Increasing Returns, Capital Utilization, and the Effects of Government Spending*

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Abstract

We show that a one-sector real business cycle model with mild increasing returns-to-scale, variable capital utilization and saddle-path stability is able to produce qualitatively realistic business cycles driven solely by disturbances to government purchases. Due to an endogenous increase in labor productivity, a positive spending shock can lead to simultaneous increases in output, consumption, investment, employment and real wage. Our analysis illustrates a close relationship between this result and the recent literature that explores indeterminacy and sunspots in real business cycle models under laissez-faire. In particular, the condition which governs the required magnitude of increasing returns for government spending shocks to generate procyclical macroeconomic effects is identical to that necessary for a laissez-faire one-sector real business cycle model to exhibit local indeterminacy.

Keywords: Increasing Returns, Capital Utilization, Government Spending Shocks, Real Business Cycles, Indeterminacy.

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1 Introduction

Recently, Devereux, Head and Lapham (DHL, 1996) have explored the macroeconomic effects of temporary and permanent changes in government spending in a one-sector real business cycle (RBC) model with increasing returns, monopolistic competition and saddle-path stability. In a parametric version of the DHL model with logarithmic utility in consumption, indivisible labor and *large* markup ratio of price over marginal cost (= 1.5), a positive government spending shock can lead to higher output, consumption, investment, employment and real wage.[1] This result is driven by the endogenous response of total factor productivity to a change in government purchases. DHL's finding is an important one since it shows that qualitatively realistic business cycles can be generated in a dynamic general equilibrium framework with only demand shocks. On the contrary, in a standard one-sector RBC model with perfect competition and constant returns-to-scale, aggregate demand shocks yield countercyclical consumption and labor productivity, which is not consistent with the U.S. data (see, for example, Aiyagari, Christiano and Eichenbaum [1992], and Baxter and King [1993]).

This paper builds on DHL and makes two points. First, to maintain comparability with previous studies that mostly adopt the standard RBC model, DHL restrict the analysis to annual parameterizations in which their economy exhibits saddle-path stability and equilibrium uniqueness.[2] However, we find that DHL's large-markup specification possesses an indeterminate steady state (a sink) and stationary sunspot equilibria at the quarterly frequency whereby the household's rate of time preference and the capital depreciation rate are lower than those in the annual formulation. This result can be understood in two stages. On the one hand, DHL demonstrate that in order to generate positive correlations between government spending shocks and macroeconomic aggregates, the markup must be sufficiently high to imply that the equilibrium wage-hours locus is positively sloped and steeper than the labor supply curve. In this paper, we show that this is also a *necessary* condition for the DHL model to exhibit local indeterminacy. By contrast, Benhabib and Farmer (1994) and Schmitt-Grohé (1997) illustrate that the same condition is necessary for a laissez-faire one-sector RBC model to display

[1] In a sequel, Devereux, Head and Lapham (2000) show that in a one-sector RBC model with sufficiently strong increasing returns (or markup), purely wasteful government purchases may raise economic welfare measured by the steady-state household utility.

[2] The notable exception is Rotemberg and Woodford (1992) who obtain similar results as DHL do in an environment with oligopolistic pricing and increasing returns. While the price-to-cost markup in DHL is a constant, it is countercyclical in the Rotemberg-Woodford model due to implicit collusion.

multiple equilibria. Therefore, in this respect, our analysis brings together two strands of the one-sector business cycle literature — determinate RBC models driven solely by disturbances to government purchases (as in DHL), and indeterminate, laissez-faire RBC models only subject to sunspot shocks (as in Benhabib and Farmer [1994], and Farmer and Guo [1994], among others).

On the other hand, we show analytically that the minimum level of increasing returns needed for local indeterminacy in the DHL economy is positively related to the household's rate of time preference and the capital depreciation rate. To understand this finding, suppose that the representative household becomes optimistic about the future of the economy and decides to invest more, thus raising next period's capital stock. If the increasing returns in the firms' production process are strong enough to yield higher labor productivity, the rate of return on today's investment will rise. As a result, agents' initial optimistic expectations become self-fulfilling. When the time step gets smaller, e.g., moving from the annual to quarterly parameterization, labor is drawn more easily out of leisure to help validate agents' optimism. Hence, indeterminacy is more likely to occur when the DHL model is examined under the quarterly parameterization.

Second, given recent empirical findings of Burnside (1996) and Basu and Fernald (1997), it is now well known that the required degree of increasing returns for the Benhabib-Farmer-Guo one-sector RBC formulation to exhibit local indeterminacy is too high to be empirically plausible for the U.S. economy. In light of this criticism, Wen (1998) shows that a laissez-faire one-sector RBC model with varying capital utilization can produce multiple equilibria and sunspot fluctuations under an empirically realistic level of increasing returns.[3] The intuition for this result is straightforward. Adding variable capital utilization raises the equilibrium elasticity of output with respect to labor hours so that lower aggregate returns-to-scale in production are needed to justify agents' self-fulfilling beliefs.

Since the condition which governs the magnitude of increasing returns needed for government spending shocks to generate procyclical macroeconomic effects is identical to that necessary for a laissez-faire one-sector RBC model to exhibit local indeterminacy, the large markup used by DHL is not empirically plausible either. To resolve this inconsistency, we

[3] Benhabib and Farmer (1996), Perli (1998), Weder (2000) and Harrison (2001) show that in a two-sector RBC model with fixed factor utilization and sector-specific externalities, the minimum degree of increasing returns needed for local indeterminacy is also less stringent.

follow Wen (1998) and incorporate variable capital utilization into the DHL model. Specifically, a more intensive utilization of capital is assumed to accelerate its rate of depreciation. In a symmetric equilibrium, the social technology displays a larger labor elasticity of output and a higher level of aggregate returns-to-scale (with respect to capital and labor inputs) than those in the original DHL economy. Consequently, varying capital utilization amplifies the effects of government spending shocks since it enriches the model's endogenous propagation mechanism by providing an additional margin to change output. In comparison with DHL, the required degree of increasing returns to produce procyclical responses of output, consumption, investment, employment, labor productivity and real wage to changes in government purchases can be substantially reduced. In particular, we show that while keeping saddle-path stability and other parameters fixed, DHL's quantitative results can be obtained at a level of returns-to-scale equal to 1.1, which lies within an empirically realistic range.

This paper is related to recent work of Benhabib and Wen (2002) who also study the business cycle effects of aggregate demand shocks in a one-sector RBC model with variable capital utilization and mild increasing returns. Our analysis differs from theirs in three aspects. First, we focus exclusively on the macroeconomic effects of exogenous government spending, whereas Benhabib and Wen also consider shocks to agents' consumption demand and self-fulfilling beliefs. Second, as in DHL, we maintain saddle-path stability throughout our analysis, and examine the impacts of temporary and permanent fiscal disturbances. By contrast, while Benhabib and Wen touch on cases with a unique equilibrium, they are mainly concerned with the parametrization that exhibits local indeterminacy and its ability to reconcile several "anomalies" which standard RBC models fail to predict for the U.S. economy. Third, we demonstrate analytically the connection between the necessary and sufficient conditions for the existence of stationary sunspot equilibria and the required level of increasing returns for a positive spending shock to be expansionary, whereas Benhabib and Wen's analyses are all quantitative.

The remainder of this paper is organized as follows. Section 2 presents a simplified version of the DHL model and examines the mechanism that accounts for the procyclical effects of exogenous government purchases. Section 3 investigates local dynamics around the steady state in the DHL economy. Section 4 analyzes the quantitative effects of temporary and permanent changes in government spending in the DHL model with variable capital utilization. Section 5 concludes.

2 The DHL Model

This section reproduces the DHL model with one simplifying modification regarding the market structure. To facilitate comparison, we maintain all other features in DHL's analysis and follow their notation as much as possible. In the DHL economy, there is an intermediate-good sector in which monopolistically competitive firms operate using capital and labor inputs. The number of these intermediate firms is endogenously pinned down by the zero-profit condition due to free entry and exit.[4] A single final good is then produced from the set of available intermediates in a perfectly competitive environment. Since the degree of aggregate returns-to-scale in production is equal to the price-to-cost markup ratio in this economy, we present the specification with perfect competition and productive externalities.[5] This simplification streamlines our exposition without affecting any result of the paper.

2.1 Firms

There is a continuum of identical competitive firms in the economy, with the total number normalized to one. Each firm produces output y_t according to a constant returns-to-scale technology

$$y_t = x_t k_t^{\alpha} h_t^{1-\alpha}, \quad 0 < \alpha < 1, \tag{1}$$

where k_t and h_t are capital and labor inputs, and x_t represents productive externalities that are taken as given by the individual firm. We postulate that externalities take the form

$$x_t = \bar{k}_t^{\alpha\chi} \bar{h}_t^{(1-\alpha)\chi}, \quad \chi \geq 0, \tag{2}$$

where $\bar{k}_t$ and $\bar{h}_t$ are the economy-wide average levels of capital and labor services, and χ denotes the degree of productive externalities. In a symmetric equilibrium, all firms take the same actions such that $k_t = \bar{k}_t$ and $h_t = \bar{h}_t$, for all t. As a result, (2) can be substituted into (1) to obtain the following aggregate production function that may display increasing returns:

$$y_t = \left[k_t^{\alpha} h_t^{1-\alpha} \right]^{(1+\chi)}. \tag{3}$$

[4] Hence, the productivity of an intermediate firm depends positively on the number of intermediate goods in use. This is referred to as "returns to specialization" in the literature.

[5] See Benhabib and Farmer (1994) for another alternative formulation that incorporates internal increasing returns at the intermediate-firm level in an imperfectly competitive market structure without free entry. As a result, this version of the model exhibits positive monopoly profits in equilibrium. Nevertheless, each of these three settings yields a reduced-form aggregate production function with the same form as (3).

When $\chi = 0$, the model collapses to a standard RBC formulation with aggregate constant returns-to-scale (zero markup).[6] As in DHL, we restrict our analysis to the case of $\alpha(1+\chi) < 1$ whereby the degree of increasing returns is not strong enough to generate endogenous growth. Under the assumption that factor markets are perfectly competitive, the first-order conditions for the firms' profit maximization problem are given by

$$r_t = \alpha \frac{y_t}{k_t}, \tag{4}$$

$$w_t = (1-\alpha) \frac{y_t}{h_t}, \tag{5}$$

where r_t is the capital rental rate and w_t is the real wage. Notice that the parameters α and $(1-\alpha)$ represent the capital and labor share of national income, respectively.

2.2 Households

The economy is populated by a unit measure of identical infinitely-lived households, each endowed with one unit of time. The representative household maximizes a discounted stream of expected utilities over its lifetime

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[\log c_t - \eta \frac{h_t^{1+\sigma}}{1+\sigma} \right], \;\; 0 < \beta < 1, \;\; \sigma \geq 0 \;\; \text{and} \;\; \eta > 0, \tag{6}$$

where E is the conditional expectations operator, β is the discount factor, c_t is consumption and σ denotes the inverse of the intertemporal elasticity of substitution for labor supply.[7] Households derive income from providing capital and labor services to firms. The only fundamental uncertainty present in the economy is an exogenous shock to government purchases.

The budget constraint faced by the representative household is

$$c_t + i_t + \tau_t = w_t h_t + r_t k_t, \tag{7}$$

where i_t is investment and τ_t is a lump-sum tax. The law of motion for the capital stock is

$$k_{t+1} = (1-\delta) k_t + i_t, \;\; k_0 \text{ given}, \tag{8}$$

[6] Notice that the level of aggregate returns-to-scale in our model, $1+\chi$, is equal to the markup ratio of price over the marginal cost in DHL's framework with zero monopoly profits. Therefore, in addition to representing the degree of productive externalities, χ can also be interpreted as the price-to-cost markup.

[7] The period utility function in DHL is given by $\log c_t + V(L - h_t)$, where L represents the time endowment and $V(\cdot)$ is a concave function. Here, for expositional simplicity, we normalize the time endowment to one unit, and specify a particular function form for V with $\sigma \equiv \left(\frac{-V''}{V'}\right)(L-h)$, where h denotes the steady-state labor hours. None of the results in the paper is sensitive to this modification.

where $\delta \in (0,1)$ is the capital depreciation rate.

The first-order conditions for the household's optimization problem are given by

$$\eta c_t h_t^{\sigma} = w_t, \tag{9}$$

$$\frac{1}{c_t} = \beta E_t[\frac{1}{c_{t+1}}(1 - \delta + r_{t+1})], \tag{10}$$

$$\lim_{t \to \infty} \beta^t \frac{k_{t+1}}{c_t} = 0, \tag{11}$$

where (9) is an intra-temporal condition that equates the household's marginal rate of substitution between consumption and leisure to the real wage. Equation (10) is the standard Euler equation for intertemporal consumption choices and (11) is the transversality condition.

2.3 Government

The government purchases goods and services, and balances its budget each period. Therefore, the period government budget constraint is $g_t = \tau_t$, where g_t represents government spending and it evolves according to

$$g_{t+1} = (1-\gamma)\, g + \gamma g_t + \varepsilon_{t+1}, \;\; g_0 \;\text{ given and }\; 0 < \gamma < 1, \tag{12}$$

where γ governs the persistence of the fiscal shock, g is the steady-state level of government purchases, and ε is an i.i.d. random error with mean zero and variance σ_{ε}^2. Finally, the aggregate resource constraint for the economy is given by

$$c_t + k_{t+1} - (1-\delta)k_t + g_t = y_t. \tag{13}$$

2.4 Effects of Government Spending

We focus on symmetric competitive equilibria which consist of a set of prices $\{r_t, w_t\}_{t=0}^{\infty}$ and quantities $\{c_t, h_t, k_{t+1}, g_{t+1}\}_{t=0}^{\infty}$ that satisfy the household's and firms' first-order conditions and the government budget constraint. Since government spending does not contribute to either production or household utility, an increase in g_t is equivalent to a pure resource drain that lowers the household's consumption through the negative wealth effect. This shifts out the labor supply curve, which causes labor hours to rise and real wage to fall. If the degree of productive externalities (or the price-to-cost markup) χ is high enough to yield a more-than-unity labor elasticity of output in the social technology, namely $(1-\alpha)(1+\chi)>1$ in (3), increased hours worked are associated with higher labor productivity. It follows that the labor demand curve will shift outward, reinforcing the initial employment effect of a higher government spending. Next, higher labor hours raise the household's projected income stream, thereby increasing its ability to consume. This in turn shifts the labor supply curve to the left. DHL (in their Figure 1*d*) show that when such a leftward shift makes the equilibrium wage-hours locus intersect the labor supply curve from below, a positive spending shock leads to a simultaneous expansion in output, consumption, investment, employment and real wage.[8]

The above mechanism can be understood by taking logarithms on both sides of the labor market equilibrium condition (5). The slope of the equilibrium wage-hours locus is given by $(1-\alpha)(1+\chi)-1$, while the slope of the labor supply curve is σ. Hence, the condition that is needed to produce DHL's main result is

$$(1-\alpha)(1+\chi)-1>\sigma, \tag{14}$$

which says that the equilibrium wage-hours locus is positively sloped and steeper than the labor supply curve. For the DHL parameterization in which the capital share of national income $\alpha=0.29$, and the inverse of the labor supply elasticity $\sigma=0$ (indivisible labor *à la* Hansen [1985]), the minimum level of productive externalities needed to satisfy condition (14) is $\chi_{\min}=0.408$. This explains why DHL adopt $\chi=0.5$ in their large-markup specification.

3 Dynamics Around the Steady State

This section shows that (14) is also a necessary condition for the DHL economy to exhibit local indeterminacy. Moreover, we demonstrate analytically that the minimum level of productive externalities needed for local indeterminacy is an increasing function of the household's rate of time preference and the capital depreciation rate. Our analysis begins by deriving the unique interior steady state of the DHL model. Let θ be the exogenously given ratio of government spending to output at the steady state (g/y). With this, it is straightforward to derive the steady-state expressions of all other variables. To examine the model's local stability properties, we completely shut down the spending shock, and then take log-linear

[8] If the equilibrium wage-hours locus intersects the labor supply curve from above, consumption will be countercyclical, which is inconsistent with the U.S. data.

approximations to the perfect-foresight version of the equilibrium conditions to obtain the following dynamical system:

$$\begin{bmatrix} \hat{k}_{t+1} \\ \hat{c}_{t+1} \end{bmatrix} = \mathbf{J}_1 \begin{bmatrix} \hat{k}_t \\ \hat{c}_t \end{bmatrix}, \quad \hat{k}_0 \text{ given,} \tag{15}$$

where hat variables denote percent deviations from their steady-state values, and $\mathbf{J}_1$ is the Jacobian matrix of partial derivatives of the transformed dynamical system. In the Appendix A, we derive the analytical expression for the Jacobian matrix $\mathbf{J}_1$.

3.1 Local Indeterminacy

The local stability of the steady state is determined by comparing the number of eigenvalues of $\mathbf{J}_1$ located inside the unit circle with the number of initial conditions (= 1). When both eigenvalues are inside the unit circle, the DHL model exhibits an indeterminate steady state and a continuum of stationary sunspot equilibria. Using the elements that make up the Jacobian matrix $\mathbf{J}_1$ derived in the Appendix A, its determinant is

$$\det(\mathbf{J}_1) = (1+\rho)\left\{1 + \frac{\chi(\rho+\delta)(1+\sigma)}{(\rho+\delta)(1+\sigma) - (1-\delta)\left[(1-\alpha)(1+\chi) - 1 - \sigma\right]}\right\}, \tag{16}$$

where $\rho \equiv \frac{1}{\beta} - 1$ is the household's rate of time preference. Local indeterminacy requires that both enigenvalues of $\mathbf{J}_1$ are less than one in modulus, which implies that $\det(\mathbf{J}_1)$ is also less than one in modulus. If $(1-\alpha)(1+\chi)-1-\sigma$ is negative, then the determinant of $\mathbf{J}_1$ is greater than one, indicating that the steady state is a saddle point. Therefore, a *necessary* condition for local indeterminacy is $(1-\alpha)(1+\chi) - 1 - \sigma > 0$, which is exactly the same as (14). In other words, the condition that governs the magnitude of increasing returns needed to generate positive relationship between government spending shocks and macroeconomic aggregates is also a necessary condition for the DHL economy to display equilibrium indeterminacy.

On the other hand, Benhabib and Farmer (1994) and Schmitt-Grohé (1997) show that for a laissez-faire one-sector RBC model to exhibit local indeterminacy, the equilibrium wage-hours locus needs to be upward sloping and cut the labor supply curve from below.[9] Together with the above examination, it follows that a one-sector RBC model under laissez-faire or with wasteful government purchases requires the same necessary condition to possess multiple

[9] While Benhabib and Farmer (1994) first derive this condition in a continuous-time RBC model, Schmitt-Grohé (1997, p. 124) obtains the same result in a discrete-time framework.

equilibria. Hence, our analysis brings together DHL and the recent sunspot literature that starts with the work of Benhabib and Farmer (1994) and Farmer and Guo (1994).

3.2 Annual versus Quarterly Parameterization

Although DHL's large-markup parameterization with $\chi = 0.5$ satisfies (14), which is also a necessary condition for local indeterminacy, saddle-path stability is maintained throughout their analysis. It turns out that this has to do with the annual parameterization adopted by DHL in which the discount factor β is set to be 0.96, and the capital deprecation rate δ equals 0.1. In this case, one eigenvalue of $\mathbf{J}_1$ lies outside and the other inside the unit circle. However, when the DHL model is calibrated at the quarterly frequency with $\beta = 0.99$ and $\delta = 0.025$, both eigenvalues of $\mathbf{J}_1$ are inside the unit circle, indicating that the steady state is indeterminate and becomes a sink.

To understand the general relationship between time frequency and local stability, we derive analytically the *necessary and sufficient* condition for local indeterminacy in the DHL model as follows:

$$\chi > \chi^* \equiv \max\left(\chi_1^*,\ \chi_4^*\right), \tag{17}$$

where the definitions of χ_1^* and χ_4^* are given in the Appendix B. Plugging DHL's parameter values of $\alpha = 0.29$, $\beta = 0.96, \delta = 0.1$, $\sigma = 0$ and the steady-state government spending to output ratio $\theta = 0.2$ into (17), we find that $\chi^* = 0.727$ for their annual specification, which is higher than $\chi_{\min} = 0.408$ that is needed to satisfy the necessary condition for multiple equilibria as in (14). On the other hand, keeping α, σ and θ fixed, $\chi^* = 0.472$ in the quarterly formulation where $\beta = 0.99$ and $\delta = 0.025$. This explains why DHL's large-markup parameterization displays saddle-path stability at the annual frequency ($\chi = 0.5 < 0.727$), but exhibits local indeterminacy at the quarterly frequency ($\chi = 0.5 > 0.472$).

Using (17), it can be shown that the minimum level of increasing returns needed for the existence of stationary sunspot equilibria is positively related to the household's rate of time preference and the capital depreciation rate, i.e., $\frac{\partial \chi^*}{\partial \rho} > 0$ and $\frac{\partial \chi^*}{\partial \delta} > 0$.[10] The intuition for this result is tantamount to understanding how indeterminacy arises in the first place. When agents

[10] Using (17), we also find that $\frac{\partial \chi^*}{\partial \theta}$ is either zero (if $\chi^* = \chi_1^*$) or positive (if $\chi^* = \chi_4^*$). In the latter case, when the government expenditure to output ratio rises, there will be less resources available for the purposes of consumption and investment. Therefore, higher increasing returns are needed to validate agents' optimism or pessimism. It turns out that $\chi^* = \chi_1^*$ in DHL's annual parameterization and its quarterly counterpart. This implies that in our analysis, the share of government spending has no effect on the required magnitude of productive externalities for local indeterminacy.

anticipate a higher future rate of return on capital, they will increase today's investment, thus raising next period's capital stock. If the external effects in the firms' production processes are strong enough, the rate of return on capital will rise because of a higher labor supply. As the time step becomes smaller, e.g., moving from the annual to quarterly parameterization, labor is drawn more easily out of leisure to help fulfill agents' optimistic expectations.[11] Therefore, indeterminacy is more likely to occur when the DHL model is analyzed under the quarterly parameterization.

4 Variable Capital Utilization

Recent empirical estimates reported by Burnside (1996) and Basu and Fernald (1997) have raised doubts on the empirical plausibility of the minimum degree of increasing returns needed for the Benhabib-Farmer-Guo one-sector RBC formulation to exhibit local indeterminacy. In light of this criticism, Wen (1998) finds that a laissez-faire one-sector RBC model with variable capital utilization can produce multiple equilibria under an empirically realistic level of increasing returns. Since the condition which governs the required magnitude of increasing returns for government spending shocks to generate procyclical macroeconomic effects is identical to that necessary for a laissez-faire one-sector RBC model to display equilibrium indeterminacy, the large markup used by DHL is not empirically plausible either.[12] In this section, we follow Wen's work and show that adding varying capital utilization to the DHL economy substantially reduces the level of productive externalities needed to obtain their results.

4.1 The Economy

We incorporate variable capital utilization *à la* Greenwood, Hercowitz and Huffman (1988) and Wen (1998) into the DHL model. In this case, the individual firm's production function becomes

$$y_t = z_t (u_t k_t)^{\alpha} h_t^{1-\alpha}, \ 0 < \alpha < 1, \tag{18}$$

[11] This is related to the importance of the discount factor and the capital depreciation rate in optimal growth models that display complicated dynamics (see, for example, Mitra [1998], and Baierl, Nishimura and Yano [1998]).

[12] This appears to be the main reason why DHL do not interpret choosing their model's parameter values as a calibration exercise (p. 246).

where u_t denotes the rate of capital utilization that is endogenously determined by the representative household, and the productive externality z_t is specified as a function of economy-wide average levels of utilized capital and labor inputs

$$z_t = (\bar{u}_t \bar{k}_t)^{\alpha\chi} \bar{h}_t^{(1-\alpha)\chi}, \quad \chi \geq 0. \tag{19}$$

In a symmetric equilibrium where $k_t = \bar{k}_t$, $h_t = \bar{h}_t$ and $u_t = \bar{u}_t$, the aggregate production function is

$$y_t = \left[(u_t k_t)^{\alpha} \ h_t^{1-\alpha} \right]^{(1+\chi)}. \tag{20}$$

Since factor markets are assumed to be perfectly competitive, the equilibrium capital rental rate r_t, and real wage w_t are given by (4) and (5), respectively.

On the other hand, the representative household maximizes its expected lifetime utilities (6), subject to

$$c_t + k_{t+1} - (1 - \delta_t) k_t + \tau_t = w_t h_t + r_t k_t, \tag{21}$$

where $\delta_t \in (0, 1)$ represents the time-varying capital depreciation rate. We postulate that δ_t takes the form

$$\delta_t = \frac{1}{\phi} u_t^{\phi}, \quad \phi > 1, \tag{22}$$

which means that more intensive capital utilization accelerates its rate of depreciation. The government's problem remains the same with g_t following the stochastic process as in (12).

The first-order conditions for the household's optimization problem are (9) that governs the labor supply decision, (10) that determines the intertemporal trade-off between consumption goods at different dates, (11) that is the transversality condition, and

$$\alpha \frac{y_t}{u_t} = u_t^{\phi-1} k_t, \tag{23}$$

which equates the marginal gain (more output) and marginal loss (higher capital depreciation) of a change in u_t. Rearranging (23) shows that the equilibrium rate of capital utilization is an increasing function of the marginal product of capital $\alpha \frac{y_t}{k_t}$ and hours worked h_t. Finally, combining (20) and (23) yields the following reduced-form social technology as a function of capital and labor inputs:

$$y_t = \alpha^{\frac{\alpha(1+\chi)}{\phi-\alpha(1+\chi)}} k_t^{\frac{\alpha(1+\chi)(\phi-1)}{\phi-\alpha(1+\chi)}} h_t^{\frac{\phi(1-\alpha)(1+\chi)}{\phi-\alpha(1+\chi)}}, \tag{24}$$

where $\frac{\alpha(1+\chi)(\phi-1)}{\phi-\alpha(1+\chi)} < 1$ to guarantee the existence of an interior steady state since this condition implies diminishing marginal product of capital. Notice that in comparison with (3), (24) displays a larger equilibrium labor elasticity of output for all $\chi \geq 0$, and a higher level of aggregate returns-to-scale with respect to k_t and h_t.

Following the discussion described in section 2.4, the condition needed generate procyclical responses of macroeconomic aggregates to government spending shocks in the current setting is

$$\frac{\phi(1-\alpha)(1+\chi)}{\phi-\alpha(1+\chi)} - 1 > \sigma. \tag{25}$$

Notice that (14) and (25) have exactly the same interpretation, that is, the equilibrium wage-hours locus is upward sloping and steeper than the labor supply curve. However, the required levels of χ that satisfy these conditions are quite different. Specifically, for the DHL parameterization in which $\alpha = 0.29$, $\sigma = 0$, together with $\phi = 1.417$ (based on $\beta = 0.96$ and the steady-state capital depreciation rate $\delta = 0.1$), the minimum level of productive externalities needed for (25) to hold is $\chi_{\min} = 0.093$, which is substantially lower than 0.408 that is required to satisfy (14) under fixed capital utilization. The intuition for this result is straightforward. Since variable capital utilization raises the equilibrium elasticity of output with respect to hours worked, it amplifies the impact of fiscal disturbances because it enriches the model's endogenous propagation mechanism by providing an additional channel to change output. As a result, only mild increasing returns are needed to generate an endogenous increase in labor productivity when the economy is subject to a positive spending shock.

Moreover, following the same procedure as in section 3.1, it can be shown that (25) is also a necessary condition for the DHL model with variable capital utilization to exhibit local indeterminacy. On the other hand, Wen (1998, p. 16) shows that the same condition is necessary for a laissez-faire one-sector RBC model with varying capital utilization to possess an indeterminate steady state. Therefore, our earlier finding that a one-sector RBC model under laissez-faire or with wasteful government spending requires the same necessary condition for local indeterminacy is robust to the addition of variable capital utilization.

4.2 Quantitative Analysis

This subsection undertakes a quantitative investigation on the effects of temporary and permanent changes in government spending in the DHL economy with variable capital utilization. We first derive the steady-state interest rate, hours worked and capital stock as follows:

$$r = \frac{1}{\beta} - (1 - \delta), \tag{26}$$

$$h = \left\{ \frac{(1-\alpha)\, r}{\eta \left[(1-\theta)\, r - \alpha\delta \right]} \right\}^{\frac{1}{1+\sigma}}, \tag{27}$$

$$k = \left[\alpha h^{(1-\alpha)(1+\chi)} r^{\frac{\alpha(1+\chi)-\phi}{\phi}} \right]^{\frac{1}{1-\alpha(1+\chi)}}, \tag{28}$$

where δ and θ are the (exogenously given) capital depreciation rate and government spending to output ratio at the steady state. Given (26)-(28), the steady-state expressions of all the remaining variables can then be easily derived. Next, we log-linearize the equilibrium conditions around the steady state to obtain the following stochastic dynamical system:

$$\begin{bmatrix} \hat{k}_{t+1} \\ \hat{c}_{t+1} \\ \hat{g}_{t+1} \end{bmatrix} = \mathbf{J}_2 \begin{bmatrix} \hat{k}_t \\ \hat{c}_t \\ \hat{g}_t \end{bmatrix} + \begin{bmatrix} \hat{k}_{t+1} - E_t\left(\hat{k}_{t+1}\right) \\ \hat{c}_{t+1} - E_t\left(\hat{c}_{t+1}\right) \\ \varepsilon_{t+1}/g \end{bmatrix}, \quad \hat{k}_0 \text{ and } \hat{g}_0 \text{ given}, \tag{29}$$

where $\mathbf{J}_2$ is the Jacobian matrix. To maintain comparability with DHL and many other existing studies, we restrict our attention to cases with a unique equilibrium.

For the quantitative analysis, we adopt the DHL parameterization with $\alpha = 0.29$, $\sigma = 0$, $\theta = 0.2$, $\beta = 0.96$, $\delta = 0.1$ and $\phi = 1.417$. In addition, the preference parameter η is set to be 3.78 so that the household spends about 30% of its time endowment in working at the steady state.[13] Regarding the degree of productive externalities, we consider two cases, (i) constant returns-to-scale or zero markup, $\chi = 0$, and (ii) mild increasing returns or small markup, $\chi = 0.1$, which satisfies condition (25) and can be characterized as empirically plausible *vis-à-vis* recent empirical findings of Burnside (1996) and Basu and Fernald (1997) for the U.S. economy.[14] In both formulations, two eigenvalues of $\mathbf{J}_2$ are located inside and one outside the

[13] Salyer (2001) shows that in the case of indivisible labor with $\sigma = 0$, there is no need to calibrate the preference parameter η since it does not appear in the Jacobian matrix $\mathbf{J}_2$, thus has no influence on the model's equilibrium characteristics.

[14] For example, the point estimate of valued-added returns-to-sacle that Burnside (1996) reports is 1.12 (standard error = 0.05). The largest estimate of aggregate returns-to-scale that Basu and Fernald (1997) obtain, after correcting reallocation of inputs across industries, is 1.03 (standard error = 0.18).

unit circle, hence the model exhibits saddle-path stability.[15] To solve for the unique rational expectations solution to (29), we iterate the unstable root of $\mathbf{J}_2$ backwards to obtain the stable branch of the saddle path, which expresses $\hat{c}_t$ as a linear function of $\hat{k}_t$ and $\hat{g}_t$.[16] Notice that the quantitative results reported below for the low-externality case ($\chi = 0.1$) are qualitatively identical to those in the original DHL economy with fixed capital utilization and large markup of $\chi = 0.5$.

4.2.1 A Permanent Change in Government Spending

Table 1 presents the effects on the steady-state labor supply, capital stock (investment), output and consumption (labor productivity and real wage) of a one-percent permanent increase in the share of government purchases, i.e., raising θ to 0.21.

Table 1: Effects of A Permanent Government Spending Shock[17]

	h	k, i	y	c, $y/h, w$
$\chi = 0$	1.71%	1.71%	1.71%	no change
$\chi = 0.1$	1.71%	1.96%	1.96%	0.25%

As is well known in the literature, a permanent increase in the share of government spending raises the steady-state labor supply because of a negative wealth effect. In addition, this response is independent of the externality parameter χ since it does not enter the steady-state expression of hours worked (see equation 27).

In the standard RBC model with constant returns-to-scale ($\chi = 0$), a higher labor supply shifts up the marginal product schedule for capital, thus increasing investment and capital accumulation along the transition path. In the long run, there will be higher levels of capital and labor inputs, but the capital to labor ratio remains unchanged (see Baxter and King [1993]). Therefore, capital, investment and output all rise by the same percentage as labor hours at the steady state. On the other hand, a constant capital to labor ratio in the long run implies that the relative factor price ratio, w/r, is fixed. Since r is invariant to movements in

[15] The required degree of externalities that satisfies the necessary and sufficient conditions for multiple equilibria is 0.143. As is discussed in section 3.2, this value is higher than that (= 0.093) needed for (25) to hold.

[16] The model here continues to display equilibrium uniqueness under both levels of χ at the quarterly frequency when $\beta = 0.99$ and $\delta = 0.025$.

[17] Since $i = \delta k$ at the steady state, investment and capital stock exhibit the same percentage responses. Moreover, in our parameterization with indivisible labor ($\sigma = 0$), consumption is proportional to labor productivity and real wage (see equations 9 and 5). Hence, c, y/h and w are listed in the same column.

θ (see equation 26), the real wage, consumption and labor productivity do not change at the steady state.

As in the above benchmark case, a higher θ raises the long-run labor supply, capital stock and aggregate output in the specification with $\chi = 0.1$. However, the quantitative results here are quite different. In particular, combined with variable capital utilization, this level of productive externalities is sufficiently strong to generate an endogenous increase in labor productivity because under this parameterization, the labor elasticity of output, $\frac{\phi(1-\alpha)(1+\chi)}{\phi-\alpha(1+\chi)}$ in the social technology (24), is greater than one. It follows that both consumption and real wage will also rise in the long run. Moreover, for all admissible levels of productive externalities, a higher steady-state labor supply leads to a more than proportional increase in the capital stock (see equation 28) and investment. Finally, to maintain the constant marginal product of capital ($= r$) at the steady state, output will be increased by the same percentage as the capital stock in the long run. Notice that for any given $\chi \geq 0$, the impact on the steady-state output, employment, capital stock, and investment to a permanent government spending shock are qualitatively identical in DHL and the current setting.

4.2.2 A Temporary Change in Government Spending

Here, we examine the macroeconomic effects of a one-time positive innovation to government purchases, described by (12), equal to 1 percent of initial (steady-state) output. As in DHL, we study two possibilities: (i) low persistence, $\gamma = 0.6$, and (ii) high persistence, $\gamma = 0.95$, of government spending shocks for each of the two returns-to-scale considered.[18]

Figure 1 plots the impulse response functions in the low-persistence ($\gamma = 0.6$) case.[19] When there is no externality ($\chi = 0$), a temporary increase in government spending, financed by lump-sum taxation, lowers the amount of net resources available to the economy. Hence, the representative household will decrease its consumption expenditure at the impact period. Since households desire to smooth consumption over time, they will also lower investment in the face of a transitory fiscal disturbance. As a result, consumption and investment both fall on impact and then return monotonically towards the steady state. In addition, the negative wealth effect shifts the labor supply curve outward, raising employment and output, and

[18] The value of $\gamma = 0.95$ is taken from Christiano and Eichenbaum (1992) based on their GMM estimation on the U.S. data, 1955:3-1983:4. For comparison purposes, DHL also consider the case with $\gamma = 0.6$ in which the effects of a temporary spending shock are relatively short lived.

[19] Given our calibration of indivisible labor, consumption, labor productivity and real wage exhibit the same percentage responses.

reducing the real wage. Finally, a higher labor supply leads to more intensive utilization of capital, which both contribute to an immediate rise in the interest rate (see equation 23). In sum, varying capital utilization does not affect the familiar qualitative results of a standard RBC model in response to a transient government spending shock with low persistence (see Aiyagari, Christiano and Eichenbaum [1992], and Baxter and King [1993]).

In the low-externality formulation with $\chi = 0.1$, the output response is about twice as large as that in the above benchmark case because increasing returns strengthens the expansionary effect of the spending shock. Moreover, the outward shift of the labor supply curve is now more than offset by an endogenous increase in labor productivity coming from externalities and variable capital utilization.[20] As a result, consumption and real wage both rise on impact, peak after three periods and then fall back gradually to the steady state. On the other hand, the effect of a higher labor supply dominates the increase in the interest rate, resulting in a sharp positive initial response of investment (see DHL's equation 35). This stimulates capital accumulation, which in turn causes the interest rate to display a cyclical convergent pattern towards the steady state. Notice that falling consumption is accompanied by the interest rate below its steady-state value.

Figure 2 presents the dynamic responses of macroeconomic aggregates in the high-persistence ($\gamma = 0.95$) case. Compared to the low-persistence configuration, the negative wealth effect of a positive transient shock to government spending is stronger, as is shown by Aiyagari, Christiano and Eichenbaum (1992). Consequently, employment rises more than before, thereby generating a higher and more persistent output response for both returns-to-scale that we examine.

When the model exhibits constant returns-to-scale ($\chi = 0$), a more responsive labor supply leads to a stronger increase in the interest rate, and a larger fall of consumption and real wage, and a *rise* in investment. To understand the latter result, consider the effects of a permanent increase in government purchases. In this case, as we have discussed in the previous subsection, the capital stock will be higher in the long run. Therefore, investment must rise on impact to build up for the higher steady-state capital stock. Similar reasoning can be applied to the current specification with highly persistent spending shocks. Since capital and labor are

[20]In other words, mild increasing returns-to-scale together with varying capital utilization raise the labor elasticity of output to be above one (see equation 24). Hence, in equilibrium, the marginal product of labor is an increasing function of employment.

complements in the production function, a higher employment leads to an initial increase in investment after a positive fiscal disturbance (see Edelberg, Eichenbaum and Fisher [1999]).

On the other hand, the impulse responses in the low-externality case ($\chi = 0.1$) are qualitatively identical to those when $\gamma = 0.6$, i.e., consumption, investment, labor productivity, real wage and the interest rate all rise immediately following the fiscal disturbance (see Benhabib and Wen [2002], Figure 3). In fact, even when the shock is purely transitory ($\gamma = 0$), a transient increase in government purchases raises the real wage, and crowd in consumption and investment due to the endogenous increase in labor productivity. This implies that in a one-sector RBC model with variable capital utilization, saddle-path stability, and a degree of productive externalities χ that satisfies (25), an increase in government purchases can generate expansionary macroeconomic effects, regardless of the persistence of the underlying spending shock.[21] By contrast, DHL obtain the same finding with $\chi = 0.5$ when capital utilization is constant and the relevant condition is (14).

Overall, as in DHL, our result that a one-sector RBC model can produce procyclical movements in macroeconomic aggregates to a permanent or temporary government spending shock is driven by an endogenous, positive response of labor productivity. In the parameterization with logarithmic utility in consumption and indivisible labor, DHL find that such response requires a large markup around 1.5. On the contrary, this subsection shows that, *ceteris paribus*, a mild increasing returns of 1.1 is strong enough to yield the same outcome because variable capital utilization amplifies the effects of a spending shock by providing an additional margin to change output.

5 Conclusion

DHL have shown that a one-sector RBC model with sufficiently strong increasing returns is capable of producing qualitatively realistic business cycles driven solely by changes in government purchases. Our analysis illustrates that their finding is closely related to a growing literature that explores indeterminacy and sunspots in the RBC framework under laissez-faire (see Benhabib and Farmer [1999] for a survey). Specifically, the condition which governs the minimum

[21] With an externality in the indeterminacy region, Benhabib and Wen (2002) show that shocks to government purchases, or to agents' preferences or self-fulfilling beliefs also generate positive correlations among macroeconomic aggregates because of the similar endogenous propagation mechanism that we have discussed. Moreover, in this case, output, investment and labor hours display hump-shaped, trend reverting responses to persistent demand shocks that mimic those observed in the U.S. data.

degree of increasing returns needed for government spending shocks to generate positive co-movements among macroeconomic aggregates is identical to that necessary for a laissez-faire one-sector RBC model to exhibit local indeterminacy. In addition, not only does variable capital utilization substantially reduce the required magnitude of aggregate returns-to-scale for the existence of stationary equilibria in a one-sector RBC economy under laissez-faire, it also enhances the endogenous propagation mechanism in the DHL model so that their results can be obtained at an empirically plausible level of increasing returns. Hence, this paper makes a parallel contribution to the demand-driven business cycle literature as Wen (1998) does to the RBC-based indeterminacy and sunspot literature.

Finally, regarding possible extensions of our analysis, it would be worthwhile to examine other frameworks that can generate an endogenous propagation mechanism similar to that we have studies in this paper. For example, incorporating public capital into the production function (as in Futagami, Morita and Shibata [1993], Glomm and Ravikumar [1994], and Turnovsky [1997]), or introducing a second sector with human capital (as in Lucas [1988], and Ladron-de-Guevara, Ortigueira and Santos [1997]) will allow us to further contrast our results with the fiscal policy literature using neoclassical growth models, and to show how they compare to a standard RBC model driven by technology shocks. We plan to pursue this project in the near future.

A Appendix

This appendix presents the equations that are used to study the model's perfect-foresight dynamics whereby government spending is nonrandom and equals to its mean, i.e., $g_t = g$, for all t. In this case, the first-order conditions that characterize the equilibrium are

$$k_{t+1} = k_t^{\alpha(1+\chi)} h_t^{(1-\alpha)(1+\chi)} + (1-\delta)k_t - c_t - g, \; k_0 \text{ given}, \tag{A.1}$$

$$\eta c_t h_t^{\sigma} = \underbrace{(1-\alpha)\, k_t^{\alpha(1+\chi)} h_t^{(1-\alpha)(1+\chi)-1}}_{w_t}, \tag{A.2}$$

$$\frac{1}{c_t} = \frac{\beta}{c_{t+1}} \left[1 - \delta + \underbrace{\alpha k_{t+1}^{\alpha(1+\chi)-1} h_{t+1}^{(1-\alpha)(1+\chi)}}_{r_{t+1}} \right], \tag{A.3}$$

Using equation (A.2), h_t can be expressed as a function of k_t and c_t where

$$h_t = \left[\frac{\eta}{(1-\alpha)} \frac{c_t}{k_t^{\alpha(1+\chi)}} \right]^{\frac{1}{(1-\alpha)(1+\chi)-1-\sigma}}. \tag{A.4}$$

Substituting (A.4) into (A.1) and (A.3), we obtain a first-order non-linear dynamical system in terms of k_t and c_t.

Next, it is straightforward to show that the model exhibits a unique interior steady state (k, c). In the neighborhood of this steady state, the equilibrium conditions can approximated by the following log-linear dynamical system:

$$\begin{bmatrix} \hat{k}_{t+1} \\ \hat{c}_{t+1} \end{bmatrix} = \underbrace{\begin{bmatrix} \lambda_1 & \lambda_2 \\ \frac{\lambda_1 \lambda_3}{\lambda_4} & \frac{1+\lambda_2\lambda_3}{\lambda_4} \end{bmatrix}}_{\mathbf{J}_1} \begin{bmatrix} \hat{k}_t \\ \hat{c}_t \end{bmatrix}. \tag{A.5}$$

The elements that make up the Jacobian matrix $\mathbf{J}_1$ are given by

$$\lambda_1 = 1 - \delta - \frac{(1+\sigma)(\rho+\delta)(1+\chi)}{(1-\alpha)(1+\chi) - 1 - \sigma}, \tag{A.6}$$

$$\lambda_2 = \delta + \frac{(\rho+\delta)\left[\theta(1-\alpha)(1+\chi) + (1-\theta)(1+\sigma)\right]}{\alpha[(1-\alpha)(1+\chi) - 1 - \sigma]}, \tag{A.7}$$

$$\lambda_3 = -\frac{(\rho+\delta)\left[\alpha(1+\chi)(1+\sigma) + (1-\alpha)(1+\chi) - 1 - \sigma\right]}{(1+\rho)[(1-\alpha)(1+\chi) - 1 - \sigma]}, \tag{A.8}$$

$$\lambda_4 = 1 - \frac{(\rho+\delta)(1-\alpha)(1+\chi)}{(1+\rho)[(1-\alpha)(1+\chi) - 1 - \sigma]}, \tag{A.9}$$

where $\rho \equiv \frac{1}{\beta} - 1$ is the household's rate of time preference, and θ denotes the steady-state government spending to output ratio.

B Appendix

This appendix derives the necessary and sufficient conditions for local indeterminacy in the DHL model. Local indeterminacy requires that both eigenvalues of the Jacobian matrix $\mathbf{J}_1$ are less than one in modulus. This is the case if and only if

$$-1 < \det(\mathbf{J}_1) < 1 \text{ and } -(1+\det(\mathbf{J}_1)) < tr(\mathbf{J}_1) < 1 + \det(\mathbf{J}_1), \tag{B.1}$$

where the determinant of $\mathbf{J}_1$ is given by (16), and the trace of $\mathbf{J}_1$ can be expressed as follows:

$$tr(\mathbf{J}_1) = 1 + \det(\mathbf{J}_1) + m, \tag{B.2}$$

$$m \equiv \frac{\frac{1}{\alpha}\left\{(\rho+\delta)(1+\sigma)[1-\alpha(1+\chi)][\alpha\delta-(\rho+\delta)(1-\theta)]-\theta(1-\alpha)(1+\chi)(\rho+\delta)^2\right\}}{(\rho+\delta)(1+\sigma)-(1-\delta)[(1-\alpha)(1+\chi)-1-\sigma]}.$$

A necessary condition for $\det(\mathbf{J}_1) < 1$ is

$$(\rho+\delta)(1+\sigma)-(1-\delta)[(1-\alpha)(1+\chi)-1-\sigma] < 0. \tag{B.3}$$

Notice that since $\rho > 0$, $\sigma \geq 0$ and $\delta \in (0,1)$, (B.3) implies condition (14) in the text. Given (B.3), $\det(\mathbf{J}_1) > -1$ if and only if

$$\chi > \chi_1^* \equiv \frac{(2+\rho)[(1+\sigma)(1+\rho)-(1-\alpha)(1-\delta)]}{(1-\alpha)(1-\delta)(2+\rho)-(\rho+\delta)(1+\sigma)(1+\rho)}, \tag{B.4}$$

which is independent of θ. Moreover, given (B.3), $\det(\mathbf{J}_1) < 1$ if and only if

$$\chi < \chi_2^* \equiv \frac{\overbrace{\rho[(1+\rho)(1+\sigma)-(1-\alpha)(1-\delta)]}^{\text{positive}}}{\underbrace{\rho(1-\alpha)(1-\delta)-(\rho+\delta)(1+\sigma)(1+\rho)}_{\text{negative}}}. \tag{B.5}$$

Since χ_2^* is negative, (B.5) is not binding. Therefore, $-1 < \det(\mathbf{J}_1) < 1$ if and only if $\chi > \chi_1^*$, i.e., when (B.4) holds.

If (B.3) holds, then $tr(\mathbf{J}_1) < 1 + \det(\mathbf{J}_1)$ if and only if

$$\chi < \chi_3^* \equiv \frac{(1-\alpha)\left\{(1+\sigma)\left[\alpha\delta - (\rho+\delta)(1-\theta)\right] - \theta(\rho+\delta)\right\}}{\alpha(1+\sigma)\left[\alpha\delta - (\rho+\delta)(1-\theta)\right] + \theta(1-\alpha)(\rho+\delta)}. \tag{B.6}$$

Next, given (B.3), $tr(\mathbf{J}_1) > -(1+\det(\mathbf{J}_1)$ if and only if

$$\chi > \chi_4^* \equiv \frac{Numerator}{Denominator}, \tag{B.7}$$

where

$$\begin{aligned} Numerator &= (\rho+\delta)(1+\sigma)\left[2\alpha(2+\rho) + \alpha(1-\alpha)\delta - (1-\alpha)(\rho+\delta)(1-\theta)\right] \\ &\quad +2\alpha(1-\delta)(2+\rho)(\alpha+\sigma) - \theta(1-\alpha)(\rho+\delta)^2, \end{aligned}$$

and

$$\begin{aligned} Denominator &= \alpha(\rho+\delta)(1+\sigma)\left[\alpha\delta - (\rho+\delta)(1-\theta) - 2(1+\rho)\right] \\ &\quad +2\alpha(1-\alpha)(1-\delta)(2+\rho) + \theta(1-\alpha)(\rho+\delta)^2. \end{aligned}$$

(B.6) is not binding since we are looking for the minimum level of χ that results in local indeterminacy. Therefore, $-(1+\det(\mathbf{J}_1)) < tr(\mathbf{J}_1) < 1+\det(\mathbf{J}_1)$ if and only if $\chi > \chi_4^*$, i.e., when (B.7) holds.

Finally, we cannot be certain whether (B.4) or (B.7) is more demanding because $\alpha, \delta, \theta, \rho$ and σ enter them in a rather complicated way. As a result, the necessary and sufficient condition for both eigenvalues of the Jacobian matrix $\mathbf{J}_1$ to be inside the unit circle is

$$\chi > \max(\chi_1^*,\ \chi_4^*), \tag{B.8}$$

which is (17) in the text.

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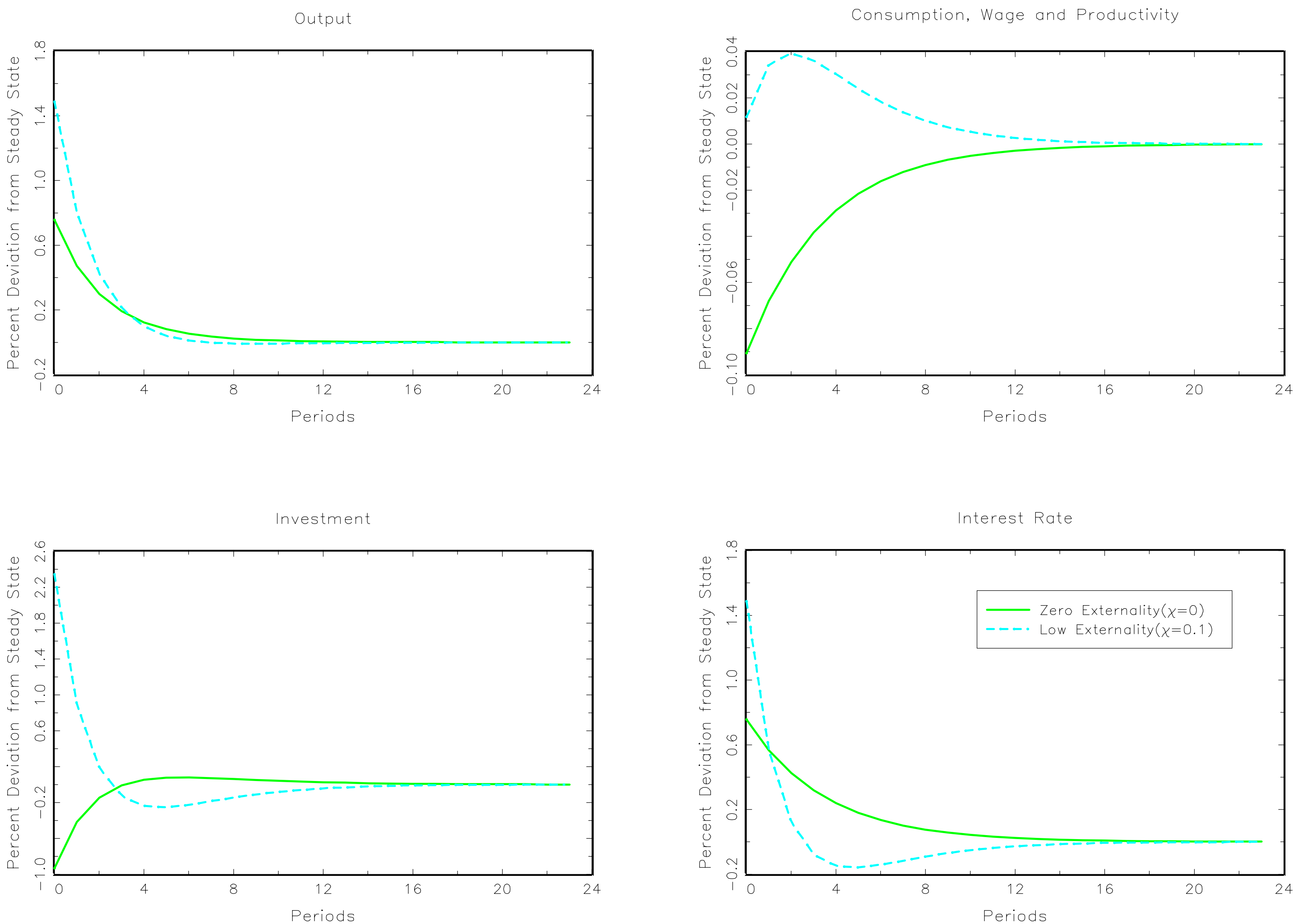

Figure 1. Response to a Temporary Shock: Low Persistence(γ=0.6)

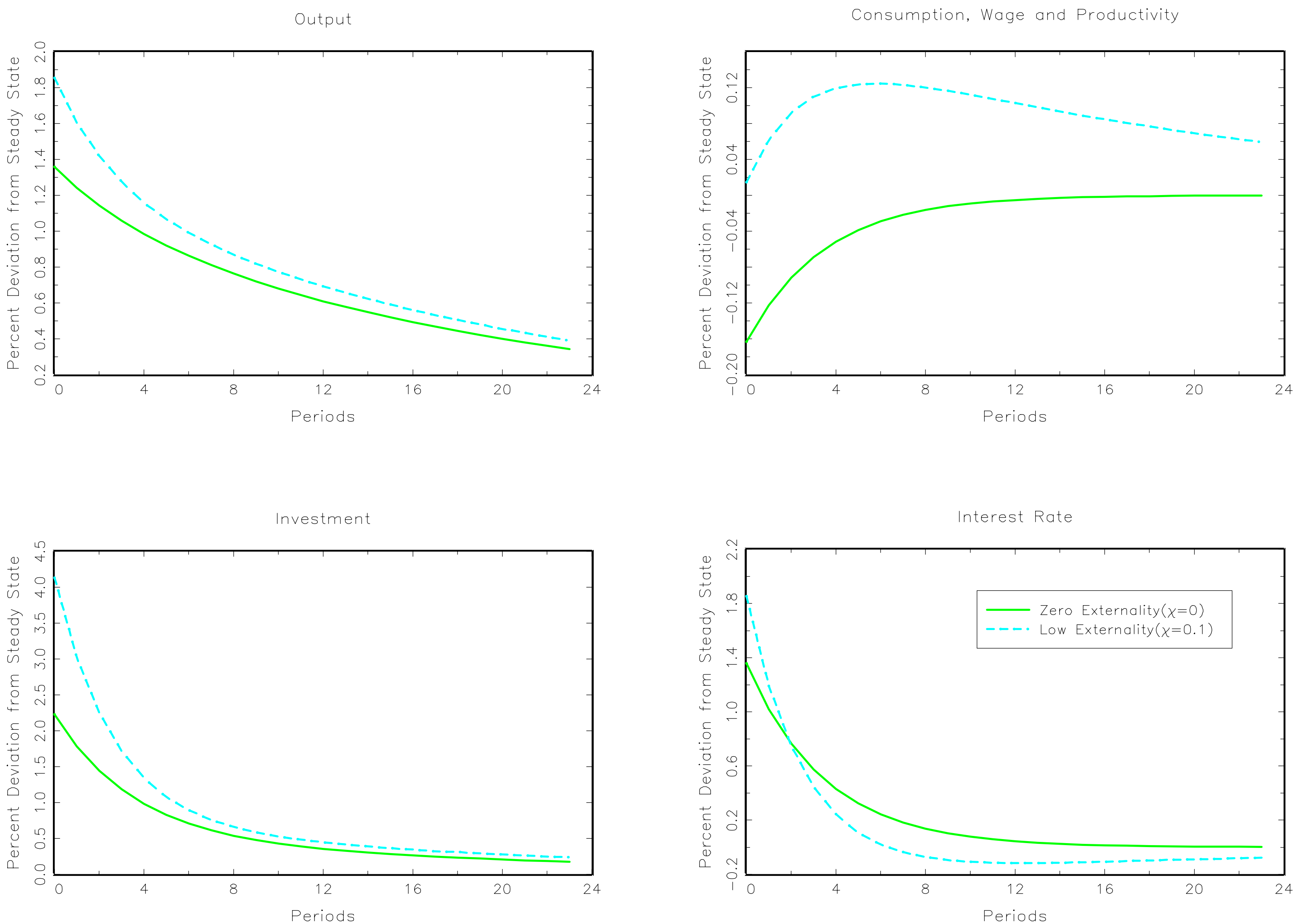

Figure 2. Response to a Temporary Shock: High Persistence(γ=0.95)