K_t. Therefore, the relative wealth $\frac{k_t}{K_t}$ is postulated to enter the household's preferences in a non-separable manner, and the parameter β measures the degree for "the spirit of capitalism" (Zou 1994, 1995, 1998). Since $\beta > 0$, the marginal utility of an individual household's own consumption increases with the economy's aggregate capital stock. It follows that the household utility (1) exhibits a negative capital externality because each agent does not take into account the external effect that his/her capital accumulation reduces the utility of everyone else's.

The budget constraint faced by the representative household is

$$c_t + i_t + \dot{m}_t = y_t - \pi_t m_t + \tau_t, \tag{2}$$

where i_t is gross investment, π_t is the inflation rate, m_t denotes the real money balances that are equal to the nominal money supply M_t divided by the aggregate price level P_t, and τ_t represents real lump-sum transfers that households receive from the monetary authority. Output y_t is produced by the following technology:

$$y_t = Ak_t, \;\; A > 0, \tag{3}$$

and the law of motion for the capital stock is given by

$$\dot{k}_t = i_t - \delta k_t, \;\; k_0 > 0 \text{ given}, \tag{4}$$

where $\delta \in (0,1)$ is the capital depreciation rate.

The representative household also faces the most generalized cash-in-advance (CIA) or liquidity constraint as follows:

$$\phi_c c_t + \phi_i i_t \leq m_t, \;\; 0 < \phi_c, \;\; \phi_i \leq 1, \tag{5}$$

where ϕ_c and ϕ_i represent the (non-zero) fractions of consumption and investment expenditures that must be financed by the household's real balances m_t.

On the monetary side of the economy, nominal money supply M_t is assumed to evolve according to

$$M_t = M_0 e^{\mu t}, \;\; M_0 > 0 \;\text{ given,} \tag{6}$$

where $\mu > 0$ is the time-invariant monetary growth rate, and the resulting seigniorage returned to households as a lump-sum transfer is given by $\tau_t = \mu m_t$.

The first-order conditions for the representative household with respect to the indicated variables and the associated transversality conditions (TVC) are

$$c_t \;:\; c_t^{-\sigma} = \lambda_{mt} + \phi_c \psi_t, \tag{7}$$

$$i_t \;:\; \lambda_{kt} = \lambda_{mt} + \phi_i \psi_t, \tag{8}$$

$$k_t \;:\; \dot{\lambda}_{kt} = (\rho + \delta)\lambda_{kt} - \beta \frac{c_t^{1-\sigma}}{k_t} - A\lambda_{mt}, \tag{9}$$

$$m_t \;:\; \dot{\lambda}_{mt} = (\rho + \pi_t)\lambda_{mt} - \psi_t, \tag{10}$$

$$\text{TVC}_1 \;:\; \lim_{t\to\infty} e^{-\rho t}\lambda_{kt} k_t = 0, \tag{11}$$

$$\text{TVC}_2 \;:\; \lim_{t\to\infty} e^{-\rho t}\lambda_{mt} m_t = 0, \tag{12}$$

where λ_{mt} and λ_{kt} are the shadow prices (or utility values) of real money balances and physical capital, respectively, and ψ_t denotes the Lagrange multiplier associated with the CIA constraint (5). Equation (7) equates the marginal benefit and marginal cost of consumption, which is the marginal utility of having an additional unit of real dollar. In addition, equations (8) and (9) together govern the evolution of physical capital over time, where the standard intertemporal consumption Euler equation is modified to reflect the marginal utility benefit from agents' status-seeking capital accumulation captured by the term $\beta \frac{c_t^{1-\sigma}}{k_t}$. Finally, equation (10) states that the marginal values of real money holdings are equal to their marginal costs.

As is common in the macroeconomics literature, we assume that the liquidity constraint (5) is always binding in equilibrium, hence $\psi_t > 0$ for all t. Furthermore, clearing in the goods and money markets imply that

$$c_t + i_t = y_t, \tag{13}$$

and

$$\dot{m}_t = (\mu - \pi_t)\, m_t. \tag{14}$$

3 Balanced Growth Path

We focus on the economy's balanced growth path (BGP) along which output, consumption, physical capital and real money balances all grow at a common positive rate denoted as θ. To facilitate the subsequent dynamic analyses, we adopt the following variable transformations: $p_t \equiv \frac{\lambda_{kt}}{\lambda_{mt}}$ and $z_t \equiv \frac{c_t}{k_t}$. Using (3), (4), (13) and (14), the economy's equilibrium inflation rate π_t is given by

$$\pi_t = \mu - A + \delta + z_t. \tag{15}$$

Moreover, equation (8) implies that the transformed variable $p_t > 1$ for all t in that $\phi_i \in (0, 1]$ and $\psi_t > 0$. With these transformed variables and equation (15), the model's equilibrium conditions can be collapsed into the following autonomous dynamical system:

$$\frac{\dot{p}_t}{p_t} = \frac{\frac{\sigma(p_t-1)}{\phi_i} + [g_2(z_t) - \sigma]\left\{\frac{A+\beta z_t\left[1+(p_t-1)\frac{\phi_c}{\phi_i}\right]}{p_t} - \delta\right\} + \sigma\left[1 - g_2(z_t)\right](A - \delta - z_t) - \sigma\mu - \rho g_2(z_t)}{\sigma - g_1(p_t)g_2(z_t)}, \tag{16}$$

$$\frac{\dot{z}_t}{z_t} = \frac{1}{\sigma}\left\{g_1(p_t)\frac{\dot{p}_t}{p_t} + \frac{A + \beta z_t\left[1 + (p_t - 1)\frac{\phi_c}{\phi_i}\right]}{p_t} - \rho - \delta\right\} - A + \delta + z_t, \tag{17}$$

where $g_1(p_t) \equiv \frac{\phi_i - \phi_c}{\phi_c(p_t-1)+\phi_i}$ and $g_2(z_t) \equiv \frac{(\phi_c-\phi_i)z_t}{(\phi_c-\phi_i)z_t+\phi_i A}$.

A balanced-growth equilibrium is characterized by a pair of positive real numbers (p^*, z^*) such that $\dot{p}_t = \dot{z}_t = 0$. It is straightforward to derive from (16) and (17) that p^* is the solution(s) to the quadratic equation

$$p^* = 1 + \phi_i\left\{\mu - A + \frac{A}{p^*} + \frac{\left\{1 + \frac{\beta}{p^*}\left[1 + (p^* - 1)\frac{\phi_c}{\phi_i}\right]\right\}\left[\rho + \delta - \frac{A}{p^*} + \sigma(A - \delta)\right]}{\sigma + \frac{\beta}{p^*}\left[1 + (p^* - 1)\frac{\phi_c}{\phi_i}\right]}\right\} \equiv f(p^*), \tag{18}$$

and the corresponding expression for z^* is given by

$$z^* = \frac{\rho + \delta - \frac{A}{p^*} + \sigma (A - \delta)}{\sigma + \frac{\beta}{p^*} \left[1 + (p^* - 1) \frac{\phi_c}{\phi_i} \right]}. \tag{19}$$

It follows that

$$\frac{dz^*}{dp^*} = \frac{A + \beta z^* (1 - \frac{\phi_c}{\phi_i})}{p^* \left\{ \beta \left[1 + (p^* - 1) \frac{\phi_c}{\phi_i} \right] + \sigma p^* \right\}} \gtrless 0, \tag{20}$$

where the denominator is positive because $\beta > 0$ and $p^* > 1$, and the numerator's sign is governed by the relative strength between the CIA-constrained proportion of consumption expenditures and the investment fraction $\frac{\phi_c}{\phi_i}$. Notice that in the previous literature without wealth-based preferences for social status ($\beta = 0$), the sign of $\frac{dz^*}{dp^*}$ is always positive (see, for example, Suen and Yip, 2005; and Chen and Guo, 2008*a*, 2008*b*). As a result, the economy exhibits two balanced-growth equilibria only when the intertemporal elasticity of substitution in consumption is strictly greater than one, *i.e.* $\sigma < 1$. However, such a parametric formulation is not consistent with the vast majority of existing empirical evidence. By contrast, section 4.3 below shows that dual BGP's can arise in our model economy when $\sigma > 1$ and $\frac{dz^*}{dp^*} < 0$.

To examine the existence and number of the economy's balanced growth path(s), we first note that equilibrium p^* can be found from the intersection(s) of $f(p^*)$, as in the right-hand side of (18), and the 45-degree line. Moreover, using $\frac{dz^*}{dp^*}$ from (20), we obtain that

$$f^{'}(p^*) = \phi_i (1 - \sigma) \underbrace{\frac{\left[A + \beta z^* (1 - \frac{\phi_c}{\phi_i}) \right]}{p^* \left\{ \beta \left[1 + (p^* - 1) \frac{\phi_c}{\phi_i} \right] + \sigma p^* \right\}}}_{\frac{dz^*}{dp^*}} \lesseqqgtr 0, \tag{21}$$

where (i) $f^{'}(p^*) = 0$ when $\sigma = 1$; (ii) $f^{'}(p^*) < 0$ when $\sigma > 1$ and $\frac{dz^*}{dp^*} > 0$ $\left(\text{or } \frac{\phi_c}{\phi_i} < 1 + \frac{A}{\beta z^*} \right)$; and (iii) $f^{'}(p^*) > 0$ when $\sigma > 1$ and $\frac{dz^*}{dp^*} < 0$ $\left(\text{or } \frac{\phi_c}{\phi_i} > 1 + \frac{A}{\beta z^*} \right)$. Next, it can be shown that

$$f''(p^*) = -f^{'}(p^*) \underbrace{\left\{ \frac{2 \left(\sigma + \beta \frac{\phi_c}{\phi_i} \right)}{\beta \left[1 + (p^* - 1) \frac{\phi_c}{\phi_i} \right] + \sigma p^*} \right\}}_{\text{positive}} \gtreqqless 0 \quad \text{when} \quad f^{'}(p^*) \lesseqqgtr 0. \tag{22}$$

As a result, $f(p^*)$ can be (i) a horizontal line, or (ii) a downward-sloping curve that is convex to the origin and $f(0) \to \infty$, or (iii) a upward-sloping and concave curve with $f(0) \to -\infty$. It follows that the economy exhibits a unique balanced-growth equilibrium in cases (i) and (ii) as $f(p^*)$ intersects the 45-degree line once in the positive quadrant. On the other hand, depending on the model's structural parameters (such as A, δ, σ, β, μ, ϕ_c and ϕ_i), the number of BGP equilibria in case (iii) can be zero, one or two.

4 Growth Effects of Money and Social Status

In this section, we examine the local stability properties of the model's balanced growth path(s) as well as the associated growth effects of money (or inflation)[3] and social status under the above-mentioned three different parametric configurations. Combining (3), (4) and (13) shows that when a balanced-growth equilibrium exists, its common positive rate of economic growth θ is given by

$$\theta = A - \delta - z^*, \tag{23}$$

thus the BGP's growth rate is negatively related to the transformed variable z^* $\left(\frac{d\theta}{dz^*} = -1\right)$. In terms of the BGP's local dynamics, we compute the Jacobian matrix J for the dynamical system (16) and (17) evaluated at (p^*, z^*). The determinant and trace of the Jacobian are

$$Det = \frac{z^* \left[1 - f'(p^*)\right] \left\{\beta \left[1 + (p^* - 1)\frac{\phi_c}{\phi_i}\right] + \sigma p^*\right\}}{\phi_i \left[\sigma - g_3(p^*)\, g_4(z^*)\right]}, \tag{24}$$

$$Tr = \frac{Det}{z^*} + \frac{Num}{\sigma - g_3(p^*)\, g_4(z^*)}, \tag{25}$$

where $f'(p^*)$ is given by (21),

$$Num \equiv \frac{1}{p^*}\left\{\left[1 - g_4(z^*)\right]\left[A + \beta z^*\left(1 - \frac{\phi_c}{\phi_i}\right)\right] + \beta z^* \left[1 - g_3(p^*)\right]\left[1 + (p^* - 1)\frac{\phi_c}{\phi_i}\right]\right\}$$
$$+ \frac{\beta}{\phi_i}\left[1 + (p^* - 1)\frac{\phi_c}{\phi_i}\right] + z^*\left[\sigma - g_3(p^*)\right],$$

$g_3(p^*) \equiv \frac{\phi_i - \phi_c}{\phi_c(p^*-1)+\phi_i}$ and $g_4(z^*) \equiv \frac{(\phi_c - \phi_i)z^*}{(\phi_c-\phi_i)z^* + \phi_i A}$.

The local stability property of a balanced-growth equilibrium is determined by comparing the eigenvalues of J that have negative real parts to the number of initial conditions in the dynamical system (16)-(17), which is zero because p_t and z_t are both jump variables. As a result, the BGP displays equilibrium uniqueness and saddle-path stability when both eigenvalues have positive real parts. If one or two eigenvalues have negative real parts, then the

[3] On the balanced growth path, its inflation rate π^* is *ceteris paribus* positively related to the monetary growth rate μ because equation (14) implies that $\mu = \pi^* + g$.

BGP is locally indeterminate (a sink) and can be exploited to generate endogenous growth fluctuations driven by agents' self-fulfilling expectations or sunspots.

Next, we take total differentiation on (23), and use the chain rule together with (18), (20) and (21) to find that the growth effect of money/inflation is given by

$$\frac{d\theta}{d\mu} = \underbrace{\frac{d\theta}{dz^*}}_{\text{negative}} \underbrace{\frac{dz^*}{dp^*}}_{\text{positive / negative}} \frac{dp^*}{d\mu}, \tag{26}$$

where

$$\frac{dp^*}{d\mu} = \frac{\phi_i}{1 - f'(p^*)} \gtreqless 0 \text{ when } f'(p^*) \lesseqgtr 1. \tag{27}$$

Generally speaking, within a dynamic general equilibrium macroeconomics model, the sign of $\frac{dp^*}{d\mu}$ depends on the relative strength of two opposing forces. On the one hand, an increase in the monetary growth rate μ leads to a higher inflation, which in turn raises the cost of real money holdings. As a result, the representative household substitutes out of real balances and into physical capital (the portfolio substitution effect). This will cause a rise in the relative shadow price of capital p^* because of a higher demand $\left(\frac{dp^*}{d\mu} > 0\right)$, thereby reducing its net (after-inflation) rate of return. On the other hand, a higher monetary growth rate μ *ceteris paribus* induces the representative household to consume less and invest more today in exchange for higher future consumption (the intertemporal substitution effect).[4] This expands the supply of physical capital, hence reducing its relative shadow price p^*. Moreover, agents' status-seeking motive further strengthens this supply effect through additional capital accumulation (see the term $\beta\frac{c_t^{1-\sigma}}{k_t}$ in equation 9). Hence, the intertemporal substitution effect leads to $\frac{dp^*}{d\mu} < 0$ and a higher net rate of return on physical capital.

With regard to the growth effect of social status, we follow the same procedure as above and find that

$$\frac{d\theta}{d\beta} = \underbrace{\left\{ \frac{\left[1 + (p^* - 1)\frac{\phi_c}{\phi_i}\right] z^*}{\beta\left[1 + (p^* - 1)\frac{\phi_c}{\phi_i}\right] + \sigma p^*} \right\}}_{\text{positive}} \frac{1}{1 - f'(p^*)} \gtreqless 0 \text{ when } f'(p^*) \lesseqgtr 1, \tag{28}$$

[4]Equation (15) shows that holding the inflation rate constant, an increase in μ leads to a lower consumption-to-capital ratio z_t. This requires an intertemporal substitution from current to future consumption, thus raising today's investment.

thus the sign of $\frac{d\theta}{d\beta}$ is theoretically ambiguous. In particular, whether wealth-induced preferences for social status enhance or harm economic growth depends on the magnitude of $f^{'}(p^*)$, which in turn is governed by ϕ_i, σ and $\frac{dz^*}{dp^*}$ (see equation 21).

4.1 When $\sigma = 1$

In this case, the representative household's period utility function becomes separable and logarithmic in c_t and $\frac{k_t}{K_t}$ where[5]

$$U_t = \log c_t + \beta \log \left(\frac{k_t}{K_t} \right), \quad \beta > 0. \tag{29}$$

Substituting $\sigma = 1$ into (18) and (19) yields that the economy possesses a unique balanced-growth equilibrium characterized by

$$p^* = 1 + \phi_i \left(\mu + \rho \right) \tag{30}$$

and

$$z^* = \frac{\rho + \phi_i \left(\mu + \rho \right) \left(A + \rho \right)}{1 + \beta + \left(\beta \phi_c + \phi_i \right) \left(\mu + \rho \right)}. \tag{31}$$

We then plug (30), (31) and $f^{'}(p^*) = 0$ into (24) and (25), and find that the determinant and trace of the model's Jacobian matrix J are both positive, indicating that none of the two eigenvalues has negative real part. As a result, the balanced-growth equilibrium exhibits saddle-path stability. This implies that given the initial capital stock k_0 and nominal money supply M_0, the period-0 consumption c_0 as well as the price level P_0 will be uniquely determined such that the economy immediately reaches its balanced-growth values of p^* and z^*, given by (30)-(31), and always stays there without the possibility of deviating transitional dynamics. Therefore, equilibrium indeterminacy and endogenous growth fluctuations can never occur in this setting.

In terms of the output-growth impact of money or inflation, we note that $\frac{dp^*}{d\mu} = \phi_i > 0$ because of a dominating portfolio substitution effect; and that

[5] In Chen and Guo's (2009) monetary endogenous growth model, the representative household has the following separable period utility function in consumption and its capital ownership:

$$U_t = \frac{c_t^{1-\sigma} - 1}{1 - \sigma} + \beta \frac{k_t^{1-\sigma} - 1}{1 - \sigma}, \quad \beta > 0 \text{ and } \sigma \geq 1,$$

and faces Stockman's (1981) liquidity constraint given by $c_t + i_t \leq m_t$. In this case, the economy's unique balanced-growth equilibrium is a saddle path, and the output-growth effect of money/inflation is always negative.

$$\frac{dz^*}{dp^*} = \frac{A(1+\beta) + \rho\beta\left(1 - \frac{\phi_c}{\phi_i}\right)}{\left[1+\beta+(\beta\phi_c+\phi_i)(\mu+\rho)\right]^2} \gtreqless 0. \tag{32}$$

Substituting these expressions into (26) leads to

$$\frac{d\theta}{d\mu} \gtreqless 0 \quad \text{when} \quad \frac{\phi_c}{\phi_i} \gtreqless 1 + \underbrace{\frac{A(1+\beta)}{\rho\beta}}_{\text{positive}}. \tag{33}$$

Intuitively, when consumption is sufficiently more liquidity-constrained relative to gross investment such that $\frac{\phi_c}{\phi_i} > 1 + \frac{A(1+\beta)}{\rho\beta}$, it will be much less stringent for the household to accumulate physical capital in response to an increase in the growth of nominal money supply. As a result, the BGP's consumption-to-capital ratio z^* may fall, which in turn generates a faster GDP growth through stimulated capital accumulation. On the contrary, the economy displays a negative growth effect of money/inflation $\left(\frac{d\theta}{d\mu} < 0\right)$ if the ratio of the CIA-constrained proportion of consumption expenditures to the investment fraction $\frac{\phi_c}{\phi_i}$ is smaller than the critical value given in (33). Notice that this negative relationship continues to hold when agents' preferences do not include a status-seeking motive ($\beta = 0$).

Next, plugging $f'(p^*) = 0$ into (28) shows that $\frac{d\theta}{d\beta} > 0$, indicating a positive growth effect of social status. When the economy exhibits a higher degree for "the spirit of capitalism", it provides strong incentives for the representative household to accumulate more physical capital that raises its social status and thus utilities. This will lower the equilibrium consumption-to-capital ratio, which in turn enhances the BGP's output growth.

4.2 When $\sigma > 1$ and $\frac{dz^*}{dp^*} > 0$

Figure 1 shows that $f(p^*)$ in this formulation is a downward-sloping and convex curve that intersects the 45-degree line once in the positive quadrant, hence there exists a unique balanced-growth equilibrium characterized by p^*. Regarding local dynamics, it is straightforward to show that the model's Jacobian matrix J possesses a positive determinant and a positive trace, indicating the BGP's saddle-path stability. Therefore, as in the previous case with $\sigma = 1$, the economy jumps onto the balanced-growth equilibrium path at the initial period, and then stays on it for all t. On the other hand, since $f'(p^*) < 0$, equation (28) shows that the BGP's output growth rate and the strength of agents' status-seeking motive are positively related $\left(\frac{d\theta}{d\beta} > 0\right)$.

Figure 1 also shows that due to a stronger portfolio substitution effect, an increase in μ shifts the locus of $f(p^*)$ to the right such that $\frac{dp^*}{d\mu} > 0$ (see equation 27). Moreover, since the liquidity-constrained ratio of consumption to investment $\frac{\phi_c}{\phi_i}$ is smaller than the threshold level give by $1 + \frac{A}{\beta z^*}$, a higher inflation rate raises the household's relative cash-in-advance cost for capital accumulation, which in turn increases the BGP's consumption-to-capital ratio z^*. This leads to a negative relationship between money/inflation and the rate of economic growth $\frac{d\theta}{d\mu} < 0$ because p^* and z^* are moving in the same direction (see equation 33).

4.3 When $\sigma > 1$ and $\frac{dz^*}{dp^*} < 0$

Figure 2 shows that $f(p^*)$ in this formulation is a upward-sloping and concave curve, hence the number of intersections between $f(p^*)$ and the 45-degree line in the positive quadrant can be zero, one or two. First, we derive the critical value of σ, denoted as $\hat{\sigma}$, at which $f(p^*)$ is tangent to the 45-degree line so that there exists a unique balanced-growth equilibrium characterized by $\hat{p}$ and $\theta(\hat{p})$. Using (21) with $f'(\hat{p}) = 1$ and (18) evaluated at $\hat{p}$, it is straightforward to show that $\hat{\sigma}$ and $\hat{p}$ are jointly determined by

$$\hat{\sigma} = \frac{\phi_i A + \beta \left\{ \left(1 - \frac{\phi_c}{\phi_i}\right) \left[\phi_i \left(A + \frac{\hat{p}-1}{\phi_i} - \delta - \rho - \mu\right) - \hat{p}\right] - \hat{p}^2 \frac{\phi_c}{\phi_i} \right\}}{\hat{p}^2 + \phi_i \left[A + \beta (A - \delta) \left(1 - \frac{\phi_c}{\phi_i}\right)\right]}, \tag{34}$$

and

$$\hat{p} = \frac{(\hat{\sigma} - 1) \left\{ 1 + \phi_i \left[-A + \mu + \left(1 + \beta \frac{\phi_c}{\phi_i}\right) \left(A - \delta + \frac{\rho + \mu + \frac{1}{\phi_i}}{\hat{\sigma} - 1}\right)\right]\right\} + \beta \left(1 - \frac{\phi_c}{\phi_i}\right)}{2 \left(\hat{\sigma} + \beta \frac{\phi_c}{\phi_i}\right)}. \tag{35}$$

Next, we note that a higher σ shifts the locus of $f(p^*)$ upwards because

$$\frac{\partial f(p^*)}{\partial \sigma} = \frac{\theta p^* \left[\sigma p^* + 1 + (p^* - 1) \frac{\phi_c}{\phi_i}\right]}{\left\{\sigma p^* + \beta \left[1 + (p^* - 1) \frac{\phi_c}{\phi_i}\right]\right\}^2} > 0. \tag{36}$$

As a result, the economy possesses no (two) balanced growth path(s) provided $\sigma < (>) \hat{\sigma}$. Since $\frac{d\theta}{dz^*} = -1$ and the BGP's p^* and z^* are now moving in the opposite direction $\left(\text{or } \frac{\phi_c}{\phi_i} > 1 + \frac{A}{\beta z^*}\text{; see equation 20}\right)$, using the chain rule yields

$$\frac{d\theta}{dp^*} = \underbrace{\frac{d\theta}{dz^*}}_{\text{negative}} \underbrace{\frac{dz^*}{dp^*}}_{\text{negative}} > 0. \tag{37}$$

Therefore, when there are two BGP equilibria, the equilibrium path with a higher utility value of physical capital, denoted as p_2^* in Figure 2, will grow faster than the other associated with p_1^*. That is, $\theta(p_2^*) > \theta(p_1^*)$. Notice that this finding, as well the subsequent results on the growth effect of money/inflation and the BGP's local dynamics, turns out to be exactly the opposite of those in Suen and Yip (2005), and Chen and Guo (2008*a*, 2008*b*) in which $\sigma < 1$, $\frac{dz^*}{dp^*} > 0$ and $\beta = 0$.

Figure 2 also shows that in this case, $f(p^*)$ shifts up in response to an increase in the growth rate of nominal money supply μ. If the economy starts at the high-growth BGP equilibrium, a stronger portfolio substitution effect raises the relative shadow price of capital $\left(\frac{dp_2^*}{d\mu} > 0\right)$, which in turn leads to a positive growth effect of money/inflation ($\frac{d\theta\left(p_2^*\right)}{d\mu} > 0$; see equation 33 with $\frac{dz^*}{dp^*} < 0$). Conversely, the intertemporal substitution effect outweighs the portfolio substitution effect along the low-growth equilibrium path. It follows that the relative price of physical capital falls $\left(\frac{dp_1^*}{d\mu} < 0\right)$, and that the BGP's output growth and money/inflation are negatively correlated ($\frac{d\theta\left(p_1^*\right)}{d\mu} < 0$).

In terms of local stability properties, we find that (after some tedious algebra) around the balanced growth path associated with p_2^*, the model's Jacobian matrix possesses a positive trace and a positive determinant. Thus, as in the cases analyzed above, this high-growth equilibrium is a saddle path. On the other hand, in the neighborhood of the BGP equilibrium associated with p_1^*, the determinant of the Jacobian is negative, indicating that one of the eigenvalues has negative real part. It follows that the low-growth equilibrium exhibits indeterminacy and sunspots. The intuition for this indeterminacy result can be understood as follows. When agents expect a higher future return on capital, they will reduce consumption and increase investment today. If the intertemporal substitution effect (from consumption to investment) is sufficiently strong, the net rate of return on physical capital will rise because of a decline in its relative shadow price p^*. As a result, agents' initial optimistic expectations become self-fulfilling. On the contrary, equilibrium indeterminacy does not occur when the portfolio substitution effect (from real balances to capital) dominates. In this case, agents' optimism leads to a lower net rate of return on capital because of the increase in its relative shadow price of capital p^*, thus preventing agents' expectations from becoming self-fulfilling.

Finally, it is straightforward to show that $f(p^*)$ shifts up when the degree for the "spirit of capitalism" β rises $\left(\frac{\partial f(p^*)}{\partial \beta} > 0\right)$. Consequently, the BGP's output growth rate and the strength of social status are positively related $\left(\frac{d\theta}{d\beta} > 0\text{; see equation 28 with } 0 < f'(p_2^*) < 1\right)$

along the high-growth equilibrium path since a higher β raises the relative shadow price of physical capital and lowers the economy's consumption-to-capital ratio. By contrast, the low-growth equilibrium path with $f'(p_1^*) > 1$ exhibits negative output-growth impact of social status due to a dominating intertemporal substitution effect.

5 Conclusion

Starting in late 2008, the Federal Reserve has substantially increased the quantity of nominal money supply in the U.S. economy to combat the economic downturn and financial crisis. Motivated by this observed monetary expansion, we systematically examine the theoretical interrelations between wealth-induced preferences for social status, the most generalized cash-in-advanced constraint and the equilibrium growth impact of money/inflation within a canonical one-sector AK model of endogenous growth. Our analysis shows that the sign for the correlation between money/inflation and output growth is governed by (i) whether the liquidity-constrained ratio of consumption to investment is higher or lower than a threshold level; and (ii) how the utility value of physical capital responds to a change in the monetary growth rate, which is determined by the relative strength of two opposing forces dubbed as the portfolio substitution effect and the intertemporal substitution effect. Moreover, this money-growth correlation, as well as the growth effect of social status, is closely related to the local stability properties of the economy's balanced growth path(s).

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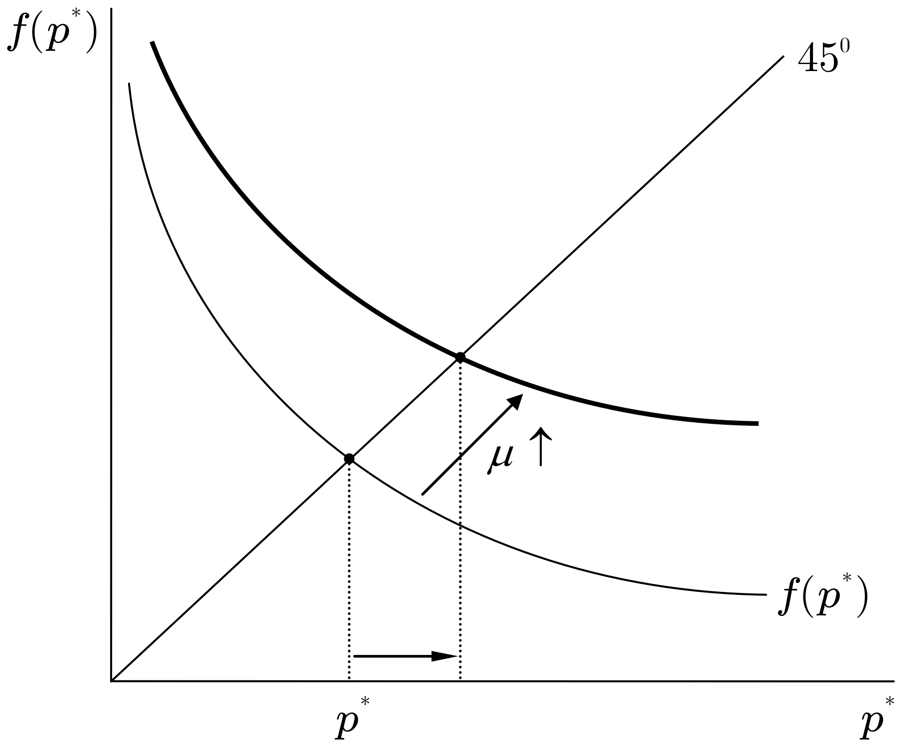

Figure 1: $\sigma > 1$ and $\dfrac{dz^*}{dp^*} > 0$

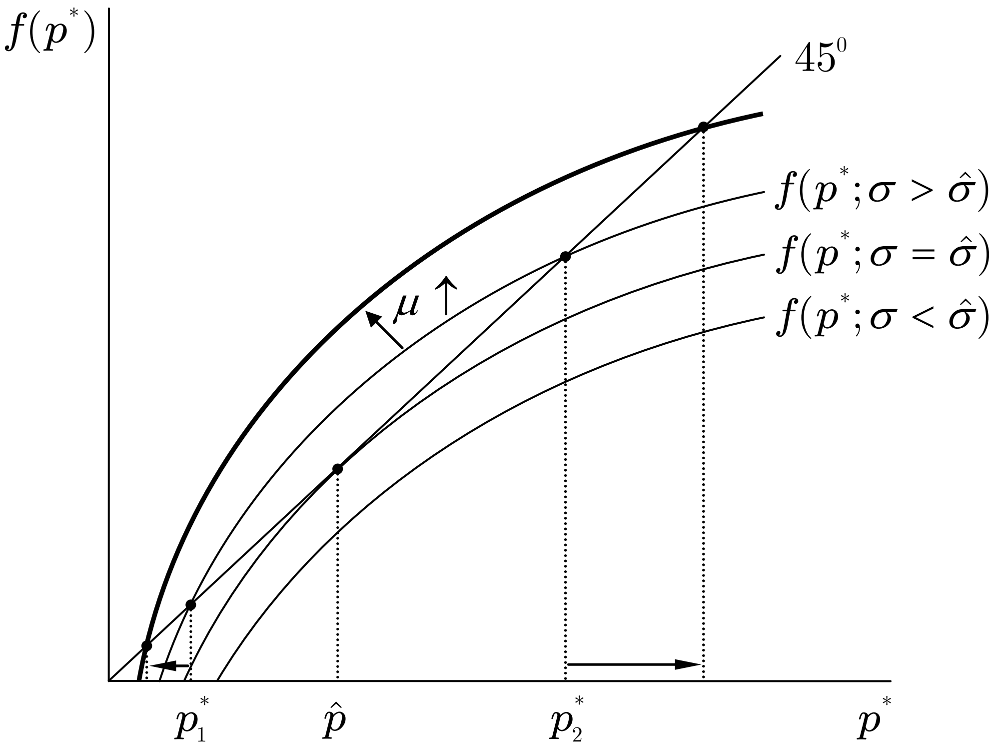

Figure 2: $\sigma > 1$ and $\dfrac{dz^*}{dp^*} < 0$