Indeterminacy with Capital Utilization and Sector-Specific Externalities

Jang-Ting Guo[*]
University of California, Riverside

Sharon G. Harrison[†]
Barnard College, Columbia University

September 15, 2000

Abstract

We show that the minimum level of increasing returns-to-scale needed for indeterminacy in a two-sector real business cycle model with variable capital utilization and sector-specific externalities is extremely small. Moreover, this value is lower than that required in several of our model's predecessors, and is quite easily within the range of empirical plausibility.

Keywords: Indeterminacy, Capital Utilization, Sector-Specific Externalities.

JEL Classification: E30, E32.

[*]Corresponding Author: Department of Economics, 1150 University Ave., University of California, Riverside, CA, 92521, (909) 787-5037, ext. 1588, Fax: (909) 787-5685, e-mail: guojt@mail.ucr.edu.

[†] Barnard College, Department of Economics, 3009 Broadway, New York, NY 10027, (212) 854-3333, Fax: (212) 854-8947, e-mail: sh411@columbia.edu.

1 Introduction

Starting with the work of Benhabib and Farmer (1994) and Farmer and Guo (1994), there is now an extensive macroeconomic literature that explores indeterminacy and sunspots in the real business cycle (RBC) model.[1] However, the degree of increasing returns-to-scale needed to generate multiple equilibria in the Benhabib-Farmer-Guo one-sector formulation is too high to be empirically plausible (see Burnside, 1996, and Basu and Fernald, 1997). In light of this criticism, recent theoretical developments have shown that in a one-sector RBC model with variable capital utilization (Wen, 1998) or in a two-sector RBC model with sector-specific externalities (Benhabib and Farmer, 1996, and Perli, 1998), the minimum level of increasing returns needed for indeterminacy is much less stringent.[2]

In this paper, we extend the Benhabib-Farmer-Wen analysis by incorporating variable capital utilization and sector-specific externalities into a two-sector RBC model with distinct consumption and investment goods. This modification allows us to further identify model features and parameters that govern the region of indeterminacy. In a calibrated version of our model with logarithmic utility and indivisible labor, we find that indeterminacy arises with returns-to-scale of about 1.02, which is lower than that required in the above-mentioned existing studies. Moreover, this value can easily be characterized as empirically plausible vis-à-vis the aggregation-corrected direct value-added estimate (1.03) for the total private U.S. economy reported in Basu and Fernald (1997).

2 The Economy

2.1 Firms

Our model incorporates variable capital utilization into the discrete-time version of the two-sector RBC framework as in Benhabib and Farmer (1996). In the consumption sector, output is produced by competitive firms using the following technology:

$$Y_{ct} = A_t \left(u_t K_{ct}\right)^{\alpha} L_{ct}^{1-\alpha}; \quad 0 < \alpha < 1; \tag{1}$$

where u_t denotes the rate of capital utilization that is endogenously determined by the representative household, and K_{ct} and L_{ct} are the capital and labor inputs used in the production

[1] See Benhabib and Farmer (1999) for an excellent survey. With the noted exceptions of Benhabib and Nishimura (1998), and Benhabib, Meng and Nishimura (2000), most studies in this literature postulate constant returns-to-scale at the individual firm level. We also maintain this assumption throughout our analysis.

[2] The two sectors in Benhabib and Farmer (1996) refer to consumption and investment. On the other hand, Perli (1998) studies market and home production sectors.

of consumption goods. In addition, A_t represents productive externalities that each individual firm takes as given, and is specified as

$$A_t = \left[\left(\bar{u}_t \bar{K}_{ct} \right)^{\alpha} \bar{L}_{ct}^{1-\alpha} \right]^{\theta}; \quad \theta > 0; \tag{2}$$

where $\bar{u}_t \bar{K}_{ct}$ and $\bar{L}_{ct}$ denote the economy-wide average capital and labor services used in producing the consumption good, and θ measures the degree of sector-specific externalities in the consumption sector.

Similarly, investment goods are produced by competitive firms using the technology

$$Y_{It} = B_t \left(u_t K_{It} \right)^{\alpha} L_{It}^{1-\alpha}; \quad \text{where} \quad B_t = \left[\left(\bar{u}_t \bar{K}_{It} \right)^{\alpha} \bar{L}_{It}^{1-\alpha} \right]^{\theta}: \tag{3}$$

Here, K_{It} and L_{It} are capital and hours worked in the investment sector, and B_t represents a productive externality that is an increasing function of the economy-wide average levels of productive capital and labor devoted to producing investment goods. Following Benhabib and Farmer (1996), the degree of sector-specific externalities in the investment sector is also measured by θ.[3]

Under the assumptions that factor markets are perfectly competitive and that capital and labor inputs are perfectly mobile across the two sectors, the first-order conditions for the firms' profit maximization problems are

$$r_t = \frac{\alpha Y_{ct}}{K_{ct}} = p_t \frac{\alpha Y_{It}}{K_{It}}; \tag{4}$$

$$w_t = \frac{(1-\alpha) Y_{ct}}{L_{ct}} = p_t \frac{(1-\alpha) Y_{It}}{L_{It}}; \tag{5}$$

where r_t is the rental rate of capital, w_t is the real wage rate, and p_t denotes the price of investment goods relative to consumption goods.

2.2 Households

There is a unit measure of identical infinitely-lived households, each of which maximizes its present discounted lifetime utility

$$\sum_{t=0}^{\infty} \beta^t \left[\log C_t - \zeta L_t \right]; \quad 0 < \beta < 1 \text{ and } \zeta > 0; \tag{6}$$

[3] Harrison (2000) considers different sizes of sector-specific externalities, and finds that indeterminacy occurs when externalities in the investment sector are sufficiently high, even when there is none for consumption. Weder (2000) obtains the same finding in a monopolistic competitive setting in which the two sectors exhibit different markups of prices over marginal costs. Our results are robust to allowing for unequal sectoral externalities or markups.

where C_t and L_t are the representative household's consumption and hours worked, and β is the discount factor.[4] The budget constraint faced by the representative household is

$$C_t + p_t I_t = Y_t = r_t K_t + w_t L_t, \tag{7}$$

where I_t is investment, Y_t is GDP and K_t is the household's capital stock. The law of motion for the capital stock is given by

$$K_{t+1} = (1 - \delta_t) K_t + I_t, \tag{8}$$

where $\delta_t \in (0, 1)$ is the time-varying capital depreciation rate. As in Wen (1998), we postulate that δ_t takes the form

$$\delta_t = \frac{1}{\gamma} u_t^{\gamma}, \quad \gamma > 1, \tag{9}$$

which states that more intensive capital utilization accelerates its rate of depreciation. We assume that there is no fundamental uncertainty present in the economy.

The first-order conditions for the household's optimization problem are

$$\zeta C_t = w_t, \tag{10}$$

$$\frac{1}{C_t} = \frac{\beta}{C_{t+1}} \left[\frac{r_{t+1} + (1 - \delta_{t+1}) p_{t+1}}{p_t} \right], \tag{11}$$

$$\alpha \frac{Y_t}{u_t} = p_t u_t^{\gamma - 1} K_t, \tag{12}$$

$$\lim_{t \to \infty} \beta^t \frac{K_{t+1}}{C_t} = 0, \tag{13}$$

where (10) equates the slope of the household's indifference curve to the real wage, and (11) is the consumption Euler equation. Moreover, (12) equates the marginal gain (more output) and marginal loss (higher capital depreciation) of a change in the capital utilization rate, and (13) is the transversality condition. Notice that our model has the versatility to subsume the following three existing studies: (i) the Benhabib-Farmer two-sector model corresponds to the case with u_t being a constant, i.e., $u_t = u$, for all t, (ii) the Wen one-sector model corresponds to the case with $p_t = 1$, for all t, and (iii) the Benhabib-Farmer-Guo one-sector model corresponds to the case with $u_t = u$ and $p_t = 1$, for all t.

[4] We adopt the period utility function with indivisible labor, as in Hansen (1985), whereby the labor supply elasticity is infinity. See Table 2 of Schmitt-Grohé (1997) which shows that the higher the labor supply elasticity, the lower the increasing returns needed to obtain indeterminacy in RBC models.

2.3 Equilibrium and Local Dynamics

We focus on symmetric perfect-foresight equilibria which consist of a set of prices $\{p_t, r_t, w_t\}_{t=0}^{\infty}$ and quantities $\{C_t, L_t, u_t, K_{t+1}\}_{t=0}^{\infty}$ that satisfies the household's and firms' first-order conditions. In addition, the aggregate consistency condition requires that $u_t = \bar{u}_t$, $K_{ct} = \bar{K}_{ct}$, $L_{ct} = \bar{L}_{ct}$, $K_{It} = \bar{K}_{It}$ and $L_{It} = \bar{L}_{It}$, for all t. The equalities of demand by households and supply by firms in the consumption and investment sectors are given by $C_t = Y_{ct}$ and $I_t = Y_{It}$. Finally, factor markets clear whereby

$$K_{ct} + K_{It} = K_t, \tag{14}$$

$$L_{ct} + L_{It} = L_t. \tag{15}$$

It is straightforward to show that our model possesses a unique interior steady state. We then take log-linear approximations to the equilibrium conditions in the neighborhood of this steady state to obtain the following dynamic system:

$$\begin{bmatrix} \hat{K}_t \\ \hat{p}_t \end{bmatrix} = J \begin{bmatrix} \hat{K}_{t+1} \\ \hat{p}_{t+1} \end{bmatrix}, \quad \hat{K}_0 \text{ given}, \tag{16}$$

where hat variables denote percent deviations from their steady-state values, and J is the Jacobian matrix of partial derivatives of the transformed dynamic system. The model exhibits saddle path stability when one eigenvalue of J lies inside and the other outside the unit circle. When both eigenvalues are outside the unit circle, the steady state is indeterminate and thus a sink. When both eigenvalues are inside the unit circle, the steady state becomes a totally unstable source.

3 Quantitative Analysis

In this section, we undertake a quantitative investigation of local stability in a calibrated version of the model. For the purpose of this paper, we are mainly interested in examining when indeterminacy results for combinations of parameters whose values are chosen based on empirically observed features of the post-war U.S. economy. Each period in the model is taken to be one quarter. As is common in the real business cycle literature, the capital share of national income, α, is chosen to be 0.3, the discount factor, β, is set equal to 0.99, and the steady-state capital depreciation rate, δ, is fixed at 0.025. The selected values of β and δ imply that $\gamma = 1.404$. Moreover, since the preference parameter ζ plays no role in the dynamics of the model, it is normalized to be 1.

It turns out that with this parameterization, our economy displays an indeterminate steady state when $0{:}0206 \cdot \mu \cdot 0{:}0262$. To put this result in perspective, Table I also reports the regions of indeterminacy for the above-mentioned three nested models. Notice that the minimum returns-to-scale $(1 + \mu)$ needed for indeterminacy is the lowest (around 1:02) in our two-sector economy with variable capital utilization and productive externalities. In addition, this value can certainly be classi…ed as empirically plausible vis-à-vis the aggregation-corrected direct value-added estimate (1:03) for the total private U.S. economy reported in Basu and Fernald (1997).

Table I: Regions of Indeterminacy	
Guo-Harrison Two-Sector Model	$0{:}0206 \cdot \mu \cdot 0{:}0262$
Benhabib-Farmer Two-Sector Model ($u_t = u$)	$0{:}0772 \cdot \mu \cdot 0{:}4307$
Wen One-Sector Model ($p_t = 1$)	$0{:}1037 \cdot \mu \cdot 2{:}3310$
Benhabib-Farmer-Guo One-Sector Model ($u_t = u$ and $p_t = 1$)	$0{:}4935 \cdot \mu \cdot 2{:}3178$

The results of Table I re‡ect two cooperating factors in producing multiple, indeterminate equilibria. On the one hand, when production takes place in two separate sectors, agents have the ability to reallocate resources between them to take advantage of increasing returns. Hence, by moving from the one-sector to the two-sector framework, the size of productive externalities required for indeterminacy falls. On the other hand, adding variable capital utilization raises the elasticity of output with respect to labor so that lower returns-to-scale are needed for self-ful…lling beliefs. As a result, the Benhabib-Farmer-Guo one-sector model requires the highest external e¤ect, whereas the model we propose needs the lowest, and therefore the most empirically plausible, increasing returns-to-scale to generate indeterminacy.

4 Conclusion

This paper has shown that in a two-sector RBC model with variable capital utilization and sector-speci…c externalities, the level of returns-to-scale needed for indeterminacy is extremely low. In particular, it is lower than that required in the three predecessors that we consider. It would be worthwhile to extend our analysis by allowing for non-separable utility and/or CES technology, both of which have been shown to be important in governing the stability of equilibria in one-sector RBC models (see Bennett and Farmer, 2000, and Wen, 2000). We plan to pursue this project in the near future.

References

[1] Basu, Susanto, and John G. Fernald. (1997). "Returns-to-Scale in U.S. Production: Estimates and Implications," Journal of Political Economy, 105, 249-283.

[2] Benhabib, Jess, and Roger E.A. Farmer. (1994). "Indeterminacy and Increasing Returns," Journal of Economic Theory, 63, 19-41.

[3] Benhabib, Jess, and Roger E.A. Farmer. (1996). "Indeterminacy and Sector-Speci…c Externalities," Journal of Monetary Economics, 37, 421-444

[4] Benhabib, Jess, and Kazuo Nishimura. (1998). "Indeterminacy and Sunspots with Constant Returns," Journal of Economic Theory, 81, 58-96.

[5] Benhabib, Jess, and Roger E.A. Farmer. (1999). "Indeterminacy and Sunspots in Macroeconomics," in J. Taylor and M. Woodford, eds., Handbook of Macroeconomics, 387-448. Amsterdam: North Holland.

[6] Benhabib, Jess, Qinglai Meng, and Kazuo Nishimura. (2000). "Indeterminacy Under Constant Returns to Scale in Multisector Economies," forthcoming in Econometrica.

[7] Bennett, Rosalind L., and Roger E.A. Farmer. (2000). "Indeterminacy with Non-Separable Utility," Journal of Economic Theory, 93, 118-143.

[8] Burnside, Craig. (1996). "Production Function Regression, Returns to Scale, and Externalities," Journal of Monetary Economics, 37, 177-201.

[9] Farmer, Roger E.A., and Jang-Ting Guo. (1994). "Real Business Cycles and the Animal Spirits Hypothesis," Journal of Economic Theory, 63, 42-72.

[10] Hansen, Gary D. (1985). "Indivisible Labor and the Business Cycle," Journal of Monetary Economics, 16, 309-327.

[11] Harrison, Sharon G. (2000). "Indeterminacy in a Model with Sector-Speci…c Externalities," forthcoming in Journal of Economic Dynamics and Control.

[12] Perli, Roberto. (1998). "Indeterminacy, Home Production and the Business Cycle: A Calibrated Analysis," Journal of Monetary Economics, 41, 105-125.

[13] Schmitt-Grohé, Stephanie. (1997). "Comparing Four Models of Aggregate Fluctuations due to Self-Ful…lling Expectations," Journal of Economic Theory, 72, 96-147.

[14] Weder, Mark. (2000). "Animal Spirits, Technology Shocks and the Business Cycle," Journal of Economic Dynamics and Control, 24, 273-295.

[15] Wen, Yi. (1998). "Capital Utilization under Increasing Returns to Scale," Journal of Economic Theory, 81, 7-36.

[16] Wen, Yi. (2000). "Understanding Self-Ful…lling Rational Expectations Equilibria in Real Business Cycle Models," forthcoming in Journal of Economic Dynamics and Control.