Capital-Labor Substitution, Equilibrium Indeterminacy, and the Cyclical Behavior of Labor Income*

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Abstract

This paper examines the quantitative relationship between the elasticity of capital-labor substitution and the conditions needed for equilibrium indeterminacy (and belief-driven fluctuations) in a one-sector growth model. Our analysis employs a "normalized" version of the CES production function so that all steady-state allocations and factor income shares are held constant as the elasticity of substitution is varied. We demonstrate numerically that higher elasticities cause the threshold degree of increasing returns for indeterminacy to decline monotonically, albeit very gradually. When the elasticity of substitution is unity (the Cobb-Douglas case), our model requires increasing returns to scale of around 1.08 for indeterminacy. When the elasticity of substitution is raised to 5, which far exceeds any empirical estimate, the threshold degree of increasing returns reduces to around 1.05. We also demonstrate analytically that labor's share of income becomes pro-cyclical as the elasticity of substitution increases above unity, whereas labor's share in postwar U.S. data is countercyclical. This observation, together with other empirical evidence, indicates that the elasticity of capital-labor substitution in the U.S. economy is actually below unity.

Keywords: *Capital-Labor Substitution, Equilibrium Indeterminacy, Capital Utilization, Real Business Cycles, Labor Income.*

JEL Classification: E30, E32.

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1 Introduction

It is well-known that the elasticity of substitution between capital and labor in production can have an important influence on transition dynamics in the standard one-sector growth model.[1] One would therefore expect this elasticity to also affect the characteristics of fluctuations near the model's steady state. Specifically, rational belief-driven business cycles or "sunspots" can arise when the steady state of the model is locally indeterminate, i.e., a sink.[2] With a few exceptions, the indeterminacy literature has mostly restricted attention to a Cobb-Douglas production technology that exhibits a unitary elasticity of substitution between capital and labor. A Cobb-Douglas specification also implies that factor income shares are constant over the business cycle. In contrast, labor's share of income in U.S. postwar data is countercyclical while capital's share is procyclical.

This paper examines two closely-related issues. First, we examine the quantitative relationship between the elasticity of capital-labor substitution and the threshold degree of increasing returns for local indeterminacy (and belief-driven fluctuations) within a one-sector growth model. Second, we examine the link between the elasticity of capital-labor substitution and the cyclical properties of the factor income shares. Following Klump and de La Grandville (2000) and Klump and Preissler (2000), we employ a "normalized" version of the standard CES production function so that all steady-state allocations and factor income shares are held constant as the elasticity of capital-labor substitution is changed. The normalization procedure identifies a family of constant-elasticity-of-substitution (CES) production functions that are distinguished only by the elasticity parameter, and not by the steady-state allocations which are used to approximate the model's local dynamics. In practical terms, the normalization procedure amounts to recalibrating the model to "match the facts" each time the elasticity parameter is varied. Klump and Saam (2006) emphasize that normalization is necessary to avoid "arbitrary and inconsistent results."

The framework for our analysis is an extended version of the one-sector real business cycle model of Guo and Lansing (2007) that includes variable capital utilization and endogenous maintenance expenditures. In the present paper, we demonstrate numerically that higher elasticities of substitution cause the threshold degree of increasing returns for equilibrium indeterminacy to decline monotonically, albeit very gradually. When the elasticity of substitution is unity (the Cobb-Douglas case considered by Guo and Lansing, 2007), the model requires increasing returns-to-scale of around 1.08 for equilibrium indeterminacy. When the

[1] See, for example, Barro and Sala-i-Martin (1995, p. 45), Klump and de La Grandville (2000), Klump and Preissler (2000), Turnovsky (2002), and Smetters (2003), among others.

[2] See Benhabib and Farmer (1999) for a survey of this literature.

elasticity of substitution is raised to 5, which far exceeds any empirical estimate, the threshold degree of increasing returns reduces to around 1.05. Intuitively, a higher elasticity of capital-labor substitution makes indeterminacy easier to obtain because it allows equilibrium labor hours to respond more freely to belief shocks, rather than being tightly coupled to utilized capital which responds more sluggishly.

Our results are qualitatively consistent with those of Pintus (2006) who also shows that higher elasticities of substitution can reduce the minimum degree of increasing returns needed for local indeterminacy in a one-sector growth model. Our results are *quantitatively* different because: (1) we allow for variable capital utilization along the lines of Wen (1998), and (2) unlike Pintus (2006), we require the curvature of the agent's separable utility function to be consistent with balanced long-run growth.[3] Both features are known to be important for influencing the threshold degree of increasing returns in this class of models. The details of the numerical computations are important, in our view, for assessing the quantitative impact of capital-labor substitution in a particular model, in order to gauge whether the threshold degree of increasing returns for indeterminacy lies within the range of empirical plausibility (see Basu and Fernald, 1997).

We demonstrate analytically that one can infer useful information about the elasticity of capital-labor substitution by examining the cyclical properties of the factor income shares. In our model, movements in the labor income share over the business cycle are linked directly to movements in the ratio of labor hours to utilized capital. Labor hours in the model are more volatile than utilized capital in response to shocks. A positive belief shock will therefore raise the ratio of labor hours to utilized capital while output increases. When the elasticity of substitution exceeds unity, a higher ratio of labor hours to utilized capital causes labor's share to become pro-cyclical, whereas labor's share in U.S. data is countercyclical. It follows that in order to match the cyclical behavior of labor's share in the data, the model requires the elasticity of substitution to be below unity.

Direct empirical estimates also indicate that the elasticity of capital-labor substitution in the U.S. economy is below unity. Using time series data for the period 1953 to 1998, Klump, McAdam, and Willman (2007) find that the elasticity of substitution is significantly below unity, with a point estimate of 0.5 to 0.6. A panel data study by Chirinko, Fazzari, and Meyer (2004) yields a precisely-estimated elasticity of approximately 0.4. In our model, an elasticity of 0.4 would push up the threshold degree of increasing returns only slightly—to a

[3]The numerical examples in Pintus (2006) employ a utility function that is close to risk-neutral in consumption, with coefficients of relative risk aversion that range from 0.04 to 0.15. In our model, the coefficient of relative risk aversion for consumption is 1.0, corresponding to the logarithmic case.

value around 1.09.

Finally, we note that other types of growth models can become more susceptible to local or global indeterminacy when the elasticity of capital-labor substitution is *below* unity, as appears to be the case for the U.S. economy. Examples include the finance-constrained (capitalist-worker) model of Grandmont, Pintus, and de Vilder (1998), and the multisector growth model of Nishimura and Venditti (2004). Another recent example is the one-sector growth model of Wong and Yip (2007) where the elasticity of substitution is not a parameter, but rather is assumed to be a decreasing linear function of the economy's aggregate capital-labor ratio.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 examines the model's local dynamics and presents some quantitative analyses. Section 4 studies the cyclical behavior of labor income. Section 5 concludes.

2 The Model

We adopt the basic setup of Guo and Lansing (2007) that allows for: (1) variable capital utilization along the lines of Wen (1998), and (2) endogenous maintenance activity along the lines of McGrattan and Schmitz (1999). We depart from the usual assumption of a Cobb-Douglas production function by introducing a "normalized" version of the standard constant-elasticity-of-substitution (CES) production function.

2.1 Households

The economy is populated by a unit measure of identical infinitely-lived households, each endowed with one unit of time. The representative household maximizes

$$\sum_{t=0}^{\infty} \beta^t \left[\log(c_t) - \frac{A n_t^{1+\gamma}}{1+\gamma} \right], \qquad A > 0, \tag{1}$$

subject to the budget constraint

$$c_t = w_t n_t + d_t, \tag{2}$$

where $\beta \in (0,1)$ is the subjective time discount factor, c_t is consumption, n_t is hours worked, $\gamma \geq 0$ is the inverse of the intertemporal elasticity of substitution in labor supply, w_t is the real wage, and d_t is dividends paid out by the firm which the household takes as given. The household's period utility function in (1) is consistent with balanced long-run growth, a feature that is commonly maintained in the modern business cycle literature.

The first-order condition for the household's optimization problem is given by

$$A c_t n_t^{\gamma} = w_t, \tag{3}$$

which equates the marginal rate of substitution between consumption and leisure to the real wage.

2.2 Firms

There are a large number of identical competitive firms, each endowed with k_0 units of capital, that produce a single final good y_t using the following linearly homogeneous technology:

$$y_t = B\left[\alpha\left(u_t k_t\right)^{\psi} + (1-\alpha)\, n_t^{\psi}\right]^{\frac{1}{\psi}} X_t, \tag{4}$$

$$B > 0, \qquad \alpha \in (0,1), \qquad \psi \equiv \frac{\sigma - 1}{\sigma}, \qquad \sigma \in (0, \infty),$$

where u_t is the endogenous rate of capital utilization and k_t is the firm's stock of physical capital. The parameter ψ depends on the elasticity of substitution σ between utilized capital $u_t k_t$ and labor hours n_t. When $\sigma = 1$ (or $\psi = 0$), we recover the usual Cobb-Douglas production technology. When $\sigma \to 0$ (or $\psi \to -\infty$), the production technology takes a Leontief formulation such that utilized capital and labor become perfect compliments. When $\sigma \to \infty$ (or $\psi \to 1$), utilized capital and labor become perfect substitutes.

As described in the appendix, our normalization procedure recalibrates the parameters B and α in equation (4) each time the elasticity of substitution σ is varied so that all steady-state allocations and factor income shares are held constant across computations. Other model parameters are also recalibrated each time that σ is varied. For expositional convenience, we omit the explicit notation $B(\sigma)$ and $\alpha(\sigma)$ where these and other parameters appear in the paper.

The symbol X_t represents a productive externality that takes the form

$$X_t = Y_t^{\frac{\eta}{1+\eta}}, \qquad \eta \geq 0, \tag{5}$$

where Y_t is the economy-wide average level of output per firm. In a symmetric equilibrium, all firms take the same actions such that $y_t = Y_t$, for all t. As a result, equation (5) can be substituted into (4) to obtain the following social technology that may display increasing returns-to-scale:

$$y_t = B\left[\alpha\left(u_t k_t\right)^{\psi} + (1-\alpha)\, n_t^{\psi}\right]^{\frac{1+\eta}{\psi}}, \tag{6}$$

where the degree of increasing returns in production is given by $1+\eta$. When $\eta = 0$, the model collapses to one with constant returns-to-scale at both the firm and social levels.

The law of motion for the capital stock is given by

$$k_{t+1} = (1-\delta_t)\, k_t + i_t, \qquad k_0 \text{ given}, \tag{7}$$

where $\delta_t \in (0,1)$ is the endogenous rate of capital depreciation and i_t is investment in new capital. We postulate that δ_t takes the form

$$\delta_t = \tau \, \frac{u_t^{\theta}}{(m_t/k_t)^{\phi}}, \qquad \tau > 0, \;\; \theta > 1, \text{ and } \; \phi \geq 0, \tag{8}$$

where m_t/k_t represents maintenance expenditures per unit of installed capital. When $\phi = 0$, we recover the depreciation technology of Wen (1998) which abstracts from maintenance activity. Our setup is motivated by the work of McGrattan and Schmitz (1999) who argue that maintenance and repair activity is "too big to ignore." When $\theta \to \infty$, the model collapses to one with constant depreciation and utilization rates.

Under the assumption that the labor market is perfectly competitive, firms take w_t as given and choose sequences of n_t, u_t, m_t, and $k_{t+1,}$ to maximize the following discounted stream of expected dividends:

$$\sum_{t=0}^{\infty} \beta^t \; (1/c_t) \; \underbrace{[y_t - w_t \, n_t - i_t - m_t]}_{d_t}, \tag{9}$$

subject to the firm's production function (4), the law of motion for capital (7), and the depreciation technology (8). Firms act in the best interests of households such that dividends in period t are valued using the household's marginal utility of consumption, as given by $1/c_t$.

The firm's first-order conditions with respect to the indicated variables are

$$n_t \quad : \qquad \frac{(1-\alpha)\, y_t}{n_t} \left[\frac{n_t^{\psi}}{\alpha \, (u_t k_t)^{\psi} + (1-\alpha) n_t^{\psi}} \right] = w_t, \tag{10}$$

$$u_t \quad : \qquad \frac{\alpha y_t}{\theta k_t} \left[\frac{(u_t k_t)^{\psi}}{\alpha \, (u_t k_t)^{\psi} + (1-\alpha) n_t^{\psi}} \right] = \delta_t, \tag{11}$$

$$m_t \quad : \qquad 1 = \phi \frac{\delta_t k_t}{m_t} \tag{12}$$

$$k_{t+1} \quad : \qquad \frac{1}{c_t} = \frac{\beta}{c_{t+1}} \left\{ \frac{\alpha y_{t+1}}{k_{t+1}} \left[\frac{(u_{t+1} k_{t+1})^{\psi}}{\alpha \, (u_{t+1} k_{t+1})^{\psi} + (1-\alpha) n_{t+1}^{\psi}} \right] + 1 - (1+\phi)\, \delta_{t+1} \right\}, \tag{13}$$

together with the transversality condition $\lim_{t\to\infty} \beta^t \; (k_{t+1}/c_t) = 0$.

By combining the household's budget constraint (2), the law of motion for physical capital (7) and the firm's dividend (9), we obtain the following aggregate resource constraint

$$y_t = c_t + k_{t+1} - (1 - \delta_t) \; k_t + m_t. \tag{14}$$

3 Analysis of Dynamics

The dimensionality of the dynamical system can be reduced as follows. First, equation (12) is used to eliminate m_t from the resource constraint (14). Second, equation (11) is used to eliminate δ_{t+1} and δ_t from the consumption Euler equation (13) and the resource constraint (14). This procedure yields the dynamical system

$$k_{t+1} = y_t \left\{ 1 - \frac{\alpha(1+\phi)}{\theta} \left[\frac{(u_t k_t)^\psi}{\alpha (u_t k_t)^\psi + (1-\alpha) n_t^\psi} \right] \right\} + k_t - c_t, \tag{15}$$

$$\frac{1}{c_t} = \frac{\beta}{c_{t+1}} \left\{ \frac{\alpha y_{t+1}}{k_{t+1}} \left(\frac{\theta - 1 - \phi}{\theta} \right) \left[\frac{(u_{t+1} k_{t+1})^\psi}{\alpha (u_{t+1} k_{t+1})^\psi + (1-\alpha) n_{t+1}^\psi} \right] + 1 \right\}, \tag{16}$$

where y_t is governed by the social technology (6). The dimensionality of the system can be further reduced by eliminating u_t and n_t, followed by u_{t+1} and n_{t+1}.

Solving equation (8) for u_t and then eliminating m_t and δ_t as before yields

$$u_t = \left(\frac{\phi^\phi}{\tau} \right)^{\frac{1}{\theta}} \left(\frac{\alpha y_t}{\theta k_t} \right)^{\frac{1+\phi}{\theta}} \left[\frac{(u_t k_t)^\psi}{\alpha (u_t k_t)^\psi + (1-\alpha) n_t^\psi} \right]^{\frac{1+\phi}{\theta}}. \tag{17}$$

The additively-separable term $\alpha (u_t k_t)^\psi + (1-\alpha) n_t^\psi$ in the denominator can be eliminated using the social technology (6). Doing so and then multiplying by k_t yields the following expression

$$u_t k_t = \left[\frac{\phi^\phi}{\tau} \left(\frac{\alpha}{\theta} \right)^{1+\phi} B^{(1+\phi)\psi} \right]^{\frac{\psi}{\theta-(1+\phi)\psi}} y_t^{\frac{[(1+\phi)(1+\eta)-\theta]\psi}{(1+\eta)[\theta-(1+\phi)\psi]}} k_t^{\frac{\theta-1-\phi}{\theta-\psi(1+\phi)}}. \tag{18}$$

Next, we combine (3) and (10), and then once again use (6) to eliminate the additively-separable term. The resulting expression for n_t is

$$n_t = \left[\frac{(1-\alpha) B^\psi}{A} \right]^{\frac{1}{1+\gamma-\psi}} y_t^{\frac{1-\psi/(1+\eta)}{1+\gamma-\psi}} c_t^{\frac{-1}{1+\gamma-\psi}}. \tag{19}$$

The final step is to substitute the above expressions for $u_t k_t$ and n_t into the social technology (6), yielding a nonlinear equation for y_t in terms of k_t and c_t only. We log-linearize this equation around the normalized steady state (described in the Appendix) and then express y_t as an approximate power function in k_t and c_t. Iterating this function ahead one period yields an analogous function for y_{t+1} in terms of k_{t+1} and c_{t+1}. The approximate power functions for y_t and y_{t+1} are then substituted back into (15), (16), (18), and (19), so that the only remaining variables in the dynamical system are k_t, c_t, k_{t+1}, and c_{t+1}.

It is also useful to derive an approximate version of the equilibrium social technology in terms k_t and n_t. To accomplish this, we substitute the expression for $u_t k_t$ from (18) into (6). We then log-linearize the resulting expression around the normalized steady state and solve for y_t. The approximate social technology is given by

$$\log\left(y_t/\,\overline{y}\,\right) \;\simeq\; \alpha_k \log\left(k_t/\,\overline{k}\,\right) \,+\, \alpha_n \log\left(n_t/\,\overline{n}\,\right), \tag{20}$$

where $\overline{y}$, $\overline{k}$, and $\overline{n}$ are the normalized steady-state quantities. The production function elasticities are given by

$$\alpha_k \;=\; \frac{\alpha\,(1+\eta)\,(\theta-1-\phi)}{\alpha\left[\theta-(1+\eta)\,(1+\phi)\right] \,+\, (1-\alpha)\left[\theta-\psi\,(1+\phi)\right]\left[\,\overline{n}\,/\,\left(\overline{u}\,\overline{k}\right)\,\right]^{\psi}}, \tag{21}$$

$$\alpha_n \;=\; \frac{(1-\alpha)\,(1+\eta)\left[\theta-\psi\,(1+\phi)\right]}{\alpha\left[\theta-(1+\eta)\,(1+\phi)\right]\left[\,\overline{n}\,/\,\left(\overline{u}\,\overline{k}\right)\,\right]^{-\psi} \,+\, (1-\alpha)\left[\theta-\psi\,(1+\phi)\right]}. \tag{22}$$

When $\psi = 0$, the above expressions are identical to those derived by Guo and Lansing (2007) for the Cobb-Douglas case. As usual, we restrict our analysis to the case of $\alpha_k < 1$, which implies that the productive externality is not strong enough to generate sustained endogenous growth. When $\eta = 0$, it is straightforward to verify that $\alpha_k + \alpha_n = 1$ for any value of ψ. Thus, when the productive externality vanishes, the model exhibits constant returns-to-scale in production. This condition ensures that the individual firm's decision problem is concave.

3.1 Local Indeterminacy

The nonlinear dynamical system consists of equations (15) and (16) expressed in terms of k_t, c_t, k_{t+1}, and c_{t+1}. The system is log-linearized around the normalized steady state to obtain:

$$\begin{bmatrix} \log\left(k_{t+1}/\,\overline{k}\,\right) \\ \log\left(c_{t+1}/\,\overline{c}\,\right) \end{bmatrix} = \mathbf{J} \begin{bmatrix} \log\left(k_t/\,\overline{k}\,\right) \\ \log\left(c_t/\,\overline{c}\,\right) \end{bmatrix}, \qquad k_0 \text{ given}, \tag{23}$$

where $\mathbf{J}$ is the Jacobian matrix of partial derivatives. The local stability properties of the steady state are determined by comparing the number of eigenvalues of $\mathbf{J}$ located inside the unit circle with the number of initial conditions. There is one initial condition represented by k_0. Hence, if both eigenvalues of $\mathbf{J}$ lie inside the unit circle, then the steady state is indeterminate (a sink) and the economy is subject to belief-driven fluctuations. This will occur if and only if

$$-1 \;<\; \det(\mathbf{J}) \;<\; 1 \quad \text{ and } \quad -\left[1+\det(\mathbf{J})\right] \;<\; \mathrm{tr}(\mathbf{J}) \;<\; 1+\det(\mathbf{J}). \tag{24}$$

For our calibration, the most-binding condition among the necessary and sufficient conditions for local indeterminacy in (24) turns out to be $\det(\mathbf{J}) + \mathrm{tr}(\mathbf{J}) > -1$.

[Figure 1 about here].

Figure 1 depicts the stability properties of the steady state as a function of the externality parameter η and the elasticity of captal-labor substitution $\sigma = 1/(1-\psi)$. At each point on the stability plot, we recalibrate the model using the normalization procedure described in the Appendix. The downward-sloping line, which separates the regions labeled "Saddle" and "Sink, " plots the minimum required value of η for local indeterminacy. Higher values of σ allow indeterminacy to occur with a smaller externality parameter. When $\sigma \to 0$, implying a Leontief specification, our model requires $\eta > 0.099$ for local indeterminacy. When $\sigma = 1$, implying a Cobb-Douglas specification, our model requires $\eta > 0.083$ for local indeterminacy, coinciding with results of Guo and Lansing (2007). When $\sigma = 5$, which far exceeds any empirical estimate for the U.S. economy, our model requires $\eta > 0.0496$ for local indeterminacy

The intuition for why higher values of σ can make equilibrium indeterminacy easier to obtain is straightforward. When agents become optimistic about the future, they will invest more today, thus raising next period's capital stock. To validate agents' optimistic expectations as a self-fulfilling prophecy, we require the next period's return on capital, net of depreciation, to rise in equilibrium. Other things being equal, a higher value of σ allows labor hours n_t to respond more strongly to belief shocks, rather than being tightly coupled to utilized capital $u_t k_t$, which responds more sluggishly. The positive response of labor hours provides a direct boost to the return on capital, allowing agents' optimistic beliefs to be validated at lower threshold degree of increasing returns.

As noted in the introduction, empirical studies indicate that the value of σ for the U.S. economy falls in the range of 0.4 to 0.6. When $\sigma = 0.4$, our model requires $\eta > 0.092$ for local indeterminacy. This threshold corresponds to a relatively mild degree of increasing returns, one that remains within the realm of empirical plausibility. For example, Basu and Fernald (1997, Table 3, col. 1, p. 268) report a returns-to-scale estimate of 1.03 (standard error = 0.18) for the U.S. private business economy.

A recent paper by Pintus (2006) also demonstrates that a higher elasticity of captal-labor substitution can reduce the minimum degree of increasing returns needed for local indeterminacy. Pintus considers a one-sector model with constant capital depreciation and utilization rates, a setup that corresponds to our model with $\theta \to \infty$. In his numerical examples, the coefficient of relative risk aversion (CRRA) for consumption in the agent's additively separable utility function is close to zero—a calibration that is not consistent with balanced growth within the model. He reports (p. 643), that when the CRRA is below 0.04 and the elasticity of captal-labor substitution exceeds 2.16, the model requires increasing returns to scale of around 1.03 for indeterminacy. In our framework where the CRRA for consumption is 1.0 (log utility) and $\sigma = 2.16$, the threshold degree of increasing returns for indeterminacy is around 1.07.

Given that our model has variable capital utilization (which, all else equal, makes it easier to obtain indeterminacy) while Pintus' model does not, the lower threshold for indeterminacy in Pintus' numerical examples can be traced to the assumption of very low curvature in the utility of consumption. The near-zero risk coefficient implies a very low welfare loss from belief-driven cycles, making these cycles more likely to occur in an optimizing framework. For any given level of risk aversion, a higher elasticity of capital-labor substitution reduces the curvature of the firm's isoquants, which also makes belief-driven cycles more likely to occur. Our results are thus qualitatively consistent with those of Pintus (2006).

4 Cyclical Behavior of Labor's Share of Income

Figure 2 plots the cyclical component of labor's share of income in U.S. data together with the cyclical component of real GDP for the period 1949.Q1 to 2004.Q4.[4] The correlation coefficient between the two series is -0.26, indicating that labor's share of income moves countercyclically.

[Figure 2 about here]

In the model, labor's share of income is given by

$$\frac{w_t n_t}{y_t} = \frac{(1-\alpha)}{\alpha \left[n_t \,/\, (u_t k_t) \right]^{-\psi} \,+\, (1-\alpha)}, \tag{25}$$

which is obtained by rearranging the firm's first-order condition (10). The above expression shows that movements in labor's share over the business cycle are linked directly to movements in the ratio $n_t/\left(u_t k_t\right)$.

For our calibration with indivisible labor $(\gamma = 0)$, labor hours n_t are more volatile than utilized capital $u_t k_t$ in response to shocks. A positive belief shock will therefore raise the ratio $n_t/\left(u_t k_t\right)$ while output y_t increases. When $\sigma > 1$, we have $\psi > 0$ such that labor's share moves in the same direction as the ratio $n_t/\left(u_t k_t\right)$ and hence is pro-cyclical. A countercyclical labor share requires $w_t n_t / y_t$ to move in the opposite direction as the ratio $n_t/\left(u_t k_t\right)$. This condition is achieved when $\sigma < 1$ such that $\psi < 0$. Intuitively, an elasticity of capital-labor substitution below unity ties labor more closely to utilized capital, thus hindering labor hours from responding as freely to positive shocks in order to generate more income.

Gomme and Greenwood (1995) and Boldrin and Horvath (1995) also document the countercyclical movement of labor's share of income in U.S. data. Both papers develop models

[4] Data on Labor's share of income is from http://www.bls.gov/data, using series ID PRS85006173. Data on real GDP is from http://research.stlouisfed.org/fred2/series/GDPC96. The cyclical components are obtained by detrending each series with the Hodrick-Prescott filter, using a smoothing parameter of 1600.

where labor contracts between workers and firms can break the direct link between the real wage and the marginal product of labor. The labor contracts can generate a countercyclical labor share even when the elasticity of capital-labor substitution is unity (the Cobb-Douglas case).

5 Conclusion

This paper highlights the quantitative link between the elasticity of capital-labor substitution and the required conditions for equilibrium indeterminacy in a one-sector growth model developed by Guo and Lansing (2007). Under the maintained assumption of balanced long-run growth, we show that higher elasticities cause the minimum degree of increasing returns needed for indeterminacy and sunspots to decline monotonically, albeit very gradually. Moreover, we show that when the elasticity of substitution exceeds unity, the model's labor share of national income becomes procyclical, which is inconsistent with the postwar U.S. data. This observation, together with other empirical evidence, indicates that the elasticity of capital-labor substitution in the U.S. economy is actually below unity. In our model, a below-unity value for the elasticity of capital-labor substitution pushes up the threshold degree of increasing returns for indeterminacy only slightly relative to the Cobb-Douglas case, such that the threshold remains empirically plausibile.

A Appendix: Normalization Procedure

The normalized steady state quantities are denoted by $\overline{n}$, $\overline{\delta}$, $\overline{k}$, $\overline{y}$, $\overline{c}$, $\overline{u}$, and $\overline{m}$. As the elasticity of capital-labor substitution σ is varied, the normalized quantities are held constant by the appropriate choice of parameters. The reference point that defines the normalized quantities is the Cobb-Douglas case with $\sigma = 1$ (or $\psi = 0$) and $B = 1$. Following Guo and Lansing (2007), straightforward computations yield.

$$\overline{n} = \left[\frac{1-\overline{\alpha}}{A - A\,\overline{\alpha}\,(1+\phi)\,/\theta}\right]^{\frac{1}{1+\gamma}}, \tag{A.1}$$

$$\overline{\delta} = \frac{\rho}{\theta - 1 - \phi}, \tag{A.2}$$

$$\overline{k} = \left[b\,\overline{\alpha}\,\overline{n}^{\mu_n} \,/\, \left(\theta\,\overline{\delta}\right)\right]^{\frac{1}{1-\mu_k}}, \tag{A.3}$$

$$\overline{y} = b\,\overline{k}^{\mu_k}\,\overline{n}^{\mu_n}, \tag{A.4}$$

$$\overline{c} = \left[1 - \overline{\alpha}\,(1+\phi)\,/\theta\right]\,\overline{y}, \tag{A.5}$$

$$\overline{u} = \left[\frac{\phi^{\phi}\;\overline{\delta}^{1+\phi}}{\tau}\right]^{\frac{1}{\theta}}, \tag{A.6}$$

$$\overline{m} = \left[\frac{\phi\,\overline{\alpha}}{\theta}\right]\overline{y}, \tag{A.7}$$

where $\rho \equiv 1/\beta - 1$ is the rate of time preference, and $\overline{\alpha} = 0.3$ such that labor's share of income $1 - \bar{\alpha}$ is 0.70 for the Cobb-Douglas case. In addition, we define the following combinations of parameters for the Cobb-Douglas case:

$$b \equiv \left[\frac{\phi^{\phi}}{\tau}\left(\frac{\overline{\alpha}}{\theta}\right)^{1+\phi}\right]^{\frac{\overline{\alpha}\,(1+\eta)}{\theta - \overline{\alpha}\,(1+\eta)(1+\phi)}}, \tag{A.8}$$

$$\mu_k \equiv \frac{\overline{\alpha}\,(1+\eta)\,(\theta - 1 - \phi)}{\theta - \overline{\alpha}\,(1+\eta)\,(1+\phi)}, \tag{A.9}$$

$$\mu_n \equiv \frac{(1-\overline{\alpha})\,(1+\eta)\;\theta}{\theta - \overline{\alpha}\,(1+\eta)\,(1+\phi)}. \tag{A.10}$$

Given a value for the externality parameter η, the elasticity of substitution σ is varied over a wide range of values. For all computations, we set $\beta = 0.99$ to obtain a quarterly real interest rate of 1 percent and $\gamma = 0$ to reflect "indivisible labor". The constant τ affects no result, so we set $\tau = 1$.

As σ takes on different values, the parameters ϕ, θ, and A are set to maintain the following calibration targets used by Guo and Lansing (2007): $\overline{m}/\,\overline{y} = 0.061$, $\overline{\delta} = 0.025$, and $\overline{n} = 0.3$. The remaining parameters for the general CES specification are α and B. As σ is varied, the parameter α is set to maintain the steady-state labor's income share at 0.7, while the parameter B is set to maintain the steady-state output level equal to the Cobb-Douglas value $\overline{y}$. In this way, all steady state quantities are maintained at the corresponding Cobb-Douglas values.

The normalization procedure can be summarized by the following calibration formulas

$$\phi = \overline{m} / \left(\overline{\delta}\, \overline{k} \right), \tag{A.11}$$

$$\theta = 1 + \phi + \rho / \overline{\delta}, \tag{A.12}$$

$$\alpha = \frac{\overline{\alpha}}{\overline{\alpha} + (1 - \overline{\alpha}) \left(\overline{u}\overline{k} / \overline{n} \right)^{\psi}} \tag{A.13}$$

$$B = \frac{\overline{y}^{\frac{1}{1+\eta}}}{\left[\alpha \left(\overline{u}\overline{k} \right)^{\psi} + (1 - \alpha)\, \overline{n}^{\psi} \right]^{\frac{1}{\psi}}} \tag{A.14}$$

$$A = \frac{(1 - \alpha)\, B^{\psi}\, \overline{y}^{\frac{-\psi}{1+\eta}} \left[1 - \overline{m} / \overline{y} - \overline{\delta}\, \overline{k} / \overline{y} \right]^{-1}}{\overline{n}^{1+\gamma-\psi}}, \tag{A.15}$$

where $\psi = (\sigma - 1) / \sigma$, $\overline{\alpha} = 0.3$, and $\overline{m}$, $\overline{\delta}$, $\overline{k}$, $\overline{n}$, and $\overline{y}$, are the steady-state quantities from the Cobb-Douglas case.

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Figure 1: Stability Properties of the Steady State

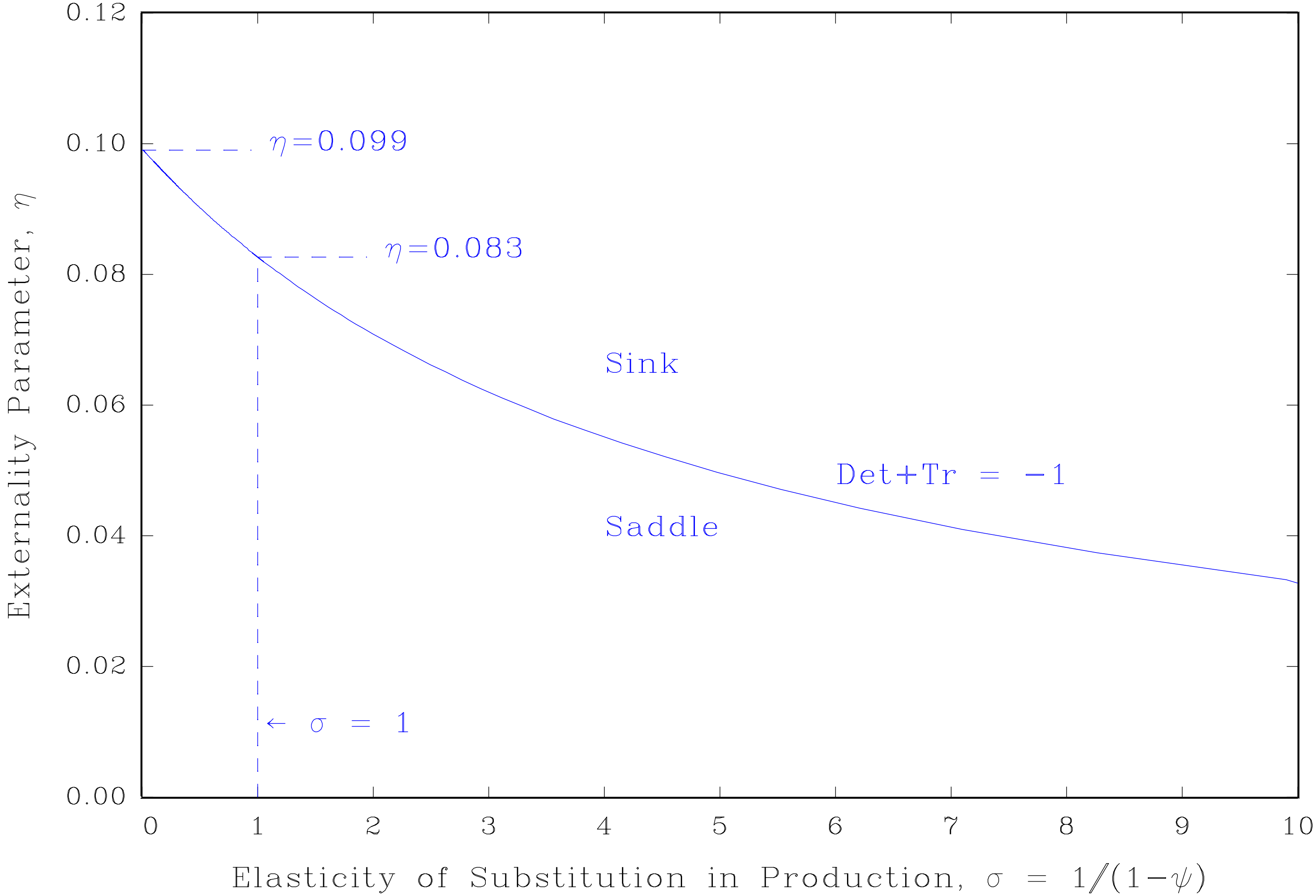

Figure 2: Cyclical Behavior of U.S. Labor Share, 1949.Q1 to 2004.Q4