Indeterminacy with No-Income-Effect Preferences and Sector-Specific Externalities*

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Abstract

We examine a two-sector real business cycle (RBC) model with sector-specific externalities in the production of distinct consumption and investment goods. In addition, the household utility is postulated to exhibit no income effect on the demand for leisure. Unlike in the one-sector counterpart, we show that equilibrium indeterminacy can result with sufficiently high returns-to-scale in the production of investment goods. We also find that the smaller the labor supply elasticity, the lower the threshold level of investment externalities needed for generating indeterminacy and sunspots. This finding turns out to be exactly the opposite of that in all existing RBC-based indeterminacy studies. Finally, in contrast to previous sunspot-driven two-sector RBC models, our no-income-effect economy is able to match the stylized facts that sectoral labor inputs are positively correlated and consumption is procyclical.

Keywords: Indeterminacy, Income Effect, Sector-Specific Externalities.

JEL Classification: E30, E32.

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1 Introduction

Starting with the work of Benhabib and Farmer (1994) and Farmer and Guo (1994), there is now an extensive macroeconomic literature that explores indeterminacy and sunspots in the real business cycle (RBC) model.[1] The original Benhabib-Farmer-Guo one-sector economy, with a Cobb-Douglas production function and sufficiently strong increasing returns, displays multiple equilibria and belief-driven business cycle fluctuations when the separable household utility is logarithmic in consumption and convex in hours worked. More recently, Meng and Yip (2008) and Jaimovich (2008) (hereafter MYJ) have shown that a one-sector RBC model, instead with non-separable preferences, always exhibits saddle-path stability and equilibrium uniqueness when there is no income effect on the demand for leisure, regardless of the degree of aggregate returns-to-scale in production. MYJ's result illustrates the critical importance of the income effect associated with the household's labor supply decision in generating indeterminacy and sunspots within one-sector RBC models.

In this paper, we build upon MYJ's analyses and examine the quantitative interrelations between equilibrium indeterminacy and the no-income-effect utility function in a two-sector RBC model. Distinct consumption and investment goods are produced with sector-specific productive externalities *a la* Benhabib and Farmer (1996) and Harrison (2001). Our main theoretical findings are summarized as follows. First, unlike in MYJ's one-sector model, our two-sector economy may exhibit an indeterminate steady state, and thus a continuum of stationary perfect-foresight equilibria, when sufficiently strong externalities are present. Intuitively, in order for equilibrium indeterminacy to occur in any dynamic general equilibrium macroeconomic model, the associated first-order conditions must continue to hold when there is a change in non-fundamental expectations. In particular, the household's intertemporal Euler equation equates the shadow value of capital sacrificed this period in order to consume – this period's marginal utility of consumption – to the discounted utility value of capital gained next period – its gross rate of return weighted by the marginal utility of next period's consumption. Therefore, upon the expectation of a higher return on capital in the future, agents will consume and work more next period. In the MYJ economy, this expectation cannot be self-fulfilled because an increase in labor hours large enough to raise the gross rate of return on capital will generate an unsustainable rise in its net return, through a higher mar-

[1] See Benhabib and Farmer (1999) for an excellent survey. With the noted exceptions of Benhabib and Nishimura (1998), and Benhabib, Meng and Nishimura (2000), most studies in this literature postulate constant returns-to-scale at the individual firm level. We also maintain this assumption throughout our analysis.

ginal utility of consumption. In our two-sector model, however, next period's net return on capital also depends positively on its relative price, which increases when future consumption rises. Therefore, a very small increase in hours worked can in fact lead to the anticipated hike in the return on capital.

Second, we find that a necessary and sufficient condition for indeterminacy and sunspots in our model is sufficiently strong increasing returns-to-scale in the investment sector. The intuition for this result is the same as in the separable preference set-up of Harrison (2001): when agents anticipate that the return on capital will increase tomorrow, they need incentive to give up consumption today for more capital accumulation. As long as they will be rewarded with productive investment, in the form of sufficient increasing returns in that sector, it will be worthwhile for them to do so.

Third, and perhaps most surprising, the degree of increasing returns-to-scale required for our model to exhibit multiple equilibria increases with the household's labor supply elasticity. In other words, the smaller the labor supply elasticity, the lower the threshold level of investment externalities needed to produce equilibrium indeterminacy. This finding is exactly the opposite of that in all existing RBC-based indeterminacy studies, where an infinitely elastic labor supply is often adopted. The reason is that, to fulfill agents' optimistic expectations and satisfy the household's intertemporal Euler equation in our model economy, movements in total labor hours across time periods must be kept small. Therefore, the smaller the labor supply elasticity, the easier indeterminacy and sunspots are to obtain, in that lower returns-to-scale in production are needed. In a calibrated version of our model, we find that equilibrium indeterminacy is rendered impossible when the labor supply elasticity is higher than 2.

The last theoretical result illustrates the empirical "tension" in our model – a very inelastic labor supply is needed to generate indeterminacy and sunspots with an empirically plausible level of investment externalities. However, such a low labor supply elasticity dampens the fluctuations in total labor hours. This motivates us to conduct a quantitative analysis. As in previous such two-sector RBC models (e.g. Benhabib and Farmer, 1996; and Harrison, 2001), the benchmark parameterization of our no-income-effect economy – with a very small labor supply elasticity and fluctuations driven solely by *i.i.d.* sunspot shocks – exhibits negatively correlated sectoral hours worked (the *labor-comovement puzzle*) and counteryclical consumption (the *consumption-cyclicality puzzle*), as well as low volatility in aggregate labor hours. However, reallocation of productive resources, caused by swings in agents' optimism and pessimism, leads to high variabilities in the relative price of investment and in the two sectoral labor inputs, which in turn generate high standard deviations of consumption and (price-weighted) investment.

We also examine several alternative parameterizations, with higher labor supply elasticity and investment externalities, in which business cycles are generated either by agents' animal spirits alone or by sunspots together with technology shocks. The simulation results are much-improved in these specifications: not only is consumption procyclical, but so are total labor hours and their sectoral components. We conclude that our model economy is able to resolve both the labor-comovement and the consumption-cyclicality puzzles. In addition, a higher volatility in aggregate hours worked moderates the magnitude of cyclical fluctuations in the relative price of investment and the two sectoral labor inputs. It follows that consumption and (price-weighted) investment are smoother than those within our benchmark formulation.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 examines the local stability properties in a calibrated version of our model economy. Section 4 discusses the simulation results from a quantitative analysis. Section 5 concludes.

2 The Economy

Our model incorporates a no-income-effect preference, as in Greenwood, Hercowitz and Huffman (1988), into Harrison's (2001) discrete-time two-sector real business cycle (RBC) model. Households live forever, and derive utility from consumption and leisure. The production side of the economy consists of two sectors, consumption and investment. For expositional simplicity, firms in each sector produce output using identical technologies, but subject to distinct sector-specific external effects.

2.1 Firms

In the consumption sector, output is produced by competitive firms using the following technology:

$$Y_{ct} = A_t K_{ct}^{\alpha} L_{ct}^{1-\alpha},\ 0 < \alpha < 1, \tag{1}$$

where K_{ct} and L_{ct} are the capital and labor inputs used in the production of consumption goods. In addition, A_t represents productive externalities that each individual firm takes as given, and is specified as

$$A_t = \left[\bar{K}_{ct}^{\alpha} \bar{L}_{ct}^{1-\alpha}\right]^{\theta_c},\ \theta_c \geq 0, \tag{2}$$

where $\bar{K}_{ct}$ and $\bar{L}_{ct}$ denote the economy-wide average capital and labor used in producing the consumption good, and θ_c measures the degree of sector-specific externalities in the consumption sector.

Similarly, investment goods are produced by competitive firms using the technology

$$Y_{It} = B_t K_{It}^{\alpha} L_{It}^{1-\alpha}, \text{ where } B_t = \left[\bar{K}_{It}^{\alpha} \bar{L}_{It}^{1-\alpha}\right]^{\theta_I}. \tag{3}$$

Here, K_{It} and L_{It} are capital and hours worked in the investment sector, and B_t represents a productive externality that is an increasing function of the economy-wide average levels of productive capital and labor devoted to producing investment goods. As in Harrison (2001), the degree of sector-specific externalities in the investment sector, denoted as θ_I, is allowed to differ from that for consumption, θ_c.

Under the assumptions that factor markets are perfectly competitive and that capital and labor inputs are perfectly mobile across the two sectors, the first-order conditions for the firms' profit maximization problems are

$$r_t = \frac{\alpha Y_{ct}}{K_{ct}} = p_t \frac{\alpha Y_{It}}{K_{It}}, \tag{4}$$

$$w_t = \frac{(1-\alpha) Y_{ct}}{L_{ct}} = p_t \frac{(1-\alpha) Y_{It}}{L_{It}}, \tag{5}$$

where r_t is the rental rate of capital, w_t is the real wage rate, and p_t denotes the price of investment relative to consumption goods.

2.2 Households

There is a unit measure of identical infinitely-lived households, each with one unit of time endowment, that maximizes its expected present discounted lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_t - \frac{\Lambda L_t^{1+\chi}}{1+\chi}) \right], \; 0 < \beta < 1, \; \chi \geq 0 \quad \text{and} \quad \Lambda > 0, \tag{6}$$

where C_t and L_t are the representative household's consumption and hours worked, β is the discount factor, and χ is the inverse of the wage elasticity of labor supply. The budget constraint faced by the representative household is

$$C_t + p_t I_t = Y_t = r_t K_t + w_t L_t, \tag{7}$$

where I_t is gross investment, Y_t is GDP and K_t is the household's capital stock. The law of motion for the capital stock is given by

$$K_{t+1} = (1-\delta)K_t + I_t, \; K_0 \text{ given}, \tag{8}$$

where $\delta \in (0,1)$ is the capital depreciation rate.

The first-order conditions for the household's optimization problem are

$$\Lambda L_t^{\chi} = w_t, \tag{9}$$

$$\frac{1}{C_t - \frac{\Lambda L_t^{1+\chi}}{1+\chi}} = \beta E_t \left\{ \left(\frac{1}{C_{t+1} - \frac{\Lambda L_{t+1}^{1+\chi}}{1+\chi}} \right) \left[\frac{r_{t+1} + (1-\delta)p_{t+1}}{p_t} \right] \right\} \tag{10}$$

$$\lim_{t \to \infty} \beta^t \frac{K_{t+1}}{C_t} = 0, \tag{11}$$

where (9) equates the slope of the household's indifference curve to the real wage. Note that this equation illustrates the lack of income effect associated with the household's labor supply decision, as C_t is missing. It follows that the income elasticity of intertemporal substitution in hours worked (or leisure) is zero. Moreover, (10) is the standard stochastic consumption Euler equation, and (11) is the transversality condition.

Our goal is to examine the local stability properties of the steady state of the above model economy. Before proceeding further, it is useful to obtain an understanding of the implications of incorporating the no-income-effect preference formulation. In particular, Benhabib and Farmer (1994, 1996), Farmer and Guo (1994) and Harrison (2001), among many others, adopt a standard period utility that is separable in consumption and hours worked:

$$U(C_t, L_t) = \log(C_t) - \frac{AL_t^{1+\chi}}{1+\chi}, \quad \chi \geq 0 \quad \text{and } A > 0; \tag{12}$$

and the resulting first-order condition for the choice of labor supply is

$$AL_t^{\chi} = \frac{w_t}{C_t}. \tag{13}$$

Here, the income and substitution effects can be separated. An increase in the real wage, holding consumption constant, will raise L_t – the substitution effect. An increase in consumption, however, leads to a fall in L_t – the income effect. In all RBC-based models, the real wage and consumption are procyclical: they move together with output. In addition, in these models, the substitution effect dominates, so that when w_t and C_t increase, so does L_t.

In our no-income-effect economy, the first-order condition for hours worked is equation (9), where there is no income effect to counteract the substitution effect. As a result, when the real wage (and consumption) increases, L_t rises by more than that in (13).

2.3 Equilibrium and Local Dynamics

We focus on symmetric perfect-foresight equilibria which consist of a set of prices $\{p_t, r_t, w_t\}_{t=0}^{\infty}$ and quantities $\{C_t, L_t, K_{t+1}\}_{t=0}^{\infty}$ that satisfies the household's and firms' first-order conditions. Moreover, the aggregate consistency condition requires that $K_{ct} = \bar{K}_{ct}$, $L_{ct} = \bar{L}_{ct}$, $K_{It} = \bar{K}_{It}$ and $L_{It} = \bar{L}_{It}$, for all t. The equalities of demand by households and supply by firms in the consumption and investment sectors are given by $C_t = Y_{ct}$ and $I_t = Y_{It}$. Finally, both the capital and labor markets clear whereby

$$K_{ct} + K_{It} = K_t, \tag{14}$$

$$L_{ct} + L_{It} = L_t. \tag{15}$$

It is straightforward to show that our model possesses a unique interior steady state. We then take log-linear approximations to the equilibrium conditions in a neighborhood of this steady state to obtain the following dynamic system:

$$\begin{bmatrix} \hat{K}_t \\ \hat{p}_t \end{bmatrix} = J \begin{bmatrix} \hat{K}_{t+1} \\ \hat{p}_{t+1} \end{bmatrix}, \quad \hat{K}_0 \text{ given}, \tag{16}$$

where hat variables denote percentage deviations from their steady-state values, and J is the Jacobian matrix of partial derivatives of the transformed dynamic system. The model exhibits saddle-path stability and equilibrium uniqueness when one eigenvalue of J lies inside and the other outside the unit circle. When both eigenvalues are outside the unit circle, the steady state is indeterminate and thus a sink. When both eigenvalues are inside the unit circle, the steady state becomes a totally unstable source.

3 Local Stability Properties

As discussed earlier, MYJ show that equilibrium indeterminacy cannot result in the one-sector version of the above model. This section examines the local stability properties in a calibrated version of our two-sector setting. It turns out that the first result clearly distinguishes our model from MYJ.

Result 1: In sharp contrast to MYJ, indeterminate equilibria result in our model with sufficiently high returns-to-scale.

The exact meaning of "sufficiently high" will be made clear below. As demonstrated by MYJ, in the one-sector version of our model, equilibrium indeterminacy is not possible regardless of the degree of aggregate returns-to-scale in production. What is different here? The answer lies in recalling the intratemporal first-order condition with respect to labor supply, (9), and comparing the intertemporal Euler equations from the two models. After rearranging the perfect-foresight version of (10), ours is

$$\frac{C_{t+1}-\frac{\Lambda L_{t+1}^{1+\chi}}{1+\chi}}{C_t-\frac{\Lambda L_t^{1+\chi}}{1+\chi}}=\beta\left[\frac{r_{t+1}+(1-\delta)p_{t+1}}{p_t}\right], \tag{17}$$

and its counterpart in the one-sector version is

$$\frac{C_{t+1}-\frac{\Lambda L_{t+1}^{1+\chi}}{1+\chi}}{C_t-\frac{\Lambda L_t^{1+\chi}}{1+\chi}}=\beta[r_{t+1}+(1-\delta)]. \tag{18}$$

In order for equilibrium indeterminacy to occur in either economy, the relevant Euler equation must be satisfied when there is a change in non-fundamental expectations. For example, starting from the steady state, upon an expected increase in the marginal return on capital, agents sacrifice consumption this period in order to invest more today. Therefore, C_t falls while K_{t+1} increases. Due to the lack of income effect, as seen in (9), L_t remains unchanged in response to the lower period-t consumption. On the other hand, a higher K_{t+1} leads to an increase in L_{t+1}, via the firms' labor demand, which in turn allow agents to consume more in period $t+1$: C_{t+1} rises. The change in L_{t+1} exerts two counteracting effects in the Euler equation. First, the bigger (smaller) the increase in L_{t+1}, the smaller (bigger) the increase of the left-hand side. Second, the bigger (smaller) the increase in L_{t+1}, the larger (smaller, or a decrease may occur) the rise in the real interest rate, r_{t+1}. In MYJ's one-sector model with (18), these two effects render the equality impossible. With C_t falling and C_{t+1} rising, a large increase in L_{t+1} is needed for r_{t+1} and the right-hand side to rise. But this would then decrease the left-hand side. At the same time, if the increase in L_{t+1} is small, keeping the left-hand side high, r_{t+1} and the right-hand side cannot rise enough.

However, in our two-sector model with (17), movements of productive resources affect the relative price of investment. In particular, as Benhabib and Farmer (1996) and Harrison (2001) have explained, due to the presence of increasing returns-to-scale, the social PPF showing the trade-off between consumption and investment is convex to the origin. This implies that shifting resources towards the production of a good raises the marginal product of each factor used in its production, and lowers the price of that good. Therefore, upon agents' optimistic expectations at period t, although total labor L_t is fixed, there is a change in the composition of hours worked as labor moves from the consumption to the investment sector.[2] Thus C_t and the relative price of investment, p_t, both fall. In addition, as discussed above, K_{t+1}, L_{t+1} and C_{t+1} all rise. Such movements of productive resources are self-fulfilling in equilibrium only if the (price-weighted) rate of return on investment, *i.e.* the right-hand side of (17), increases. Contrary to MYJ, this can in fact happen here, even though the real interest rate r_{t+1} may not rise (or rise enough), because the shift toward consumption in period $t+1$ also raises p_{t+1}. It follows that to maintain the equality in (17), small movements in L_{t+1} or large movements in C_{t+1} that can sufficiently raise its left-hand side are called for.

Our second result is reminiscent of Harrison (2001):

Result 2: A necessary and sufficient condition for equilibrium indeterminacy is a sufficiently high value of θ_I, the degree of productive externality in investment.

The reason is the same as in Harrison (2001), in which preferences are separable. When agents expect the return on capital to increase tomorrow, they need incentive to give up consumption today for more capital accumulation. As long as they will be rewarded with productive investment, in the form of increasing returns in that sector, it will be worthwhile for them to do so. In other words, it is the return on capital that agents care about, and so their expectation of its increase is fulfilled when there are sufficiently high returns-to-scale in the production of investment goods.

Result 2 allows us to set $\theta_c = 0$ from now on. We calibrate the rest of the model economy using parameter values consistent with post-war US data. Each period in the model is taken to be one quarter. As is common in the real business cycle literature, the capital share of national income, α, is chosen to be 0.3; the discount factor, β, is set equal to 0.99; and the capital depreciation rate, δ, is fixed at 0.025. We then examine the model's local stability properties for different combinations of χ and θ_I. In each parametric configuration, the preference parameter Λ is set to ensure that the steady-state labor hours is equal to 1/3. Figure 1 illustrates the local stability properties of our model, and our third result:

[2] Since the economy's aggregate capital stock K_t is a predetermined state variable, it (as well as its sectoral components K_{ct} and K_{It}) remain unaffected at the period of the sunspot shock.

Result 3: Below $\chi = 0.5$ (labor supply elasticity above 2), equilibrium indeterminacy is not possible (not shown).[3] However, as long as $\chi \geq 0.5$, a sufficiently high externality in investment results in local indeterminacy. We denote the threshold value of the externality $\theta_I^{\min}$.

The intuition for this result lies in reconsidering (17). Repeating the experiment, starting from the steady state, upon an expected increase in the marginal return on capital, agents will invest more today. C_t and p_t fall; and $K_{t+1.}$, C_{t+1} and p_{t+1} increase. With no income effect, L_t remains unchanged, but w_{t+1} and L_{t+1} increase, which in turn raises the right-hand side of the Euler equation, (17). Therefore, the left-hand side must increase equally; and this requires a small increase in L_{t+1}. Looking again at (9), we see that the higher the χ (or the lower the labor supply elasticity), the smaller the increase in L_{t+1}. It turns out that for $\chi < 0.5$, the labor supply elasticity is "too high", thus L_{t+1} rises too much. For $\chi \geq 0.5$, and lower labor supply elasticity, a smaller change in L_{t+1} results.

The above finding also helps us to understand our fourth result, which explains the negative slope of the curve in Figure 1:

Result 4: Equilibrium indeterminacy becomes easier to obtain, in the sense that lower investment externalities are needed, as the labor supply elasticity falls. That is, when χ increases, $\theta_I^{\min}$ decreases $\left(\frac{\partial \theta_I^{\min}}{\partial \chi} < 0\right)$.

This result is exactly the opposite of that in every other RBC-based indeterminacy model.[4] What it means is that as the household's labor supply becomes less elastic, lower returns-to-scale in investment are required for equilibrium indeterminacy. Intuitively, just like with χ, the change in L_{t+1} falls as θ_I increases. This is because higher returns-to-scale imply that the same output gain, and increase in the real wage, can be achieved with a smaller rise in inputs. Therefore, since increases in both χ and θ_I lead to smaller changes in L_{t+1}, the higher the χ, the lower the returns-to-scale needed to keep the increase in L_{t+1} small enough to satisfy (17).

[3] For $\chi < .5$, the equilibrium can be either (locally) determinate or unstable. In particular, for each $\chi < .5$, the model's steady state is a saddle point below a critical value of θ_I, and is a source above it. This critical value of θ_I increases with χ.

[4] For example, Benhabib and Farmer (1996, p. 433), demonstrate the positive relationship between χ and $\theta_I^{\min}$ in a two-sector model with the separable utility function (12).

4 Quantitative Analysis

Figure 1 shows that when the household's labor supply elasticity takes on the highest possible value for which the economy possesses multiple equilibria ($1/\chi = 2$), the associated threshold level of investment externalities, $\theta_I^{\min}$, is 0.7. This value is higher than existing empirical estimates. On the other hand, $\theta_I^{\min}$ can be lowered to around 0.3 if the labor supply elasticity is reduced to about 1/6. In other words, there is an empirical "tension" in our model – a very inelastic labor supply (a high value of χ) is needed to generate indeterminacy and sunspots with a reasonable degree of returns-to-scale in production.[5] In terms of quantitative implications, such a low labor supply elasticity dampens the fluctuations in aggregate hours worked. By contrast, this is not an issue in the two-sector model with separable preferences, (e.g. Benhabib and Farmer, 1996; and Harrison, 2001) because an infinitely elastic labor supply ($\chi = 0$) is adopted and $\theta_I^{\min}$ increases with χ (see footnote 4).

This motivates us to examine the quantitative properties of our model economy. In particular, we are interested in whether the model, driven either solely by *i.i.d.* belief shocks or by both sunspot and technology shocks, can produce economically significant fluctuations with an empirically plausible degree of investment externalities. This numerical simulation analysis also enables us to explore the following stylized facts that a belief-driven two-sector RBC model traditionally has difficulty matching: (i) sectoral labor inputs are positively correlated (the *labor-comovement puzzle*), and (ii) consumption is procyclical (the *consumption-cyclicality puzzle*).

With regard to the labor-comovement puzzle, it is well-known that in a two-sector RBC model that includes an income effect on leisure demand (Benhabib and Farmer, 1996; and Harrison, 2001), the labor inputs into the two production sectors comove negatively. Recall the first-order condition for labor supply in this case, (13). Substituting w_t from (5) and L_t from (15), we can rewrite it as

$$(L_{ct} + L_{It})^{\chi} L_{ct} = \frac{(1-\alpha)}{A}, \tag{19}$$

where the right-hand side is a constant. Hence, L_{ct} and L_{It} must be negatively correlated. However, this shortcoming may be overturned within our no-income-effect model. Rewriting the relevant first-order condition, (9), yields

$$(L_{ct} + L_{It})^{\chi} L_{ct} = \frac{(1-\alpha)\, C_t}{\Lambda}. \tag{20}$$

[5] Thanks to an anonymous referee for pointing this out to us.

Notice that the right-hand side is no longer a constant, hence L_{ct} and L_{It} could move together.[6]

In terms of the consumption-cyclicality puzzle, Benhabib and Farmer (1996) demonstrate it when business cycles are driven solely by sunspot shocks. When agents become optimistic about the future of the economy, they move productive resources out of the consumption sector and into investment. As a result, consumption moves in the opposite direction of output in a self-fulfilling equilibrium. However, Harrison (2001) shows that consumption becomes less countercyclical, or even procyclical, when the degree of investment externalities, θ_I, rises. It is therefore worthwhile to investigate the performance of our no-income-effect model with respect to this puzzle.

In the following subsections, we first examine the model's quantitative predictions when business cycles are driven by only sunspot shocks, and then we add in productivity disturbances.

4.1 Sunspot Shocks Only

Each period in the model is taken to be one quarter. In addition to calibrating α, β, δ and Λ under the same rationale described in the preceding section, we seek to choose empirically plausible values of χ and θ_I in our benchmark parameterization. There is an extensive literature in labor economics that has provided estimates of the (Frisch) wage elasticity of labor supply at the micro level.[7] For example, Altonji (1986, Table 1) finds that the estimated intertemporal labor supply elasticity is 0.067, with a standard error of 0.08. Drawing on this result, we choose $\chi = 15$. With respect to θ_I, Basu and Fernald (1997, Table 3) report an aggregation-corrected point estimate for returns-to-scale in the durables manufacturing industry of 1.33 (standard error = 0.11). Using durables as a proxy for the investment goods in our model, we set $\theta_I = 0.3$. In addition, we maintain the assumption that $\theta_c = 0$. Finally, the standard deviation of the driving *i.i.d.* sunspot or belief shocks, $\{v_t\}$, is calibrated to match the volatility of detrended output observed in the actual data, which is taken to be 2.1%. These mean-zero shocks enter the intertemporal consumption Euler equation (10).

Table 1 presents the simulation results from our benchmark parameterization. The statistics reported in columns 2-3 are sample means from 1,000 simulations of length 1,000. As in previous sunspot-driven two-sector RBC models (Benhabib and Farmer, 1996; and Harrison,

[6] Thanks to an anonymous referee for pointing this out to us.

[7] The estimated elasticities are typically higher for females (close to 1) than for males (smaller than 0.3). See, for example, the excellent surveys of Pencavel (1986), Killingsworth and Heckman (1986) and Blundell and McCurdy (1999) for more details.

2001), in our no-income-effect economy: (i) sectoral hours worked are negatively correlated ($\text{corr}(L_c, L_I) < 0$) and (ii) consumption and its labor input (L_c) move in the opposite direction of GDP. Hence, our benchmark specification does not resolve either the labor-comovement or the consumption-cyclicality puzzle. On the other hand, we see: (iii) the relative price of investment goods (p) is countercyclical, which implies a downward-sloping sectoral aggregate supply curve. This result is not inconsistent with empirical evidence (Shea, 1996).

Table 1 also shows that the very small value of the household's labor supply elasticity generates an extremely low volatility in total hours worked (L). However, reallocation of productive resources, caused by swings in agents' optimism and pessimism, leads to high variabilities in the relative price of investment (p) and in the two sectoral labor inputs (L_c and L_I). Since L_c and L_I both exhibit high volatilities, and they are highly negatively correlated, their large movements almost offset each other to produce very small changes in total labor L. Moreover, these large movements in sectoral inputs and the relative price result in high standard deviations of consumption and (price-weighted) investment.

To understand the high variability of consumption, we log-linearize the first-order condition for labor supply (20) to obtain

$$\widehat{C}_t = \chi \widehat{L}_t + \widehat{L}_{ct}, \tag{21}$$

where hat variables represent percentage deviations from their respective steady-state values. Since total labor and its consumption component move together, *i.e.* $\text{corr}(L, L_c) > 0$, the variance of consumption is higher than those of the right-hand-side variables. Although the standard deviation of aggregate hours worked is quite low, the large value of χ and high volatility of L_c dominate. Consequently, consumption becomes highly variable.

Table 1: Benchmark – Sunspots Only, $\chi = 15$ and $\theta_I = 0.3$

	std.(%)	corr. with output
Output (Y)	2.1	
Consumption (C)	9.1	-0.14
Investment (price-weighted, pI)	32.6	0.14
Relative price (p)	9.8	-0.14
Total labor (L)	0.02	-0.01
Labor in consumption (L_c)	8.9	-0.14
Labor in investment (L_I)	32.7	0.14
$\text{Corr}(L_c, L_I)$	-1.00	

Table 2 presents the simulation results from an alternative parameterization. In particular, we increase the household's labor supply elasticity by setting $\chi = 0.6$, which is close to the lower bound (0.5) of its admissible range (see Figure 1). This in turn raises the volatility of total labor as well as the $\theta_I^{\min}$ that is needed for indeterminacy and sunspots. The reported results use $\theta_I = 0.8$. We acknowledge that this value is too large to be considered empirically plausible, but our goal here is to find parameter combinations that resolve the labor-comovement and consumption-cyclicality puzzles when business cycles are caused by agents' animal spirits alone.

In comparison with Table 1, the overall results in this alternative specification are much-improved. Both puzzles are now resolved. Not only is consumption procyclical, but so are aggregate hours worked and their sectoral components. As a result, our model is able to generate a positive comovement between the sectoral labor inputs. On the other hand, a higher volatility in aggregate hours worked moderates the magnitude of cyclical fluctuations in the relative price of investment and the two sectoral labor inputs. Consequently, consumption and (price-weighted) investment are smoother than those under our benchmark parameterization.

Table 2: Sunspot Shocks Only, $\chi = 0.6$ and $\theta_I = 0.8$

	std.(%)	corr. with output
Output (Y)	2.1	
Consumption (C)	2.1	0.95
Investment (price-weighted, pI)	3.1	0.68
Relative price (p)	2.5	-0.68
Total labor(L)	1.3	1.00
Labor in consumption (L_c)	1.4	0.89
Labor in investment(L_I)	2.6	0.52
Corr(L_c, L_I)	0.09	

4.2 Sunspot and Technology Shocks

The quantitative results from the previous subsection imply that with sunspots alone, allowing a larger elasticity for labor supply does not resolve the procyclicality issue in consumption and sectoral labor inputs unless the degree of investment externalities is "too high" from an empirical plausibility perspective. One approach, as in Benhabib and Farmer (1996), that may alleviate this difficulty is to adopt empirically plausible returns-to-scale in production and incorporate a productivity disturbance whose innovations are postulated to be correlated with sunspot shocks. In this case, sunspots can be interpreted as overreactions to changes in firms' production technologies and they may raise the cyclical correlation between output and consumption.

With the above considerations, we choose $\theta_I = 0.44$, the highest possible value that is regarded as empirically plausible. This is one standard error above Basu and Fernald's (1997) point estimate for returns-to-scale in the durables manufacturing industry. Given this particular value of θ_I, we read from the curve in Figure 1 in order to maintain equilibrium indeterminacy, and choose $\chi = 1.6$.

We introduce a multiplicative economy-wide technology shock z_t to the production functions for consumption (1) as well as for investment (3). As is standard in the real business cycle literature, technology shocks are postulated to evolve according to

$$\log(z_{t+1}) = \rho \log(z_t) + \varepsilon_{t+1}, \quad 0 < \rho < 1 \text{ and } z_0 \text{ given,} \tag{22}$$

where the persistence parameter ρ is set to be 0.95. Following Benhabib and Farmer (1996), we also assume that innovations to the firms' productivity $\{\varepsilon_t\}$ and the *i.i.d.* sunspot shock $\{v_t\}$ are perfectly correlated and have the same standard deviation, calibrated to produce the same output volatility (2.1%) as in Tables 1 and 2.

Table 3 presents the simulation results from this specification. In this framework, as in our benchmark parameterization, agents' optimistic expectations lead to reallocation of productive resources towards the investment sector, which yields a movement along the social PPF with consumption falling as output increases. However, at the same time, a positive productivity disturbance shifts the social PPF to the right, thereby raising output as well as consumption and investment. This expansionary effect is further strengthened by the perfectly positive correlation between the shocks. Our numerical simulations show that the latter effect dominates. Hence, in contrast to Table 1, total hours worked, consumption and its labor input now all move in the same direction as GDP. However, L_c and L_I remain negatively correlated.[8] As discussed earlier, L_c and L_I are both highly variable because of reallocation of hours worked across the two production sectors. Consequently, a positive correlation between sectoral labor inputs requires total employment L $(= L_c + L_I)$ to be sufficiently volatile, a

[8] Although not reported, we have simulated the benchmark parameterization ($\chi = 15$ and $\theta_I = 0.3$) driven by (i) technology shocks alone and (ii) sunspot plus technology shocks. Compared to Table 1, employment in the consumption sector remains countercyclical and the two sectoral labor inputs continue to be negatively correlated in both cases. However, consumption stays countercyclical in (i), but becomes procyclical in (ii) because of additional movements in the capital stock induced by dual shocks.

feature that does not occur in this setting.[9]

Table 3: Sunspot and Technology Shocks, $\chi = 1.6$ and $\theta_I = 0.44$		
	std.(%)	corr. with output
Output (Y)	2.1	
Consumption (C)	3.6	0.48
Investment (price-weighted, pI)	11.9	0.29
Relative price (p)	5.2	-0.26
Total labor(L)	0.8	0.99
Labor in consumption (L_c)	3.1	0.14
Labor in investment(L_I)	11.7	0.19
Corr(L_c, L_I)	-0.94	

In order to resolve the above labor-comovement puzzle under dual driving shocks, the household's labor supply elasticity is raised to the same level used in Table 2, *i.e.* $\chi = 0.6$. We then look for the minimum value of θ_I such that the economy displays local indeterminacy and positive correlation between L_c and L_I. As it turns out, this requires that $\theta_I = 0.73$; and Table 4 reports the resulting simulation outcomes.[10] Although this degree of investment externalities is lower than that previously adopted in the sunspot-only specification ($\theta_I = 0.8$ in Table 2), it is nevertheless empirically implausible. Therefore, the statistics reported in Tables 2 and 4 should be viewed more from a methodological perspective as illustrating the rather robust empirical "tension" between χ and $\theta_I^{\min}$ in our no-income-effect two-sector RBC model. This conflict is highlighted by our finding that the correlation between sectoral labor inputs is, *ceteris paribus,* an increasing function of the degree of investment externalities.

Table 4: Sunspot and Technology Shocks, $\chi = 0.6$ and $\theta_I = 0.73$		
	std.(%)	corr. with output
Output (Y)	2.1	
Consumption (C)	2.1	0.93
Investment (price-weighted, pI)	3.3	0.65
Relative price (p)	2.4	-0.64
Total labor(L)	1.3	1.00
Labor in consumption (L_c)	1.4	0.85
Labor in investment(L_I)	2.9	0.50
Corr(L_c, L_I)	0.008	

[9]The simulation results in Table 3 remain qualitatively unchanged when this formulation ($\chi = 1.6$ and $\theta_I = 0.44$) is driven solely by sunspots or by technology shocks. In particular, the two sectoral labor inputs are negatively correlated.

[10]Note that L_c and L_I.are negatively correlated when business cycles are driven by sunspots alone or only by productivity disturbances in this version of our model ($\chi = 0.6$ and $\theta_I = 0.73$).

5 Conclusion

This paper extends MYJ's analyses, and examines the stability effect of incorporating no-income-effect preferences into a two-sector real business cycle model with sector-specific externalities. While indeterminacy and sunspots are impossible in the MYJ one-sector economy, their result is overturned here because movements of factors of production affect the relative price of investment in our two-sector setting. In addition, due to the non-separability of consumption and leisure in the household's utility, the key to generating equilibrium indeterminacy is small changes in total hours worked. Hence, indeterminacy and sunspots are easier to obtain with a lower labor supply elasticity. Our quantitative analysis shows that the benchmark specification of our no-income-effect two-sector RBC model is subject to the labor-comovement and consumption-cyclicality puzzles. However, both puzzles are resolved under alternative parameterizations in which the household's labor supply elasticity as well as the degree of investment externalities are raised.

This paper can be extended in several directions. Specifically, it would be interesting to incorporate a naturally countercyclical sector, like "home production" (Benhabib, Rogerson and Wright, 1991; and Perli, 1998) or "search" (Merz, 1995; and Andolfatto, 1996) or "schooling – human capital accumulation" (Perli and Sakellaris, 1998; and Maffezzoli, 2000), into our indeterminate RBC model. In response to changes in agents' non-fundamental expectations, such a sector provides labor to the market economy during expansions and absorbs labor during recessions, which in turn can help generate procyclical series of consumption and its sectoral labor input. Moreover, it would be worthwhile to examine the robustness of our simulation results under a generalized non-separable utility *a la* Jaimovich (2008) that allows for different degrees of income effect on the demand for leisure. Introducing this flexible preference formulation to the analysis will further enhance our understanding of the quantitative interrelations between income effects in the utility function and the empirical plausibility of equilibrium indeterminacy. We plan to pursue these research projects in the near future.

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Figure 1: Local Stability Properties