‘Regular’ Choice and the Weak Axiom of Stochastic Revealed Preference[•]

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Abstract: We explore the relation between two 'rationality conditions' for stochastic choice behavior: regularity and the weak axiom of stochastic revealed preference (WASRP). We show that WASRP implies regularity, but the converse is not true. We identify a restriction on the domain of the stochastic choice function, which suffices for regularity to imply WASRP. When the universal set of alternatives is finite, this restriction is also necessary for regularity to imply WASRP. Furthermore, we identify necessary and sufficient domain restrictions for regularity to imply WASRP, when the universal set of alternatives is finite and stochastic choice functions are all degenerate. Results in the traditional, deterministic, framework regarding the relation between Chernoff’s Condition and the Weak Axiom of Revealed Preference follow as special cases. Thus, general conditions are established, under which regularity can substitute for WASRP as the axiomatic foundation for a theory of choice behaviour.

KEYWORDS: Stochastic Choice, Regularity, Chernoff Condition, Weak Axiom of Revealed Preference, Weak Axiom of Stochastic Revealed Preference, Complete Domain, Incomplete Domain.

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1. Introduction

A large literature exists on stochastic choice behavior of an agent.[1] This literature has advanced a number of rationality, or consistency, restrictions for such choice behavior. Perhaps the best known and intuitively most compelling of these restrictions is the so-called 'regularity' (RG) condition. This simply postulates that the probability of choosing from any subset of a feasible set of options cannot rise if the feasible set is expanded. RG is the stochastic generalization of the familiar Chernoff's Condition (CC) for deterministic choice functions, which specifies that contracting the feasible set without removing the chosen element should make no difference to choice behaviour.

Given its weakness, transparency and intuitive appeal, RG recommends itself as the natural axiomatic foundation for the theory of choice behaviour. Yet, the empirical and predictive implications of the traditional theory of choice behaviour are typically derived instead from the Weak Axiom of Revealed Preference (WARP), whether singly or in conjunction with other conditions. Recently, Bandyopadhyay, Dasgupta and Pattanaik (2004, 2002, 1999) have introduced the generalized stochastic version of this rationality postulate, the weak axiom of stochastic revealed preference (WASRP), and analyzed its implications in the context of consumers' behavior.[2]

A priori, RG and WASRP appear to have very different focus. While RG specifies consistency restrictions for choice behavior when the feasible set is contracted or expanded, it does not explicitly spell out any such restriction across two feasible sets, neither of which includes the other. WASRP, however, is formulated explicitly to cover such cases. In doing so, WASRP appears intuitively more demanding, and less transparent, than RG.

A natural question to ask, therefore, is: what is the relationship between these two rationality postulates? In particular, if it can be shown that RG implies WASRP, at least for a large class of cases, then the justification for WASRP would be significantly enhanced. Even more importantly, for that class of cases, one would be able to replace WASRP by RG (or WARP by CC) as the foundational rationality (i.e. consistency) postulate, thereby strengthening the a priori intuitive appeal of the standard theory, without any loss of its predictive or empirical content.

We address this issue in this paper. We first show that WASRP necessarily implies RG, regardless of the domain of the stochastic choice function. However, if the stochastic choice function is not defined for some subsets of the universal set of alternatives, then it can violate WASRP while satisfying RG. Thus, in this sense, WASRP is stronger than RG. We then identify a restriction on the domain of the stochastic choice function, under which RG turns out to be equivalent to WASRP. This restriction allows the possibility that the domain of the stochastic choice function is 'incomplete', i.e., not defined for some subsets of the universal set of alternatives. A corollary of our results is that, for

[1] See Barbera and Pattanaik (1986), Block and Marschak (1960), Cohen (1980), Corbin and Marley (1974), Falmagne (1978), Fishburn (1973, 1977, 1978), Halldin (1974), Luce (1958, 1959, 1977), Luce and Suppes (1965), Manski (1977), Marschak (1960), Quandt (1956), Sattath and Tversky (1976) and Yellott (1977).

[2] We define RG and WASRP formally in Section 2 below.

every stochastic choice function with a complete domain, WASRP and regularity are equivalent properties. We proceed to show that our domain restriction is also necessary for RG to imply WASRP, when the universal set is finite. Lastly, we provide a necessary and sufficient domain restriction under which RG and WASRP are equivalent, when the universal set is finite and stochastic choice functions are constrained to be degenerate. Results in the traditional, deterministic, framework regarding the relation between Chernoff's Condition (or Sen's Condition α) and the Weak Axiom of Revealed Preference follow as special cases of our general analysis.

Section 2 introduces the notion of a stochastic choice function and the two rationality properties for stochastic choice, namely, RG and WASRP, which constitute the focus of this paper. In Section 3, we consider the relationship between RG and WASRP when no restrictions are imposed on the domain of the social choice function. Sections 4 and 5 explore the relationship between RG and WASRP under alternative restrictions regarding the domain of the stochastic choice function. Section 4 deals with the general case where stochastic choice functions may be degenerate, but are not constrained to be so. Section 5 deals with the special case where the universal set of alternatives is assumed to be finite and stochastic choice functions are constrained to be degenerate. Section 6 discusses some applications of our analysis and concludes. Proofs are relegated to the Appendix.

2. Stochastic choice functions, Regularity, and the Weak Axiom of Stochastic Revealed Preference

2.1. *Stochastic choice functions*

Let X denote the (non-empty) universal set of alternatives, and let χ denote the class of all non-empty subsets of X. Let $\Im$ be the class of all non-empty subsets of χ. Thus, an element of $\Im$ is a non-empty class of non-empty subsets of X.

Definition 2.1. Let $Z \in \Im$. A *stochastic choice function* (SCF) over Z is a function which, for every $A \in Z$, specifies exactly one finitely additive probability measure over the class of all subsets of A.

If F is an SCF over $Z \in \Im$ and $A \in Z$, then the probability measure specified by F for A will be denoted by p_{FA}. When there is no ambiguity about the SCF, F, we shall write simply p_A rather than p_{FA}. Given an SCF F over $Z \in \Im$ and given $A \in Z$, for every subset B of A, $p_A(B)$ is to be interpreted as the probability that the agent's choice from the set A will lie in B. When B contains exactly one element, say *x*, we write $p_A(x)$ rather than $p_A(\{x\})$.

Definition 2.2. An SCF is said to have a *complete domain* iff its domain is χ. An SCF has an *incomplete domain* iff its domain is a proper subset of χ.

Definition 2.3. An SCF F over $Z \in \mathfrak{I}$ is *degenerate* iff, for all $A \in Z$, there exists $x \in A$ such that $p_A(x) = 1$.

Definition 2.4. Let $Z \in \mathfrak{I}$. A *deterministic choice function* (DCF) over Z is a function which, for every $A \in Z$, specifies exactly one alternative in A.

Our notions of complete and incomplete domains for DCFs are exactly analogous to those for SCFs, as specified in Definition 2.2 above.

Remark 2.5. Let $Z \in \mathfrak{I}$. Let F be a degenerate SCF over Z and f be a DCF over Z. We say that F induces f iff, for all $A \in Z$, $f(A) = x$, where $p_A(x) = 1$; and we say that f induces F iff, for all $A \in Z$, $p_A(x) = 1$, where $x = f(A)$. It is clear that every degenerate SCF induces a DCF, which, in turn, induces the degenerate SCF under consideration. Similarly, every DCF induces a degenerate SCF, which, in turn, induces the DCF under consideration.

2.2. *Regularity and the Weak Axiom of Stochastic Revealed Preference*

Definition 2.6. Let $Z \in \mathfrak{I}$.

(i) An SCF F over Z satisfies *regularity* (RG) iff, for all $A, B, C \in Z$ such that $C \subseteq B \subseteq A$, $p_B(C) \geq p_A(C)$.

(ii) A DCF f over Z satisfies *Chernoff's Condition* (CC) iff, for all $A, B \in Z$ such that $B \subseteq A$, $(B - \{f(B)\}) \subseteq (A - \{f(A)\})$.

RG is arguably the weakest rationality property of stochastic choices discussed in the literature. It stipulates that, if we start with $B \in Z$, and if, by adding some alternatives to B, we arrive at a new set $A \in Z$, then the probability that the agent's choice will lie in a subset C of B cannot increase when we pass from the feasible set B to the feasible set A. CC, first introduced by Chernoff (1954), and later discussed by Sen (1969), who called it α, requires that an alternative that is rejected in a set B cannot be chosen, when the set B is expanded by adding new alternatives.

Remark 2.7. It is clear that a degenerate SCF F satisfies RG iff the DCF induced by F satisfies CC, and, conversely, a DCF f satisfies CC iff the degenerate SCF induced by f satisfies RG.

Definition 2.8. Let $Z \in \mathfrak{I}$.

(i) An SCF F over Z satisfies the *weak axiom of stochastic revealed preference* (WASRP) iff, for all $A, B \in Z$,

$$p_B(C) - p_A(C) \leq p_A(A - B) \text{ for all } C \subseteq A \cap B. \tag{2.1}$$

(ii) A DCF f over Z satisfies the *weak axiom of revealed preference* (WARP) iff, for all $A, B \in Z$, and, for all distinct $x, y \in X$, if $x, y \in A \cap B$ and $f(A) = x$, then $f(B) \neq y$.

WARP, introduced by Samuelson in the context of competitive consumers' choices,[3] and reformulated by Houthakker (1950) for the general choice context, is a very familiar property of DCFs and hardly needs any explanation. WASRP was introduced by Bandyopadhyay, Dasgupta and Pattanaik (1999). The intuition behind WASRP is as follows. Suppose, initially, A is the set of all available alternatives, and $C \subseteq A$. Then $p_A(C)$ is the probability that the agent's choice from A lies in C. Now suppose that the set of available alternatives changes to B, but $C \subseteq B$. If, at all, the new choice probability for *C*, i.e. $p_B(C)$, is greater than $p_A(C)$, then this increase must be due to the fact that the alternatives in *C* face 'less competition' to the extent that the alternatives in A - B are no longer available. Hence $p_B(C) - p_A(C)$ must not be greater than the choice probability for $A - B$, i.e. $p_A(A - B)$, in the initial situation.

Remark 2.9. A degenerate SCF F satisfies WASRP iff the DCF induced by F satisfies WARP. Conversely, a DCF f satisfies WARP iff the degenerate SCF induced by f satisfies WASRP.

Remark 2.10. It is known that a stochastic choice function, which is rationalizable in terms of stochastic orderings,[4] necessarily satisfies RG and WASRP, though neither WASRP nor RG necessarily implies rationalizability in terms of stochastic orderings.[5] Therefore, just as the violation of CC or WARP enables us to identify deterministic choices that could not have been induced by a preference ordering, violation of RG or WASRP allows us to identify stochastic choice behavior that could not have been induced by stochastic preference orderings.

Remark 2.11. As is the case with CC and WARP in the deterministic context, RG and WASRP have significant consequences in both normative and positive economics. For example, Pattanaik and Peleg (1986) show that a stochastic social choice function that satisfies RG in addition to certain other plausible conditions is characterized by weighted random dictatorship for every non-empty proper subset of the universal set of alternatives. On the other hand, WASRP has important implications in the context of the theory of stochastic choice behavior of consumers (Bandyopadhyay,

[3] See, for example, Samuelson (1948).

[4] Let Q be the set of all orderings over X (i.e., the set of all reflexive, connected and transitive binary relations over X). For all subsets A $(A \neq \emptyset)$ and B of X, let $Q(A, B)$ be the set of all $J \in Q$ such that J has a unique greatest element in A and this unique greatest element in A belongs to B. An SCF F with domain Z is *rationalizable in terms of stochastic orderings* iff there exists a finitely additive probability measure q defined over the class of all subsets of Q such that, for every $A \in Z$ and every subset B of A, $p_A(B) = q(Q(A, B))$.

[5] Bandyopadhyay, Dasgupta and Pattanaik (1999) show that an SCF which is rationalizable in terms of stochastic orderings must satisfy WASRP, and that the converse is not necessarily true. Block and Marschak (1960) show that rationalizability of the SCF in terms of stochastic orderi