Bias-Corrected Production Frontiers: Application to Productivity Growth and Convergence

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Abstract

Data envelopment analysis (DEA), originally developed to study production efficiency of micro-level organizations, has recently been used to adopt a production-frontier approach to the analysis of international macroeconomic growth and convergence patterns. DEA methods, however, are known to provide biased estimates of the production frontier and efficiency. We employ recent results from the statistics/econometrics literature (and more recent, more extensive, and more accurate data than previously employed) to construct the unbiased convex production frontier and then use this construction to decompose productivity growth into components attributable to technological change (shift of the production frontier), efficiency change (movements toward or away from the frontier), physical capital deepening, and human capital accumulation. Using this decomposition, we analyze the evolution of the world productivity distribution, especially the shift from uni-modality to bimodality over the 1965–2000 period.

Keywords: Convergence, Growth, Data Envelopment Analysis, Nonparametric, Production Frontier

JEL: C14, Semiparametric and Nonparametric Methods O57, N10, Macroeconomics and Monetary Economics; Growth and Fluctuations: General, International, or Comparative

1 Introduction

Nonparametric production-frontier methods—envelopment of input and output quantity data in the "smallest" or "tightest fitting" convex set (or convex cone)—have been extensively employed over the last several decades in many areas of economics. The principal objective of these methods has been to construct efficiency scores for individual "decision making units" (firms and other types of organizations).

Most of the applications of these methods, commonly referred to as Data Envelopment Analysis (DEA), have employed micro-level data. In recent years, however, these methods have been used to analyze macroeconomic growth and convergence. Färe et al. (1994b) constructed a nonparametric production frontier for OECD countries in order to decompose productivity improvement into two components: technological change (shifts of the frontier in the region of input-output space occupied by a given country) and improvements (or deteriorations) in efficiency (distance of a country from the frontier). Kumar and Russell (2002) constructed a worldwide frontier (for countries in the Penn World Table) and decomposed labor productivity growth into components attributable to technological change, changes in efficiency, and capital deepening (movements along the frontier). Los and Timmer (2005) adopted the same framework but referred to these three components of productivity change, respectively, as "localized innovation, assimilation of knowledge spillovers, and creating potential for appropriate technology spillovers through investment" (page 518). Kumar and Russell went on to use this decomposition to analyze the evolution of the worldwide distribution of productivity, most importantly the transformation over time from a uni-modal to a bimodal distribution. Los and Timmer used their decomposition to "focus in depth on growth 'miracles' and 'disasters' " (page 524).

Henderson and Russell (2005) incorporated human capital into the Kumar-Russell structure, thus decomposing labor productivity growth into four components, including human capital accumulation. Badunenko et al. (2008) updated the Kumar-Russell study, discovering an apparent structural change in the growth process in the 1990s. A significant number of studies adopting the framework of these papers has been published (or written) in recent years (many of them studying particular regions of the world or provinces within countries), but the papers cited above are particularly relevant to the analysis of this paper.

The principal advantages of the nonparametric production-frontier approach are the elimination of the need to to specify a functional form for the technology, to maintain neutrality of technological change, or to make assumptions about market structure or the absence of market imperfections. The approach can also handle data sets with variables so numerous as to pose insurmountable problems for parametric econometric analysis.

The basic DEA approach has two fundamental drawbacks. First, as a putatively deterministic method, it ignores stochastic phenomena like measurement error and hence does not allow for confidence intervals and other types of statistical inference. Failure to take account of stochastic factors is especially important in this case because the DEA construction of the frontier can be very sensitive to outliers.[1] As pointed out by Simar and Wilson (1998, 2000), owing to measurement error, a frontier constructed by DEA methods should be treated as an *estimate* of the frontier based on a single sample drawn from some unknown population.

A second (related) problem with the basic DEA approach to the construction of production frontiers is that this estimate, while consistent,[2] is biased. This is because the construction, at best, identifies a lower bound on the true frontier; that is, it identifies a "best practice" frontier, not the "true" frontier.

Simar and Wilson (1998, 2000) and others have taken us a long way toward rectification of these problems. Using bootstrap methods, they design a procedure for consistent estimation of the production frontier and the efficiency scores, with standard errors and confidence intervals. The starting point of our analysis is that these statistical procedures can also be used to reconstruct the frontier after the bootstrapping procedure has been completed. In particular, we propose a method to restore a convex frontier after correcting for the inherent bias of the basic DEA procedure. Specifically, we use DEA to obtain the efficiency scores, then use the Simar-Wilson method to correct for the bias, then con-

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[1] Koop et al. (1999) state that "the sensitivity of DEA to outliers is no doubt one of the weaknesses of the DEA approach. In particular, it is difficult to present some measure of uncertainty (*e.g.*, confidence intervals) using DEA methods."

[2] See Kneip et al. (1998) for a proof of consistency of the DEA estimator, as well as Kneip et al. (2008) for its limiting distribution.

struct the unbiased convex frontier, and finally evaluate the original data to obtain the unbiased convex efficiency scores under the assumption of convexity of the technology.[3]

We use the Henderson and Russell (2005), hereafter HR, study as a stepping stone and compare our unbiased frontier decomposition with the results of their study. Along the way, we make several other contributions to this literature. First, we increase the HR sample of countries studied by nearly a third (substantially increasing coverage of the African continent). Second, we extend their data, which ends at 1990, to the year 2000. Finally, we employ the most recently available—and most reliable—data on educational attainment across both developed and developing countries (Cohen and Soto (2007)); these data are used to construct human capital indexes.

Our results confirm the HR finding that technological change is decidedly non-neutral, with all technological progress taking place among richer countries. Although efficiency change was the primary factor causing bimodality in HR, we find that efficiency is *the* cause of the emergence of bimodality in the distribution of output per worker. Whereas capital accumulation played the major role in the shift of the distribution of output per worker in HR, it is found to be less important here; instead, both technology and human capital accumulation significantly contributed to the shift of the distribution. Further analysis shows that the contributions differ significantly between groups of countries and over time. As did HR, we find that during the 1965-1990 period, OECD productivity growth was generated by both physical capital accumulation and technological change. During the 1990's, however, OECD countries made huge strides in the state of technological, whereas physical capital accumulation did not appear to be as important. During the same time period, non-OECD economies made relatively large investments in physical capital.

The remainder of this paper is organized as follows. The second section describes the standard DEA estimator, the bias-corrected efficiency scores, the construction of the unbiased convex frontier, and the estimation of the unbiased efficiency scores under the assumption of convexity. Section 3 discusses the data. The fourth section provides the

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[3] We are not the first to implement the Simar-Wilson estimation procedure; Enflo and Hjertstrand (2008) use this method to obtain unbiased estimates of efficiency scores using European regional data, but they do not construct the unbiased convex frontier; consequently, the frontier implicit in their analysis is highly non-convex. More on this later.

results for the convergence study. Section 5 reports on the results of a battery of robustness checks for that study. The final section concludes.

2 Efficiency measurement revisited

2.1 Data envelopment analysis

Our technology contains four macroeconomic variables: aggregate output and three aggregate inputs: labor, physical capital, and human capital. Let $\langle Y_{it}, K_{it}, L_{it}, H_{it}\rangle$, $t = 1, 2, \ldots, T$, $i = 1, 2, \ldots, N$, represent T observations on these four variables for each of the N countries. As is standard in the macroeconomic literature, we assume that human capital enters the technology as a multiplicative augmentation of physical labor input, so that our NT observations can be written as $\langle Y_{it}, K_{it}, \hat{L}_{it}\rangle$, $t = 1, 2, \ldots, T$, and $i = 1, 2, \ldots, N$, where $\hat{L}_{it} = L_{it}H_{it}$ is the amount of labor input measured in *efficiency* units in country i at time t. Utilizing the "sequential production set" formulation of Diewert (1980) to preclude implosion of the frontier over time, we construct the convex, free-disposal, constant-returns-to-scale technology in period t, using all the data up to that point in time, as

$$\mathcal{T}_t = \left\{ \left\langle Y, \hat{L}, K \right\rangle \in \Re^3_+ \mid Y \leq \sum_{\tau \leq t} \sum_i z_{i\tau} Y_{i\tau},\ \hat{L} \geq \sum_{\tau \leq t} \sum_i z_{i\tau} \hat{L}_{i\tau},\ K \geq \sum_{\tau \leq t} \sum_i z_{i\tau} K_{i\tau},\ z_{i\tau} \geq 0 \ \forall\, i, \tau \right\}, \tag{1}$$

where $z_{i\tau}$ are the activity levels. Figure 1 schematically presents the idea of constructing the frontier under the non-implosion assumption in a hypothetical two-dimensional case with two periods, a base period and a current period.

The Farrell (1957) (output-based) efficiency score for country i at time t is defined by

$$\mathrm{TE}_{it} = E(Y_{it}, \hat{L}_{it}, K_{it}) = \min \left\{ \lambda \mid \left\langle Y_{it}/\lambda, \hat{L}_{it}, K_{it} \right\rangle \in \mathcal{T}_t \right\}. \tag{2}$$

This score is the inverse of the maximal proportional amount that output Y_{it} can be expanded while remaining technologically feasible, given the technology and input quantities. It is less than or equal to unity and takes the value of unity if and only if the it observation is on the period-t production frontier. In our special case of a scalar output, the output-based efficiency score is simply the ratio of actual to potential output evaluated at the actual input quantities.

Figure 1 here

2.2 Bias-corrected efficiency scores

The technology in equation (2) is necessarily *estimated* with error, and since DEA constructs a lower bound on the "true" technology, this estimate is biased: the estimated $\widehat{\mathcal{T}}_t$ is likely to be a subset of some unknown true technology in period t, $\mathcal{T}_t$. Hence, the efficiencies *estimated* relative to $\widehat{\mathcal{T}}_t$ are too optimistic. The bootstrap procedures proposed by Simar and Wilson (1998, 2000) and Kneip et al. (2008) to correct the bias uses the idea that the known distribution of the difference between estimated and bootstrapped efficiency scores mimics the unknown distribution of the difference between the true and the estimated efficiency scores. This relationship facilitates estimation of the bias and confidence intervals for the individual estimated efficiency scores.

Under some reasonable assumptions (see Kneip et al. (1998)), the DEA estimator $\widehat{\mathrm{TE}}_{it}\left(\langle Y_{it}, \hat{L}_{it}, K_{it}\rangle \mid \widehat{\mathcal{T}}_t\right)$ at any fixed point $\langle Y_{it}, \hat{L}_{it}, K_{it}\rangle$ defined in equation (2) is consistent, and its sampling distribution can be estimated via a consistent bootstrap analog, as shown in Kneip et al. (2008). Specifically, using the original data set $\{\langle Y_{it}, \hat{L}_{it}, K_{it}\rangle\}_{i=1}^n$ at time t, we can obtain a bootstrap sample $\{\langle Y_{it}^s, \hat{L}_{it}^s, K_{it}^s\rangle\}_{i=1}^n$ at time t and use the DEA estimator to obtain the analog of the time-t technology set,

$$\begin{aligned}\widehat{\mathcal{T}}_t^s = \Big\{\langle Y, \hat{L}, K\rangle \in \Re_+^3 \mid Y \le \sum_i z_{it} Y_{it}^s,\ \hat{L} \ge \sum_i z_i \hat{L}_{it}^s,\\ K \ge \sum_i z_i K_{it}^s,\ z_i \ge 0\, \forall\, i\Big\}.\end{aligned} \tag{3}$$

We then calculate a consistent bootstrap estimate of technical efficiency at the same fixed point $\langle Y_{it}, \hat{L}_{it}, K_{it}\rangle$ using

$$\widehat{\mathrm{TE}}_{it}^s\left(\langle Y_{it}, \hat{L}_{it}, K_{it}\rangle \mid \widehat{\mathcal{T}}_t^*\right) = \min\left\{\lambda \mid \left\langle Y_{it}/\lambda, \hat{L}_{it}, K_{it}\right\rangle \in \widehat{\mathcal{T}}_t^s\right\}.$$

Intuitively, the unknown distribution of the difference between the true and estimated efficiency score is approximated by the distribution of the difference between the estimated and the bootstrapped efficiency score, which, in principal, is known. This relationship allows for consistent estimation of the bias and of confidence intervals for $\widehat{\mathrm{TE}}_{it}\left(\langle Y_{it}, \hat{L}_{it}, K_{it}\rangle \mid \widehat{\mathcal{T}}_t\right)$ at any fixed point $\langle Y_{it}, \hat{L}_{it}, K_{it}\rangle$.

In practice, the bootstrap distribution at time t can be approximated by generating S bootstrap samples,[4] $\{\langle Y^s_{it}, \hat{L}^s_{it}, K^s_{it}\rangle\}^n_{i=1}$, $s = 1, 2, \ldots, S$. Technical efficiency is then re-estimated S times, yielding $\widehat{\mathrm{TE}}^s_{it}\left(\langle Y_{it}, \hat{L}_{it}, K_{it}\rangle \mid \widehat{\mathcal{T}}^s_t\right)$, $s = 1, 2, \ldots, S$. The bootstrap bias estimate and confidence interval for the DEA estimate of $\widehat{\mathrm{TE}}_{it}\left(\langle Y_{it}, \hat{L}_{it}, K_{it}\rangle \mid \widehat{\mathcal{T}}_t\right)$ can then be obtained from the distribution of $\widehat{\mathrm{TE}}^s_{it}\left(\langle Y_{it}, \hat{L}_{it}, K_{it}\rangle \mid \widehat{\mathcal{T}}^s_t\right)$.[5]

2.3 Unbiased efficiency scores

As suggested by Henderson and Zelenyuk (2007), Enflo and Hjertstrand (2008) use the Simar-Wilson bootstrapping approach to obtain bias-corrected efficiency scores. They go on to construct a bias-corrected frontier and use it to decompose productivity growth within the Kumar and Russell (2002) growth-accounting framework. However, the frontier they construct by simply marking up each observation by the estimated efficiency factor and connecting the dots (presumably of non-dominated vectors) can be highly non-convex. In our view, this is not the best approach to growth accounting using the bootstrapping approach. As the fundamental writings in in this area frequently point out,[6] the axiomatic foundation of the DEA approach to efficiency estimation is rooted in activity analysis, attributable primarily to Koopmans (1951) and further developed by Afriat (1972). A salient (and compelling) axiom of activity analysis is the additivity axiom, which says that any number of feasible activities (or processes) can be constructed as a linear combination of "basic" processes. Basic processes, existential in activity analysis, are determined by individual, observed ⟨input, output⟩ vectors in DEA. Implementation of Koopmans' additivity axiom then generates a convex technology. We do not think it

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[4] S should be rather large.

[5] Specifically, two types of sampling procedures can be used: (i) sub-sampling from $\{\langle Y_{it}, \hat{L}_{it}, K_{it}\rangle\}^n_{i=1}$ and (ii) sampling from the smooth kernel-based density estimate of $\widehat{\mathrm{TE}}_{it}\left(\langle Y_{it}, \hat{L}_{it}, K_{it}\rangle | \widehat{\mathcal{T}}_t\right)$. Kneip et al. (2008) show that the first procedure is consistent, while the second one, as was suggested and outlined in Simar and Wilson (1998), is an approximation of the first procedure. Given our rather small sample, we adopt the second approach.

[6] See, *e.g.*, the comprehensive treatments of DEA and related production-frontier methods in Färe et al. (1985), Färe et al. (1994a), and Charnes et al. (1994) as well as the pioneering paper by Farrell (1957).

consistent with the salience of convexity in DEA analysis to adopt a non-convex technology in growth accounting predicated on the activity-analysis approach to constructing the production frontier.

Our approach, then, is to build on the Enflo-Hjerstrand construction by first generating a new sample of observed quantities augmented by the efficiency factor and then employing the additivity axiom to construct the convex hull of this sample. This approach is illustrated with an (artificial) two-dimensional sample in Figure 2. The artificial data points are represented by the empty rhombi, and the DEA frontier is given by the dashed kinked line. The filled rhombi are the bias-adjusted observations, and the piecewise linear curve connecting these points is the "bias-adjusted frontier." We then take the convex free-disposal hull of these points to obtain the "bias-adjusted convex frontier," shown as the solid kinked line.

Figure 2 here

The mechanics of constructing the unbiased convex frontier in a cross-sectional case are as follows:

(i). run DEA on the data set $\{\langle Y_i, \hat{L}_i, K_i\rangle\}_{i=1}^{n}$ to obtain the technical efficiency scores, $\text{TE}_i,\ i = 1, \ldots, n$;

(ii). using the Simar and Wilson (1998, 2000) method, obtain the bias-corrected scores $\text{TE}_i^{\text{corr}},\ i = 1, \ldots, n$, via a bootstrap on $\{\langle Y_i, \hat{L}_i, K_i\rangle\}_{i=1}^{n}$;

(iii). obtain the unbiased convex frontier and efficiency scores by running DEA on $\left\{\langle Y_i/\text{TE}_i^{\text{corr}}, \hat{L}_i, K_i\rangle\right\}_{i=1}^{n}$.

The procedure for the construction of the unbiased convex frontier under the assumption of non-implosion requires some additional notation. We describe the procedure for the special case, which we do employ below, of two comparison periods, a base period, denoted b, and a current period, denoted c. The procedure takes the following steps:

(i). run DEA on the base-period data set $\{\langle Y_{ib}, \hat{L}_{ib}, K_i\rangle\}_{i=1}^{n}$ to obtain the base-period technical efficiency scores, $\text{TE}_{ib},\ i = 1, \ldots, n$;

(ii). run DEA on the current-period data set $\{\langle Y_{ic}, \hat{L}_{ic}, K_i\rangle\}_{i=1}^n$ to obtain the current-period technical efficiency scores, $\mathrm{TE}_{ic},\ i = 1, \ldots, n$;

(iii). using the Simar and Wilson (1998, 2000) method, obtain the base-period bias-corrected scores $\mathrm{TE}_{ib}^{\mathrm{corr}},\ i = 1, \ldots, n$, via a bootstrap on $\{\langle Y_{ib}, \hat{L}_{ib}, K_{ib}\rangle\}_{i=1}^n$;

(iv). using the Simar and Wilson method, obtain the current-period bias-corrected scores $\mathrm{TE}_{ic}^{\mathrm{corr}},\ i = 1, \ldots, n$, via a bootstrap on $\{\langle Y_{ic}, \hat{L}_{ic}, K_{ic}\rangle\}_{i=1}^n$;

(v). obtain the base-period unbiased convex frontier and corresponding efficiency scores by running DEA on $\left\{\langle Y_{ib}/\mathrm{TE}_{ib}^{\mathrm{corr}}, \hat{L}_{ib}, K_{ib}\rangle\right\}_{i=1}^n$.

(vi). obtain the current-period unbiased convex frontier and concomitant efficiency scores by running DEA on $\left\{\langle Y_{ib}/\mathrm{TE}_{ib}^{\mathrm{corr}}, \hat{L}_{ib}, K_{ib}\rangle\right\}_{i=1}^n \bigcap \left\{\langle Y_{ic}/\mathrm{TE}_{ic}^{\mathrm{corr}}, \hat{L}_{ic}, K_{ic}\rangle\right\}_{i=1}^n$.

The last step prevents implosion of the frontier. The procedure can easily be extended to incorporate additional previous periods.

We might note that there is an alternative way to construct a convex frontier using the unbiased estimates of efficiency scores. The alternative is to adjust only the efficient data points in the basic DEA for the estimated bias. This frontier would coincide with the unbiased convex frontier unless some inefficient country in the original DEA calculation has a standard error so large that it overtakes the unbiased convex country after correcting for the bias.[7] Given that such cases occur with positive measure, it is important to emphasize this point.[8]

For convenience (and owing to the lack of imagination to come up with a better term), we refer to the unbiased convex frontier as the "true" frontier and to the concomitant efficiency estimate as the "true" efficiency score in the analysis that follows.

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[7] It is not exactly clear in Enflo and Hjertstrand (2008) how they construct their frontier. However, the authors graciously allowed us to view their data. We were able to replicate their results and found that they employ the bias-corrected frontier and efficiency scores. The confusion occurs because they plot the bias-corrected frontier obtained obtained using the alternative method (their Figure 1). They do not, however, evaluate the scores with respect to that frontier. That is, they adopt a non-convex technology and thus have higher efficiency scores than are obtained using our method. In contrast, our procedure of constructing a convex technology deals with the presence of inefficient points that define the unbiased frontier. as hypothetically displayed in Figure 2a.

[8] In fact, this phenomenon occurs in one of the robustness exercises described in Section 5.

2.4 Quadripartite decomposition

The HR growth accounting approach decomposes productivity growth into components attributable to (1) changes in efficiency (technological catch-up), (2) technological change (shifts in the frontier), (3) capital deepening (increases in the capital-labor ratio), and (4) human capital accumulation. Under the assumption of constant returns to scale, the production frontier can be represented in $\langle \hat{k}, \hat{y} \rangle$ space by a function $\hat{y}(\hat{k})$, where $\hat{y} = Y/\hat{L}$ and $\hat{k} = K/\hat{L}$ are the ratios of output and capital, respectively, to effective labor. The "true" potential outputs per efficiency unit of labor in the base period and the current period are defined, respectively, by $\overline{y}_b(\hat{k}_b) = \hat{y}_b/e_b^{\text{true}}$ and $\overline{y}_c(\hat{k}_c) = \hat{y}_c/e_c^{\text{true}}$, where e_b^{true} and e_c^{true} are the values of the efficiency scores in the respective periods as calculated in Steps (v) and (vi) above. The use of e_b^{true} and e_c^{true} in this last step is where our decomposition departs from that of HR. Dividing the current period ratio of output to effective labor ratio by that in the base period yields

$$\frac{\hat{y}_c}{\hat{y}_b} = \frac{e_c^{\text{true}}}{e_b^{\text{true}}} \cdot \frac{\overline{y}_c(\hat{k}_c)}{\overline{y}_b(\hat{k}_b)}. \tag{4}$$

Let $\tilde{k}_c = K_c/(L_c H_b)$ denote the ratio of capital to labor measured in efficiency units under the counterfactual assumption that human capital had not changed from its base period and let $\tilde{k}_b = K_b/(L_b H_c)$ denote the ratio of capital to labor measured in efficiency units under the counterfactual assumption that human capital were equal to its current-period level. Then $\overline{y}_b(\tilde{k}_c)$ and $\overline{y}_c(\tilde{k}_b)$ are the "true" potential output per efficiency unit of labor at $\tilde{k}_c$ and $\tilde{k}_b$ using the base-period and current-period technologies, respectively. By multiplying the numerator and denominator of Eq. (4) alternatively by $\overline{y}_b(\hat{k}_c)\overline{y}_b(\tilde{k}_c)$ and $\overline{y}_c(\hat{k}_b)\overline{y}_c(\tilde{k}_b)$, we obtain two alternative decompositions of the growth of $\hat{y}$

$$\frac{\hat{y}_c}{\hat{y}_b} = \frac{e_c^{\text{true}}}{e_b^{\text{true}}} \cdot \frac{\overline{y}_c(\hat{k}_c)}{\overline{y}_b(\hat{k}_c)} \cdot \frac{\overline{y}_b(\tilde{k}_c)}{\overline{y}_b(\hat{k}_b)} \cdot \frac{\overline{y}_b(\hat{k}_c)}{\overline{y}_b(\tilde{k}_c)}, \tag{5}$$

and

$$\frac{\hat{y}_c}{\hat{y}_b} = \frac{e_c^{\text{true}}}{e_b^{\text{true}}} \cdot \frac{\overline{y}_c(\hat{k}_b)}{\overline{y}_b(\hat{k}_b)} \cdot \frac{\overline{y}_c(\hat{k}_c)}{\overline{y}_c(\tilde{k}_b)} \cdot \frac{\overline{y}_c(\tilde{k}_b)}{\overline{y}_c(\hat{k}_b)}. \tag{6}$$

The growth of productivity, $y_t = Y_t/L_t$, can be decomposed into the growth of output per efficiency unit of labor and the growth of human capital as follows:

$$\frac{y_c}{y_b} = \frac{H_c}{H_b} \cdot \frac{\hat{y}_c}{\hat{y}_b}. \tag{7}$$

Combining Eqs. (5) and (6) with (7), we obtain

$$\begin{aligned} \frac{y_c}{y_b} &= \frac{e_c^{\text{true}}}{e_b^{\text{true}}} \cdot \frac{\overline{y}_c(\hat{k}_c)}{\overline{y}_b(\hat{k}_c)} \cdot \frac{\overline{y}_b(\tilde{k}_c)}{\overline{y}_b(\hat{k}_b)} \cdot \left[\frac{\overline{y}_b(\hat{k}_c)}{\overline{y}_b(\tilde{k}_c)} \cdot \frac{H_c}{H_b} \right] \\ &=: EFF \times TECH^c \times KACC^b \times HACC^b, \end{aligned} \tag{8}$$

and

$$\begin{aligned} \frac{y_c}{y_b} &= \frac{e_c^{\text{true}}}{e_b^{\text{true}}} \cdot \frac{\overline{y}_c(\hat{k}_b)}{\overline{y}_b(\hat{k}_b)} \cdot \frac{\overline{y}_c(\hat{k}_c)}{\overline{y}_c(\tilde{k}_b)} \cdot \left[\frac{\overline{y}_c(\tilde{k}_b)}{\overline{y}_c(\hat{k}_b)} \cdot \frac{H_c}{H_b} \right] \\ &=: EFF \times TECH^b \times KACC^c \times HACC^c. \end{aligned} \tag{9}$$

Eqs. (8) and (9) decompose the growth of labor productivity over the two periods into the change in efficiency (EFF), technological change (TECH), the change in the capital-labor ratio (KACC), and human capital accumulation (HACC). The decomposition in Eq. (5) measures technological change by the shift in the frontier in the output direction at the current-period ratio of capital to effective labor, whereas the decomposition in Eq. (6) measures technological change by the shift in the frontier in the output direction at the base-period ratio of capital to effective labor. Similarly, Eq. (8) measures the effect of physical and human capital accumulation along the base-period frontier, whereas Eq. (9) measures the effect of physical and human capital accumulation along the current-period frontier.

These two decompositions do not yield the same results (that is, the decomposition is path dependent)unless technological change is neutral. This ambiguity is resolved by adopting the "Fisher Ideal" decomposition, defined as the geometric average of the two measures of the effects of technological change, capital deepening, and human capital accumulation and obtained mechanically by multiplying the numerator and denominator of Eq. (4) by $\left(\overline{y}_b(\hat{k}_c)\overline{y}_b(\tilde{k}_c)\right)^{1/2}\left(\overline{y}_c(\hat{k}_b)\overline{y}_c(\tilde{k}_b)\right)^{1/2}$:

$$\begin{aligned}\frac{y_c}{y_b} &= EFF \times (TECH^b \cdot TECH^c)^{1/2} \times (KACC^b \cdot KACC^c)^{1/2} \times (HACC^b \cdot HACC^c)^{1/2} \\ &=: EFF \times TECH \times KACC \times HACC. \end{aligned} \tag{10}$$

3 Data

The data used for output, physical capital and labor are derived from the Penn World Tables (PWT), Version 6.2 (Heston et al. (2006)). The number of workers is obtained as RGDPCH * POP/RGDPWOK, where RGDPCH is per capita GDP computed via the chain method, POP is the population, and RGDPWOK is real GDP per worker. The measure of output is calculated as RGDPWOK multiplied by the number of workers; the resulting output is in international dollars. Real aggregate investment in international dollars is computed as RGDPL * POP * KI, where RDGPL is the real GDP computed via the Laspeyres index and KI is the investment share of real GDP.

The major difference between the measurement of capital stock in HR using PWT version 5.6 versus PWT version the measurement of capital stock in our study using PWT version 6.2 lies in the disaggregation of investment. In Version 5.6, the investment series is disaggregated into five components: machinery, transportation equipment, residential construction, business construction, and other construction. Different depreciation rates (see Hulten and Wykoff (1996)) are then employed in the perpetual inventory method. Here we do not have this level of disaggregation and are forced to use a common depreciation rate, 0.06. Following standard practice, we compute the initial capital stock, K_0, as $I_0/(g+\delta)$, where I_0 is the value of the investment series in the first year it is available, and g is the average geometric growth rate for the investment series between the first year with available data and 1970 (see Caselli and Feyrer (2007)).

For human capital, we depart from HR and employ the Cohen and Soto (2007) education data. These data are widely believed to be more accurate and more reliable for empirical studies than the Barro and Lee (2001) data (see de la Fuente and Doménech (2006)). Using the improved data, we follow HR and adopt the Hall and Jones (1999) construction of human capital, which in turn is based on the Psacharopoulos (1994) survey of wage equations evaluating the returns to education. Specifically, let ϵ_{jt} represent the average number of years of education of the adult population in country j at time t and define labor in efficiency units in country j at time t by

$$\hat{L}_{jt} = H_{jt}L_{jt} = h(\epsilon_{jt})L_{jt} = \exp^{\phi(\epsilon_{jt})} L_{jt} \tag{11}$$

where ϕ is a piecewise linear function, with a zero intercept and a slope of 0.134 through the fourth year of education, 0.101 for the next four years, and 0.068 for education beyond the eighth year. Clearly, the rate of return to education (where ϕ is differentiable) is

$$\frac{\partial \ln h(\epsilon_{jt})}{\partial \epsilon_{jt}} = \phi'(\epsilon_{jt}) \tag{12}$$

and $h(0) = 1$.

There are three main differences between our sample and that of HR. First, we consider a longer and more recent time period, 1965−2000. Second, our sample is larger by approximately one third. Most notably, we have a much better representation of the African continent.[9] Many of these additional countries can be added because of the increased availability of education data in Cohen and Soto (2007) as compared to Barro and Lee (2001). For completeness, a section on robustness includes a check to see if the results of the paper are driven by the inclusion of the additional countries, by the additional time periods or by changing the data set on years of education rather than by our proposed new method of constructing the production frontier..

4 Empirical results

4.1 Efficiency scores

Figure 3 superimposes the "true" production frontiers for 1965 and 2000. The frontiers shown here correspond to the solid one in Figure 2. For purposes of comparison, we present both the DEA "best practice" and the unbiased "true" frontier in Figure 4. While the 1965 and 2000 DEA frontiers are both clearly biased downward, the bias is negligible at very low capital-labor ratios.

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[9] Unlike HR and many studies in this area, we adopt the view of Caselli (2005) that some oil-rich countries are among the most productive in the world and should be retained in the sample.

One fact that emerges immediately from Figure 3 is the non-neutrality of technological change. Up to a capital-labor ratio of approximately 6,000 (Nicaragua in 1965), the 1965 and 2000 frontiers are coincident, but for higher levels of capitalization, the 2000 frontier shifts upward dramatically. This is basically the same result as found in HR, indicating, not surprisingly, that almost all technological change occurs at high levels of capitalization.

Figure 3 here

Figure 4 here

Country-specific estimates of standard DEA efficiency scores (eff_b and eff_c), bias-corrected efficiency scores (eff_b^{bc} and eff_c^{bc}), and "true" efficiency scores (eff_b^{true} and eff_c^{true}) are listed in Table 1. Information in this table and in Figures 3 and 4 can be combined to extract some salient features of the formation of the 1965 and 2000 frontiers.

First note that the productivity levels of Cameroon, Jordan, Mozambique, Nicaragua, and Niger, all with basic DEA efficiency scores of 1.00, define the best practice frontier in 1965 at low levels of capitalization; Venezuela's productivity determines the frontier at higher levels of capitalization (see the last kink in the 1965 best practice frontier in Figure 4). But only four of these economies—Cameroon, Jordan, Mozambique, and Nicaragua—define the "true" frontier in 1965: the bias correction of the efficiency scores drops Niger and Venezuela below the "true" frontier.

As noted above, the frontiers in 1965 and 2000 are coincident up to Nicaragua's value of $\hat{k}$; this implies that the four economies defining the "true" frontier in 1965 continue to form the "true" frontier in 2000 (at their $\hat{y}$ and $\hat{k}$ values for 1965) at relatively low capital-labor ratios. Two countries from 2000, Ireland and Singapore, define the "true" 2000 frontier at higher levels of capitalization.

Table 1 here

Our approach to measuring the "true" frontier does not imply that the economy that defines the unbiased frontier is also the most efficient. Indeed, in 1965, Venezuela turns out to be the most efficient economy, followed by New Zealand, Nicaragua, the Netherlands, Cameroon, Jordan, Niger, the United States, and Switzerland. In 2000, the sequence of the ten most efficient economies in descending order are Singapore, Ireland, Belgium, the United States, Mauritius, Norway, France, Austria, and the Netherlands. Of note are Singapore and Ireland, each of which has considerably higher efficiency scores in 2000 than in 1965.

While HR found that Spain and Mauritius were 100% efficient in 1965, our new method ranks them 27th and 28th, respectively (with the efficiency of each near 0.65). Guatemala, Mexico and Paraguay are also not as efficient as they are in HR. Each of these countries is very close to the frontier in HR, with efficiency scores of 0.96, 0.99, and 0.98, respectively. In our 1965 dataset, Guatemala has a DEA efficiency score of 0.53, a bias-corrected score of 0.47, and a "true" efficiency score of 0.46. Similar comparisons hold for Mexico and Paraguay.

The current-period results, 1990 in HR and 2000 in this paper, of course are not directly comparable, but several interesting findings arise. Each of the efficient countries in 1990 (Italy and Paraguay) fall below the frontier in 2000. Similar declines occur for Mexico and Mauritius. Of all the countries, Ireland may be the most compelling story, showing 82% efficiency in 1990 and defining the frontier for highly capitalized countries in 2000. The Irish ascendancy in the 1990s can be partially explained by Ireland's superb performance in the high-tech manufacturing sector during those years (see Margaritis et al. (2007)).

Over time, the mean efficiency index declines slightly over the 1965–2000 period. Figure 5 plots the distributions of the "true" efficiency index in 1965 and 2000.[10] This picture suggests that a portion of the mass near the top of the distribution was shifted away from the frontier.

Figure 5 here

4.2 Quadripartite decomposition

Table 2 reports the country-specific components of the decomposition of the growth rate of output per worker from 1965 to 2000. The first column shows each country's productivity growth, and the the last four show contributions to productivity growth of the

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[10] For all estimated distributions, we employ a Gaussian kernel and the Sheather and Jones (1991) method for choice of the optimal bandwidth.

four factors: efficiency change, technological change, physical capital accumulation, and human capital accumulation.[11]

On average, the contribution of efficiency change is negligible, that of technological change is significant but that of human capital accumulation is almost twice as large, and the contribution of physical capital deepening is twice that of human capital accumulation.[12] These relative contributions correspond roughly to those in HR (using a very different sample), suggesting that correction for the bias of basic DEA does not have a large effect on the relative contributions.[13]

Table 3 reports mean changes in productivity and the four components of productivity change for six groups of countries. OECD and the original EU formation (EU-15) countries experienced productivity gains slightly above the world average, primarily because of faster rates of technological progress and greater efficiency gains. It is interesting to note that physical capital accumulation for OECD countries was smaller than average. We elaborate on this comparison in the next subsection.

Table 3 here

The phenomenal growth rates of the Asian Tigers are attributable primarily to well-above-average contributions of efficiency gains (Singapore) and very large technological changes (Japan and Singapore), although the contributions of physical and human capital accumulations are above average as well.

Productivity in the remaining Asian economies grew as fast as in OECD countries, primarily because of phenomenal rates of physical capital accumulation, especially in India, Indonesia, Nepal, and Syria. Given that the unbiased frontier remained the same for low capital-labor ratios, the extraordinary physical capital performance of the remaining Asian economies moved these observations further from the frontier, which translates into large declines in efficiency.

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[11] Because of compounding, the contributions of individual components do not, of course, sum to the total productivity change.

[12] The mean contributions do not compound to the total productivity change because of the difference between the sum of the logs and the log of the sum.

[13] The corresponding contributions in HR are 0.7, 7.1, 40.5, and 16.6. We examine the robustness of these comparisons in Section 5.

The poor African performance is attributable to large efficiency losses,[14] with the average state of technology almost unchanged. The weak Latin American performance is attributable to even larger efficiency losses than those of Africa, a lack of technological progress, and, especially, low capital accumulation.

Figure 6 contains plots of productivity growth and the four productivity-component growth rates against output per worker in 1965, along with GLS regression lines. Panel A is a standard growth convergence equation; the statistical insignificance of the beta coefficient does not support beta-convergence. Panel B, showing the relationship between the contribution of efficiency to productivity growth and the initial level of productivity, evinces no clear pattern, with many negative as well as positive changes. The regression slope coefficient is statistically insignificant, suggesting that technological catch-up has done little, if anything, to lower income inequality across countries; apparently, technology transfer has benefited relatively rich countries about as much as relatively poor countries. The (statistically significant) positive regression slope coefficient in Panel C indicates that relatively wealthy countries have benefited much more from technological progress than have less-developed countries. Although panel D indicates a wide dispersion of contributions from capital deepening, the negative slope is statistically significant, suggesting that the international pattern of capital accumulation has contributed to convergence. Finally, Panel E demonstrates a wide dispersion of contributions from human capital accumulation as well, but here the slope is statistically insignificant, suggesting that human capital accumulation has done little to contribute to convergence. Of course, each of these interpretations is based on first-moment characterizations of the productivity distribution and is therefore vulnerable to the Quah (1993, 1996, 1997) critique; for that reason, we place much more emphasis on the analyses of the shifts in the productivity distribution in the next subsection.

Figure 6 here

4.3 Analysis of the world distribution of income per worker

A plot of the distributions of output per worker across the 70 countries in our sample in 1965 and 2000 appears in Figure 7. The solid (dashed) curve is the estimated 1965 (2000)

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[14] The average efficiency loss would be even larger if not for the large efficiency improvement of the Ghana economy

distribution of output per worker. The first thing to note is that the distribution is apparently unimodal in 1965 and bimodal in 2000. This observation, confirmed by results of the calibrated Silverman's test for multimodality in lines 1 and 2 of Table 4,[15] extends the finding of HR: the evolving bimodal distribution they found in 1990 holds true through the end of 2000. Following HR, we aim to explain this feature of the change in the productivity distribution from 1965 to 2000, along with the apparent shift to the right of the distribution, in terms of the four components of the decomposition of productivity changes.

Figure 7 here

Table 4 here

Our analysis of the change in the productivity distribution exploits, in addition to the calibrated Silverman test for multimodality, nonparametric methods to test formally for the statistical significance of differences between (actual and counterfactual) distributions. Specifically, we follow HR and choose the test developed by Li (1996) and further studied by Fan and Ullah (1999) to test the null hypothesis, H_0: $f(x) = g(x)$ for all x, against the alternative, H_1: $f(x) \neq g(x)$ for some x. This test, which works with either independent or dependent data, is often used, for example, when testing whether income distributions across two regions, groups, or times are the same.[16]

Table 5 here

Using the quadripartite decomposition of productivity growth, we can explore the role of each of the four components in the transformation of the productivity distribution over the sample period. For this purpose, we rewrite (10) as follows:

$$y_c = (EFF \times TECH \times KACC \times HACC) \times y_b. \tag{13}$$

Accordingly, the labor productivity distribution in the current period (2000) can be constructed by consecutively multiplying the labor productivity in the base period (1965) by each of the four components. To isolate the impact of each component, we create counterfactual distributions by introducing each of the components in sequence. For instance,

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[15] For further details, see Hall and York (2001), Henderson et al. (2008), and Silverman (1981).

[16] For further details, see Fan and Ullah (1999), Li (1996), and Pagan and Ullah (1999).

we assess the shift of the labor productivity distribution attributable solely to efficiency changes by examining the counterfactual distribution of the variable,

$$y^E = EFF \times y_b, \tag{14}$$

assuming no capital deepening, no technological change, and no human capital accumulation. This counterfactual distribution is shown, along with the actual distributions in the base and current periods, as a dotted curve in Panel A of Figure 8. The corresponding vertical lines are inserted at the means of the distributions. The effect of efficiency changes alone is striking: these changes shifted a considerable proportion of the probability mass from the middle of the distribution to the bottom and the top of the distribution, creating an apparently bimodal distribution. Line 3 of Table 4 provides statistical confirmation of this apparent fact: efficiency changes alone account for the shift to bimodality. While this tendency was also observed by HR during the 1965−1990 period (*p-value* = 0.091),[17] Table 4 provides much more convincing evidence that efficiency change alone can account for the shift from uni-modality to bimodality. The Li-test, however, rejects the null hypothesis that efficiency change is solely responsible for moving the 1965 distribution to that of 2000: line 2 of Table 5 decisively rejects the hypothesis that the counterfactual distribution of EFF$\times y_b$ is identical to the actual 2000 productivity distribution.

Figure 8 here

The counterfactual distribution of the variable,

$$y^{EK} = (EFF \times KACC) \times y_b = KACC \times y^E, \tag{15}$$

drawn in Panel B of Figure 8, isolates the joint effect of efficiency changes and capital deepening on the base-period distribution. Introduction of capital deepening does not seem to affect the distribution materially. In fact, the mean of the counterfactual distribution is only slightly higher than the mean of the 1965 distribution. This finding might seem to contradict the fact (from Table 2) that capital accumulation accounts for a large

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[17] The *p-value* in HR is obtained using the standard Silverman test, which has been shown to be conservative in the sense that the true asymptotic level is less than the nominal one. When employing the calibrated Silverman test with the original HR data and set-up, the *p-value* is equal to 0.028. Therefore the null that the counterfactual distribution incorporating only efficiency changes is uni-modal is decisively rejected using the calibrated test for the HR sample as well.

portion of the change in mean productivity over the 1965–2000 period. The reason for this apparent inconsistency is that the last line of Table 2, the unweighted means of productivity change and its components, provides the average importance of capital deepening across countries. But the capital accumulation incorporated into the distribution is essentially weighted by the initial level of output per worker. Recall that physical capital accumulation of OECD economies was relatively small among OECD economies and relatively large among non-OECD economies. The simple average across countries provides an incorrect impression that the total amount of physical capital accumulation had a major impact on the shift in the world-wide distribution. Row 8 of Table 4 and row 7 of Table 5 also indicate that physical capital accumulation did not supplement efficiency change in shifting the productivity distribution. Physical capital accumulation does not appear to exert the driving power found by HR during the shorter 1965−1990 period (using different methods and a different sample of countries.

To incorporate the additional effect of human capital accumulation on the counterfactual distribution, multiply y^{EK} by $HACC$ to obtain

$$y^{EKH} = (EFF \times KACC \times HACC) \times y_b = HACC \times y^{EK}. \tag{16}$$

The resulting counterfactual distribution is shown in Panel C of Figure 8. The distribution remains bimodal (row 15 of Table 4), but the Li-test rejects the hypothesis that the three effects in conjunction move the 1965 income distribution closer to the 2000 income distribution (row 14 of Table 5). The effect of the last component, technological change, can be deduced from comparing the counterfactual distribution of y^{EKH} and the actual distribution in 2000. This finding is quite different from that of HR, who find that efficiency change and physical capital accumulation come close to explaining the transformation of the distribution from during the 1965–1990 period and that these two components along with human capital fully account for the change.

The comparison suggests the possibility of a structural change in the 1990s, one in which technological change became an important force in explaining the evolution of the distribution of productivity. Badunenko et al. (2008) came to this conclusion by focusing on the 1990s within the Kumar and Russell (2002) framework. This result calls for introducing the components in a different sequence to scrutinize the effect of technological change. For example, when technological change is introduced after physical and human capital accumulation, it moves the 1965 distribution closer to the 2000 distribution (see

Table 5). The last row of Table 4, however, tells us that these three components, neither separately nor in combination, lead to bimodality. Again, it seems that efficiency changes, and efficiency changes alone, account for the transformation to bimodality, a result that is very different from that of HR using an inferior method (and, of course, a different sample).

Implementation of other sequencing combinations (see Figures 10 and 11) indicate that the results are not sensitive to changes in the sequencing order. The introduction of efficiency change always leads to bimodality of the income distribution. Only in combination with efficiency change is a counterfactual distribution statistically bimodal. With respect to the transformation of the distribution, technological change brings the 1965 income distribution to that of 2000, but it does so statistically only in combination with human capital accumulation. If either technological change or human capital accumulation is not present in the sequence, the counterfactual distribution significantly differs from the 2000 distribution.

5 Robustness checks

In addition to our main objective of correcting for the bias in DEA studies of growth and convergence, we also aim to improve on earlier studies, particularly the HR paper, by extending the sample to encompass additional countries and a longer time period and by employing an improved international data set on years of schooling (used to construct human capital indexes). We have found, compared to the HR results, a much greater role of efficiency changes in driving the shift to bimodality and a greater role of technological change and human capital accumulation, as compared to physical capital accumulation, on the mean increase in productivity and the overall transformation of the distribution. This section reports on some robustness tests (described in more detail in appendices available at the authors' websites) aimed at determining which of the above improvements is generating these differences. These robustness exercises shed light on some other issues as well (*e.g.*, the effect of changing the sample size).

5.1 HR sample for the 1965−2000 period

To assess the possibility that the differences between our results and those of HR are attributable to the inclusion of considerably more countries in the sample, we re-run the analysis using only countries included in the HR study. We have 1965–2000 data (including the newer education data) for all but seven of the countries in the HR sample.[18] Appendix A provides the results of this exercise.

In traditional DEA, decreasing the number of countries can only lead to increases or no changes in efficiency of extant countries, since the frontier can be no higher with fewer countries in the sample. In our approach, however, two effects occur: one is the traditional effect on the frontier, and the other is a decrease in efficiency owing to the larger standard errors attributable to having fewer countries in the sample. As it turns out, shrinking the sample increases average efficiency from 0.67 to 0.81 in 1965 and from 0.66 to 0.73 in 2000.

None of the countries forming our 1965 frontier are in the HR sample. Those forming the "true" 1965 frontier using the HR sample with the extended sample period are Guatemala, Mauritius, New Zealand, Paraguay, Syria, and the United States. Interestingly , Mauritius helps form the "true" frontier, even though it is not on the best-practice frontier. The "true" 2000 frontier is formed by the 1965 observations for these same countries, along with the 2000 value for Ireland.

This robustness exercise yields two important findings. First, the decrease in the number of countries in the sample changes the conclusion about the cause of the polarization of the productivity distribution: with the smaller sample, capital accumulation as well as efficiency changes can each account solely for the shift to bimodality. As compared to the HR results, this finding indicates that efficiency playing the sole role in the shift to bimodality seems to be the result of the increase in the number of countries in the sample (especially the increased number of poorer African economies).

Another important finding of this robustness exercise is that the effect of physical capital accumulation on the productivity distribution is stronger in the smaller sample. This effect, however, is not the same as in HR: here, we need both physical capital accumulation and technological change to (significantly) explain the change in the distribution, whereas in HR it could be explained by physical capital accumulation alone. This result

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[18] The omitted countries, none of which determines the shape of the production frontier in 1965 or 2000, are Germany, Hong Kong, Iceland, Israel, Sri Lanka, Sierra Leone, and Taiwan.

suggests that, during the 1990's, technological change was a greater factor than in the earlier years, confirming a similar finding by Badunenko et al. (2008) in the context of the Kumar and Russell (2002) growth-accounting framework.

5.2 Full sample for the 1965−1990 period

Given the results in the previous sub-section, we also investigate the effects of the components of productivity using our sample of countries over the shorter time period, 1965–1990. In doing so, we address the question of whether the conclusions reached by HR are robust to expansion of the number of countries. The full set of results appears in Appendix B.

A principal finding is that, for this shorter time period (as well as for the full time period), efficiency change alone can account for the shift to bimodality. Also, physical capital accumulation, much greater over this shorter time period, was a statistically significant contributor to the overall shift in the income distribution, along with technological change. Combining these two robustness exercises, we can infer that the major fall in the importance of efficiency and the rise in the importance of technological change happened during the final decade. The major contribution of capital deepening occurred during the 1965−1990 period, leaving technological change as the dominating factor in the 1990s.

5.3 Barro and Lee (2001) data set

Another reason our results differ from those of HR may be because we use the more precise schooling data of Cohen and Soto (2007). We report in Appendix C the results of re-running the analysis using the Barro and Lee (2001) data. The results are very similar. If anything, using the Barro and Lee (2001) data set only emphasizes the importance of technological change and the deflated role of physical capital deepening during the 1990s.

5.4 Standard DEA approach

To assess the effect on the results of employing the improved method of constructing the frontier, we carry out the analysis using the full sample of countries for the full time period, with the more recent data on years of education, but using the standard DEA method. Appendix D gives the results.

Using the standard method does not significantly affect the results: even though the efficiency scores differ greatly for some countries, changes in efficiency and technology are largely unaffected. Human and physical capital accumulations mainly reflect movements along the frontier, but since the observations are not changed by the change in the method of constructing the frontier, these two components are also largely unchanged.

6 Conclusion

In this paper we suggest methods to identify the "true" convex frontier in DEA efficiency estimation. Specifically, we use standard DEA methods to obtain the standard DEA efficiency scores, correct for the bias in these scores, use the unbiased efficiency scores to construct the "true" frontier, and evaluate the original data to obtain the unbiased efficiency scores predicated on a convex technology.

We employ these methods to obtain the HR quadripartite decomposition of labor productivity and analyze international macroeconomic convergence using this decomposition. Along the way, we make several additional contributions to this literature. (1) We increase the sample of countries studied by nearly a third. (2) We extend the panel to include data up to the year 2000. (3) We employ the most recently available and reliable data on educational attainment across countries.

Consistently with the HR results, we find that technological change is decidedly non-neutral, with all technological advances taking place among richer countries. We find that efficiency change is *the* factor explaining the emergence of two modes in the distribution of output per worker. Whereas capital accumulation played the major role in the shift of the distribution of output per worker in the HR study, it was found to be less important here. Instead, both technological change and human capital accumulation were found to be significant factors in the shift of the distribution. Robustness exercises suggest that the revision of the stylized facts emerging from this study are attributable not to the improved method of constructing the production frontier but rather to the extension of the sample period to 2000. It appears that the 1990s were structurally different from the 1965–1990 period, a fact that has been noted earlier by Badunenko et al. (2008).

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7 Tables and figures

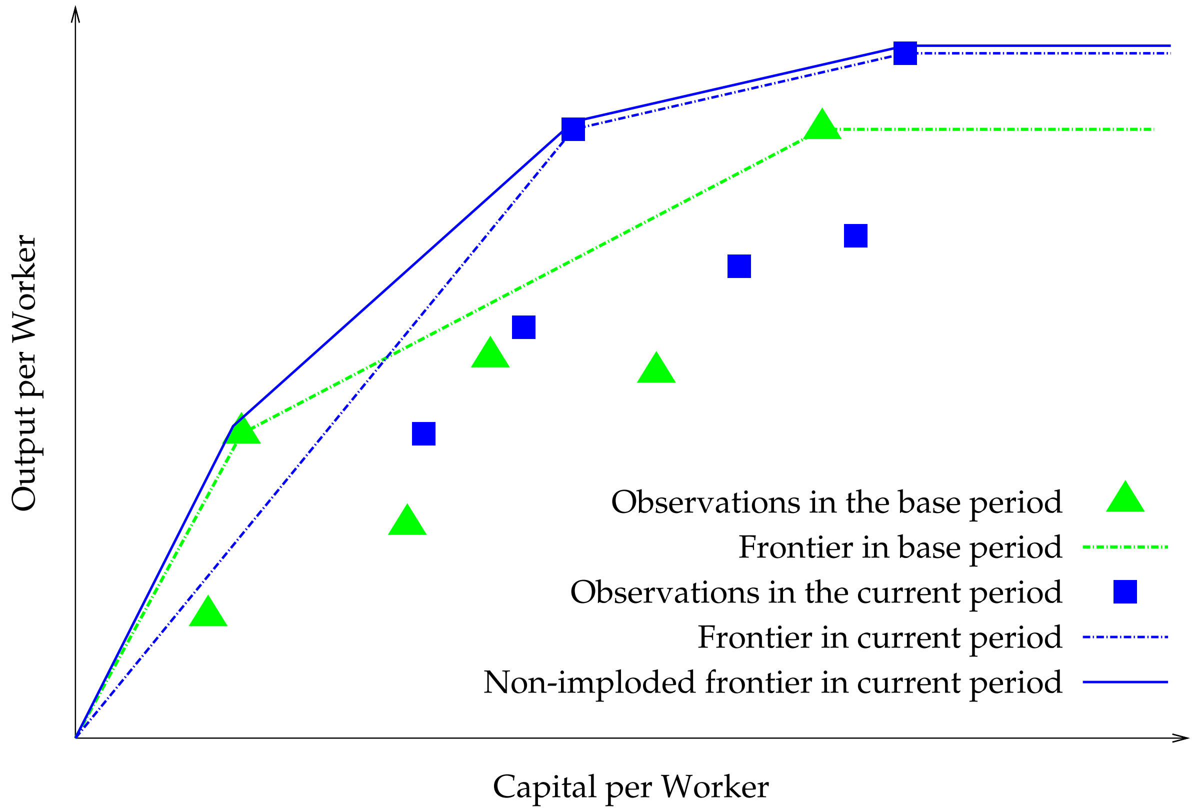

Figure 1: Constructing the current-period frontier under the non-implosion assumption.

Note that the 'true' frontier envelops observations from both the base and current period.

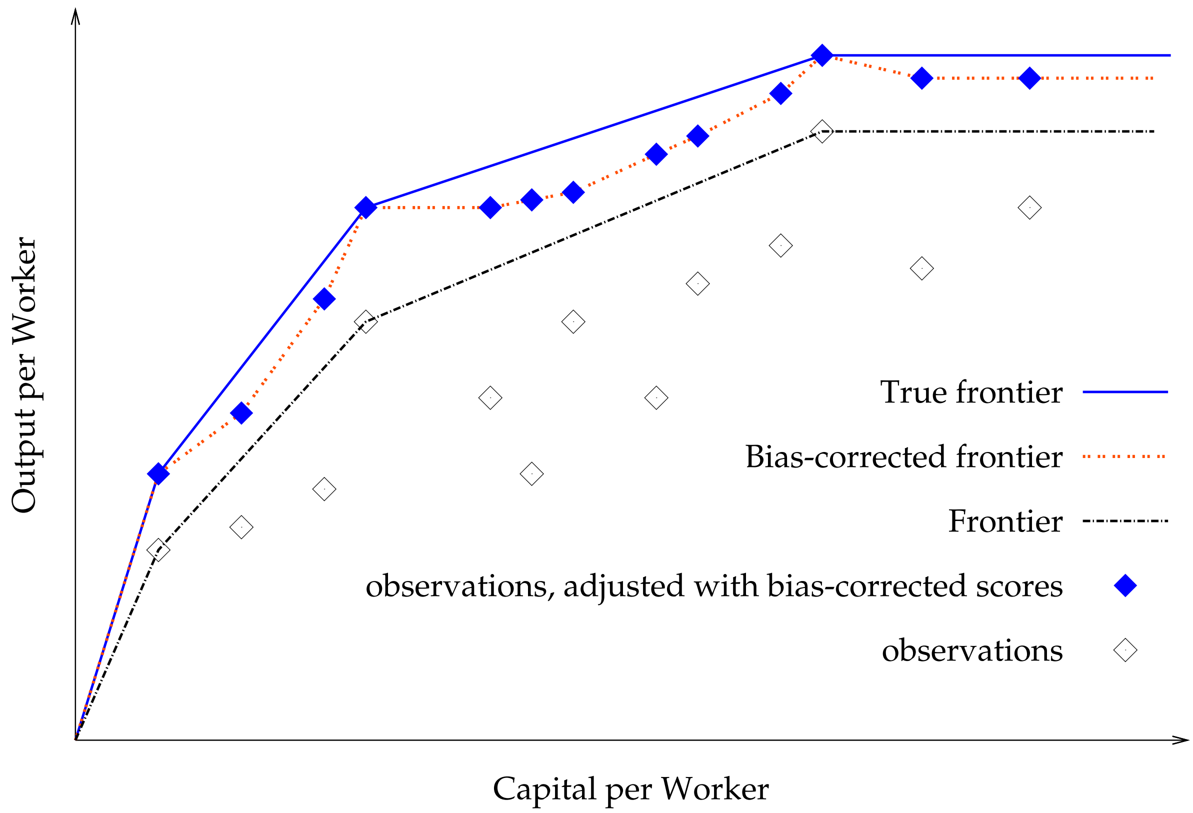

Figure 2: Constructing the “true” frontier

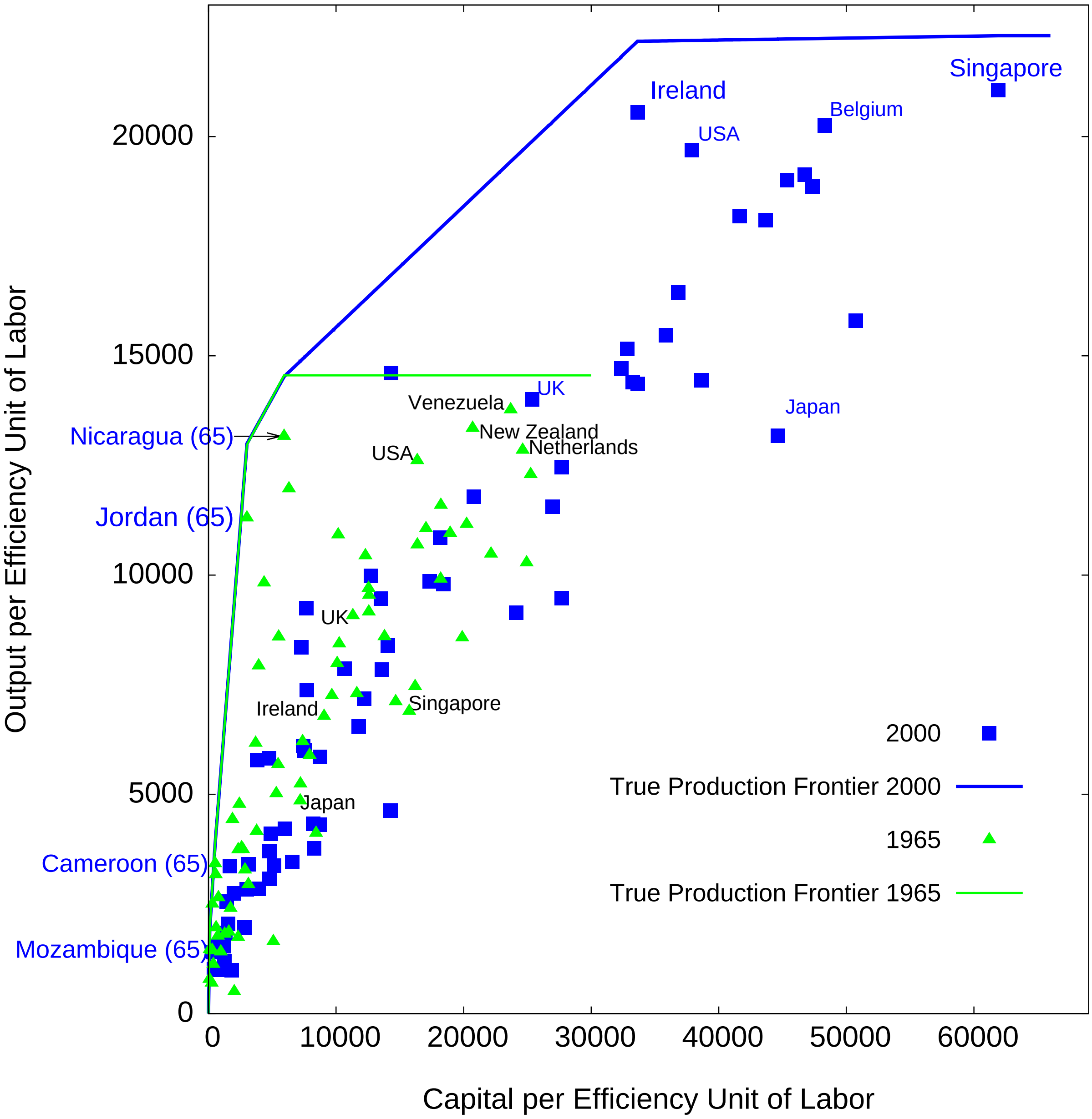

Figure 3: Estimated "true" best-practice world production frontiers and scatterplots of the data for 1965 and 2000

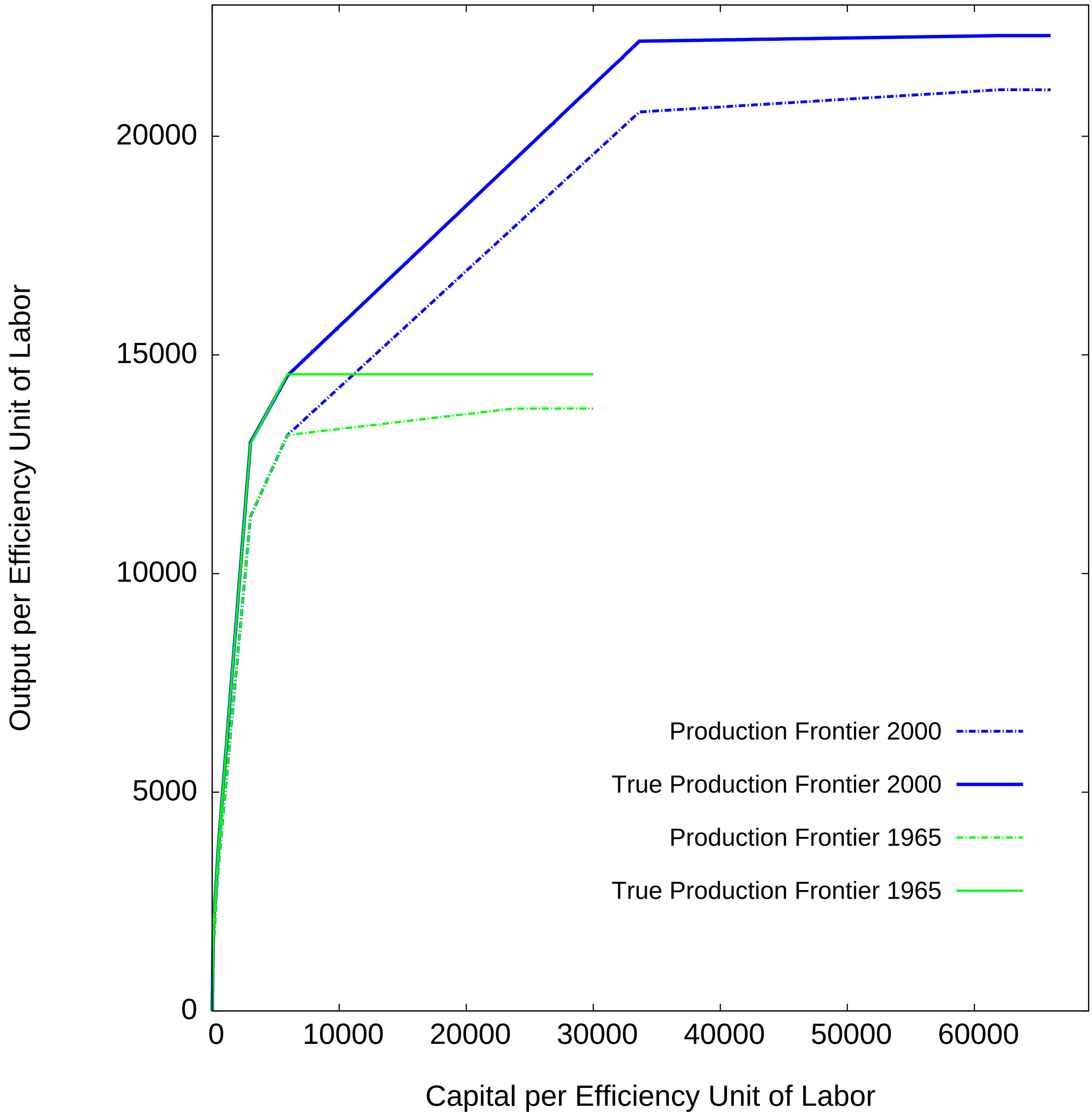

Figure 4: Estimated "true" and DEA best-practice world production frontiers for 1965 and 2000

Table 1: Efficiency scores, 1965and2000

#	Country	eff_b	eff_b^{bc}	eff_b^{true}	eff_c	eff_c^{bc}	eff_c^{true}
1	Algeria	0.908	0.828	0.822	0.545	0.534	0.498
2	Argentina	0.780	0.746	0.717	0.661	0.649	0.611
3	Australia	0.729	0.699	0.681	0.698	0.658	0.658
4	Austria	0.640	0.613	0.591	0.907	0.866	0.855
5	Belgium	0.805	0.771	0.752	0.973	0.929	0.918
6	Benin	0.431	0.390	0.379	0.310	0.298	0.271
7	Bolivia	0.353	0.317	0.310	0.252	0.227	0.219
8	Brazil	0.445	0.415	0.405	0.445	0.437	0.407
9	Cameroon	1.000	0.881	0.881	0.474	0.454	0.414
10	Canada	0.853	0.818	0.796	0.704	0.665	0.663
11	Chile	0.552	0.529	0.512	0.608	0.598	0.562
12	Colombia	0.441	0.401	0.397	0.450	0.436	0.408
13	Costa Rica	0.808	0.732	0.716	0.678	0.658	0.616
14	Denmark	0.817	0.783	0.760	0.751	0.713	0.708
15	Dominican Republic	0.507	0.459	0.441	0.541	0.525	0.491
16	Ecuador	0.310	0.291	0.283	0.310	0.302	0.282
17	El Salvador	0.667	0.601	0.588	0.470	0.450	0.419
18	Finland	0.628	0.601	0.589	0.700	0.667	0.659
19	France	0.791	0.758	0.734	0.915	0.875	0.863
20	Ghana	0.061	0.056	0.053	0.327	0.309	0.287
21	Greece	0.546	0.521	0.501	0.616	0.598	0.576
22	Guatemala	0.525	0.471	0.461	0.489	0.462	0.431
23	Honduras	0.362	0.323	0.315	0.248	0.237	0.221
24	India	0.313	0.288	0.274	0.299	0.272	0.261
25	Indonesia	0.535	0.478	0.471	0.267	0.256	0.239
26	Iran	0.820	0.779	0.750	0.508	0.496	0.473
27	Ireland	0.511	0.481	0.466	1.000	0.941	0.941
28	Italy	0.764	0.728	0.720	0.873	0.834	0.823
29	Jamaica	0.396	0.367	0.360	0.259	0.249	0.235
30	Japan	0.367	0.340	0.333	0.635	0.607	0.599
31	Jordan	1.000	0.869	0.869	0.319	0.305	0.289
32	Kenya	0.273	0.252	0.238	0.209	0.200	0.183
33	Korea	0.332	0.306	0.290	0.499	0.484	0.467
34	Malawi	0.315	0.271	0.266	0.257	0.239	0.226
35	Malaysia	0.380	0.341	0.330	0.594	0.583	0.549
36	Mali	0.397	0.348	0.346	0.310	0.297	0.271
37	Mauritius	0.713	0.682	0.656	0.949	0.933	0.872
38	Mexico	0.633	0.601	0.579	0.516	0.507	0.474
39	Mozambique	1.000	0.758	0.758	0.410	0.379	0.361
40	Nepal	0.597	0.519	0.514	0.184	0.168	0.160

(*continued on next page*)

Table 1 (*Continued*)

#	Country	eff_b	eff_b^{bc}	eff_b^{true}	eff_c	eff_c^{bc}	eff_c^{true}
41	Netherlands	0.933	0.887	0.883	0.879	0.840	0.828
42	New Zealand	0.977	0.933	0.917	0.657	0.636	0.615
43	Nicaragua	1.000	0.904	0.904	0.328	0.315	0.293
44	Niger	1.000	0.870	0.866	0.274	0.263	0.240
45	Norway	0.747	0.710	0.706	0.920	0.879	0.868
46	Panama	0.393	0.358	0.353	0.484	0.475	0.443
47	Paraguay	0.573	0.527	0.499	0.441	0.428	0.401
48	Peru	0.511	0.490	0.474	0.314	0.306	0.286
49	Philippines	0.303	0.267	0.264	0.298	0.286	0.266
50	Portugal	0.600	0.569	0.548	0.745	0.706	0.701
51	Romania	0.292	0.267	0.256	0.274	0.266	0.249
52	Senegal	0.889	0.790	0.783	0.407	0.391	0.356
53	Singapore	0.528	0.507	0.489	1.000	0.946	0.946
54	South Africa	0.666	0.606	0.599	0.617	0.597	0.560
55	Spain	0.684	0.655	0.629	0.798	0.758	0.751
56	Sweden	0.817	0.781	0.766	0.727	0.690	0.684
57	Switzerland	0.893	0.849	0.844	0.758	0.722	0.715
58	Syria	0.625	0.567	0.548	0.341	0.324	0.297
59	Tanzania	0.708	0.551	0.537	0.338	0.308	0.295
60	Thailand	0.191	0.174	0.166	0.301	0.295	0.276
61	Trinidad & Tobago	0.724	0.693	0.027	0.688	0.674	0.036
62	Tunisia	0.469	0.435	0.426	0.666	0.654	0.611
63	Turkey	0.408	0.371	0.356	0.421	0.410	0.383
64	Uganda	0.616	0.534	0.526	0.515	0.467	0.449
65	United Kingdom	0.680	0.648	0.624	0.763	0.744	0.712
66	United States	0.933	0.895	0.866	0.955	0.910	0.900
67	Uruguay	0.546	0.516	0.498	0.623	0.612	0.572
68	Venezuela	1.000	0.951	0.946	0.547	0.538	0.503
69	Zambia	0.130	0.118	0.116	0.131	0.125	0.115
70	Zimbabwe	0.259	0.226	0.225	0.239	0.227	0.211
	Average	0.729	0.693	0.674	0.708	0.677	0.662

Table 2: Efficiency scores and percentage change of tripartite decomposition indexes, 1965−2000

#	Country	productivity change	EFF−1 × 100	TECH−1 × 100	KACC−1 × 100	HACC−1 × 100
1	Algeria	14.9	−39.4	4.5	8.9	66.7
2	Argentina	24.9	−14.8	16.7	6.4	18.2
3	Australia	72.8	−3.4	35.3	13.3	16.7
4	Austria	160.6	44.8	31.5	17.2	16.7
5	Belgium	125.2	22.0	36.8	13.6	18.7
6	Benin	46.8	−28.4	0.0	88.8	8.6
7	Bolivia	0.5	−29.3	0.0	8.1	31.5
8	Brazil	67.6	0.6	7.0	6.5	46.3
9	Cameroon	42.8	−53.0	0.0	169.8	12.6
10	Canada	52.9	−16.8	35.1	13.4	19.9
11	Chile	70.4	9.6	19.6	3.8	25.3
12	Colombia	43.7	2.8	1.3	5.9	30.3
13	Costa Rica	28.5	−14.0	1.5	10.3	33.4
14	Denmark	65.5	−6.9	34.3	14.1	15.9
15	Dominican Republic	109.7	11.3	1.6	54.2	20.2
16	Ecuador	45.8	−0.3	4.7	2.3	36.4
17	El Salvador	2.6	−28.8	0.0	10.5	30.3
18	Finland	119.4	12.0	37.3	13.5	25.8
19	France	121.7	17.6	34.1	15.7	21.6
20	Ghana	303.8	437.5	0.0	−31.3	9.4
21	Greece	108.3	14.8	23.3	14.5	28.5
22	Guatemala	33.8	−6.7	0.0	12.0	28.0
23	Honduras	11.9	−30.0	0.0	36.0	17.5
24	India	170.3	−4.8	0.0	128.6	24.1
25	Indonesia	196.1	−49.2	0.0	300.5	45.6
26	Iran	41.2	−36.9	19.6	15.1	62.7
27	Ireland	260.5	102.1	25.9	20.6	17.5
28	Italy	131.1	14.3	39.7	12.6	28.5
29	Jamaica	−10.6	−34.8	1.7	1.7	32.5
30	Japan	223.5	79.6	24.3	22.8	17.9
31	Jordan	−35.3	−66.8	0.1	40.0	39.0
32	Kenya	−2.4	−23.2	0.0	13.0	12.5
33	Korea	513.6	61.4	18.0	142.7	32.8
34	Malawi	86.7	−15.0	0.0	91.8	14.5
35	Malaysia	328.2	66.3	10.7	71.4	35.7
36	Mali	83.7	−21.8	0.0	125.1	4.4
37	Mauritius	125.7	33.0	13.6	4.9	42.5
38	Mexico	31.4	−18.2	10.8	5.4	37.5
39	Mozambique	31.4	−52.4	0.0	162.8	5.0

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Table 2 (*Continued*)

#	Country	productivity change	EFF−1 × 100	TECH−1 × 100	KACC−1 × 100	HACC−1 × 100
40	Nepal	98.7	−68.9	0.0	399.3	28.2
41	Netherlands	66.5	−6.2	42.0	9.0	14.7
42	New Zealand	11.1	−33.0	32.8	7.4	16.2
43	Nicaragua	−51.6	−67.6	0.0	2.1	46.2
44	Niger	−31.0	−72.3	0.0	137.3	5.0
45	Norway	124.8	22.9	42.6	9.4	17.3
46	Panama	99.1	25.7	5.5	11.1	35.1
47	Paraguay	68.9	−19.8	1.4	81.0	14.7
48	Peru	−12.0	−39.7	10.7	−2.5	35.2
49	Philippines	50.3	0.8	0.0	28.8	15.8
50	Portugal	168.4	27.9	26.4	20.5	37.8
51	Romania	235.8	−2.7	2.1	189.3	16.9
52	Senegal	4.6	−54.6	0.0	110.5	9.4
53	Singapore	341.8	93.5	33.1	19.6	43.5
54	South Africa	27.2	−6.6	1.2	5.4	27.6
55	Spain	133.6	19.4	29.7	18.7	27.0
56	Sweden	54.0	−10.8	36.3	10.7	14.4
57	Switzerland	41.7	−15.3	43.1	7.8	8.5
58	Syria	72.0	−45.8	0.0	162.9	20.8
59	Tanzania	54.6	−45.0	0.0	170.4	3.9
60	Thailand	324.0	66.2	7.3	78.5	33.2
61	Trinidad & Tobago	49.7	−4.2	19.2	8.5	20.8
62	Tunisia	133.1	43.3	7.3	7.6	40.9
63	Turkey	132.4	7.7	2.5	65.5	27.2
64	Uganda	24.4	−14.7	0.0	29.2	12.8
65	United Kingdom	95.7	14.2	21.7	13.2	24.3
66	United States	80.2	3.8	34.1	14.1	13.5
67	Uruguay	66.0	14.7	10.2	5.0	25.0
68	Venezuela	−24.2	−46.9	23.1	−3.7	20.3
69	Zambia	−20.0	−1.5	0.0	−30.0	16.0
70	Zimbabwe	47.5	−6.4	0.0	25.9	25.1
	Average	90.2	1.6	13.2	46.8	24.7

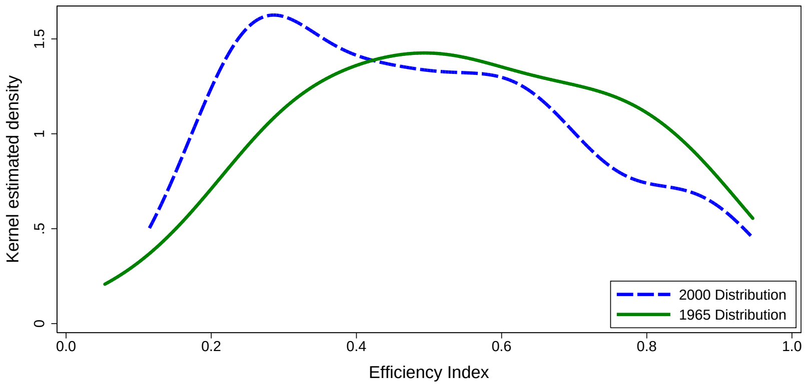


Figure 5: Distributions of efficiency indices, 1965 and 2000

Table 3: Mean percentage changes of the quadripartite decomposition indices (country groupings), 1965–2000

Country Group	Productivity Change	EFF−1 ×100	TECH−1 ×100	KACC−1 ×100	HACC−1 ×100
OECD [3a]	110.2	10.2	31.2	19.8	21.3
Non-OECD	52.0	−19.1	4.7	43.3	25.3
Asian Tigers [3b]	344.3	75.9	25.7	53.4	30.9
Latin America [3c]	26.6	−17.5	6.2	13.1	27.7
Africa [3d]	43.6	−19.4	1.6	49.1	17.6
EU 15 [3e]	118.4	16.2	33.5	15.3	22.1
Asia [3f]	114.8	−32.0	4.7	122.5	35.6

[3a] Australia, Austria, Belgium, Canada, Denmark, Finland, France, Greece, Ireland, Italy, Japan, Korea, Mexico, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, Turkey, United Kingdom, and United States.

[3b] Japan, Korea, and Singapore.

[3c] Argentina, Bolivia, Brazil, Chile, Colombia, Costa Rica, Dominican Republic, Ecuador, El Salvador, Guatemala, Honduras, Jamaica, Nicaragua, Panama, Paraguay, Peru, Philippines, Trinidad & Tobago, Uruguay, and Venezuela.

[3d] Algeria, Benin, Cameroon, Ghana, Kenya, Malawi, Mali, Mauritius, Mozambique, Niger, Senegal, South Africa, Tanzania, Tunisia, Uganda, Zambia, and Zimbabwe.

[3e] Austria, Belgium, Denmark, Finland, France, Greece, Ireland, Italy, Netherlands, Portugal, Spain, Sweden, and United Kingdom.

[3f] India, Indonesia, Iran, Jordan, Malaysia, Nepal, Syria, and Thailand.

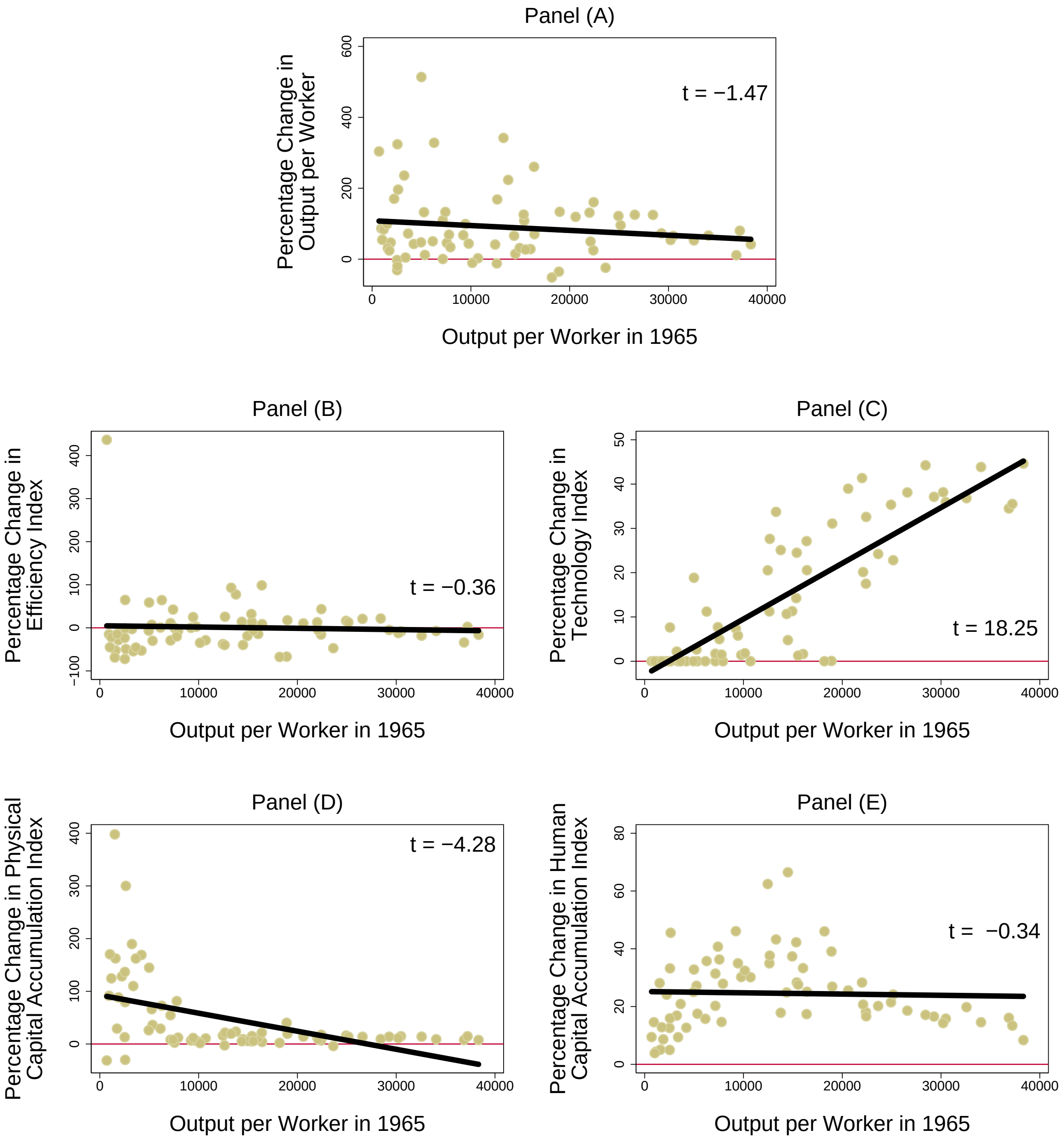


Figure 6: Percentage change (from 1965 to 2000) in output per worker and four decomposition indexes, plotted against output per worker in 1965

Note: Each panel contains a GLS regression line.

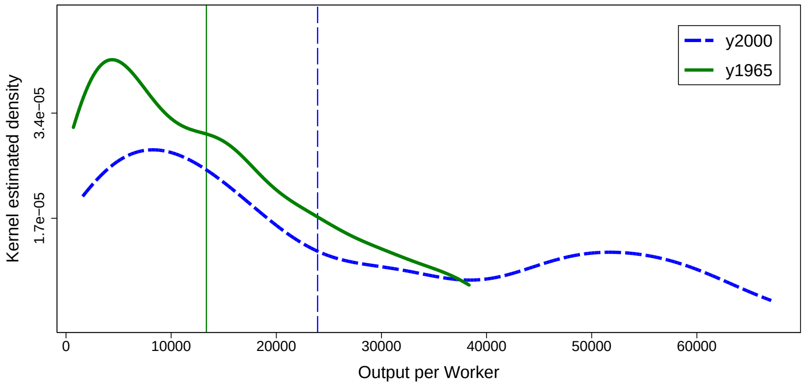


Figure 7: Actual output-per-worker distributions

Table 4: Modality tests (*p-values*)

	Distribution	H_0: One Mode H_A: More Than One Mode
1	$f(y_{1965})$	0.650
2	$f(y_{2000})$	0.009
3	$f(y_{1965} \times EFF)$	0.037
4	$f(y_{1965} \times TECH)$	0.587
5	$f(y_{1965} \times KACC)$	0.828
6	$f(y_{1965} \times HACC)$	0.191
7	$f(y_{1965} \times EFF \times TECH)$	0.011
8	$f(y_{1965} \times EFF \times KACC)$	0.044
9	$f(y_{1965} \times EFF \times HACC)$	0.039
10	$f(y_{1965} \times TECH \times KACC)$	0.401
11	$f(y_{1965} \times TECH \times HACC)$	0.401
12	$f(y_{1965} \times KACC \times HACC)$	0.757
13	$f(y_{1965} \times EFF \times TECH \times KACC)$	0.006
14	$f(y_{1965} \times EFF \times TECH \times HACC)$	0.008
15	$f(y_{1965} \times EFF \times KACC \times HACC)$	0.075
16	$f(y_{1965} \times TECH \times KACC \times HACC)$	0.739

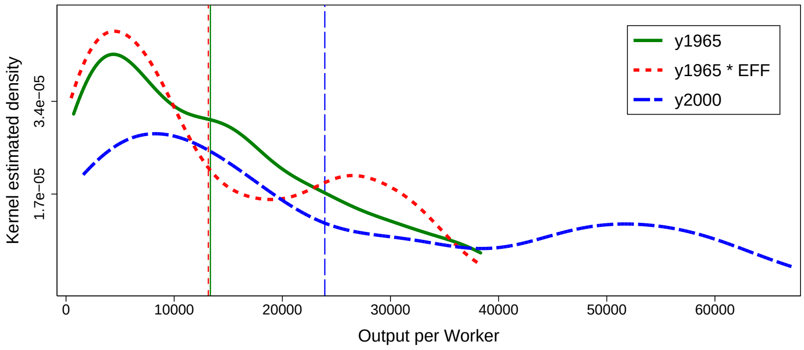
Notes: We used the bootstrapped calibrated Silverman tests for multimodality due to Hall and York (2001) with 1000 bootstrap replications.

Table 5: Distribution hypothesis tests (*p-values*)

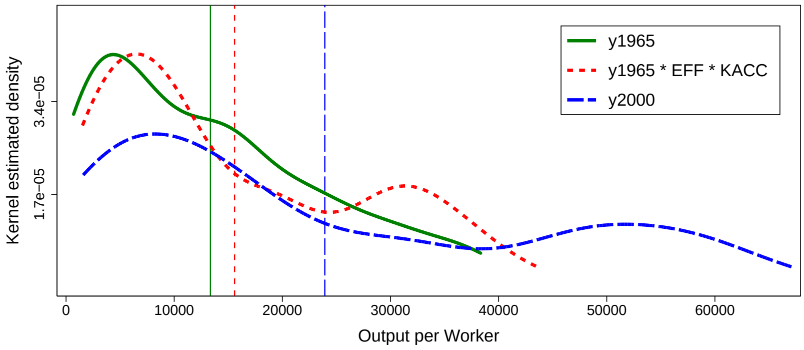
	Distribution	Li Test
1	$g(y_{2000})$ vs. $f(y_{1965})$	0.000
2	$g(y_{2000})$ vs. $f(y_{1965} \times EFF)$	0.000
3	$g(y_{2000})$ vs. $f(y_{1965} \times TECH)$	0.001
4	$g(y_{2000})$ vs. $f(y_{1965} \times KACC)$	0.000
5	$g(y_{2000})$ vs. $f(y_{1965} \times HACC)$	0.000
6	$g(y_{2000})$ vs. $f(y_{1965} \times EFF \times TECH)$	0.002
7	$g(y_{2000})$ vs. $f(y_{1965} \times EFF \times KACC)$	0.000
8	$g(y_{2000})$ vs. $f(y_{1965} \times EFF \times HACC)$	0.000
9	$g(y_{2000})$ vs. $f(y_{1965} \times TECH \times KACC)$	0.007
10	$g(y_{2000})$ vs. $f(y_{1965} \times TECH \times HACC)$	0.229
11	$g(y_{2000})$ vs. $f(y_{1965} \times KACC \times HACC)$	0.000
12	$g(y_{2000})$ vs. $f(y_{1965} \times EFF \times TECH \times KACC)$	0.023
13	$g(y_{2000})$ vs. $f(y_{1965} \times EFF \times TECH \times HACC)$	0.671
14	$g(y_{2000})$ vs. $f(y_{1965} \times EFF \times KACC \times HACC)$	0.008
15	$g(y_{2000})$ vs. $f(y_{1965} \times TECH \times KACC \times HACC)$	0.686

Notes: We used the bootstrapped Li (1996) Tests with 5000 bootstrap replications and the Sheather and Jones (1991) bandwidth.

(A) Effect of Efficiency Change



(B) Effect of Capital Deepening



(C) Effect of Human Capital

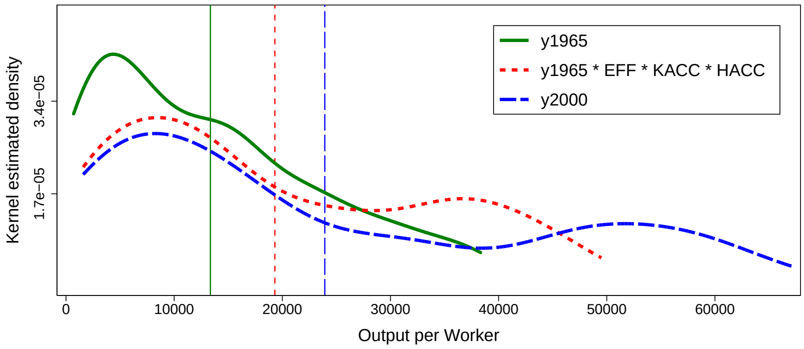
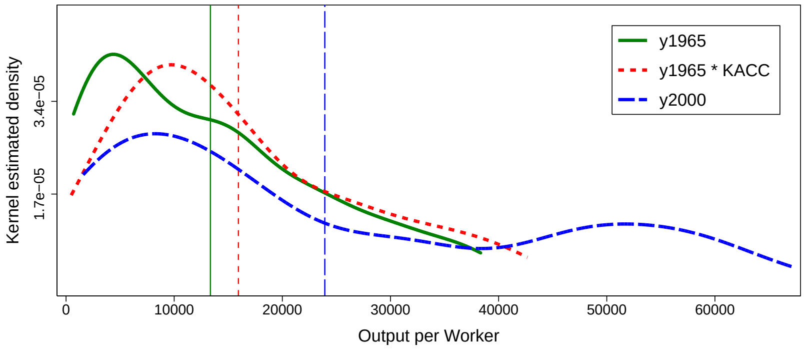


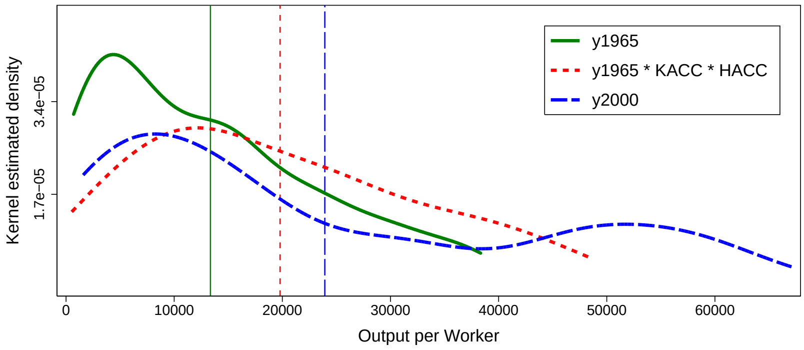
Figure 8: Counterfactual Distributions of Output per Worker. Sequence of introducing effects of decomposition: EFF, KACC, and HACC

In each panel, the solid curve is the estimated 1965 distribution and the solid vertical line represents the 1965 mean value. The dashed curve is the estimated 2000 distribution and the dashed vertical line represents the 2000 mean value. The dotted curves in each panel are the counterfactual distributions isolating, sequentially, the effects of efficiency change, capital deepening, and human capital accumulation on the 1965 distribution, and the dotted vertical line represents the respective counterfactual mean.

(A) Effect of Capital Deepening



(B) Effect of Human Capital



(C) Effect of Technological Change

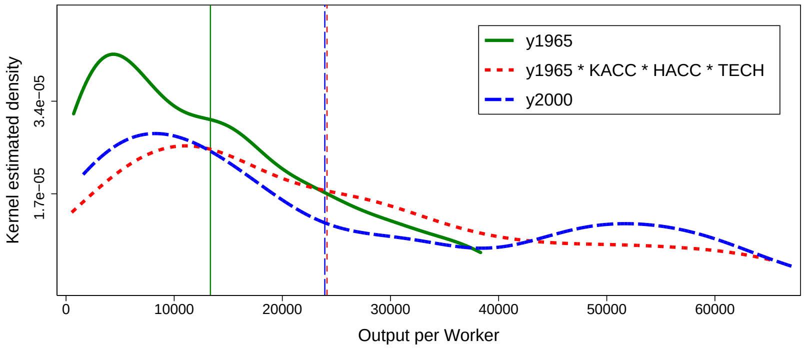
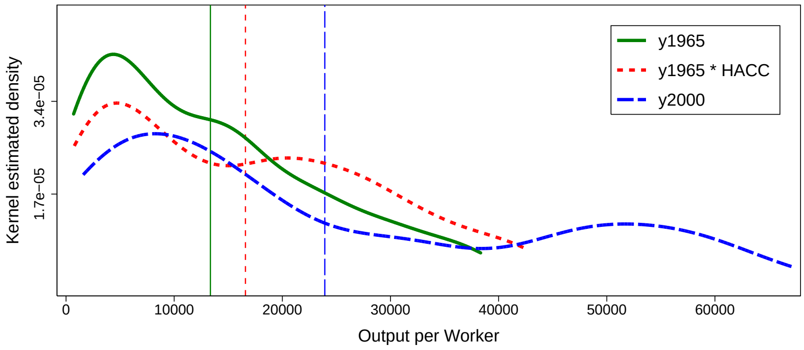
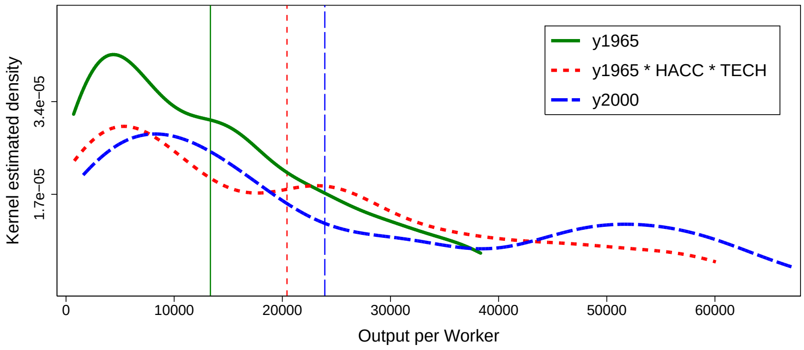


Figure 9: Counterfactual Distributions of Output per Worker. Sequence of introducing effects of decomposition: KACC, HACC, and TECH

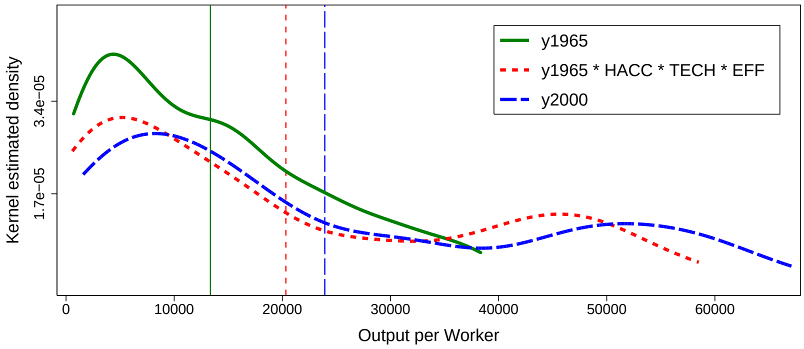
In each panel, the solid curve is the estimated 1965 distribution and the solid vertical line represents the 1965 mean value. The dashed curve is the estimated 2000 distribution and the dashed vertical line represents the 2000 mean value. The dotted curves in each panel are the counterfactual distributions isolating, sequentially, the effects of capital deepening, human capital accumulation, and technological change on the 1965 distribution, and the dotted vertical line represents the respective counterfactual mean.



(A) Effect of Human Capital



(B) Effect of Technological Change

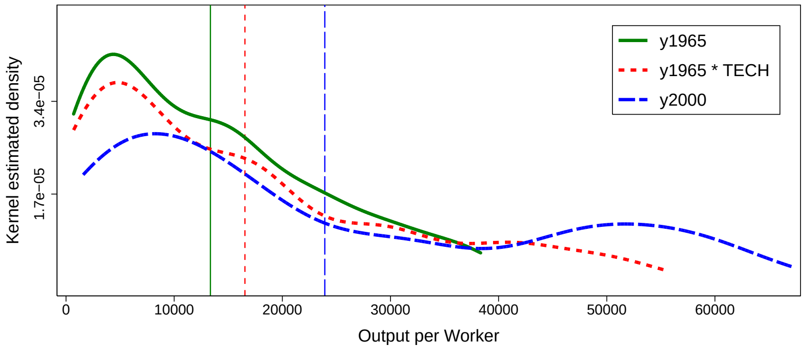


(C) Effect of Efficiency Change

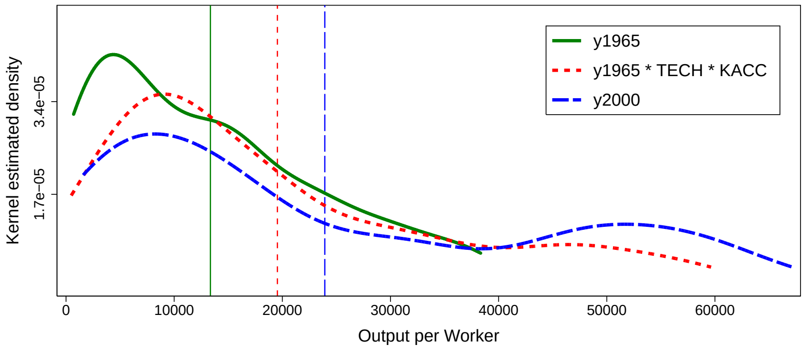
Figure 10: Counterfactual Distributions of Output per Worker. Sequence of introducing effects of decomposition: HACC, TECH, and EFF

In each panel, the solid curve is the estimated 1965 distribution and the solid vertical line represents the 1965 mean value. The dashed curve is the estimated 2000 distribution and the dashed vertical line represents the 2000 mean value. The dotted curves in each panel are the counterfactual distributions isolating, sequentially, the effects of human capital deepening, technological change, and efficiency change on the 1965 distribution, and the dotted vertical line represents the respective counterfactual mean.

(A) Effect of Technological Change



(B) Effect of Capital Deepening



(C) Effect of Efficiency Change

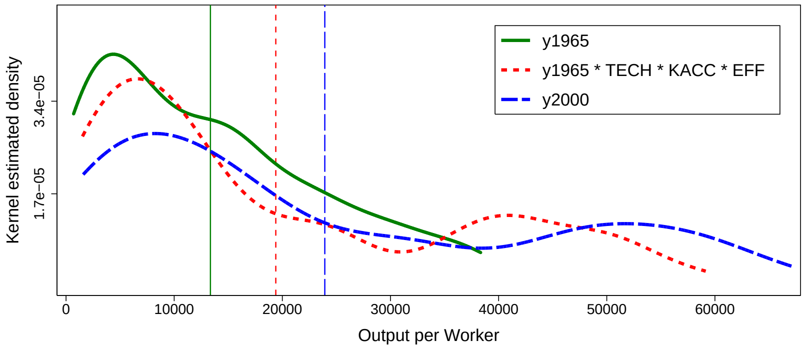


Figure 11: Counterfactual Distributions of Output per Worker. Sequence of introducing effects of decomposition: TECH, KACC, and EFF

In each panel, the solid curve is the estimated 1965 distribution and the solid vertical line represents the 1965 mean value. The dashed curve is the estimated 2000 distribution and the dashed vertical line represents the 2000 mean value. The dotted curves in each panel are the counterfactual distributions isolating, sequentially, the effects of technological change, capital deepening, and efficiency change on the 1965 distribution, and the dotted vertical line represents the respective counterfactual mean.