Macroeconomic (In)stability under Real Interest Rate Targeting*

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Abstract

We show that in a one-sector monetary endogenous growth model under real interest rate targeting, the local stability properties of the economy's balanced growth path depend crucially on the exact formulation of the cash-in-advance constraint and the degree of productive externalities. In particular, when a positive fraction (including 100%) of gross investment is subject to the liquidity constraint, the model exhibits indeterminacy and sunspots if and only if the equilibrium wage-hours locus is positively sloped and steeper than the labor supply curve. On the other hand, when real money balances are required only for the household's consumption purchases, the economy always displays saddle-path stability and equilibrium uniqueness, regardless of the strength of productive externalities.

Keywords: Real Interest Rate Targeting; Endogenous Growth; Cash-in-Advance Constraint; Indeterminacy.

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1 Introduction

In recent years, there has been a voluminous literature that explores the interrelations between local (in)determinacy of steady-state or balanced-growth-path equilibria and various monetary policy rules within dynamic general equilibrium macroeconomic models. The types of operational policy regimes that have been extensively analyzed include money growth targeting, nominal interest rate targeting, inflation targeting and nominal income targeting, among others.[1] However, the stability effects of real interest rate targeting are largely ignored in the existing studies. Such a gap is particularly glaring because of its use as the main monetary policy instrument in developing countries like Brazil (Clifton, 1990) and Chile (Corbo and Fischer, 1994). Here, we address this research question by examining the equilibrium impact of a pure real-interest-rate peg in a monetary endogenous growth model with productive externalities and a generalized cash-in-advance constraint.

This paper builds upon the endogenous growth version of Benhabib and Farmer's (1994) one-sector representative agent model, a commonly adopted theoretical framework to investigate indeterminacy and sunspots in the real business cycle literature. This not only provides us a useful analytical benchmark, but also facilitates comparison with previous work that studies the macroeconomic effects of alternative monetary policy rules within the same endogenous growth framework. Money is incorporated into our analysis through a generalized cash-in-advance (CIA) or liquidity constraint, as in Wang and Yip (1992), where the household's entire consumption expenditures and a non-negative fraction of gross investment must be financed by its real money balances. Moreover, for analytical simplicity in this first study, we postulate that policymakers set the real interest rate to be a constant over time. This is achieved by adjusting the growth rate of nominal money supply to whatever level that is needed to maintain the target value of the real interest rate.

As it turns out, our model economy possesses a unique balanced growth path (BGP) along which labor hours are stationary, and output, consumption, capital and real money balances all grow at a common positive rate. In addition, we show that the BGP's local dynamics depend crucially on the exact formulation of the cash-in-advance constraint and the degree of productive externalities. Specifically, when the CIA constraint is applicable to a

[1] See, for example, Fukuda (1997), McCallum (1999), Clarida, Galí and Gertler (2000), Benhabib, Schmitt-Grohé and Uribe (2001), Dupor (2001), Meng (2002), Walsh (2003), Woodford (2003), Itaya and Mino (2003, 2004), Lai, Chen and Shaw (2005), Davig and Leeper (2007), and articles in the edited volumes of Taylor (1999) and Bernanke and Woodford (2005), among many others.

positive proportion (including 100%) of the household's investment purchases, the economy displays local indeterminacy and endogenous growth fluctuations *if and only if* the equilibrium wage-hours locus slopes upwards and is steeper than the labor supply curve. Perhaps quite surprisingly, this finding implies that under the postulated pure real-interest-rate peg, our monetary endogenous growth model with a generalized liquidity constraint exhibits the same local stability properties as those in the original Benhabib-Farmer non-monetary economy under laissez-faire.

The intuition for the above indeterminacy result is straightforward. Start from a particular equilibrium path, and suppose that agents become optimistic about the future of the economy. Acting upon this belief, households will reduce consumption and increase investment today, thus substituting out of real financial assets and into physical capital (Tobin's portfolio substitution effect). This in turn raises the relative shadow price of capital because of a higher demand. For a given nominal interest rate, this also shifts out the labor supply curve in light of lower consumption spending. If the external effects in the firms' production process are strong enough to make the equilibrium wage-hours locus intersect the labor supply curve from below, a positive sunspot shock leads to simultaneous expansions in GDP, consumption, investment, hours worked and labor productivity. Moreover, due to a higher employment, the marginal product of capital and its relative shadow price both will rise, hence validating agents' initial optimistic expectations.

On the other hand, when real money balances are required only for the household's consumption expenditures, we find that the BGP's shadow prices of capital and real financial wealth are equal, and that the equilibrium level of labor hours remains time invariant as a function of the (constant) real interest rate and model parameters. Therefore, when agents become optimistic and decide to raise their investment today, the mechanism described earlier that makes for multiple equilibria, *i.e.* movements of hours worked caused by the household's portfolio substitution, is completely shut down, regardless of the strength of productive externalities. It follows that our model *always* exhibits saddle-path stability and equilibrium uniqueness in this setting. In particular, starting from the initial capital stock, the economy immediately jumps onto its balanced growth path and stays there without any possibility of deviating transitional dynamics. As a corollary, the same stability/uniqueness result will be obtained if our analysis starts with fixed labor supply in the household's utility function, no matter whether gross investment is subject to the liquidity constraint or not.

This paper is related to recent work of Vegh (2002) who also studies the stability effects of a pure real-interest-rate peg in infinite-horizon representative agent models. Our analysis differs from his in three aspects. First, Vegh's model does not include capital accumulation, variable labor supply or sustained endogenous growth. In addition, his model is not truly a general equilibrium formulation in that output is determined solely by the economy's demand side. Second, Vegh introduces money through a pecuniary transaction cost technology, whereas we adopt the approach of a generalized cash-in-advance constraint. Third, the inflation rate is sticky in Vegh's framework because it is assumed to a predetermined variable. By contrast, prices are fully flexible and the inflation rate is endogenously determined in our examination.

The remainder of this paper is organized as follows. Section 2 describes the model and analyzes the equilibrium conditions. Section 3 derives the the local dynamics of the economy's balanced growth path, and examines the stability effects of real interest rate targeting under two different formations of the cash-in-advance constraint. Section 4 concludes.

2 The Economy

We incorporate the government and a generalized cash-in-advance constraint *a la* Wang and Yip (1992) into the endogenous growth version of Benhabib and Farmer's (1994) one-sector representative agent model. To facilitate comparison, we follow previous studies and adopt the same preference and technology formulations as those in the Benhabib-Farmer economy.[2] Households live forever, and derive utility from consumption and leisure. The production side consists of a social technology with increasing returns-to-scale that are caused by the presence of productive externalities. The government issues nominal money supply and public debt to finance its expenditures on interest payments and lump-sum transfers. We assume that there are no fundamental uncertainties present in the economy.

2.1 Firms

There is a continuum of identical competitive firms, with the total number normalized to one. Each firm produces output y_t according to a Cobb-Douglas production function

$$y_t = x_t k_t^{\alpha} h_t^{1-\alpha}, \;\; 0 < \alpha < 1, \tag{1}$$

[2] See, for example, Fukuda (1997), and Itaya and Mino (2003, 2004), among others.

where k_t and h_t are capital and labor inputs, respectively, and x_t represents productive externalities that are taken as given by the individual firm. We postulate that externalities take the form

$$x_t = K_t^{1-\alpha} H_t^{(1-\alpha)\eta}, \ \ \eta > 0, \tag{2}$$

where K_t and H_t denote the economy-wide levels of capital and labor services. In a symmetric equilibrium, all firms make the same decisions such that $k_t = K_t$ and $h_t = H_t$, for all t. As a result, (2) can be substituted into (1) to obtain the following social production function that displays increasing returns-to-scale:

$$y_t = \ k_t \, h_t^{(1-\alpha)(1+\eta)}. \tag{3}$$

Notice that the economy exhibits sustained endogenous growth because the aggregate technology (3) displays linearity in capital. Under the assumption that factor markets are perfectly competitive, the first-order conditions for the firm's profit maximization problem are given by

$$u_t = \alpha \frac{y_t}{k_t}, \tag{4}$$

$$w_t = (1-\alpha)\, \frac{y_t}{h_t}, \tag{5}$$

where u_t is the capital rental rate and w_t is the real wage.

2.2 Households

The economy is populated by a unit measure of identical infinitely-lived households, each has one unit of time endowment and maximizes a discounted stream of utilities over its lifetime

$$\int_0^\infty \left\{ \log c_t - A \frac{h_t^{1+\gamma}}{1+\gamma} \right\} e^{-\rho t} dt, \ \ A > 0, \tag{6}$$

where c_t is the individual household's consumption, $\gamma \geq 0$ denotes the inverse of the intertemporal elasticity of substitution in labor supply, and $\rho > 0$ is the discount rate.

The budget constraint faced by the representative household is given by

$$c_t + i_t + \dot{m}_t \, + \dot{b}_t = u_t k_t + w_t h_t + r_t b_t - \pi_t m_t + \tau_t, \tag{7}$$

where i_t is gross investment, r_t is the real interest rate on government bonds b_t,[3] π_t is the inflation rate, and τ_t represents lump-sum transfers (expressed in real terms) that households receive from the government. The variable m_t denotes the real money balances that equal the nominal money supply M_t divided by the price level P_t. Investment adds to the stock of physical capital according to the law of motion

$$\dot{k}_t = i_t - \delta k_t, \quad k_0 > 0 \text{ given,} \tag{8}$$

where $\delta \in (0,1)$ is the capital depreciation rate.

As in Wang and Yip (1992), the representative household also faces the following generalized cash-in-advance (CIA) or liquidity constraint:

$$c_t + \phi i_t \leq m_t, \quad \phi \in [0,1], \tag{9}$$

that is, all consumption purchases and a fraction ϕ of gross investment must be financed by the household's real balances m_t; the remaining fraction $(1-\phi)$ of investment goods are acquired through barter.

The first-order conditions for the representative household with respect to the indicated variables and the associated transversality conditions (TVC) are

$$c_t : \quad 1/c_t = \lambda_{at} + \psi_t, \tag{10}$$

$$h_t : \quad A h_t^{\gamma} = \lambda_{at} w_t, \tag{11}$$

$$i_t : \quad \lambda_{kt} = \lambda_{at} + \phi \psi_t, \tag{12}$$

$$k_t : \quad \dot{\lambda}_{kt} = (\rho + \delta)\lambda_{kt} - u_t \lambda_{at}, \tag{13}$$

$$m_t : \quad \dot{\lambda}_{at} = (\rho + \pi_t)\lambda_{at} - \psi_t, \tag{14}$$

$$b_t : \quad \dot{\lambda}_{at} = (\rho - r_t)\lambda_{at}, \tag{15}$$

$$\text{TVC}_1 : \quad \lim_{t\to\infty} e^{-\rho t}\lambda_{at} a_t = 0, \tag{16}$$

$$\text{TVC}_2 : \quad \lim_{t\to\infty} e^{-\rho t}\lambda_{kt} k_t = 0, \tag{17}$$

where λ_{at} and λ_{kt} are the shadow prices of real financial assets a_t ($\equiv m_t + b_t$) and physical capital, respectively, and ψ_t represents the Lagrange multiplier for the CIA constraint (9).

[3] The variable r_t can also be interpreted as the before-inflation-adjusted interest rate on indexed government bonds. It is straightforward to show that all the results reported in section 3, which analyzes the stability effects of a pure real-interest-rate peg, remain unaffected under this alternative interpretation.

As is common in the literature, we assume that the liquidity constraint (9) is strictly binding in equilibrium, thus $\psi_t > 0$ for all t. Equation (10) states that the marginal benefit of consumption equals its marginal cost, which is the marginal utility of having an additional unit of real financial wealth. Moreover, equation (11) equates the slope of the representative household's indifference curve to the real wage, and equations (12) and (13) together govern the evolution of physical capital over time. Finally, equations (14) and (15) equate the marginal utility values of real money and bonds holdings to their respective marginal costs.

2.3 Government

The government issues nominal money supply, which is postulated to grow at a rate of μ_t, and public debt to finance its spending on interest payments and lump-sum transfers. Hence, the real flow budget constraint for the government is given by

$$\dot{m}_t + \dot{b}_t = r_t b_t - \pi_t m_t + \tau_t. \tag{18}$$

By combining equations (3), (4), (5), (7), (8), and (18), we obtain the following aggregate resource constraint for the economy:

$$c_t + \dot{k}_t + \delta k_t = k_t\, h_t^{(1-\alpha)(1+\eta)}. \tag{19}$$

In addition, clearing in the money market implies that

$$\dot{m}_t = (\mu_t - \pi_t)\, m_t. \tag{20}$$

3 Real Interest Rate Targeting

We focus on the economy's balanced growth path (BGP) along which labor hours are stationary, and output, consumption, capital and real money balances all exhibit a common, positive constant growth rate denoted by θ. Moreover, the real interest rate on government bonds is set to be a constant where $r_t = r > 0$ for all t. Under this policy regime, the central bank adjusts the money growth rate μ_t to whatever level that is needed to maintain the target value of the real interest rate. To facilitate the analysis of perfect-foresight dynamics, we make the following transformation of variables: $p_t \equiv \lambda_{kt}/\lambda_{at}$ and $z_t \equiv c_t/k_t$. With this transformation, the model's equilibrium conditions can be expressed as an autonomous pair of differential equations

$$\frac{\dot{p}_t}{p_t} = r + \delta - \frac{\alpha}{p_t}\left[\frac{\phi(1-\alpha)}{Az_t(p_t - 1 + \phi)}\right]^{\frac{(1-\alpha)(1+\eta)}{1+\gamma-(1-\alpha)(1+\eta)}}, \tag{21}$$

$$\frac{\dot{z}_t}{z_t} = z_t - \rho - \frac{(r+\delta)(1-\phi)}{p_t - 1 + \phi} + \left(\frac{1+\alpha-\phi-p_t}{p_t - 1 + \phi}\right)\left[\frac{\phi(1-\alpha)}{Az_t(p_t - 1 + \phi)}\right]^{\frac{(1-\alpha)(1+\eta)}{1+\gamma-(1-\alpha)(1+\eta)}}. \tag{22}$$

Given the above dynamical system (21) and (22), the BGP equilibrium is characterized by a pair of positive real numbers (p^*, z^*) that satisfy $\dot{p}_t = \dot{z}_t = 0$.[4] It can be shown that our model economy possesses a unique balanced growth path on which the common (positive) growth rate of GDP, consumption, capital and real money balances is $\theta = r - \rho$. Next, we compute the Jacobian matrix J of the system defined by (21) and (22), evaluated at (p^*, z^*). The trace and determinant of the Jacobian are

$$Tr = r + \delta + z^* + \frac{(r+\delta)p^*}{\alpha}\left[\frac{(1-\alpha)(1+\eta)}{1+\gamma-(1-\alpha)(1+\eta)}\right], \tag{23}$$

$$Det = \frac{r+\delta}{1+\gamma-(1-\alpha)(1+\eta)}\left\{(1-\alpha)(1+\eta)\left[\frac{(r+\delta)p^*}{\alpha} + \frac{(1-\phi)z^*}{p^*-1+\phi}\right] + (1+\gamma)z^*\right\}. \tag{24}$$

The BGP's local stability properties are determined by comparing the eigenvalues of J that have negative real parts to the number of initial conditions in the dynamical system (21)-(22), which is zero because p_t and z_t are both jump variables. As a result, the balanced growth path exhibits saddle-path stability and equilibrium uniqueness when both eigenvalues have positive real parts. If one or two eigenvalues have negative real parts, then the BGP equilibrium is locally indeterminate (a sink) and can be exploited to generate endogenous growth fluctuations driven by agents' self-fulfilling expectations or sunspots.

[4] Using equation (12), the transformed state variable p_t can be rewritten as $p_t = 1 + \phi\frac{\psi_t}{\lambda_{at}}$, which is not strictly smaller than one because $\phi \in [0,1]$ and $\psi_t > 0$. It follows that $p^* \geq 1$ on the economy's balanced growth path. We have verified that this condition holds for all the cases that are examined in the subsequent analyses.

3.1 When $0 < \phi \leq 1$

In this case, money holdings are required for the household's entire consumption purchases and a positive fraction (including 100%, as in Stockman, 1981) of its investment expenditures. With p^* from solving (21) and (22), it is straightforward to show that the corresponding BGP expressions for the inflation rate π^* and the growth rate of nominal money supply μ^* are

$$\pi^* = \frac{p^* - 1}{\phi} - r \quad \text{and} \quad \mu^* = \frac{p^* - 1}{\phi} - \rho, \tag{25}$$

where $p^* > 1$ (see footnote 4). The remaining endogenous variables on the economy's balanced growth path can then be easily derived. Since r, δ, η, $z^* > 0, 0 < \alpha < 1$, $\gamma \geq 0$, $p^* \geq 1$ and $0 < \phi \leq 1$, the determinant (24) is negative if and only if

$$(1 - \alpha)(1 + \eta) - 1 > \gamma. \tag{26}$$

Moreover, given the first three terms of (23) are positive, the trace passes through plus infinity to minus infinity as $(1 - \alpha)(1 + \eta) - (1 + \gamma)$ moves past zero from the negative to the positive region. Therefore, the condition reported in (26) not only guarantees a negative determinant, but also leads to a negative trace, indicating the presence of one eigenvalue with negative real part. This implies that (26) is the *necessary and sufficient* condition for our model to exhibit equilibrium indeterminacy and belief-driven fluctuations. On the contrary, when the condition of (26) is not satisfied, the trace (23) and determinant (24) of the Jacobian matrix J are both positive. It follows that the economy's BGP equilibrium is a (locally determinate) saddle path.

The intuition for the above (in)determinacy result can be understood as follows. First, using (14) and (15), it can be shown that the equilibrium Lagrange multiplier on the liquidity constraint (9) under the postulated pure real-interest-rate peg is

$$\psi_t = \lambda_{at}(r + \pi_t), \tag{27}$$

where $r + \pi_t$ is the nominal interest rate R_t through the Fisher condition. We then substitute (10) and (27) into the household's labor supply decision (11) to obtain

$$Ac_t h_t^\gamma = \frac{w_t}{1 + R_t}, \tag{28}$$

where w_t is determined by equation (5), and $(1 + R_t)$ is the CIA-generated monetary wedge that raises the effective price of consumption goods.

Next, start from a particular equilibrium path, and suppose that agents become optimistic about the future of the economy. Acting upon this belief, households will invest more and consume less today, thus substituting out of real financial assets and into physical capital (Tobin's portfolio substitution effect). This in turn raises the relative shadow price of capital p^* because of a higher demand. For a given nominal interest rate, this also shifts out the labor supply curve, which causes labor hours to rise and real wage to fall. If the degree of productive externalities η is strong enough to yield a more-than-unity equilibrium labor elasticity of output in the social technology, namely $(1-\alpha)(1+\eta) > 1$ in (3), increases in hours worked are associated with a higher labor productivity. It follows that the labor demand curve will shift outward, reinforcing the initial employment effect generated by lower consumption spending. Subsequently, higher labor hours raise the household's projected income stream, thereby increasing its ability to consume. This in turn shifts the labor supply curve to the left. When such a leftward shift makes the equilibrium wage-hours locus intersect the labor supply curve from below, a positive sunspot shock leads to simultaneous expansions in output, consumption, investment, hours worked and labor productivity. Moreover, due to a higher employment, the marginal product of capital and its relative shadow price both will rise, hence validating agents' initial optimistic expectations.

To further understand the preceding mechanism, substituting the aggregate production function (3) into the logarithmic version of firms' labor demand condition (5) indicates that the slope of the equilibrium wage-hours locus is equal to $(1 - \alpha)(1+\eta) - 1$. In addition, taking logarithms on both sides of (28) shows that the slope of the household's labor supply curve is $\gamma(\geq 0)$. Therefore, the condition that is needed to produce indeterminacy and sunspots, given by (26), states that the equilibrium wage-hours locus is positively sloped and steeper than the labor supply curve. By contrast, if the strength of productive externalities η is not sufficiently high so that the equilibrium wage-hours locus is flatter than the labor supply curve, consumption will be countercyclical, which is inconsistent with the actual data. As a result, agents' expectations cannot become self-fulfilling in equilibrium.

Perhaps quite surprisingly, (26) turns out to be exactly the same necessary and sufficient condition that leads to equilibrium indeterminacy in either the no-growth or the endogenous-growth version of Benhabib and Farmer's (1994) model.[5] Hence, the above (in)determinacy result shows that given identical preference and technology specifications, our monetary endogenous growth model with a pure real-interest-rate peg exhibits the same local (in)stability properties as those in the original Benhabib-Farmer non-monetary economy under laissez-faire. On the other hand, in a similar endogenous growth model where money reduces pecuniary transaction costs or time spent shopping, Itaya and Mino (2003) find that (26) is a necessary condition for the economy to possess dual balanced growth paths (one is determinate and the other is indeterminate) under a constant money growth policy.

3.2 When $\phi = 0$

In this case, the cash-in-advance constraint (9) is applicable exclusively to the household's real consumption expenditures, as in Clower (1967) and Lucas (1980), among many others. Substituting $\phi = 0$ into (12) shows that the relative price (in utility terms) of capital to real financial assets $p_t = \lambda_{kt}/\lambda_{at} = 1$ for all t, which in turn implies that the dynamical system (21) and (22) becomes degenerate. Nevertheless, the model's equilibrium conditions can be collapsed into a single differential equation in z_t as follows:

$$\frac{\dot{z}_t}{z_t} = z_t - \rho - \frac{(1-\alpha)(r+\delta)}{\alpha}. \tag{29}$$

We then linearize (29) around the balanced growth path and find that its local dynamics is governed by

$$z^* = \rho + \frac{(1-\alpha)(r+\delta)}{\alpha} > 0. \tag{30}$$

Consequently, the economy exhibits saddle-path stability and equilibrium uniqueness because there is no initial condition associated with (29).

The intuition for the above determinacy result is straightforward. Combining equations (3), (4), (13) and (15), together with $r_t = r$ and $\lambda_{kt} = \lambda_{at}$, we find that the equilibrium level of labor hours remains at a constant level

[5] It is now well known that the minimum level of increasing returns-to-scale needed to satisfy the Benhabib-Farmer condition for equilibrium indeterminacy as in (26) is too high to be empirically plausible (see, for example, Burnside 1996; Basu and Fernald, 1997). However, subsequent work has shown that in a one-sector real business cycle model with variable capital utilization (Wen, 1998) and/or maintenance expenditures on existing capital (Guo and Lansing, 2007), or in a two-sector real business cycle model with sector-specific externalities (Benhabib and Farmer, 1996; Perli, 1998; Weder, 2000; Harrison, 2001), the required degree of increasing returns to generate local indeterminacy is much less stringent. To maintain comparability with previous studies like Fukuda (1997) and Itaya and Mino (2003, 2004), among others, we employ the one-sector representative agent formulation with constant capital utilization and without maintenance activities.

$$h_t = \left(\frac{r+\delta}{\alpha}\right)^{\frac{1}{(1-\alpha)(1+\eta)}}, \text{ for all } t. \quad (31)$$

Therefore, when agents become optimistic and decide to raise its investment today, the mechanism described in the previous sub-section that makes for multiple equilibria, *i.e.* movements of hours worked caused by the household's portfolio substitution, is completely shut down, regardless of the degree of productive externalities η. This implies that starting from the initial capital stock k_0, the economy immediately jumps onto its balanced growth path characterized by z^* as in (30), and always stays there without the possibility of deviating transitional dynamics.[6] As a result, equilibrium indeterminacy and endogenous growth fluctuations can never occur in this setting. Notice that the same stability/uniqueness result will be obtained if our analysis starts with fixed labor supply in the household's utility function (6), no matter whether any gross investment is subject to the liquidity constraint or not.

As it turns out, our findings are in sharp contrast to previous studies that also examine the stability effects of alternative monetary policy rules in the same endogenous growth model where consumption alone is subject to the CIA constraint. For example, Fukuda (1997) shows that under a regime of money growth targeting, there may be two balanced growth paths, one of which is determinate and the other indeterminate, when the degree of labor externalities is sufficiently strong to satisfy the condition reported in (26). Moreover, Itaya and Mino (2004) find that in addition to (26), whether the nominal interest rate rule is active or passive plays an important role in affecting the BGP's number and associated local stability properties.

4 Conclusion

Real interest rate targeting has been used as the main monetary policy instrument in some developing countries during the 1980's and 1990's. To our knowledge, this paper is the first theoretical study that examines the associated stability effects within a standard dynamic general equilibrium macroeconomic model. We show that under the regime of a pure real-interest-rate peg, the local stability properties of the economy's balanced growth path depend

[6]It can be shown that when $\phi = 0$, the BGP expressions for the inflation rate π^* and the growth rate of nominal money supply μ^* are given by

$$\pi^* = \frac{1-\alpha}{Az^*}\left(\frac{r+\delta}{\alpha}\right)^{\frac{(1-\alpha)(1+\eta)-(1+\gamma)}{(1-\alpha)(1+\eta)}} - 1 - r \quad \text{and} \quad \mu^* = \frac{1-\alpha}{Az^*}\left(\frac{r+\delta}{\alpha}\right)^{\frac{(1-\alpha)(1+\eta)-(1+\gamma)}{(1-\alpha)(1+\eta)}} - 1 - \rho.$$

where z^* is given by (30).

crucially on the exact formulation of the cash-in-advance constraint and the degree of productive externalities. Specifically, when a positive fraction of gross investment is subject to the liquidity constraint, the necessary and sufficient condition for indeterminacy and sunspots is identical to that in Benhabib and Farmer's (1994) non-monetary model under laissez-faire. That is, the equilibrium wage-hours locus must be positively sloped, and steeper than the labor supply curve. On the other hand, when the CIA constraint is applied exclusively to the transactions of consumption goods, the economy always exhibits saddle-path stability and equilibrium uniqueness, regardless of the strength of productive externalities.

In this paper, we have explored the stability effects of real interest rate targeting in its simplest possible form where there is no feedback mechanism. It would be worthwhile to investigate a richer policy rule that allows the real interest rate to change in response to lagged inflation (Clifton, 1990), the inflation gap and/or the output gap (Vegh, 2002), among others. This will allow us to examine the robustness of our results, and further identify model features and parameters that govern the number and local dynamics of the economy's balanced growth path(s) under real interest rate targeting. We plan to pursue this research project in the near future.

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