Balanced-Budget Rules and Macroeconomic (In)stability*

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Abstract

It has been shown that under perfect competition and constant returns-to-scale, a one-sector real business cycle model may exhibit indeterminacy and sunspots when income tax rates are determined by a balanced-budget rule with a pre-set level of government expenditures. This paper shows that indeterminacy disappears if the government finances endogenous public spending and transfers with fixed income tax rates. Under this type of balanced-budget formulation, the economy exhibits saddle-path stability and equilibrium uniqueness, regardless of the source of government revenue and/or the existence of lump-sum transfers.

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1 Introduction

Using a one-sector infinite-horizon representative agent model with perfectly competitive markets and a constant returns-to-scale technology, Schmitt-Grohé and Uribe (1997) explore the interrelations between (local) stability of equilibria and a balanced-budget rule whereby constant government expenditures are financed by proportional taxation on labor/total income. It turns out that for empirically plausible values of labor and capital income tax rates, the economy can exhibit an indeterminate steady state and a continuum of stationary sunspot equilibria.[1] Under this type of balanced-budget constraint, when agents become optimistic about the future of the economy and decide to work harder and invest more, the government is forced to lower the tax rates as total output rises. This countercyclical tax policy will help fulfill agents' initial optimistic expectations, thus leading to indeterminacy of equilibria and endogenous business cycle fluctuations.[2]

This paper extends Schmitt-Grohé and Uribe's analysis by considering a different balanced-budget rule that is commonly used in the real business cycle (RBC) literature.[3] Specifically, we allow for endogenous public spending and transfers financed by separate fixed tax rates on labor and capital income. With this modification, we show that the economy exhibits saddle-path stability, regardless of whether tax revenues are returned to households as lump-sum transfers. Moreover, equilibrium uniqueness continues to hold when the source of government revenue is a constant tax rate applied to labor or total income. Under our balanced-budget formulation, constant tax rates together with diminishing returns to productive inputs reduce the higher returns from belief-driven labor and investment spurts, and in turn prevent agents' expectations from becoming self-fulfilling.[4] Therefore, as in a standard RBC framework under laissez-faire, the economy does not display endogenous business cycles driven by agents' animal spirits. Overall, our analysis illustrates that Schmitt-Grohé and Uribe's indeterminacy result depends crucially on a fiscal policy where the tax rate decreases with the household's taxable

[1] See Benhabib and Farmer (1999) for other mechanisms that generate indeterminacy and sunspots in real business cycle models.

[2] The appendix of Schmitt-Grohé and Uribe (1997) demonstrates a close formal correspondence between the equilibrium conditions of their model and those in Rotemberg and Woodford (1992) and Galí (1994*a*) with variations in the "effective fiscal markup".

[3] See, for example, Greenwood and Huffman (1991), Cooley and Hansen (1992) and Galí (1994*b*), among many others.

[4] While Schmitt-Grohé and Uribe (1997) mention this reformulation in passing, they do not systematically investigate cases with a unique equilibrium. In particular, they do not explore the specification in which all tax revenues are returned to households as lump-sum transfers.

income.[5]

The remainder of the paper is organized as follows. Section 2 describes the model and equilibrium conditions. Section 3 analyzes the perfect-foresight dynamics of the model, and discusses the necessary and sufficient condition for saddle-path stability. Section 4 concludes.

2 The Model

This paper incorporates a different formulation of the government budget constraint into Schmitt-Grohé and Uribe's one-sector real business cycle model in continuous time. In particular, our balanced-budget rule consists of endogenous government purchases and transfers, separate fixed tax rates applied to labor and capital income, and a depreciation allowance.

2.1 Households

The economy is populated by a unit measure of identical infinitely-lived households. Each household is endowed with one unit of time and maximizes

$$\int_0^\infty e^{-\rho t}\left\{\log C_t - A\frac{H_t^{1+\gamma}}{1+\gamma}\right\}dt,\ \ A>0, \tag{1}$$

where C_t and H_t are the individual household's consumption and hours worked, $\gamma \geq 0$ denotes the inverse of the intertemporal elasticity of substitution in labor supply, and $\rho \in (0,1)$ is the subjective discount rate.[6] We assume that there are no fundamental uncertainties present in the economy.

The budget constraint faced by the representative household is given by

$$\dot{K}_t = (1-\tau_h)w_tH_t + (1-\tau_k)\left(r_t-\delta\right)K_t - C_t + T_t,\ \ K_0>0 \text{ given}, \tag{2}$$

where K_t is the household's capital stock, w_t is the real wage, r_t is the rental rate of capital, $\delta \in (0,1)$ is the capital depreciation rate, and $T_t \geq 0$ is the lump-sum transfers. The variables τ_h and τ_k represent the *constant* tax rates applied to labor and capital income, respectively, and $\tau_k\delta K_t$ denotes the depreciation allowance built into the U.S. tax code. We require that $\tau_h, \tau_k \geq 0$ to rule out the existence of income subsidies that could only be financed by lump-sum taxation. We also require that $\tau_h, \tau_k < 1$ so that households have incentive to supply labor and capital services to firms.

[5] This resembles a "regressive" tax schedule, as postulated in Guo and Lansing (1998).

[6] For analytical tractability, Schmitt-Grohé and Uribe (1997) examine the model with indivisible labor as described by Hansen (1985) and Rogerson (1988). In this formulation, the household utility is linear in hours worked, i.e., $\gamma = 0$.

The first-order conditions for the household's optimization problem are

$$AC_t H_t^{\gamma} = (1 - \tau_h)\, w_t, \tag{3}$$

$$\frac{\dot{C}_t}{C_t} = (1 - \tau_k)\,(r_t - \delta) - \rho, \tag{4}$$

$$\lim_{t \to \infty} e^{-\rho t} \frac{K_t}{C_t} = 0, \tag{5}$$

where (3) equates the slope of the representative household's indifference curve to the after-tax real wage, (4) is the consumption Euler equation and (5) is the transversality condition.

2.2 Firms

There is a continuum of identical competitive firms, with the total number normalized to one. The representative firm produces output Y_t, using capital and labor as inputs, with a constant returns-to-scale Cobb-Douglas production function

$$Y_t = K_t^{\alpha}\, H_t^{1-\alpha}, \;\; 0 < \alpha < 1. \tag{6}$$

Under the assumption that factor markets are perfectly competitive, the firm's profit maximization conditions are given by

$$r_t = \alpha \frac{Y_t}{K_t}, \tag{7}$$

$$w_t = (1 - \alpha)\, \frac{Y_t}{H_t}. \tag{8}$$

2.3 Government

The government chooses the tax/transfer policy $\{\tau_h, \tau_k, T_t\}$, and balances its budget at each point in time. Hence, the instantaneous government budget constraint is

$$G_t + T_t = \tau_h w_t H_t + \tau_k (r_t - \delta) K_t, \tag{9}$$

where G_t denotes government spending on goods and services. Finally, the aggregate resource constraint for the economy is given by

$$C_t + \dot{K}_t + \delta K_t + G_t = Y_t. \tag{10}$$

3 Analysis of Dynamics

As in Schmitt-Grohé and Uribe (1997), our benchmark specification postulates that *no* tax revenue is returned to households as lump-sum transfers, i.e., $T_t = 0$, for all t. To facilitate the analysis of perfect-foresight dynamics, we make the following logarithmic transformation of variables: $k_t \equiv \log(K_t)$ and $c_t \equiv \log(C_t)$. With this transformation, the model's equilibrium conditions can be expressed as an autonomous pair of differential equations

$$\dot{k}_t = [1 - \alpha\tau_k - (1-\alpha)\tau_h]\, e^{\lambda_0 + \lambda_1 k_t + \lambda_2 c_t} - \delta(1-\tau_k) - e^{c_t - k_t}, \tag{11}$$

$$\dot{c}_t = \alpha(1-\tau_k)\, e^{\lambda_0 + \lambda_1 k_t + \lambda_2 c_t} - \rho - \delta(1-\tau_k), \tag{12}$$

where

$$\lambda_0 = \frac{(1-\alpha)\log\left[\frac{(1-\alpha)(1-\tau_h)}{A}\right]}{\alpha+\gamma}, \quad \lambda_1 = \frac{-\gamma(1-\alpha)}{\alpha+\gamma} \text{ and } \lambda_2 = \frac{-(1-\alpha)}{\alpha+\gamma}.$$

It is straightforward to show that the above dynamical system possesses a unique interior steady state. We can then compute the Jacobian matrix of the dynamical system (11) and (12) evaluated at the steady state. The trace and the determinant of the Jacobian are given by

$$Tr = \{[1 - \alpha\tau_k - (1-\alpha)\tau_h]\lambda_1 + \alpha(1-\tau_k)\lambda_2\} X_1 + X_2, \tag{13}$$

$$Det = \frac{-\alpha(1-\alpha)(1-\tau_k)(1+\gamma)}{\alpha+\gamma} X_1 X_2, \tag{14}$$

where

$$X_1 = \frac{\rho + \delta(1-\tau_k)}{\alpha(1-\tau_k)} \text{ and } X_2 = \frac{\rho[1 - \alpha\tau_k - (1-\alpha)\tau_h] + \delta(1-\alpha)(1-\tau_k)(1-\tau_h)}{\alpha(1-\tau_k)}.$$

Notice that since $0 < \alpha, \rho, \delta < 1, 0 \leq \tau_h, \tau_k < 1$ and $\gamma \geq 0$, we have X_1, $X_2 > 0$ and $Det < 0$. This means that the eigenvalues of the dynamical system (11) and (12) are of opposite sign, which in turn implies that the model exhibits saddle-path stability and equilibrium uniqueness.

The intuition for this result is tantamount to understanding how indeterminacy arises in Schmitt-Grohé and Uribe's model with perfect competition and constant returns in production. Start from a steady-state equilibrium, and suppose that the future return on capital is

expected to increase. Without taxes, indeterminacy can not occur since a higher capital stock is associated with a lower rate of return under constant returns-to-scale. However, a balanced-budget rule with countercyclical income taxes can cause the after-tax return on capital to rise, thus validating agents' initial optimistic expectations.

By contrast, the above mechanism that makes for multiple equilibria is eliminated under our balanced-budget formulation that consists of fixed tax rates and endogenous public spending. Intuitively, constant tax rates together with diminishing marginal products of productive inputs will reduce the higher anticipated returns from belief-driven labor and investment spurts, thus preventing agents' expectations from becoming self-fulfilling. In the Appendix, we further show that our economy is formally equivalent to Rotemberg and Woodford's (1992) oligopolistic model with perfect collusion or to Galí's (1994*a*) monopolistically competitive model with a constant elasticity of demand. In both cases, the price-to-cost markup is a constant over time, which in turn rules out indeterminacy and sunspots.

Moreover, we find that saddle-path stability remains regardless of the existence of lump-sum transfers. When *all* tax revenues are returned to households as lump-sum transfers, i.e., $G_t = 0$, for all t, it can be shown that the expression for the determinant becomes

$$Det = \frac{-(1-\alpha)(1+\gamma)\left[\rho + \delta(1-\alpha)(1-\tau_k)\right]}{(\alpha+\gamma)} X_1, \tag{15}$$

which is also negative. In this case with no government purchases, but lump-sum transfers financed through distortionary income taxation, the combination of fixed tax rates and diminishing returns to capital and labor again eliminates the possibility of endogenous fluctuations.[7]

Finally, we find that the above determinacy result is robust to changes in the source of government revenue. Following Schmitt-Grohé and Uribe (1997), we examine the following two policy rules: no capital income taxation ($\tau_k = 0$); and an income tax ($\tau_h = \tau_k = \tau$) without depreciation allowance. In each case, as in the preceding analysis, we solve two versions of the model: $T_t = 0$ or $G_t = 0$. It turns out that the saddle-path stability property in our benchmark economy is robust to all four of these alternative specifications.

To help understand the intuition for the robustness of our result, consider the consumption Euler equation (in discrete time for ease of interpretation) as follows:

[7] Saddle-path stability continues to hold when we allow for the simultaneous existence of government spending and lump-sum transfers, that is, $G_t \neq 0$ and $T_t \neq 0$. Here, we focus on the cases where one or the other is zero, to remain comparable with Schmitt-Grohé and Uribe's analysis.

$$\frac{C_{t+1}}{C_t} = \beta[1 - \delta + (1 - \tau_{t+1})r_{t+1}], \tag{16}$$

where β denotes the discount factor and τ_{t+1} is an income tax rate.[8] Households' optimistic expectations that lead to higher investment raise the left-hand side of this equation, but result in a lower before-tax return on capital r_{t+1} due to diminishing marginal products. Since the tax rate is a constant under our balanced-budget rule ($\tau_t = \tau$, for all t), the right-hand side of (16) falls. As a result, agents' expectations cannot be fulfilled in this specification, regardless of the details of the government's policies. By contrast, in Schmitt-Grohé and Uribe's model, countercyclical taxes $\left(\frac{\partial \tau_t}{\partial y_t} < 0\right)$ can increase the right-hand side of (16), thus validating the initial optimistic expectations. Overall, our analysis illustrates that under perfect competition and constant returns-to-scale, Schmitt-Grohé and Uribe's indeterminacy result depends crucially on a balanced-budget requirement whereby the tax rate decreases with the household's taxable income.

4 Concluding Remarks

Schmitt-Grohé and Uribe (1997) show that a standard one-sector real business cycle model can exhibit indeterminacy and sunspots if tax rates are determined by a balanced-budget rule with a pre-set level of public spending. This paper complements their analysis by examining a slightly different formulation of the government budget constraint. We find that as long as tax rates are fixed and the firm's production function displays diminishing marginal products, the economy exhibits saddle-path stability and equilibrium uniqueness, regardless of the source of government revenue and/or the existence of lump-sum transfers.

[8] In the case of separate tax rates on capital and labor income, τ_{t+1} will be replaced by τ^k_{t+1} in (16) and the subsequent discussion also holds true.

A Appendix

This appendix shows that the equilibrium conditions of our economy are formally equivalent to those in Rotemberg and Woodford's (1992) "implicit collusion" model and Galí's (1994*a*) "composition of aggregate demand" (CAD) model with a constant price-to-cost markup. As in Schmitt-Grohé and Uribe (1997), we consider the case of an income tax ($\tau_h = \tau_k = \tau$) without lump-sum transfers and depreciation allowance in discrete time. Therefore, the balanced-budget rule is given by

$$G_t = \tau Y_t. \tag{A.1}$$

The following equilibrium conditions hold in all three models:

$$AC_t H_t^{\gamma} = (1-\tau)\, w_t, \tag{A.2}$$

$$\frac{C_{t+1}}{C_t} = \beta[1-\delta+(1-\tau)\, r_{t+1}], \tag{A.3}$$

$$\tilde{Y}_t = C_t + K_{t+1} - (1-\delta)\, K_t, \tag{A.4}$$

where β is the discount factor and $\tilde{Y}_t$ represents disposal income. Notice that in the implicit collusion and CAD models, $\tilde{Y}_t = Y_t$ because there is no government ($\tau = 0$). However, in the balanced-budget model,

$$\tilde{Y}_t = Y_t - G_t = (1-\tau)\, Y_t. \tag{A.5}$$

This implies that our economy can be written as a standard one-sector RBC model with a constant in front of the production function. In this case, indeterminacy and sunspots cannot occur because of diminishing marginal products of capital and labor inputs.

Moreover, in all three models, the factor demands are determined by

$$(1-\alpha)\, \frac{Y_t}{H_t} = \mu_t\, (1-\tau)\, w_t, \tag{A.6}$$

and

$$\alpha \frac{Y_t}{K_t} = \mu_t\, (1-\tau)\, r_t. \tag{A.7}$$

In our balanced-budget model, μ_t represents the wedge between marginal products and after-tax factor prices induced by income taxation. Using $(A.1)$ and $(A.5)$, it can be shown that μ_t is a constant where

$$\mu_t = 1 + \frac{G_t}{\tilde{Y}_t} = \frac{1}{1-\tau}, \text{ for all } t. \tag{A.8}$$

On the other hand, in the CAD and implicit collusion models with $\tau = 0$, μ_t represents a markup of the price over marginal cost due to imperfect competition. In the CAD model, the markup is generally a function of the investment share of output. When the elasticities of substitution in production and consumption are equalized, the markup is a constant where (see Gal , pp. 79 and 84)

$$\mu_t = f\left(\frac{K_{t+1} - (1-\delta)\, K_t}{Y_t}\right) = \mu, \text{ for all } t. \tag{A.9}$$

The above equations describe the complete set of equilibrium conditions in the balanced-budget and CAD models.

In the implicit collusion model, the markup is generally a function of the ratio of the present discounted value of future profits Π_t to current output Y_t. Under perfect collusion in each industry, the markup is a constant where (see Rotemberg and Woodford [1992], pp. 1162 and 1165)

$$\mu_t = f\left(\frac{\Pi_t}{Y_t}\right) = \mu, \text{ for all } t. \tag{A.10}$$

In addition to the above equations, the implicit collusion model has another condition that describes how Π_t evolves over time.

Finally, as shown in Figures 2 and 4 of Schmitt-Grohé (1997), under constant returns-to-scale, the implicit collusion and CAD models do not exhibit indeterminacy and sunspots when the price-to-cost markup remains time-invariant.

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