# Globally-Stabilizing Fiscal Policy Rules*

Jang-Ting Guo†
University of California, Riverside

Kevin J. Lansing‡
Federal Reserve Bank of San Francisco

May 8, 2003

Forthcoming in *Studies in Nonlinear Dynamics and Econometrics*

**Abstract**

This paper demonstrates how fiscal policy rules can be designed to eliminate all forms of endogenous fluctuations in a one-sector growth model with increasing returns-to-scale. When the policy rules are implemented, agents' optimal decisions depend only on the current state of the economy and not on any expected future states. This property shuts down the mechanism for expectations-driven fluctuations. The proposed policy rules ensure a globally unique and stable equilibrium, regardless of the degree of increasing returns.

Keywords: *Fiscal Policy, Global Stability, Endogenous Fluctuations, Business Cycles.*

JEL Classification: E32, E62, H21.

*For helpful comments and suggestions, we thank an anonymous referee, Costas Azariadis, Jess Benhabib, Russell Cooper, Roger Farmer, Randall Wright and seminar participants at many places. Of course, we take full responsibility for any errors.

†Corresponding author. Department of Economics, 4128 Sproul Hall, University of California, Riverside, CA, 92521-0427, (909) 827-1588, Fax: (909) 787-5685, E-mail: guojt@mail.ucr.edu.

‡Research Department, Federal Reserve Bank of San Francisco, P.O. Box 7702, San Francisco, CA 94120-7702, (415) 974-2393, Fax: (415) 977-4031, E-mail: kevin.j.lansing@sf.frb.org.

# 1 Introduction

In the last several years, there has been growing interest in models that can exhibit indeterminacy of equilibrium.[1] In the one-sector growth models of Benhabib and Farmer (1994) and Farmer and Guo (1994), agents' expectations of the rate of return on investment can become self-fulfilling when the degree of increasing returns in production becomes sufficiently strong. This in turn allows the economy to exhibit business cycle fluctuations without any changes in the underlying fundamentals (preferences, technology, or endowments). By emphasizing shifts in expectations as the driving force of the business cycle, these models create an opportunity for Keynesian-type stabilization policies that are designed to suppress "animal spirits" or "sunspots." This paper demonstrates analytically how fiscal policy rules can be designed to eliminate all forms of endogenous fluctuations in the Benhabib-Farmer-Guo model. When the proposed policy rules are implemented, the economy exhibits a globally unique and stable equilibrium, regardless of the degree of increasing returns.

This paper builds on the results of Guo and Lansing (2002), where we explore the quantitative implications of government fiscal policy in the Benhabib-Farmer-Guo model. Starting from a laissez-faire economy that possesses an indeterminate steady state, we show that the introduction of a constant capital tax or subsidy can lead to various forms of endogenous fluctuations, including stable 2-, 4-, 8- and 10-cycles, quasiperiodic orbits and chaos. We also show that the use of local steady-state analysis to detect the presence of multiple equilibria in this class of models can be misleading. For a plausible range of capital tax rates, the log-linearized dynamical system exhibits saddle-point stability, suggesting a unique equilibrium, while the true nonlinear model exhibits global indeterminacy. This result implies that stabilization policies that are designed to suppress endogenous fluctuations near the steady state may not prevent sunspots, cycles, or chaos in regions away from the steady state. Overall, the results presented in Guo and Lansing (2002) illustrate the importance of using a model's nonlinear equilibrium conditions to fully investigate global dynamics.

Using the same basic framework of Guo and Lansing (2002), this paper shows how the model's nonlinear equilibrium conditions can be used to design a globally-stabilizing fiscal policy that suppresses all forms of endogenous fluctuations. Our approach is to solve for policy rules that map the equilibrium allocations of the original nonlinear economy into those of a wholly fictional construct known as the "pseudo economy." This methodology is simply an

[1] See Benhabib and Farmer (1999) for a survey of this extensive literature.

indirect method of solving for a set of target equilibrium allocations in the original economy.[2] The pseudo economy is chosen to exhibit the important property that the equilibrium allocations depend solely on the current state of the economy and not on any expected future states. Specifically, the pseudo economy's combination of logarithmic utility and Cobb-Douglas forms for production and capital accumulation generates exactly offsetting income and substitution effects of the interest rate movements that govern the investment decision. When these effects cancel out, agents' expectations of future returns can never be self-fulfilling. When the proposed policy rules are implemented, the equilibrium allocations of the original economy coincide exactly with the (unique) equilibrium allocations of the pseudo economy. In this way, the policy rules ensure a globally unique and stable equilibrium, regardless of the degree of increasing returns.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 describes our approach for constructing a set of globally-stabilizing fiscal policy rules. Section 4 concludes. An appendix extends our approach to a two-sector model.

## 2 The Model

The description of the model closely follows the presentation in Guo and Lansing (2002) with one exception. Here we allow for the possibility that tax revenues may used to finance non-productive government purchases, whereas Guo and Lansing (2002) assume that all tax revenues are returned to households in the form of income subsidies or lump-sum transfers.

### 2.1 Firms

There is a continuum of identical competitive firms in the economy. For convenience, the number of firms is normalized to one. Each firm produces output $y_t$ according to a Cobb-Douglas production function

$$y_t = z_t k_t^{\theta} h_t^{1-\theta}, \quad 0 < \theta < 1, \tag{1}$$

where $k_t$ and $h_t$ are capital and labor inputs, and $z_t$ represents a productive externality that is taken as given by the individual firm. We postulate that the externality takes the form

$$z_t = \left( K_t^{\theta} H_t^{1-\theta} \right)^{\eta}, \quad \eta \geq 0, \tag{2}$$

[2] A related approach is used by Becker (1985) to solve for equilibrium allocations in a growth model that is subject to capital taxation. Other examples of the pseudo-economy approach can be found in Stokey, Lucas and Prescott (1989, Chapter 18) and Salyer (1996).

where $K_t$ and $H_t$ denote the economy-wide levels of capital and labor services. In a symmetric equilibrium, all firms take the same actions such that $k_t = K_t$ and $h_t = H_t$, for all $t$. As a result, (2) can be substituted into (1) to obtain the following social technology:

$$y_t = k_t^{\alpha_1} h_t^{\alpha_2}, \quad \alpha_1 + \alpha_2 \geq 1, \tag{3}$$

where $\alpha_1 \equiv \theta(1+\eta)$ and $\alpha_2 \equiv (1-\theta)(1+\eta)$. When $\eta > 0$, the social technology exhibits increasing returns to scale. We restrict our attention to the case of $\alpha_1 < 1$, which implies that the externality is not strong enough to generate sustained endogenous growth. Under the assumption that factor markets are perfectly competitive, the first-order conditions of the individual firm's profit maximization problem are given by

$$r_t = \frac{\theta y_t}{k_t}, \tag{4}$$

$$w_t = \frac{(1-\theta)\, y_t}{h_t}, \tag{5}$$

where $r_t$ is the capital rental rate and $w_t$ is the real wage.

## 2.2 Households

The economy is populated by a unit measure of identical infinitely-lived households, each endowed with one unit of time, who maximize a discounted sum of expected utilities over their lifetime

$$\sum_{t=0}^{\infty} \beta^t \left[ \log c_t - \frac{A h_t^{1+\gamma}}{1+\gamma} \right], \quad A > 0, \tag{6}$$

where $\beta \in (0,1)$ is the discount factor, $c_t$ is consumption, $h_t$ is hours worked and $\gamma \geq 0$ denotes the inverse of the intertemporal elasticity of substitution in labor supply. We assume that there are no fundamental uncertainties present in the economy.

The budget constraint faced by the representative household is

$$c_t + i_t = (1 + s_{kt})\, r_t k_t + (1 + s_{ht})\, w_t h_t - T_t, \tag{7}$$

where $i_t$ is investment, $s_{kt}$ and $s_{ht}$ are the subsidy (tax) rates applied to capital and labor incomes, and $T_t$ is a lump-sum tax (transfer). The fiscal policy variables $s_{kt}$, $s_{ht}$, and $T_t$ are determined outside of the household's control. The law of motion for the capital stock is

$$k_{t+1} = (1-\delta) k_t + i_t, \quad k_0 = \bar{k}_0, \tag{8}$$

where $\delta \in (0,1)$ is the depreciation rate.

The first-order conditions for the household's optimization problem are given by

$$A c_t h_t^{\gamma} = (1 + s_{ht}) w_t, \tag{9}$$

$$\frac{1}{c_t} = \frac{\beta}{c_{t+1}} \left[ 1 - \delta + (1 + s_{kt+1}) r_{t+1} \right], \tag{10}$$

$$\lim_{t \to \infty} \beta^t \frac{k_{t+1}}{c_t} = 0, \tag{11}$$

where (9) is an intra-temporal condition that equates the household's marginal rate of substitution between consumption and leisure to the after-subsidy real wage. Equation (10) is the standard Euler equation for intertemporal consumption choices, and (11) is the transversality condition.

### 2.3 Government

The government chooses a sequence of fiscal policy variables $\{s_{kt}, s_{ht}, T_t, g_t\}_{t=0}^{\infty}$ where $g_t$ represents government purchases of goods and services that do not contribute to either production or household utility. The government budget constraint is

$$g_t = T_t - s_{kt}\, r_t k_t - s_{ht}\, w_t h_t, \tag{12}$$

where we assume that the budget is balanced each period. By specifying a set of policy rules for $s_{kt}$, $s_{ht}$, and $T_t$, we also implicitly specify a policy rule for $g_t$.

Finally, the aggregate resource constraint for the economy is given by

$$c_t + k_{t+1} - (1 - \delta) k_t + g_t = y_t. \tag{13}$$

### 2.4 Indeterminacy of Equilibria

It is well-known that one-sector growth models require strong increasing returns to exhibit indeterminacy. In particular, Guo and Lansing (2002) demonstrate that a calibrated version of the present model requires returns-to-scale of about 1.6 or greater $(1 + \eta > 1.6)$ to exhibit local indeterminacy under laissez-faire. Basu and Fernald (1997) note that returns-to-scale estimates reported in the literature vary dramatically depending on the type of data used, the level of aggregation, and the estimation method. In attempting to account for the wide range of estimates, Basu and Fernald (1997) demonstrate that while the average U.S. industry exhibits approximately constant returns-to-scale, the aggregate private business economy can appear to exhibit large increasing returns. The largest aggregate estimate they obtain is 1.72 with a standard error of 0.36. When the aggregate returns-to-scale estimation procedure is corrected to account for reallocation of inputs across industries, they find that the aggregate estimates shrink considerably and are close to the industry results. The largest corrected aggregate estimate they obtain is 1.03 with a standard error of 0.18. Despite these findings, Basu and Fernald (1997, Section V) note that the *uncorrected* aggregate estimates may actually be more appropriate for calibrating models (such as ours) that abstract from production heterogeneity and assume a single representative firm.

In what follows, we assume that the degree of increasing returns is strong enough to induce local indeterminacy under laissez-faire. We acknowledge, however, that this assumption could be challenged on empirical grounds. To the extent that one objects to the notion of strong increasing returns, the analysis below should be viewed more from a methodological perspective as illustrating a particular approach to achieving global stabilization of a nonlinear dynamical system. We also note that multi-sector growth models can exhibit indeterminacy with a much lower (and hence more realistic) degree of increasing returns.[3] In the Appendix, we show how our approach can be applied in a two-sector model.

## 3 Globally-Stabilizing Fiscal Policy

Two features of the model economy suggest a role for government intervention. First, the laissez-faire equilibria are not Pareto optimal due to the presence of the externality which introduces a wedge between the social and private marginal products of capital and labor. Second, our assumption of strong increasing returns implies that the decentralized economy is subject to expectations-driven fluctuations. One would naturally wish to solve the problem of a social planner who seeks to maximize the utility of the representative household (6), subject to the aggregate resource constraint (13). The first-best allocations provide an important benchmark for judging the desirability of stabilization policy. Recently, Dupor and Lehnert (2002) have used numerical techniques to establish some features of the first-best allocations in a special case of (6) where the disutility of labor is linear, i.e., $\gamma = 0$. They show that the solution of the planner's problem is generally characterized by endogenous cycles in which the optimal labor supply undergoes discontinuous jumps as capital varies over the cycle. Hence, a

[3] See, for example, Benhabib and Farmer (1996), Perli (1998), Benhabib and Nishimura (1998), and Benhabib, Meng, and Nishimura (2000).

fiscal policy that eliminates all forms of endogenous fluctuations in the decentralized economy would not be considered optimal. Such a policy could still be welfare-improving, however, if the policy simultaneously addressed the wedge between social and marginal products of capital and labor.[4] Any welfare analysis would also have to take into account the transition dynamics that are induced when the policy is implemented (or announced). In light of the many complicating issues surrounding a welfare analysis of stabilization policy, we restrict our attention here to issues of feasibility.

In Guo and Lansing (2002, section 4.2), we describe various control mechanisms that can eliminate local indeterminacy for the model described in section 2. In particular, we show how the log-linearized model can be used to design a state-contingent capital subsidy/tax scheme that ensures saddle-point stability of the steady state. By selecting a locally unique equilibrium, such a policy prevents expectations-driven fluctuations near the steady state. There is an important caveat, however, to a stability analysis based solely on a log-linear approximation. Guo and Lansing (2002) show that in the present model, a locally determinate steady state (a saddle point) does not preclude the possibility of expectations-driven fluctuations in regions away from the steady state. This means that stabilization policies designed using the approximating model may prove unsuccessful when introduced into the true nonlinear model. To avoid such an outcome, we now derive a set of fiscal policy rules that ensure globally stability of the model economy.[5]

## 3.1 A Pseudo Economy

Consider a wholly fictional construct known as the "pseudo economy" in which a representative household maximizes lifetime utility (6), subject to the modified constraints

$$c_t + \widehat{i}_t = r_t k_t + w_t h_t, \tag{14}$$

$$k_{t+1} = \widehat{B}\, k_t^{1-\widehat{\phi}} \;\; \widehat{i}_t^{\,\widehat{\phi}}, \quad k_0 \; = \bar{k}_0, \tag{15}$$

where $r_t$ and $w_t$ are again given by (4) and (5), $\widehat{i}_t$ is investment in the pseudo economy, $\widehat{\phi} \in (0, 1]$ denotes the elasticity of next period's capital stock with respect to investment,

[4] Cassou and Lansing (1997) conduct a welfare analysis of stabilization policy in an endogenous growth model with a productive externality. Unlike here, however, their model exhibits a unique equilibrium. They show that (suboptimal) stabilization policies can still be welfare-improving if the steady-state values of the policy variables are chosen to address the externality.

[5] A recent paper by Benhabib, Schmitt-Grohé, and Uribe (2003) shows how monetary policy rules can be designed to ensure global stablity in a sticky-price model without capital.

and $\widehat{B} > 0$ can be interpreted as an efficiency index. We use the "hat" symbol ($\widehat{\ }$) as a reminder that the given variable or parameter is specific to the pseudo economy and has no direct counterpart in the original economy. As we shall see, however, the hat variables and parameters in the pseudo economy can be interpreted as elements of the fiscal policy that is employed in the original economy. The above maximization problem differs from that of the original economy in two respects: (i) the pseudo economy is in laissez-faire, and (ii) the law of motion for capital (15) is Cobb-Douglas instead of linear. The Cobb-Douglas formulation can be viewed as reflecting adjustment costs as in Lucas and Prescott (1971).[6] When $\widehat{B} = 1$ and $\widehat{\phi} = 1$, equation (15) implies that capital depreciates completely each period, whereas $0 < \widehat{\phi} < 1$ indicates that capital is long lasting.

The first-order conditions for the pseudo economy are

$$A c_t h_t^{\gamma} = w_t, \tag{16}$$

$$\frac{\widehat{i}_t}{c_t} = \frac{\beta}{c_{t+1}} \left[ (1 - \widehat{\phi})\, \widehat{i}_{t+1} + \theta\, \widehat{\phi}\, y_{t+1} \right], \tag{17}$$

together with the transversality condition (11).

To solve for the equilibrium allocations, we substitute the budget constraint (14) into (15), and combine them with the consumption Euler equation (17) to obtain the following first-order linear difference equation in the output-consumption ratio:

$$\frac{y_t}{c_t} = \beta \left( \rho + \widehat{\phi} \right) + \varphi\, \frac{y_{t+1}}{c_{t+1}}, \tag{18}$$

where $\varphi \equiv \beta \left[ 1 - (1-\theta)\widehat{\phi} \right]$ and $\rho \equiv \beta^{-1} - 1$. Since $\rho > 0$, $\theta \in (0,1)$, and $\widehat{\phi} \in (0,1]$, we have $0 < \varphi < 1$. Therefore, (18) is a forward-looking stable dynamic equation. Iterating (18) into the future, we find that the only solution to this equation is

$$\frac{y_t}{c_t} = \beta \left( \rho + \widehat{\phi} \right) \sum_{j=0}^{\infty} \varphi^j \ = \ \frac{\rho + \widehat{\phi}}{\rho + (1-\theta)\, \widehat{\phi}}, \text{ for all } t, \tag{19}$$

that is, agents consume a constant fraction $\lambda \equiv \frac{\rho + (1-\theta)\, \widehat{\phi}}{\rho + \widehat{\phi}}$ of output each period. It follows from (14) that the investment allocation in the pseudo economy is given by

$$\widehat{i}_t = (1 - \lambda) y_t, \text{ for all } t. \tag{20}$$

[6] Following Kim (2003), equation (15) can be viewed as a special case of a more general specification where $k_{t+1} = \widehat{B} \left[ \left( 1 - \widehat{\phi} \right) k_t^{1-\sigma} + \widehat{\phi} \left( \widehat{i}_t \,/\, \widehat{\phi} \right)^{1-\sigma} \right]^{1/(1-\sigma)}$. The Cobb-Douglas form, which corresponds to the case of $\sigma = 1$, has been used in applied work by Hercowitz and Sampson (1991), Kocherlakota and Yi (1997), and Cassou and Lansing (1998), among others. The linear form of (8) can be recovered by imposing $\sigma = 0$, $\widehat{B} = 1$, and $\widehat{\phi} = \delta$.

Intuitively, the combination of logarithmic utility and Cobb-Douglas forms for production and capital accumulation generates exactly offsetting income and substitution effects of the interest rate movements that govern the investment allocation. As a result, the household only needs to observe the current state of the economy in order to decide how much to consume and invest. This is the crucial feature of the pseudo economy that gives rise to closed-form decision rules.[7]

Substituting (19) into (16), we obtain

$$h_t = \left(\frac{1-\theta}{\lambda A}\right)^{\frac{1}{1+\gamma}}, \tag{21}$$

which shows that work effort is constant in the pseudo economy despite the household's willingness to substitute labor hours intertemporally. Finally, substituting (21) into the social technology (3) yields the following expression for total output:

$$y_t = k_t^{\alpha_1}\left(\frac{1-\theta}{\lambda A}\right)^{\frac{\alpha_2}{1+\gamma}}. \tag{22}$$

The fact that $y_t$, $c_t$ and $\widehat{i}_t$ in the pseudo economy are uniquely pinned down by the *current-period* economic fundamentals has two important implications. First, the steady state in the pseudo economy is a saddle point regardless of the degree of increasing returns in production. Second, shifts in household expectations about future rates of return have no effect on the equilibrium allocations. Consequently, the pseudo economy can never exhibit local or global indeterminacy.

## 3.2 Solving for the Policy Rules

The allocations in the pseudo economy represent a potentially desirable benchmark for a policymaker who wishes to stabilize the original economy against all forms of endogenous fluctuations. We now proceed to derive a set of policy rules for the original economy that will implement the allocations of the pseudo economy. We recognize, of course, that other policy rules might accomplish the same objective by mapping into a different pseudo economy. For our approach to be successful, the pseudo economy must exhibit the important property that the investment decision does not depend on expected future returns. If the investment decision in the pseudo economy depends on agents' expectations about future returns, then these

[7] Kurz (1968) and Chang (1988) provide a number of examples of preference and technology combinations in optimal growth models for which equilibrium decisions do not depend on the future. In the terminology of Blackorby et al. (1973), these examples exhibit the property that "the future is functionally separable from the present."

expectations could become self-fulfilling when the productive externality is strong enough. In this case, the policy rules that implement the allocations of the pseudo economy would not guarantee a globally unique and stable equilibrium in the original economy.

**Proposition 1.** *The following state-contingent policy rules will stabilize the original nonlinear economy against all forms of endogenous fluctuations and yield a globally unique and stable equilibrium:*

$$s_{ht} = 0, \tag{23}$$

$$s_{kt} = \frac{(1-\lambda)\widehat{B}^{\frac{1}{\widehat{\phi}}}}{\theta\beta}\left(\frac{k_{t-1}}{k_t}\right)^{\frac{1-\widehat{\phi}}{\widehat{\phi}}} - \left(\frac{1-\delta}{\theta}\right)\frac{k_t}{y_t} - 1, \tag{24}$$

$$\frac{T_t}{y_t} = 1-\lambda-\theta+\frac{(1-\lambda)\widehat{B}^{\frac{1}{\widehat{\phi}}}}{\beta}\left(\frac{k_{t-1}}{k_t}\right)^{\frac{1-\widehat{\phi}}{\widehat{\phi}}} - \widehat{B}(1-\lambda)^{\widehat{\phi}}\left(\frac{k_t}{y_t}\right)^{1-\widehat{\phi}}, \tag{25}$$

*where* $y_t$ *is given by (22) and* $\lambda \equiv \frac{\rho+(1-\theta)\widehat{\phi}}{\rho+\widehat{\phi}}$. *The policy parameters* $\widehat{B} > 0$ *and* $\widehat{\phi} \in (0,1]$ *control the steady-state values of* $s_{kt}$ *and* $\frac{T_t}{y_t}$.

*Proof*: Given the initial capital stock $\bar{k}_0$, we solve for fiscal policy rules that make the equilibrium allocations $\{c_t, h_t, y_t, k_{t+1}\}_{t=0}^{\infty}$ identical for the two economies. By equating the (nonlinear) equilibrium conditions (7), (9) and (10) in the original economy to the corresponding conditions (14), (16) and (17) in the pseudo economy, we obtain three equations that can be solved for the three unknowns $s_{ht}$, $s_{kt}$, and $\frac{T_t}{y_t}$. We eliminate $\widehat{i}_t$ from these equations using the law of motion (15). Given that conditions (14), (16) and (17) define a globally unique set of allocations in terms of current-period fundamentals (as demonstrated in section 3.1), the above policy rules will rule out indeterminacy, local or global, for any degree of increasing returns that satisfies $\alpha_1 < 1$. ■

An expression for the state-contingent government spending policy implied by (23)-(25) can be derived by comparing the resource constraint (13) with the equilibrium version of (14) to obtain

$$g_t = \widehat{i}_t - i_t. \tag{26}$$

Equation (26) shows that $g_t$ measures the amount of additional investment that must be undertaken in the pseudo economy to produce the same sequence of capital stocks as in the original economy.[8] Hence, $g_t$ can be interpreted as a measure of the implicit adjustment costs

[8] Recall that the two economies have the same sequence of capital stocks by construction.

associated with the Cobb-Douglas capital accumulation equation (15). This helps to explain why the policy rules (24) and (25) depend on $k_t$ and $k_{t-1}$.

The policy parameters $\widehat{B}$ and $\widehat{\phi}$ influence the original economy only through the policy rules (24) and (25). The following proposition illustrates one way in which the values of $\widehat{B}$ and $\widehat{\phi}$ could be chosen by the policymaker.

**Proposition 2.** *The following settings for the policy parameters $\widehat{B}$ and $\widehat{\phi}$ achieve the laissez-faire steady state of the original economy:*

$$\widehat{B} = \delta^{-\delta}, \tag{27}$$

$$\widehat{\phi} = \delta. \tag{28}$$

*Proof*: The steady-state versions of (8) and (10) in the original economy with $s_{kt} = s_{ht} = 0$ imply $i/k = \delta$ and $i/y = \theta\delta/\left(\rho+\delta\right)$. The steady-state versions of (15) and (20) in the pseudo economy imply $i/k = \widehat{B}^{\frac{-1}{\widehat{\phi}}}$ and $i/y = \theta\widehat{\phi}/\left(\rho+\widehat{\phi}\right)$. Equating $i/k$ and $i/y$ in the two economies yields two equations that can be solved for $\widehat{B}$ and $\widehat{\phi}$. Substituting these parameter values into (23)-(26) and letting $t \to \infty$ yields $s_{ht} = s_{kt} = \frac{T_t}{y_t} = 0$ which implies laissez-faire. ■

Proposition 2 shows that by appropriate choice of $\widehat{B}$ and $\widehat{\phi}$, the policymaker can avoid the need for intervention as $t \to \infty$. The policymaker must credibly commit to intervene, however, in the event that the economy is ever perturbed away from the steady state. In such an event, the policy rules defined by (23)-(26) require the government to implement a sophisticated combination of fiscal variables over time.

Finally, we note that the policy rules defined by (23)-(26) have been designed solely for the purpose of achieving global stabilization; they do not address the productive externality. It is straightforward, however, to extend Propositions 1 and 2 to the case where the steady state values of $s_{ht}$, $s_{kt}$, and $\frac{T_t}{y_t}$ are chosen to eliminate the wedge between the social and marginal products of capital and labor. In this case, as $t \to \infty$, we would have $s_{ht} = s_{kt} = \frac{T_t}{y_t} = \eta$.[9]

## 4 Conclusion

This paper demonstrates how fiscal policy rules can be designed to ensure a globally unique and stable equilibrium in a one-sector growth model with increasing returns-to-scale. The policy rules map the model's nonlinear equilibrium allocations into those of a wholly fictional pseudo economy in which agents' optimal decisions depend only on the current state of the

[9] See Guo and Lansing (2002, Proposition 1).

economy and not on any expected future states. The fact that equilibrium decision rules are independent of expectations is the crucial feature that shuts down the mechanism for belief-driven business cycle fluctuations.

# A Appendix

This appendix extends our global stabilization approach to Harrison's (2001) two-sector model. Unlike Benhabib and Farmer (1996) who postulate uniform externalities across sectors in the production of consumption and investment goods, Harrison (2001) shows that local indeterminacy can occur if the productive externality in the investment-goods sector is sufficiently strong, even if there is no externality in the consumption-goods sector.

## A.1 A Two-Sector Model

The economy consists of two distinct production sectors. In the consumption sector, output is produced by competitive firms using the following constant returns-to-scale technology:

$$y_{ct} = k_{ct}^{\theta} h_{ct}^{1-\theta}, \quad 0 < \theta < 1, \tag{A.1}$$

where $k_{ct}$ and $h_{ct}$ are capital and labor inputs used in the production of consumption goods. Similarly, investment goods are produced by competitive firms using the technology

$$y_{it} = x_t k_{it}^{\theta} h_{it}^{1-\theta}, \text{ where } \; x_t = \left( K_{it}^{\theta} H_{it}^{1-\theta} \right)^{\eta}, \quad \eta > 0. \tag{A.2}$$

Here, $k_{it}$ and $h_{it}$ are capital and hours worked in the investment sector, and $x_t$ represents a sector-specific productive externality. In a symmetric equilibrium, all firms in the investment sector take the same actions such that $k_{it} = K_{it}$ and $h_{it} = H_{it}$, for all $t$.

Under the assumptions that factor markets are perfectly competitive and that capital and labor inputs are perfectly mobile across the two sectors, the first-order conditions for the firms' profit maximization problems are

$$r_t = \frac{\theta y_{ct}}{k_{ct}} = \frac{\theta p_t y_{it}}{k_{it}}, \tag{A.3}$$

$$w_t = \frac{(1-\theta)\, y_{ct}}{h_{ct}} = \frac{(1-\theta)\, p_t y_{it}}{h_{it}}, \tag{A.4}$$

where $p_t$ denotes the relative price of investment to consumption goods. Since firms use identical technologies and face equal factor prices across the two sectors, the fractions of capital and labor used in the consumption sector are the same,

$$\frac{k_{ct}}{k_t} = \frac{h_{ct}}{h_t} \equiv \mu_t, \tag{A.5}$$

where $k_t$ and $h_t$ are the economy's total capital and labor hours, respectively. Moreover, market clearing in the capital and labor markets requires that $k_{ct} + k_{it} = k_t$ and $h_{ct} + h_{it} = h_t$.

The representative household maximizes its lifetime utility (6), subject to

$$c_t + p_t i_t = (1 + s_{kt})\, r_t k_t + (1 + s_{ht})\, w_t h_t - T_t, \quad k_0 = \bar{k}_0, \tag{A.6}$$

where investment $i_t$ is defined by the linear capital accumulation equation (8). The first-order conditions for the household's optimization problem are

$$A c_t h_t^{\gamma} = (1 + s_{ht}) w_t, \tag{A.7}$$

$$\frac{1}{c_t} = \frac{\beta}{c_{t+1}} \left[ \frac{(1-\delta)\, p_{t+1} + (1 + s_{kt+1}) r_{t+1}}{p_t} \right], \tag{A.8}$$

and the transversality condition (11). The government budget constraint is again given by (12). Finally, the aggregate resource constraint for the economy is

$$c_t + p_t[k_{t+1} - (1-\delta)k_t] + g_t = y_t. \tag{A.9}$$

## A.2 A Pseudo Economy

Consider a pseudo economy in which the representative household maximizes (6), subject to the modified budget constraint

$$c_t + p_t \widehat{i}_t = r_t k_t + w_t h_t, \quad k_0 = \bar{k}_0, \tag{A.10}$$

where investment $\widehat{i}_t$ is defined by the Cobb-Douglas capital accumulation equation (15). The first-order conditions for the household's problem are

$$A c_t h_t^{\gamma} = w_t, \tag{A.11}$$

$$\frac{\widehat{i}_t}{c_t} = \frac{\beta}{c_{t+1}} \left[ \frac{(1-\widehat{\phi}) p_{t+1} \widehat{i}_{t+1} + \theta \widehat{\phi} y_{t+1}}{p_t} \right], \tag{A.12}$$

and the transversality condition (11). Following the same procedure as in section 3.1, it is straightforward to show that the equilibrium decision rules in the pseudo economy are given by

$$h_t = \left(\frac{1-\theta}{\mu A}\right)^{\frac{1}{1+\gamma}}, \tag{A.13}$$

$$c_t = \mu k_t^{\theta}\left(\frac{1-\theta}{\mu A}\right)^{\frac{1-\theta}{1+\gamma}}, \tag{A.14}$$

$$\widehat{i}_t = \left[(1-\mu)\, k_t^{\theta}\left(\frac{1-\theta}{\mu A}\right)^{\frac{1-\theta}{1+\gamma}}\right]^{1+\eta}, \tag{A.15}$$

$$p_t = \left[(1-\mu)\, k_t^{\theta}\left(\frac{1-\theta}{\mu A}\right)^{\frac{1-\theta}{1+\gamma}}\right]^{-\eta}, \tag{A.16}$$

where $\mu_t = \mu = \frac{\rho+(1-\theta)\widehat{\phi}}{\rho+\widehat{\phi}}$ for all $t$. Using (A.5) and the market-clearing conditions for the capital and labor markets, we can obtain the closed-form decision rules for $k_{ct}, h_{ct}, k_{it}$ and $h_{it}$ accordingly. Therefore, the pseudo economy is completely immune to local and global indeterminacy because its equilibrium allocations are uniquely pinned down by current-period fundamentals.[10]

## A.3 Solving for the Policy Rules

Given the initial capital stock $\bar{k}_0$, we solve for a fiscal policy that makes equilibrium price $\{p_t\}_{t=0}^{\infty}$ and allocations $\{c_t, \mu_t, h_t, k_{t+1}, k_{ct}, h_{ct}, k_{it}, h_{it}\}_{t=0}^{\infty}$ identical for the two economies. By equating the equilibrium conditions (A.6), (A.7) and (A.8) in the original two-sector model to the corresponding conditions (A.10), (A.11) and (A.12) in the pseudo economy, we obtain three equations that can be solved for the three unknowns $s_{ht}$, $s_{kt}$, and $\frac{T_t}{y_t}$. We eliminate $\widehat{i}_t$ from these equations using the law of motion (15). Government spending on goods and services $g_t$ is determined by (12). Given that (A.10), (A.11) and (A.12) define a globally unique set of allocations (as demonstrated above), the proposed policy rules will stabilize the two-sector economy against all forms of endogenous fluctuations.

[10] Our pseudo economy approach cannot be applied to the multi-sector models of Benhabib and Nishimura (1998) and Benhabib, Meng and Nishimura (2000) which exhibit unequal factor intensities in producing different goods. In these models, the fractions of capital and labor used in a given sector are not constant over time. This feature precludes derivation of closed-form expressions for the equilibrium allocations in a multi-sector version of the pseudo economy.

## References


[1] Basu, S. and J.G. Fernald. (1997). "Returns-to-Scale in U.S. Production: Estimates and Implications," *Journal of Political Economy*, 105, 249-283.

[2] Becker, R.A. (1985). "Capital Income Taxation and Perfect Foresight," *Journal of Public Economics*, 26, 147-168.

[3] Benhabib, J. and R.E.A. Farmer. (1994). "Indeterminacy and Increasing Returns," *Journal of Economic Theory*, 63, 19-41.

[4] Benhabib, J. and R.E.A. Farmer. (1996). "Indeterminacy and Sector-Specific Externalities," *Journal of Monetary Economics*, 37, 421-443.

[5] Benhabib, J. and K. Nishimura. (1998). "Indeterminacy and Sunspots with Constant Returns," *Journal of Economic Theory*, 81, 58-96.

[6] Benhabib, J. and R.E.A. Farmer. (1999). "Indeterminacy and Sunspots in Macroeconomics," in J. Taylor and M. Woodford, eds., *Handbook of Macroeconomics, 387-448,* Amsterdam: North Holland.

[7] Benhabib, J., Q. Meng and K. Nishimura. (2000). "Indeterminacy Under Constant Returns to Scale in Multisector Economies," *Econometrica*, 68, 1541-1548.

[8] Benhabib, J., S. Schmitt-Grohé and M. Uribe. (2003). "Backward-Looking Interest Rate Rules, Interest-Rate Smoothing, and Macroeconomic Instability," National Bureau of Economic Research, Working Paper 9558.

[9] Blackorby, C., D. Nisssen, D. Primont and R.R. Russell. (1973). "Consistent Intertemporal Decision Making," *Review of Economic Studies,* 40, 239-248.

[10] Cassou, S.P. and K.J. Lansing. (1997). "Welfare, Stabilization, or Growth: A Comparison of Different Fiscal Objectives," in J. Hairault, P. Hénin and F. Portier, eds., *Business Cycles and Macroeconomic Stability,* 55-78*.* Boston, MA: Kluwer Academic Publishers.

[11] Cassou, S.P. and K.J. Lansing. (1998). "Optimal Fiscal Policy, Public Capital, and the Productivity Slowdown," *Journal of Economic Dynamics and Control*, 22, 911-935.

[12] Chang, F.-R. (1988). "The Inverse Optimal Problem: A Dynamic Programming Approach," *Econometrica*, 56, 147-172.

[13] Dupor, W.D. and A. Lehnert (2002)."Increasing Returns and Optimal Oscillating Labor Supply," Federal Reserve Board, Finance and Economics Discussion Series Paper 2002-32.

[14] Farmer, R.E.A. and J.-T. Guo. (1994). "Real Business Cycles and the Animal Spirits Hypothesis," *Journal of Economic Theory*, 63, 42-72.

[15] Guo, J.-T. and K.J. Lansing. (2002). "Fiscal Policy, Increasing Returns, and Endogenous Fluctuations," *Macroeconomic Dynamics*, 6, 633-664.

[16] Harrison, Sharon G. (2001). "Indeterminacy in a Model with Sector-Specific Externalities," *Journal of Economic Dynamics and Control*, 25, 747-764.

[17] Hercowitz, Z. and M. Sampson. (1991). "Output Growth, the Real Wage, and Employment Fluctuations," *American Economic Review*, 81, 1215-1237.

[18] Kocherlakota, N.R. and K.-M. Yi. (1997). "Is There Endogenous Long Run Growth? Evidence from the U.S. and the U.K.," *Journal of Money, Credit and Banking*, 29, 235-262.

[19] Kim, J. (2003). "Functional Equivalence Between Intertemporal and Multisectoral Investment Adjustment Costs," *Journal of Economic Dynamics and Control*, 27, 533-549.

[20] Kurz, M. (1968). "On the Inverse Optimal Problem," in H.W. Kuhn and G.P. Szegö, eds., *Mathematical Systems Theory and Economics I.*, 189-202. Berlin: Springer-Verlag.

[21] Lucas, R.E., Jr. and E.C. Prescott. (1971). "Investment Under Uncertainty," *Econometrica,* 39, 659-681.

[22] Perli, R. (1998). "Indeterminacy, Home Production and the Business Cycle: A Calibrated Analysis," *Journal of Monetary Economics,* 41, 105-125.

[23] Salyer, K.D. (1996). "Interpreting a Stochastic Monetary Growth Model as a Modified Social Planner's Problem," *Journal of Economic Dynamics and Control*, 20, 681-689.

[24] Stokey, N.L., R.E. Lucas, Jr. and E.C. Prescott. (1989). *Recursive Methods in Economic Dynamics.* Cambridge, MA: Harvard University Press.