

## **The Inverse Relationship between Farm Size and Productivity: Refocusing the Debate**

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**Abstract:** The relationship between farm size and productivity is a recurrent topic in economics, almost as old as the discipline itself. This paper emphasizes the importance of productivity measures in the inverse relationship literature. First, we seek to clarify the common measures, their relationships, and their advantages and limitations in empirical work. Second, we argue that most of the existing literature inappropriately uses partial measures such as land productivity. Lastly, using a pseudo-panel of farms that are aggregated at the municipality and farm size levels we provide new empirical evidence on the inverse relationship between farm size and both land productivity and total factor productivity in Brazil over the period 1985-2006. The empirical exercise highlights the importance of choice of measure when testing the inverse relationship. The inverse relationship between size and land productivity is alive and well. The relationship between total factor productivity and size, in contrast, has evolved with modernization during this period and has become increasingly U-shaped.

## I. Introduction\*

The relationship between farm size and productivity is a recurrent topic in economics, almost as old as the discipline itself. John Stuart Mill observed an inverse relationship as early as 1848, a relationship that he later posited had changed due to increasing capital intensity of farming (Lipton, 2009). The issue appeared in the works of Marx, of Lenin and Chayanov in the early 20<sup>th</sup> century, and has captivated agricultural and development economists for over fifty years. Debate around the nature and causes of this relationship continues despite a mountain of empirical analysis, posing a puzzling question for modern neoclassical economics (Binswanger et al., 1995; Eastwood et al., 2010). Conventional economic wisdom expects resources to be allocated such that returns to land are equalized within and between household farms, however the empirical research on developing countries contradicts this and frequently identifies an inverse relationship. Policy-makers in developing countries have engaged the debate, as an inverse relationship between farm size and productivity indicates a role for small farms in development strategies and the potential for land reform to generate improvements in equity and efficiency simultaneously.

Of course, harnessing such a relationship requires correct interpretation of the empirical observations as well as an understanding of its causes, the channels through which it operates, and the factors that condition its strength. A number of theoretical explanations have emerged to explain this phenomenon as resulting from household heterogeneity and/or market failure. Sen's (1966) formalization of the dual labor market hypothesis was among the first, positing that surplus labor generated a wage gap between capitalist and peasant farming, differences in the intensity of labor used, and an inverse relationship. Another approach highlights the role of endowments and credit constraints in generating distinct farmer classes, land-labor ratios, and an inverse relationship (Eswaran and Kotwal, 1986). These results are further strengthened with the consideration of moral hazard and the costs of monitoring hired labor (Feder, 1985). Risk aversion and the differing responses to price risk by net buyers and sellers provides yet another explanation (Barrett, 1996). Measurement error (Lamb, 2003; Carletto et al., 2013) and omitted variables, namely soil quality, are two empirical issues that could lead to an observed inverse relationship (Bhalla and Roy, 1988; Benjamin 1995; Assunção and Braido, 2007). The simplest explanation—decreasing returns to scale (DRS) in the production function—might also play a role, although the empirical literature typically assumes, or fails to reject, constant returns to scale.

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Attempts to sort out the role and relative importance of these mechanisms have been mixed, generating inconsistent results and a body of literature with often conflicting conclusions. Adding to the confusion is the variety of productivity measures and empirical approaches that have been used to identify the inverse relationship, potentially obscuring the comparability, interpretation, and implications of the empirical literature. As with Sen (1962), Deolalikar (1981), Bhalla and Roy (1988) and Benjamin (1995), much of the early literature used yield (output per unit of land) as the measure of productivity. Conditioning yield on input use by estimating a production function, as done in the recent work of Assunção and Braido (2007) and Barrett et al. (2010), is a second commonly used approach that generates an alternative measure of productivity. Still others employ value added per unit of land (Heltberg, 1998), profit per unit of land (Bardhan, 1973; Benjamin, 1995; Carletto et al., 2013), profit (Heltberg, 1998; Lamb, 2003), or technical efficiency (Carter, 1984; Helfand and Levine, 2004).

The variety of productivity measures utilized presents a challenge to interpretation and integration of the empirical literature. Conceptual clarity on the implications of competing measures is needed, as is some clarity on how they relate to each other and to farm size. Despite the recognition that partial measures of productivity such as yield are flawed (Berry and Cline, 1979; Binswanger et al., 1995), they continue to be used and highlighted, often alongside alternative measures. Given that much of this empirical literature continues to work with partial productivity measures, *one must at least question the validity of the stylized fact that in developing countries there is an inverse relationship between productivity and farm size*. Bardhan (1973) and Berry and Cline (1979) are two examples where an alternative choice of productivity measure leads to a direct relationship between farm size and productivity. How much the choice of productivity measure matters for interpreting the existing empirical evidence is open to question. A better understanding of how these partial measures relate to broader measures of productivity will enable more accurate interpretation of the empirical evidence underlying the inverse relationship.

The lack of focus on total factor productivity and profit rate as measures of productivity is a curious feature of the inverse relationship literature, especially given the early and widespread acknowledgement of their superiority over partial measures. As Barrett (1996) notes, the inverse relationship literature “habitually, perhaps cavalierly,” uses physical yields and productivity synonymously. In fact, empirical studies assessing the productivity-farm size relationship in the developed world, such as Garcia et al. (1982) and Alvarez and Arias (2004), almost exclusively use measures of technical efficiency or total factor productivity. Not only is a comprehensive productivity measure such as

total factor productivity a theoretically preferred measure, it is also likely to be the measure most relevant to policy-makers formulating development strategies in relatively poor economies where poverty alleviation is a more pressing need. With this in mind, in this paper we argue that the inverse relationship literature needs to shift its focus from yield to total factor productivity.

This paper emphasizes the importance of productivity measures in the inverse relationship literature. First, we seek to clarify the common measures, their relationships, and their advantages and limitations in empirical work. Second, we reassess the empirical literature in this light, arguing that most of the existing inverse relationship literature inappropriately uses partial measures of productivity. In some cases this may be justifiable if, in the given context, it can be argued that the partial measures in use are reasonable proxies for total factor productivity. Unfortunately this is seldom done. Even when more appropriate measures are used it is often not made clear or emphasized, indicating confusion in the literature and hampering interpretation. Lastly, using a pseudo-panel of farms that are aggregated at the municipality and farm size levels we provide new empirical evidence on total factor productivity and the inverse relationship in Brazil over the period 1985-2006. The empirical exercise highlights the importance of choice of measure when testing the inverse relationship, a point for which there is scant supporting empirical literature. It also highlights how the relationship between total factor productivity and size has evolved with modernization during this period, shedding some light on the issues raised by Mill over 150 years ago.

## **II. Measures of Agricultural Productivity**

Farm size may be related to a broad range of economic outcomes, such as employment, poverty, inequality, food security, and growth. While these are important issues connected to the role of farm size in a developing economy, here, as with the rest of the literature on the inverse relationship, we focus specifically on the concept of productivity.

### *The Unconditional Relationship between Yield and Farm Size*

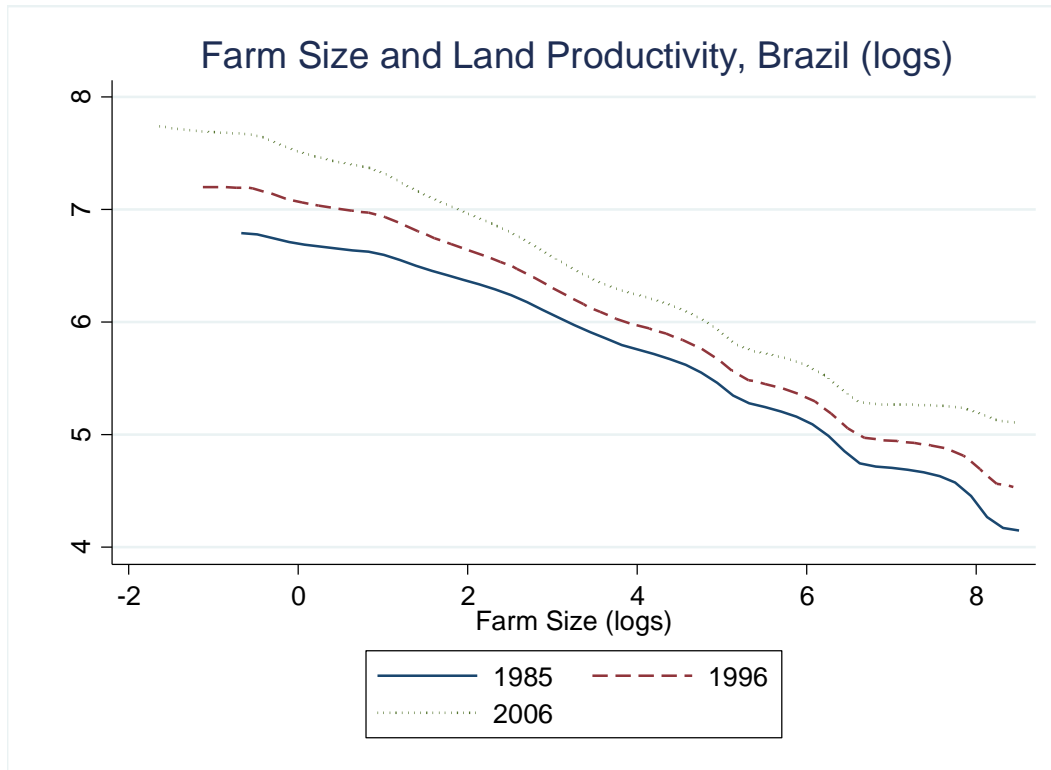
Yield is by far the most commonly used measure of productivity in the literature on the inverse relationship (IR). Even when alternative productivity measures are used, the relationship between gross

yield and farm size is often a starting point because it is a benchmark for the expansive existing literature on the IR. Yield,  $q$ , is the partial measure of land productivity:

$$Yield = \frac{Q}{A} = q = \psi_u(A) \tag{1}$$

where  $A$  is the area of the farm,  $Q$  is an index of agricultural output,  $q$  is agricultural output per unit of land, and  $\psi_u(A)$  connotes that yield may be a function of farm size. In a world where farm size and yield are unrelated we have  $\frac{\partial \psi_u(A)}{\partial A} = 0$ . However, the regularity with which empirical work has found  $\frac{\partial \psi_u(A)}{\partial A} < 0$  has led to the stylized fact that they are inversely related, and is precisely what has led to the abundant interest in the relationship and its explanations. Figure 1 below displays this relationship using data from Brazil for the years 1985, 1996, and 2006. While the relationship is potentially non-linear and may not be monotonic, for now we focus on the first order approximation.

**Figure 1: Farm Size and Land Productivity, Brazil**



Note: Smoothed as a local polynomial regression with bandwidth of 1.25 and Epanechnikov kernel.

The relationship captured by  $\psi_u(A)$  is unconditional ( $u$ ) in the sense that it is the simple bivariate relationship between yield and farm size. Factors that may be causing or influencing this relationship have

not been controlled for. Using yield as a measure of productivity is inherently limited—as would be any partial measure of productivity—whenever there is more than one factor of production. If other factors vary systematically with farm size then we might expect to see an IR due to more intensive input use by small farms, implying that our focus on the relationship between land productivity and the size of the farm may be misplaced. Similarly, analysis using different partial productivity measures may result in conflicting policy recommendations. Indeed, Sen’s (1962) seminal contribution revealed precisely this type of systematic relationship between the intensity of labor use and farm size, leading to his formal exposition of the dual labor market hypothesis (Sen 1966).

### *The Conditional Relationship between Yield and Farm Size*

It is curious that the unconditional relationship between yield and farm size has remained such a focal point in the IR literature. A more appropriate approach is to use a conditional relationship between yield and farm size, where the relationship is conditioned on a vector of controls,  $X(A)$ , that may be correlated with both yield and farm size:

$$q = g(X(A), \psi_c(A)) = \psi_u(A) \quad (2)$$

The conditional ( $c$ ) relationship,  $\psi_c(A)$ , should differ from the unconditional relationship to the extent that the conditioning controls explain the unconditional IR. For example, the impact of varying input intensities can be controlled for by including the inputs as controls, household heterogeneity could be controlled for with household fixed effects, market failure controlled for with regional fixed effects, and omitted variables such as soil quality could be added. This is a useful approach for exploring the theoretical channels that explain the IR and is a strategy commonly used by researchers (Assunção and Braido, 2007; Barrett et al., 2010).

As discussed above, partial measures of productivity such as yield are potentially misleading when there are other factors of production. At the very least, understanding any relationship between productivity and farm size requires empirical analysis that controls for the intensity with which other factors of production are used. A natural way to handle this is to include those factor intensities as conditioning variables. For exposition, assume that land, labor ( $L$ ), and capital ( $K$ ) are the only factors of production and that their intensities, labor per unit of land and capital per unit of land, are given by  $l$  and  $k$ , respectively. If, for ease of exposition, these are the only controls then (2) becomes:

$$q = g(k(A), l(A), \psi_c(A)) = \psi_u(A) \quad (3)$$

From equation (3) we see that the IR as identified by the unconditional relationship between yield and farm size,  $\frac{\partial \psi_u(A)}{\partial A}$ , is composed of the relationship between capital intensity and farm size, labor intensity and farm size, and any conditional relationship between farm size and yield,  $\frac{\partial \psi_c(A)}{\partial A}$ . When differences in the use of other factors of production are controlled for, the conditional relationship between farm size and productivity captures a more comprehensive measure of productivity. Unlike the unconditional relationship, differences in factor intensities are not driving the conditional relationship.

Exploring (3) highlights the ambiguity in how the yield and farm size relationship, as captured by  $\psi_u(A)$ , is related to the more general productivity and farm size relationship captured by  $\psi_c(A)$ . Differentiating (3) with respect to farm size shows:

$$\left(\frac{\partial g}{\partial k}\right)\left(\frac{\partial k}{\partial A}\right) + \left(\frac{\partial g}{\partial l}\right)\left(\frac{\partial l}{\partial A}\right) + \left(\frac{\partial g}{\partial \psi_c}\right)\left(\frac{\partial \psi_c}{\partial A}\right) = \left(\frac{\partial \psi_u}{\partial A}\right) \quad (4)$$

The unconditional and conditional relationships between farm size and productivity are positively related, as a change in some factor underlying the conditional relationship will also impact the unconditional relationship. Assuming output per unit of land is increasing in capital and labor per unit of land, which is to say that  $\frac{\partial g}{\partial k}$  and  $\frac{\partial g}{\partial l}$  are both positive, it is plausible for the conditional relationship to be positive ( $\frac{\partial \psi_c}{\partial A} > 0$ ) even if the unconditional relationship is negative ( $\frac{\partial \psi_u}{\partial A} < 0$ ) if  $\frac{\partial k}{\partial A}$ ,  $\frac{\partial l}{\partial A}$ , or both are negative. Similarly, if  $\frac{\partial k}{\partial A}$ ,  $\frac{\partial l}{\partial A}$ , or both are positive it may be that  $\frac{\partial \psi_c}{\partial A} < 0$  even when  $\frac{\partial \psi_u}{\partial A} > 0$ . In short, an unconditional IR is neither a necessary nor a sufficient condition for an inverse relationship between a broader measure of productivity and farm size as captured by  $\psi_c(A)$ .

When, as in (3), the conditional relationship includes all factors of production as controls, the approach is equivalent to estimating a production function. Such analysis potentially isolates the relationship between farm size and a comprehensive measure of productivity, but the interpretation must be done with care because the functional form chosen for the empirical analysis imposes, implicitly or explicitly, a functional form on production. In the case of a production function, as we show below, this conditional relationship is often interpreted as total factor productivity. Before returning to this issue let us introduce an alternative and commonly used approach to measuring the IR, one that uses variations of profit as a measure of productivity.

### *Profit and Value-Added per Unit of Land*

There are two variants of this approach. Start with the measure of profit:

$$\Pi = Q - p_L L - p_K K - p_A A = \phi(A) \quad (5)$$

where  $p_L$  is the price of labor,  $p_K$  the price of capital, and  $p_A$  the price of land. In expression (5) the output quantity index,  $Q$ , is constructed using prices in the aggregation process, making it interpretable as the value of output. Profit has been written as some function,  $\phi(A)$ , of the size of the farm. While it is certainly possible that there is no relationship between the level of profit and farm size, it is natural to think about the level of profit as the product of output and profit per unit of output:  $Q \frac{\Pi}{Q}$ . Regardless of whether or not the profit per unit of output rises or falls with size, we would expect the level of output to dominate in the determination of the level of profit. A large farm that produces 1000 units of output should generate more profit than a small farm that produces 10 units. The level of profit, then, is not a particularly good measure for comparing the productivity of farms of different sizes.

It is not profits per se that matter, but rather profitability. This requires the transformation of the profit level into a profit rate. One way to do this is to calculate profit per unit of land, as has been done by Carletto et al. (2013), an approach which has alternatively been referred to as net yield:

$$\text{net yield} = \pi_A = \frac{\Pi}{A} = q - p_L l - p_K k - p_A = q - c = \phi_n(A) \quad (6)$$

where  $c$  is the cost per unit of land incurred in producing  $q$  units of output per unit of land. Profit per unit of land is a productivity measure that controls for the levels of other inputs additively, providing an improvement over gross yield. Measures of value added are similar, however they fall short of profit measures as they control only for intermediate inputs and not the complete set of factors of production. Despite being an improvement over gross yield and value-added, profit per unit of land is itself problematic because it is a partial measure of profitability. The shortcoming here is similar to that of gross yield. Suppose net yield displays a systematic inverse relationship with farm size, i.e.  $\frac{\partial \phi_n(A)}{\partial A} < 0$ . This information is of limited use, as the partial measure of profitability does not effectively control for the profitability with which other inputs are used:

$$\pi_A = \frac{\Pi}{A} = \frac{\Pi K}{K A} = \pi_K * (k) \quad (7)$$

Here we see that the profit per unit of land can be rewritten as the product of profit per unit of capital,  $\pi_K$ , and capital intensity. An observed inverse relationship between profit per unit of land and farm size



could be associated with declining capital intensity as farm size increases, even if profit per unit of capital is increasing. If this were true, then the use of one partial profitability measure or the other would give opposite policy recommendations. Profit per unit of land would suggest that policymakers focus on small farms, whereas profit per unit of capital would indicate the reverse. Clearly these are imperfect measures for deriving policy recommendations surrounding agricultural productivity and, as with other partial measures, a more comprehensive measure is preferred.

### *TFP and Profit Rate*

Comprehensive measures, of either productivity or profitability, are the appropriate means to measure the efficiency of resource use. Binswanger et al. (1995), along with Berry and Cline (1979), have argued that total factor productivity (TFP) or the profit rate are the preferable and correct choices for productivity measures when discussing farm size. We concur. When policy implications are derived from empirically observed relationships between productivity and farm size, as has been done regarding land reform and the future of small farms, it is important to consider the productivity of all inputs, not just land productivity. This is especially true in the IR literature because poverty alleviation is a pressing need in developing economies.

Total factor productivity is a comprehensive productivity measure defined as the ratio of output to all inputs used. If, as before, land, labor, and capital are the three factors of production and the quantity indices are constructed using prices, TFP can be written as:

$$TFP = \frac{Q}{Input\ Quantity\ Index} = \varphi(A) \quad (8)$$

TFP effectively captures the productivity with which all inputs are used in the production process. It measures units of output for every unit of input used, regardless of its type, and in this sense is a comprehensive measure of productivity. If this measure is a function of farm size, i.e.  $\frac{\partial \varphi(A)}{\partial A} \neq 0$ , then there is an unambiguous difference in how productively farms of different sizes utilize assets in agricultural production, and policymakers receive clear information on the relationship between productivity and farm size. An understanding of the determinants of  $\varphi(A)$  would enable effective policy design. Although this is widely acknowledged, an explicit focus on TFP is seldom the approach of empirical analyses of the IR in developing economies.

TFP can be written as a function of a profit rate, where profit is normalized by the total value of inputs used in production:

$$TFP = \frac{\Pi}{p_L L + p_K K + p_A A} + 1 \quad (9)$$

The profit rate identified in (9) is the rate most meaningful for assessing agricultural productivity in that it is consistent with TFP. Whereas TFP looks only at the gross value of production, the profit rate effectively considers the net value of production. Any IR found using this profit rate should also be found using TFP as the productivity measure.

An alternative interpretation of this profit rate is that it is normalizing by the total value of assets used in agricultural production. Consider, for instance, a potential farmer with financial capital looking to invest in farming. If there are competitive factor markets then the value of the capital used to rent land, physical capital, and labor to run the operation is identically equal to the costs of production. Binswanger et al. (1995) advocate normalizing profit by “capital invested” or “assets.” It is unclear what exactly is meant by this because the most concrete empirical application (Rosenzweig and Binswanger, 1993) normalizes by household wealth, a concept potentially quite different from that of capital invested. Although normalizing by household wealth may be appropriate in some circumstances, it is not when comparing the productivity of small and large farming operations. Farm households may hold wealth for a variety of reasons, including for agricultural production, consumption, a store of value, or for investment in non-agricultural productive endeavors. Capital used in agricultural production is a subset of household wealth, so if other forms of wealth and farm size are positively related then an IR may emerge even when profit per unit of invested capital is constant across farm size. Normalizing profits by household wealth is clearly problematic in such circumstances.

### *Linking TFP and Partial Measures*

It is instructive to analyze the relationship between TFP and the other measures used. Consider first the relationship between TFP and the conditional relationship captured when analyzing a production function, which was claimed to reflect a comprehensive measure of productivity. Assume that the production function is homogenous of degree  $t$  so that  $f(\lambda L, \lambda K, \lambda A) = \lambda^t f(L, K, A)$ . Constant returns to scale (CRS) holds if  $t = 1$ , while decreasing returns (DRS) are observed if  $t < 1$  and increasing returns (IRS) if  $t > 1$ . Setting  $\lambda = \frac{1}{A}$  implies that the production function can be written as:

$$f(l, k, 1) = A^{-t} f(L, K, A) \quad (10)$$

and dividing output by farm size gives us yield:

$$q = \frac{f(L, K, A)}{A} = A^{t-1} f(l, k, 1) \quad (11)$$

As an example, assume a standard Cobb-Douglas production function where  $T$  is the unobserved measure of total factor productivity and production is a function of labor, capital and land:

$$f(L, K, A) = TL^\alpha K^\beta A^\gamma \quad (12)$$

If, as is often presumed, CRS holds, then farm size disappears from the right hand side of (12) after dividing through by farm size. If not, then the natural log of the production function takes the form:

$$\ln q = (t - 1)\ln A + \ln T + \alpha \ln l + \beta \ln k \quad (13)$$

If, as in (8), there exists a relationship between total factor productivity and size,  $\varphi(A)$ , we have:

$$\ln q = (t - 1)\ln A + \ln \varphi(A) + \alpha \ln l + \beta \ln k \quad (14)$$

From (14) it is clear that the conditional relationship identified in (3),  $\psi_c(A)$ , is composed of the relationship between TFP and farm size as well as any deviations from CRS in the production function.

Equation (14) highlights three features of the production function approach that are often overlooked in the literature. First, if CRS holds then the conditional relationship,  $\psi_c(A)$ , captures the relationship between TFP and farm size,  $\varphi(A)$ . Second, if CRS does not hold then it will be difficult to empirically differentiate whether a conditional relationship is driven by non-CRS, a relationship between TFP and farm size, or both. In effect, if  $\ln A$  is excluded from the estimation,  $\psi_c(A)$  will capture deviations from CRS *and* any other conditional relationship that might exist between productivity and size. This highlights the importance of explicitly investigating returns to scale in any given empirical context and, if this is not done, to interpret any observed conditional relationship as potentially coming from non-CRS. Third, if the addition of any controls other than the factor intensities explain differences in the conditional relationship, which is composed of non-CRS and the TFP-farm size relationship, they should be interpreted as such.

Thus, when empirical research estimates a production function to explore the relationship between farm size and productivity they are, in effect, estimating the relationship between farm size and TFP, not farm size and yield. This point is almost always left undiscussed. Indeed, most empirical work that

takes this approach shows the unconditional relationship first, followed by a series of regressions of production functions that include other controls, followed by interpretation of the two approaches as if they were exploring the same relationship. However, as has been shown, the conditional and unconditional relationships are by no means the same. They can plausibly take different signs, and will almost certainly have different magnitudes.

In a similar fashion, the relationship between TFP and both output per unit of land (gross yield) and profit per unit of land can be explored. TFP is a unit-less measure, but multiplying and dividing by  $\frac{1}{A}$  allows the measure to be rewritten as:

$$TFP = \frac{q}{p_L l + p_K k + p_A} = \frac{q}{c} = \varphi(A) \quad (15)$$

From the expression above TFP can be thought of as yield normalized by costs per unit of land,  $c$ . Whereas profit per unit of land is linear in  $q$  and  $c$ , effectively controlling for the costs per unit of land additively, TFP is log-linear and controls for costs per unit of land multiplicatively:

$$\ln TFP = \ln q - \ln c \quad (16)$$

How then does the relationship between TFP and farm size,  $\varphi(A)$ , relate to the relationship between profit per unit of land and farm size,  $\phi_n(A)$ ? From (15), TFP can be rewritten as a function of net yield and cost per unit of land by adding and subtracting  $\frac{c}{c}$ :

$$TFP = \frac{q - p_L l - p_K k - p_A}{p_L l + p_K k + p_A} + 1 = \frac{\pi_A}{c} + 1 \quad (17)$$

Taking a derivative with respect to farm size:

$$\frac{\partial TFP}{\partial A} = \left( \frac{\partial \pi_A}{\partial A} \right) \left( \frac{1}{c} \right) - \frac{\left( \frac{\partial c}{\partial A} \right) \pi_A}{c^2} \quad (18)$$

In similar fashion, taking the derivative of (15) with respect to  $A$  yields:

$$\frac{\partial TFP}{\partial A} = \left( \frac{\partial q}{\partial A} \right) \left( \frac{1}{c} \right) - \frac{\left( \frac{\partial c}{\partial A} \right) q}{c^2} \quad (19)$$

Generalizing (18) and (19) by allowing  $x \in \{ \pi_A, q \}$ , and employing a little bit of algebra (see Appendix A):

$$\frac{\partial x}{\partial A} = \left( \frac{\partial TFP}{\partial A} \right) c \left[ \frac{\varepsilon_{x,A}}{\varepsilon_{x,A} - \varepsilon_{c,A}} \right] \quad (20)$$

where  $\varepsilon_{x,A}$  is the elasticity of partial productivity measure  $x$  (profit or output per unit of land) with respect to size of the farm. If there is an empirically observed inverse relationship between the partial measure and farm size such that  $\frac{\partial x}{\partial A} < 0$ , then we know  $\varepsilon_{x,A}$  is negative. This implies that one of two possibilities must hold:

$$(i) \quad \frac{\partial TFP}{\partial A} < 0 \text{ and } \varepsilon_{x,A} < \varepsilon_{c,A}$$

$$(ii) \quad \frac{\partial TFP}{\partial A} > 0 \text{ and } \varepsilon_{x,A} > \varepsilon_{c,A}$$

If (i) is true then an IR between a partial measure and farm size reflects an IR between productivity and farm size as measured by TFP. If this is true then either costs per unit of land are increasing in farm size or they are decreasing, but slower than the rate at which output per unit of land is decreasing. If (ii) is true then use of a partial measure is generating an incorrect indication about the productivity and farm size relationship, and TFP is actually directly related to farm size! However, this requires that  $0 > \varepsilon_{x,A} > \varepsilon_{c,A}$ . In such a case, costs per unit of land are negatively related to farm size and they are relatively more elastic than output per unit of land with respect to farm size. This discussion highlights the conclusion that an IR between a partial measure of productivity and farm size is neither necessary nor sufficient for the existence of an IR between farm size and a comprehensive measure of productivity such as TFP.

The conditions set out in (i) and (ii) provide a framework for interpreting empirical identification of an IR using output or profit per unit of land and drawing conclusions on the relationship between TFP and farm size. For example, consider an economy with a relatively undeveloped and traditional agricultural sector where production relies predominantly on land and labor as inputs. If an IR between yield and farm size exists in such a world, the underlying mechanisms are likely to be Sen's dual labor market hypothesis and the monitoring costs associated with large farms hiring wage labor. Larger farms must incur additional supervision costs to maintain a given labor intensity, implying that  $\varepsilon_{c,A}$  is positive and (i) holds. Even if the labor intensity is falling with farm size, the employment of some additional labor requires additional supervision and costs per unit of land will likely fall slower than will yield, implying that  $\varepsilon_{c,A}$  is negative but less so than  $\varepsilon_{q,A}$ . TFP, as with yield, will have an inverse relationship with farm size according to (i). Yield may be an adequate proxy for measuring the relationship between farm size and productivity in this type of economy.

In a second stage of agricultural development, advances in agricultural technology lead to mechanization, making capital a more important class of inputs. Such a stage is often characterized by struggling institutions and imperfection in credit and labor markets. If, given the former, larger farms have

greater access to physical capital and credit than they can more readily substitute away from labor towards capital, avoiding costly monitoring and moving towards a more efficient mix of factors of production. If labor market imperfections still exist then smaller farms will continue to employ labor more intensively even as they employ capital less so, implying it is quite possible that an inverse relationship between yield and farm size continues to exist even if the improved access of larger farms to capital attenuates the relationship. However, because the influence of monitoring costs and the dual labor market hypothesis dwindles as large farms substitute capital for labor, and because credit market imperfections mean larger farms have cost advantages in the capital market, it may now mean that costs per unit of land are falling as farm size increases. In such a scenario one would not be surprised if condition (ii) holds and a direct relationship between TFP and farm size is observed even if an IR continues to exist for yield. Unlike in the first example, mechanization and an increasingly important role for capital in agricultural production has rendered yield a misleading measure for identifying the relationship between farm size and productivity for this economy.

It is possible to imagine a third stage of agricultural development that could realign the relationships between TFP, yield and farm size. Mechanization likely becomes increasingly important so access to capital remains critical. As institutions improve, distortions in the labor and capital markets begin to disappear. Other sources of economies of scale begin to come into play – larger farms have better access to managerial and technical expertise, they may continue to have advantages in their access to credit, and they have advantages in marketing and distributing products. These new factors generate a direct relationship between TFP and farm size as better access to capital makes new technology more readily available to larger farms and better access to technical expertise increases productivity, and the inverse relationship between yield and farm size may disappear because of these considerations. In this light, a focus on the choice of productivity measure helps to reconcile disparate empirical results in the IR literature – as agricultural sectors develop, the relative importance of different institutional failures in the productivity and farm size relationship changes, and as economies develop, mechanization and technological change become more important, and institutions improve the agricultural sector begins to exhibit economies of scale. This hypothetical discussion highlights how partial measures of productivity may or may not serve as appropriate proxies for comprehensive productivity measures in the context of the productivity and farm size relationship, and how this likely depends upon the stage of agricultural development and the mechanisms driving any relationship.

### **III. The Stochastic Frontier Approach**

Stochastic frontier production functions have commonly been used in the agricultural economics literature on developed countries, and in much of the literature on productivity that does not specifically focus on agriculture. There is little reason to think that an IR found while using an average production function must also be found when estimating a stochastic production frontier. Whereas the former looks at the average farm, implicitly assuming the technical efficiency of all farms and treating deviations from the production function as random, the latter looks at farms on the frontier of production possibilities and allows for technical and scale inefficiencies as well as stochastic deviations from the production frontier. Just as the choice of metric may matter for the IR debate and the resulting policy considerations, how that metric is analyzed may matter as well. It is certainly possible that the most efficient large farms exhibit higher TFP than the most efficient small farms, but that the average large farm has lower TFP than the average small farm simply because it is, on average, less technically efficient. Any systematic differences in technical change, or the changes in scale efficiency and technical efficiency among small and large farmers will contribute to the relationship between farm-size and productivity. A shift towards TFP analysis using the stochastic frontier approach would benefit the development economics literature on the IR by identifying the contribution of technical change, change in technical inefficiency, and change in scale efficiency to changes in the IR over time.

### **IV. Empirical Analysis**

We now turn to an empirical analysis of Brazilian agriculture, illustrating the conceptual discussion surrounding choice of productivity measure and highlighting its importance when studying the relationship between farm size and agricultural productivity. The intention here is not to explain the relationship by controlling for its potential determinants, but rather to use the Brazilian experience to highlight how the choice of measure can change the observed relationship, how the relationship between measures changes over time, and how this differs across stages of agricultural development. In addition, this analysis generates novel estimates of TFP in Brazilian agriculture for distinct farm size classes for the country as a whole and for each of five broad regions.

## *Data*

The data come from the 1985, 1995/1996, and 2006 rounds of the Brazilian agricultural census. For confidentiality reasons the data is aggregated to five farm size classes for each municipality in Brazil.<sup>1</sup> The changing composition of Brazil's municipalities requires the construction of minimum comparable areas (AMCs) to generate a dataset of geographic units that is consistent over time. This process aggregates the 5,548 Brazilian municipalities in 2006 (the year with the greatest number of municipalities) into 3,861 AMCs. For each farm size class of each AMC we generate a "representative farm" that characterizes the behavior of farms of that class in that AMC, an approach that effectively ignores heterogeneity among the farms within an AMC farm size class. For a more detailed account of the database construction, see Helfand et al. (2015).

We begin with 47,365 representative farms of AMC farm size classes containing three or more farms. Due to concern about the comparability of the largest Brazilian farms we remove all representative farms in the Northeast and South over 4,000 ha and all of those over 5,000 ha in the remaining three macro regions, resulting in 84 of the observations being dropped. We identify yield outliers in two stages. First, we simply cut the top and bottom 1% of the yield distribution from the sample. Second, using a quadratic specification we regress yield on farm size with AMC fixed effects and survey year dummy variables. From this regression we identify all representative farms with yield greater than four standard deviations from the conditional mean as outliers. Together, the data cleaning exercises remove 2.3% of the initial sample.

Table A1 in Appendix B displays the results of the data cleaning, for each stage of the process, for each of five macro-regions of Brazil — the North, Northeast, Southeast, South, and the Center-West. By focusing on regional analyses we are able to compare and contrast the regions' relationships between farm size and productivity. As a starting point, Figures 2 through 7 graph yield by farm size class for Brazil and each of its five macro-regions, capturing the unconditional relationship between yield and farm size. Despite considerable regional heterogeneity in their agricultural activities, each region mirrors the country as a whole in displaying a strong inverse relationship between yield and farm size that appears to be growing stronger over time.

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<sup>1</sup> These classes are 0-5 ha, 5-20 ha, 20-100 ha, 100-500 ha, and 500+ ha. To protect the confidentiality of the farms, the Brazilian Institute of Geography and Statistics (IBGE) requires that each aggregate observation have at least 3 farms.



Figure 2

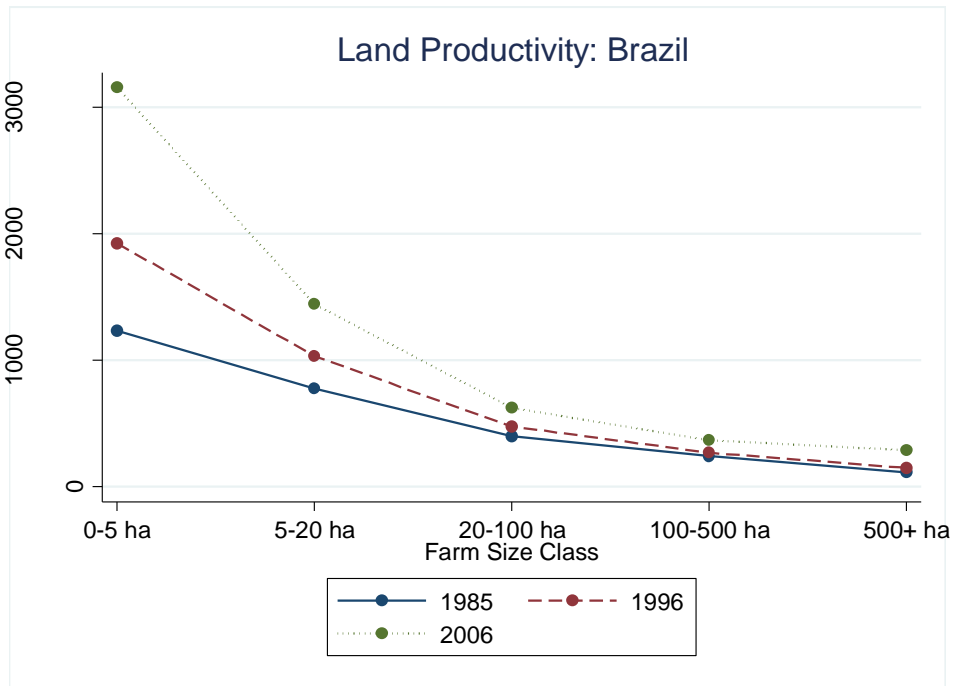


Figure 3

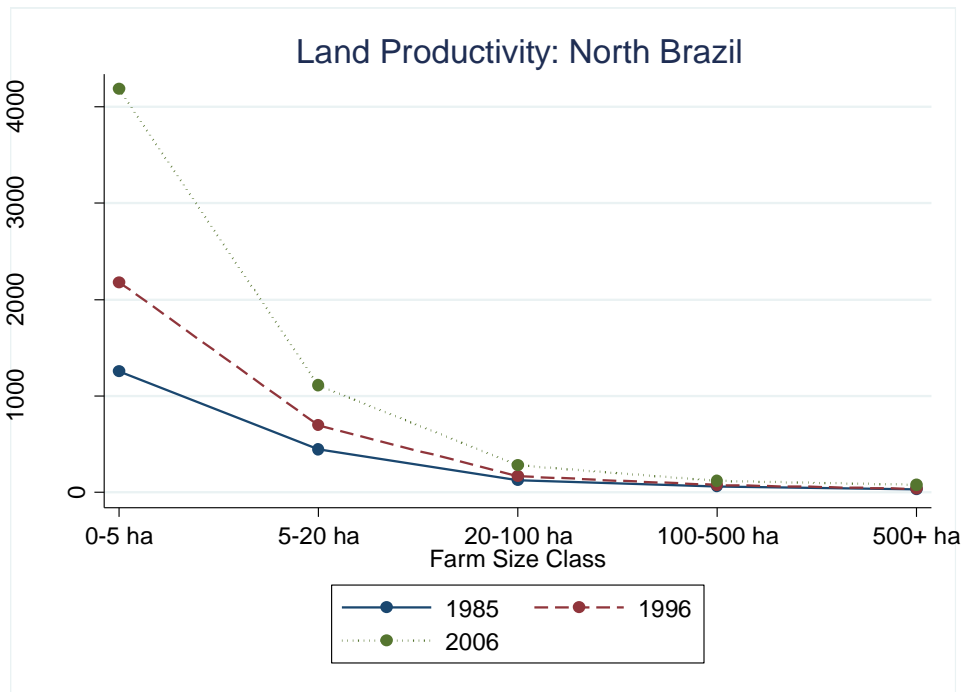


Figure 4

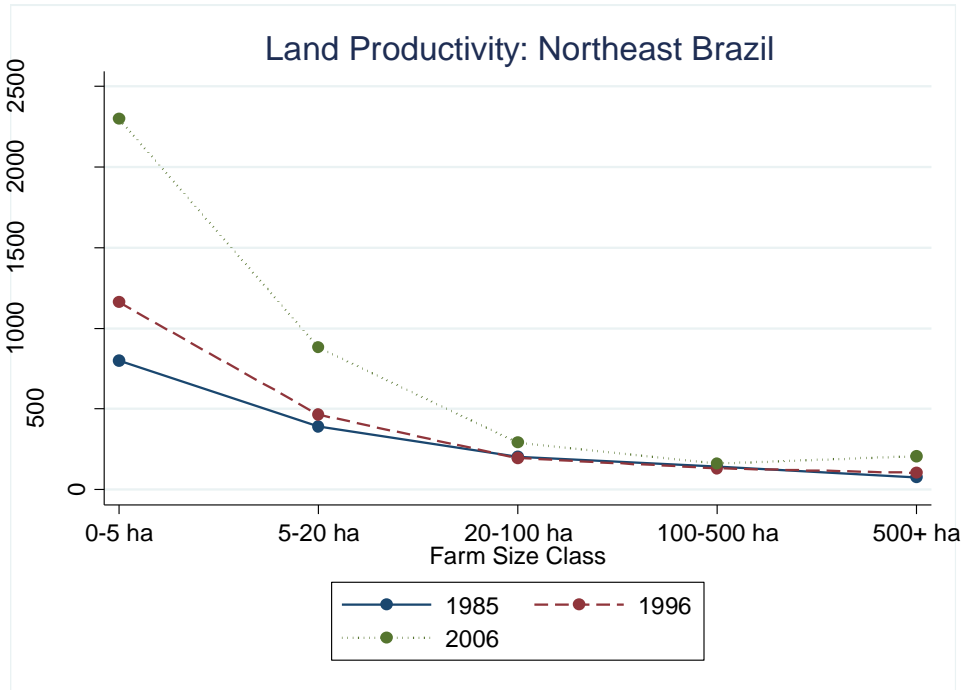


Figure 5

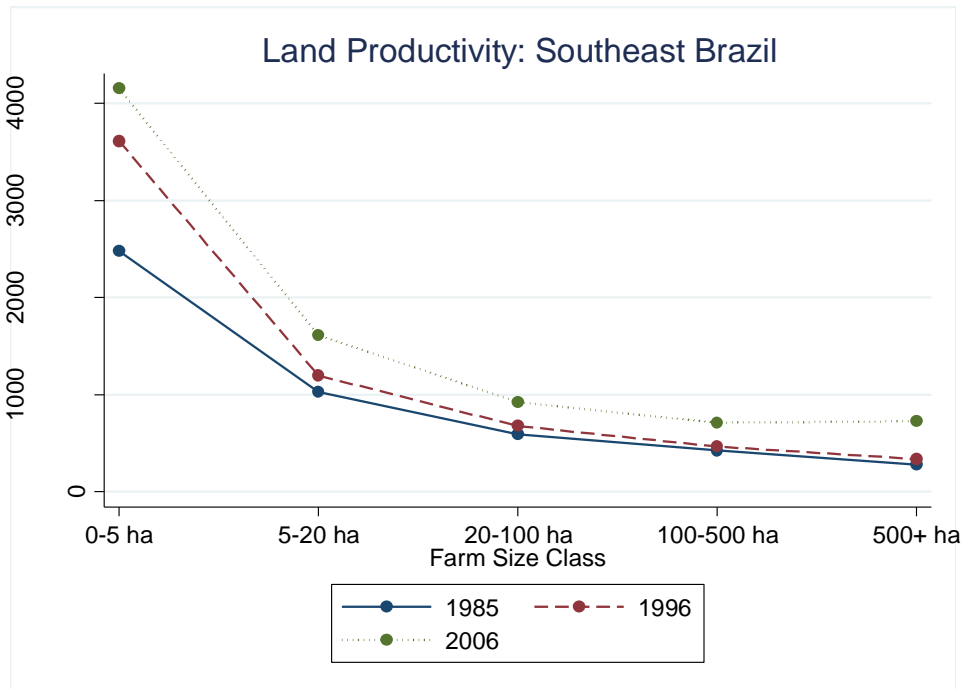


Figure 6

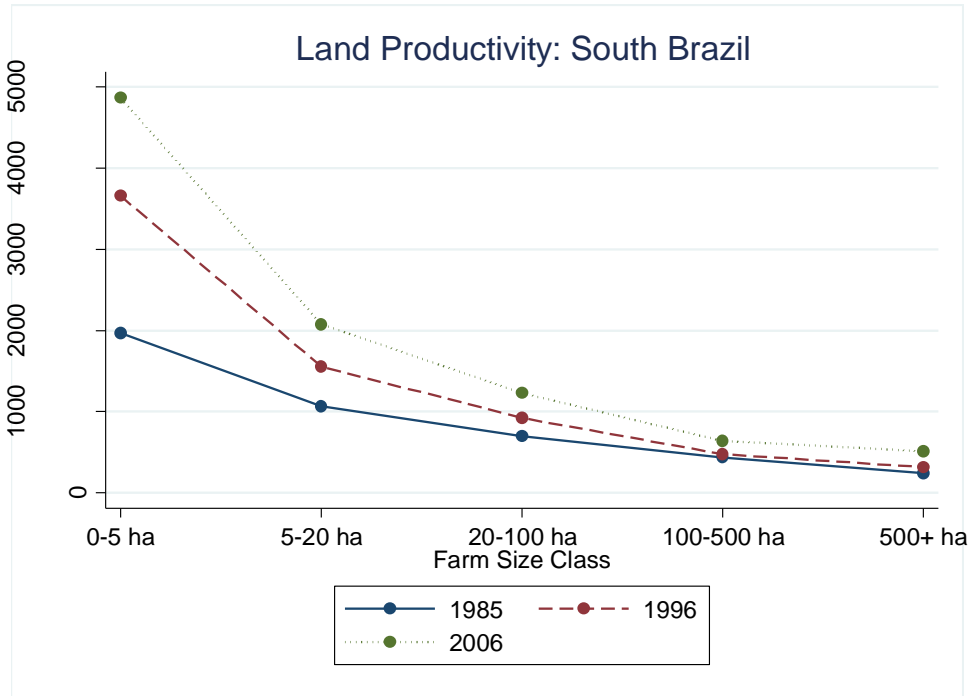
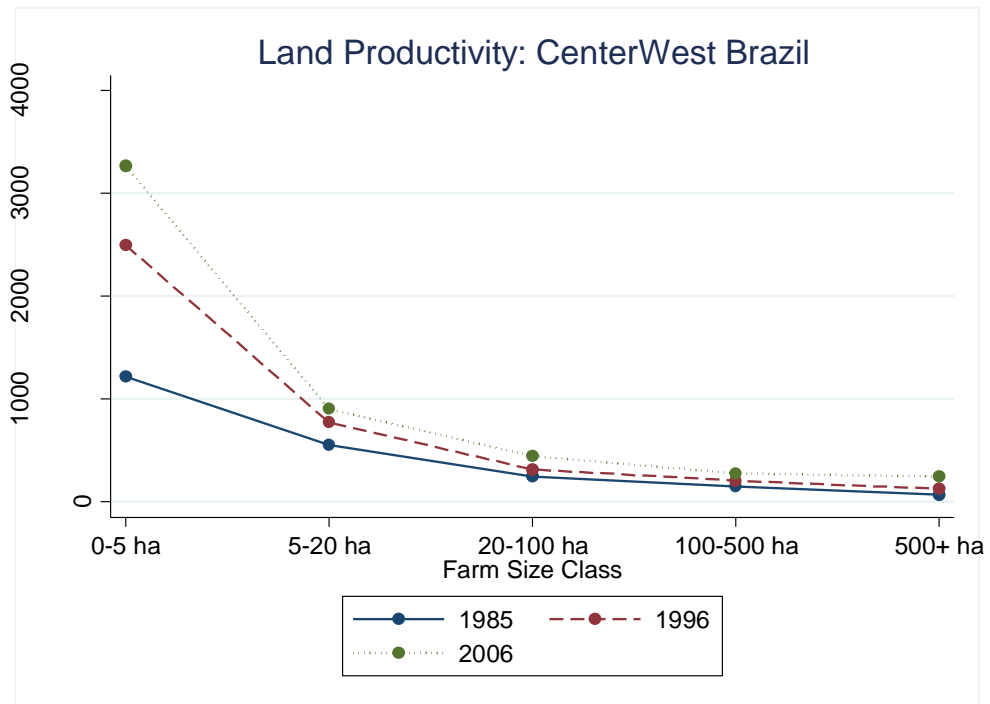
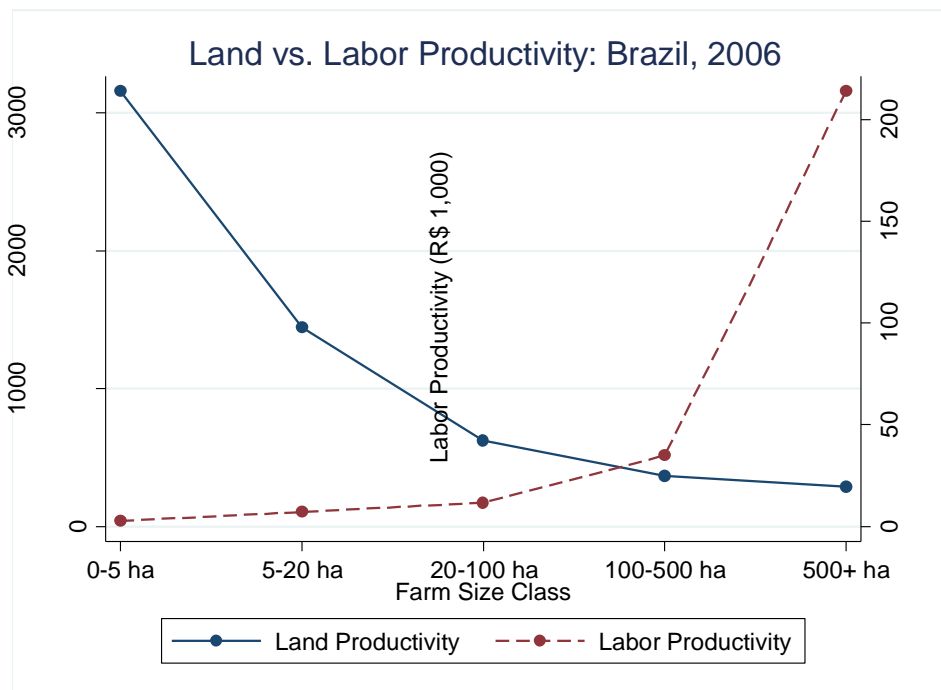


Figure 7



These figures clearly indicate an inverse relationship between farm size and productivity when measured using yield. However, as shown in (4) above, partial measures of productivity can be misleading and, in some cases, can generate conflicting results. This is shown in Figure 8 below, contrasting land productivity with the agricultural productivity of family labor in Brazil in 2006. Analysis of the farm size and productivity relationship using this partial measure of labor productivity would indicate a direct relationship and suggest that larger farms are more productive than are their smaller counterparts. This lays bare the need for a more comprehensive measure of productivity when attempting to identify and discuss any IR between farm size and productivity

**Figure 8**



*Empirical Methodology*

Consistent with the existing IR literature and the discussion above, we assume a Cobb-Douglas technology and estimate an average production function. This approach allows for the easy isolation of TFP, our preferred measure of productivity, which is allowed to vary across farm size and across time using dummy variables for farm size classes and survey rounds, respectively.

As a precursor to our main analysis we estimate the returns to scale in agriculture using a Cobb-Douglas production function, first for Brazil and then for each macro region separately. While CRS is a frequent assumption employed in the empirical identification of an IR (and many studies have found that returns to scale in agriculture are approximately constant), non-CRS have been found, particularly where the data used is less aggregated and more flexible production functions are allowed (Bardhan 1973). Equation (14) displays the difficulty in differentiating between DRS and an IR between productivity and farm size generated by some other mechanism. It is not clear how to do this in an empirical setting, but at the very least the issue indicates that careful interpretation of results is needed – unless CRS can be confirmed in the data, any observed IR (DR) includes any deviations from CRS. As with our main analysis, we treat the three survey rounds as a pseudo panel of representative farms at the AMC – farm size class level. For each region we estimate the following:

$$\ln Y_{mst} = \beta_0 + \beta_1 \ln K_{mst} + \beta_2 \ln L_{mst} + \beta_3 \ln X_{mst} + \beta_4 \ln A_{mst} + \theta_t + \lambda_m + \varepsilon_{mst} \quad (21)$$

where  $Y_{mst}$  is the value of output of the representative farm of size class  $s$  in AMC  $m$  in survey year  $t$  and  $K_{mst}$  is capital,  $L_{mst}$  is family labor,  $X_{mst}$  is purchased intermediates, and  $A_{mst}$  is the area used by that representative farm. Hired labor is included in purchased intermediates. The model includes survey year dummy variables,  $\theta_t$ , and AMC dummy variables,  $\lambda_m$ . Constant returns to scale holds if the sum of the coefficients on the factors of production,  $\beta_1$  through  $\beta_4$ , is equal to one. Increasing returns to scale will hold if they sum to greater than one, and decreasing returns if less than one. Returns to scale are estimated with and without AMC fixed effects. Standard errors are clustered at the AMC level and, because the number of farms represented by each representative farms varies, each observation is weighted by the number of farms it represents.

In estimating the relationship between farm size and productivity we continue using the pseudo panel with AMC fixed effects, errors clustered at the AMC level, and representative farms weighted by the number of farms they represent. However, output and all inputs are now normalized by area,  $A_{mst}$ . By estimating the model using intensities we effectively impose CRS, forcing any deviation from CRS into the estimated relationship between farm size and productivity. Survey year specific dummy variables for the five farm size classes,  $\delta_{st}$ , are used to flexibly capture the farm size and productivity relationship. The farm size class 0-5 ha in 1985 is excluded and used as a reference. While this structure allows the farm size and productivity relationship to change over time the technology coefficients are assumed to remain constant, implying a time invariant agricultural technology. The estimated equation takes the form:

$$\ln y_{mst} = \beta_0 + \beta_1 \ln k_{mst} + \beta_2 \ln l_{mst} + \beta_3 \ln x_{mst} + \delta_{st} + \lambda_m + \varepsilon_{mst} \quad (22)$$

where  $y_{mst}$  is aggregate output per unit of land and  $k_{mst}$ ,  $l_{mst}$ , and  $x_{mst}$  are the factors of production per unit of land. As before,  $\lambda_m$  are AMC dummies and  $\varepsilon_{mst}$  is our standard error term clustered at the AMC level. With the above approach the TFP of a farm size bin in a given period is calculated by:

$$TFP_{st} = e^{\beta_0 + \delta_{st}} \quad (23)$$

These estimates of TFP provide the foundation for analyzing the farm size and productivity relationship.

### *Empirical Results*

Table 2 displays the technology coefficients obtained by estimating the agricultural production function with logged levels without controlling for the farm size and productivity relationship, the implied RTS, and the p-value obtained when testing the null hypothesis that CRS holds in each region of analysis. This table shows that in all but one region, the Center-West, CRS is rejected in favor of DRS at the 10% level of significance, suggesting that once CRS is imposed on the production function any negative relationship observed between farm size and productivity is due in part to deviations from CRS. While not shown here, a comparison of the estimated RTS with and without AMC fixed effects reveals that RTS are consistently overstated when the data is treated as a repeated cross-section. This result is concordant with the literature's finding that endogeneity concerns may potentially lead to overestimates of RTS (Eastwood et al. 2010), highlighting the advantage of exploiting panel data when estimating a production function to assess the IR.

Table 3 shows the technology coefficients for all regions resulting from the estimation of the productivity and farm size relationship after imposing CRS through the use of intensities.<sup>2</sup> In accordance with expectations, purchased inputs are the most important factor of production in all regions. These include, among other things, seeds, fertilizer, animal feed and hired labor. Similarly, family labor is most important in the two poorest regions of the country—the North and Northeast—and least important in the Center-West, the most modern agricultural region dominated by large commercial farms.

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<sup>2</sup> As long as AMC fixed effects are included the technology coefficients are similar to those resulting from alternative specifications (not shown here) that do not include farm size class dummies (this includes not controlling for farm size, controlling for farm size linearly, and controlling for farm size with a quadratic).

**Table 2. Estimated Returns to Scale**

	Brazil	North	Northeast	Southeast	South	Center-West
Farm Size	0.123*** (0.008)	0.076*** (0.022)	0.112*** (0.012)	0.177*** (0.012)	0.179*** (0.012)	0.187*** (0.039)
Capital	0.161*** (0.012)	0.200*** (0.039)	0.188*** (0.018)	0.121*** (0.015)	0.093*** (0.017)	0.027 (0.041)
Family Labor	0.054 (0.030)	0.144 (0.121)	0.177*** (0.052)	0.060 (0.031)	0.011 (0.041)	-0.012 (0.123)
Intermediates	0.482*** (0.011)	0.358*** (0.040)	0.416*** (0.016)	0.533*** (0.019)	0.537*** (0.020)	0.599*** (0.042)
Constant	3.211*** (0.070)	3.874*** (0.255)	3.727*** (0.096)	3.204*** (0.099)	3.233*** (0.130)	3.540*** (0.397)
AMC FE	Yes	Yes	Yes	Yes	Yes	Yes
RTS	0.82	0.78	0.89	0.89	0.82	0.80
RTS p-value	0.000	0.072	0.033	0.000	0.000	0.148
R-squared	0.94	0.84	0.87	0.93	0.94	0.94
N	46515	1888	16465	17742	7382	3038

Standard errors in parentheses: \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ .

Dependent variable is logged output; all independent variables are logged.

**Table 3. Estimated Technology Coefficients**

	Brazil	North	Northeast	Southeast	South	Center-West
Capital per ha	0.168*** (0.012)	0.176*** (0.039)	0.184*** (0.018)	0.129*** (0.015)	0.137*** (0.018)	0.175*** (0.039)
Family Labor per ha	0.237*** (0.019)	0.303*** (0.066)	0.253*** (0.032)	0.190*** (0.027)	0.176*** (0.029)	0.149*** (0.045)
Intermediates per ha	0.467*** (0.011)	0.304*** (0.039)	0.416*** (0.017)	0.503*** (0.021)	0.513*** (0.023)	0.426*** (0.040)
Constant	3.091*** (0.063)	4.093*** (0.244)	3.657*** (0.083)	3.285*** (0.104)	2.953*** (0.130)	3.627*** (0.294)
AMC FE	Yes	Yes	Yes	Yes	Yes	Yes
Farm Size Dummies	Yes	Yes	Yes	Yes	Yes	Yes
R-squared	0.92	0.95	0.90	0.91	0.92	0.90
N	46515	1888	16465	17742	7382	3038

Standard errors in parentheses: \* p<0.10, \*\* p<0.05, \*\*\* p<0.01.

Dependent variable is logged output; all independent variables are logged; all variables normalized by farm size.



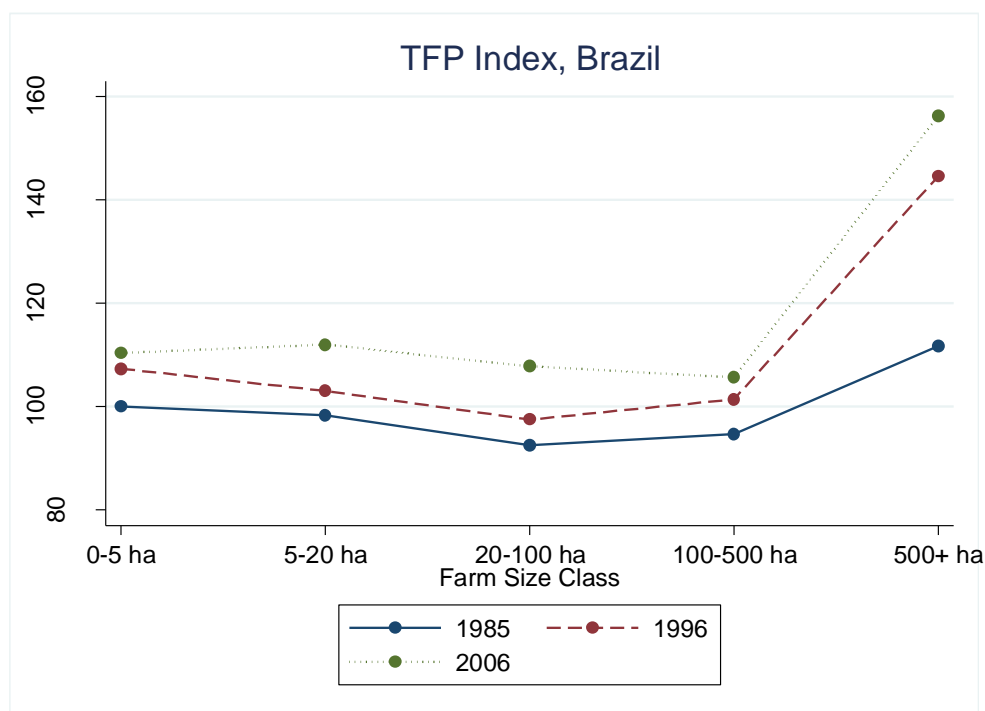
Estimates of TFP are calculated from the survey year specific farm size class dummies according to (23). A TFP index is then calculated for each farm size class in each period using the farm size class 0-5 ha in 1985 as a base level set to 100. Figure 9 graphs these indices in each time period for Brazil as a whole. The use of TFP as a productivity measure provides a very different picture of the farm size and productivity relationship than does the use of land or labor productivity. Over the first four farm size classes TFP is moderately inverse, but relatively flat over all three years. In contrast, for the largest class of farms TFP is notably higher, and increasingly so over time. This is suggestive of a non-linear relationship between farm size and productivity becoming positive over time, standing in stark contrast to the inverse relationship observed between yield and farm size.

Table 4 contains percentage deviations from the 0-5 ha farm size class for each of the other farm size classes in Brazil, along with p-values from Wald tests of the significance of each individual class' deviations in 1985, 1996, and 2006. Column 1 of Table 8 contains the results from the joint significance tests, testing the null hypothesis that the class dummy variables are jointly equal to zero. If we reject the null, we conclude that inclusion of the dummy variables is statistically significant in explaining variation in TFP. We conduct two sets of joint hypotheses because the largest farms, 500+ ha, appear so different than the remainder: first, we test if all farms greater than 5 ha have coefficients that are jointly equal to zero; second, we test if farms greater than 5 ha but less than 500 ha have coefficients are jointly equal to zero. These tests verify that while differences for any one farm size class from 5-500 ha may not be statistically meaningfully different from the smallest farms in any given period, jointly there is an inverse relationship between farm size and TFP in the 0-500 range in 1985 and 1996 that becomes statistically insignificant by 2006. Thus, between zero and 500 ha there is an IR that disappears over time. The largest farms, in contrast, begin to differentiate themselves from the rest over the sample period. The farm size class 500+ ha has statistically higher TFP in both 1995 and 2006, having TFP that is 35% and 42% higher than the smallest farms in those periods, respectively. Overall, this is suggestive of a U-shaped relationship between farm size and TFP rotating up over time.

In comparison to the international literature surrounding the IR, the Brazilian data used here represent a very heterogeneous group of farms and span a much greater range of farm sizes. A more accurate comparison group for the international literature might be farms less than 100 ha, which indeed make up approximately 90% of all Brazilian farms. Restricting our analysis to this subset of farms would not eliminate the differences in the observed farm size and productivity relationship that result from different productivity measures – the use of yield would still show a marked inverse relationship while the

use of TFP would reveal that a negative relationship has disappeared over the period of analysis. Perhaps more importantly, our inclusion of the largest farm size class displays a potential shortcoming of assessing this relationship using only the smaller range of the farm size distribution: in the Brazilian case the largest farms have notably higher productivity than the smaller farm size classes and their exclusion would result in an incomplete understanding of the farm size and productivity relationship.

**Figure 9**



**Table 4. Percentage Difference in TFP Relative to 0-5 ha Farms in each Period: Brazil**

	1985	1996	2006
5-20 ha	-1.72 (0.535)	-3.98 (0.136)	1.49 (0.598)
20-100 ha	-7.48 (0.135)	-9.16* (0.072)	-2.28 (0.641)
100-500 ha	-5.34 (0.509)	-5.48 (0.530)	-4.25 (0.602)
500 + ha	11.68 (0.381)	34.65** (0.018)	41.55*** (0.007)
N	46515	46515	46515

P-values from significance tests are in parentheses: \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ .

Turning to the regional analysis, Figures 10-14 display different experiences in the farm size and TFP relationship and how it has changed over time. Tables 5-7 show percentage deviations from farms 0-

5 ha for each other farm size class' TFP in each period, along with a significance test. Table 8 contains the joint significance tests for each region and each year, as described above. Over the first four farm size classes, the point estimates in Tables 5 through 7 reveal an inverse relationship in the North in 1985 that becomes stronger in the later periods. The joint significance tests in Table 8 reveal that the inverse relationship over the first four farm size classes is statistically significant in 1985 and 1996 but not 2006, despite the appearance of it becoming stronger in 2006, becomes statistically insignificant by 2006. However, farm size classes 3 and 4 are consistently less productive than 0-5 ha farms, being 34% and 48% less productive in 2006, respectively, suggesting a persistent inverse relationship between TFP and farm size over the 0-500 ha range. Similar to Brazil as a whole, the largest class of farms in the North appears quite different than the smaller farm size classes. Unlike Brazil, the point estimates suggest that the largest farms have TFP around 25% to 35% below that of their smallest counterparts. The joint significance tests on all farms greater than 5 ha indicates that the inverse relationship in the North is statistically significant and persistent over time and that the IR in this region is robust to choice of productivity measure. It is not, however, a linear relationship, but rather an emerging U-shaped inverse relationship.

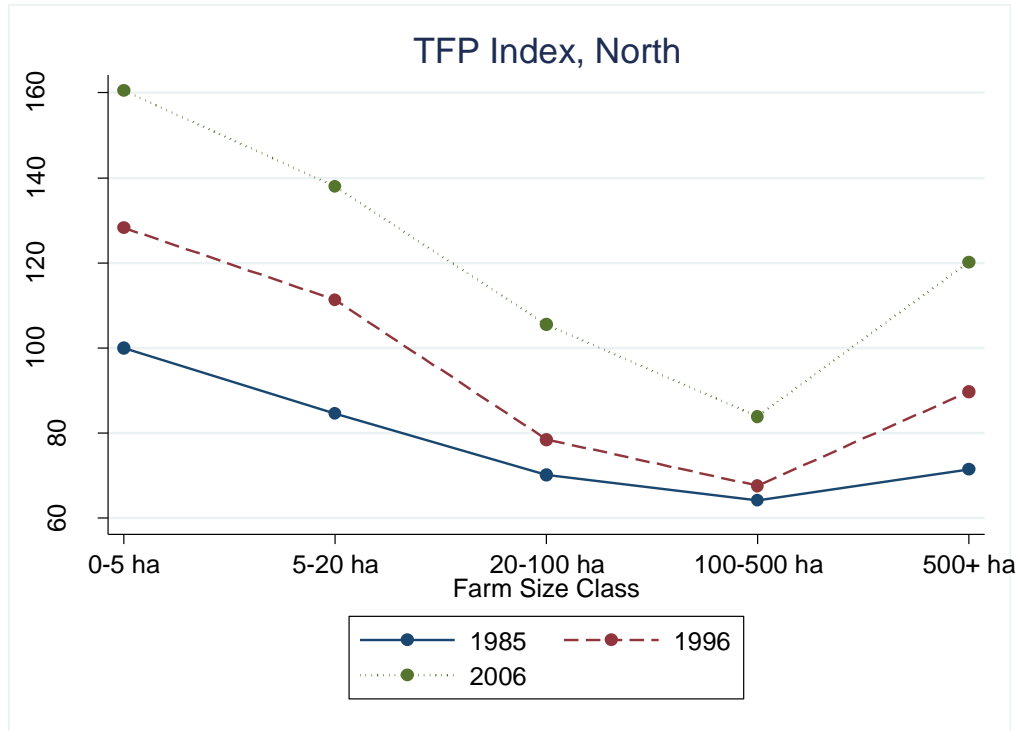
The experience in the Northeast appears qualitatively similar, with a persistent inverse relationship over 0-500 ha and a 500+ ha farm size class establishing a U-shaped inverse relationship over time. While all joint significance tests in the Northeast are highly significant, just two of the individual farm size class differences are in any period. Thus, in both the North and Northeast a strong IR identified using yield as a productivity measure and an inverse relationship is consistently found using TFP that is non-linear and U-shaped.

As with the North and the Northeast, the South and the Southeast have seen a qualitatively similar change to the farm size and TFP relationship in those regions. In both regions a relatively weak inverse relationship over the range 0-500 ha has flattened over time, if not becoming positive, and these relationships are jointly significant. In 1985, both regions had the largest farm size class being approximately 25% more productive than the smallest although the difference is statistically insignificant. This difference grows over time, becoming statistically significant, and in 2006 the largest farm size class in the South and Southeast is 38% and 49% more productive than the smallest when measured using TFP. Once again, the relationship appears non-linear and U-shaped. However, in contrast to the North and Northeast, in the South and Southeast this relationship is overall positive and, driven by relatively rapid growth among the largest farm size classes, becoming increasingly so. This contrasts sharply to the persistent IR found in these regions using yield as a measure of productivity.

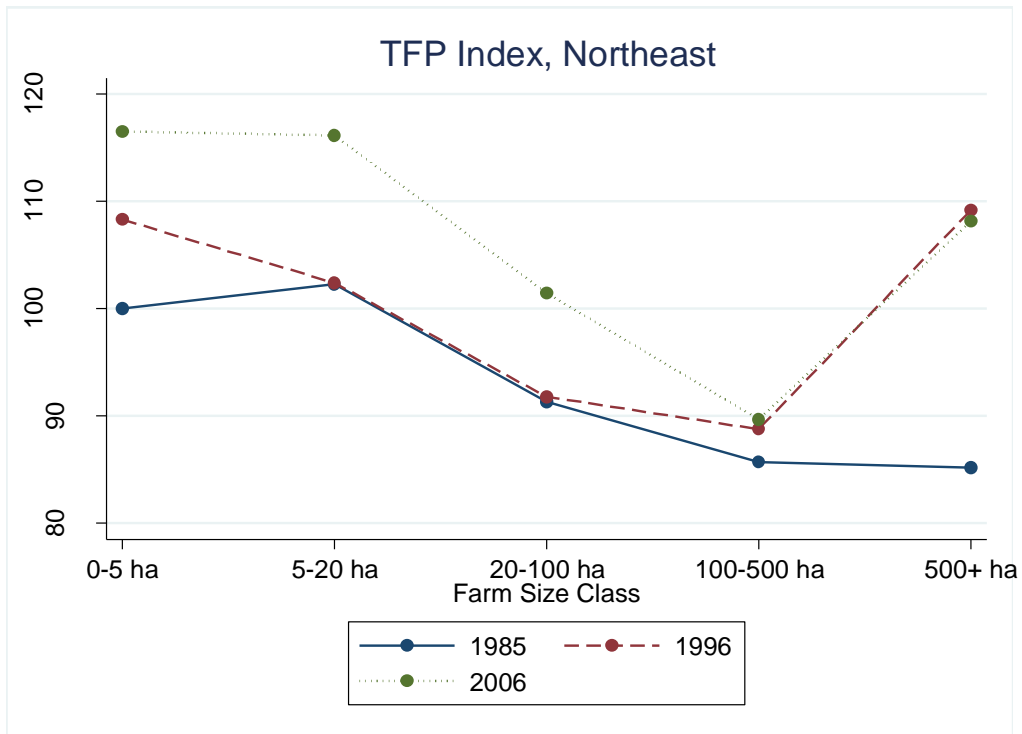
The Center-West tells a similar yet more extreme story. The strong inverse relationship over the range 0-500 ha in 1985, being jointly significant and statistically significant for individual farm size classes as well, weakens and becomes statistically flat by 2006. The point estimates show that the largest farms in the Center-West were 45% less productive than the smallest farms in 1985, and by 2006 they were 14% more productive, albeit statistically insignificantly so. Once again a U-shape emerges, driven by rapid growth of the productivity of larger farms. This is the clearest case of a strong inverse relationship becoming reversed over the 21 year period under study. Using yield to measure the farm size and productivity relationship completely misses this transformation, revealing the limitation of yield as a productivity measure in a rapidly modernizing agricultural region such as the Center-West during this time period.

The regional analysis supports that of the overall analysis of Brazil in highlighting the importance of assessing the full range of farm sizes when considering the farm size and productivity relationship. In each region the largest farms, those of 500 or more hectares, saw larger productivity gains over the 1985 to 2006 period and displayed higher TFP than farms 20-500 ha in 2006. Similarly, the regional analysis complements that of the country as a whole in displaying how the IR measured by yield relates to the IR measured by our preferred productivity measure, TFP. In some regions, such as the North and Northeast, yield adequately proxies for TFP even if it misses the distinction between the largest farms and those that are smaller. However, in others, such as the South, Southeast, and most clearly in the Center-West, using yield fails to capture a changing relationship between farm size and productivity and in some cases inappropriately characterizes a positive relationship as a negative one.

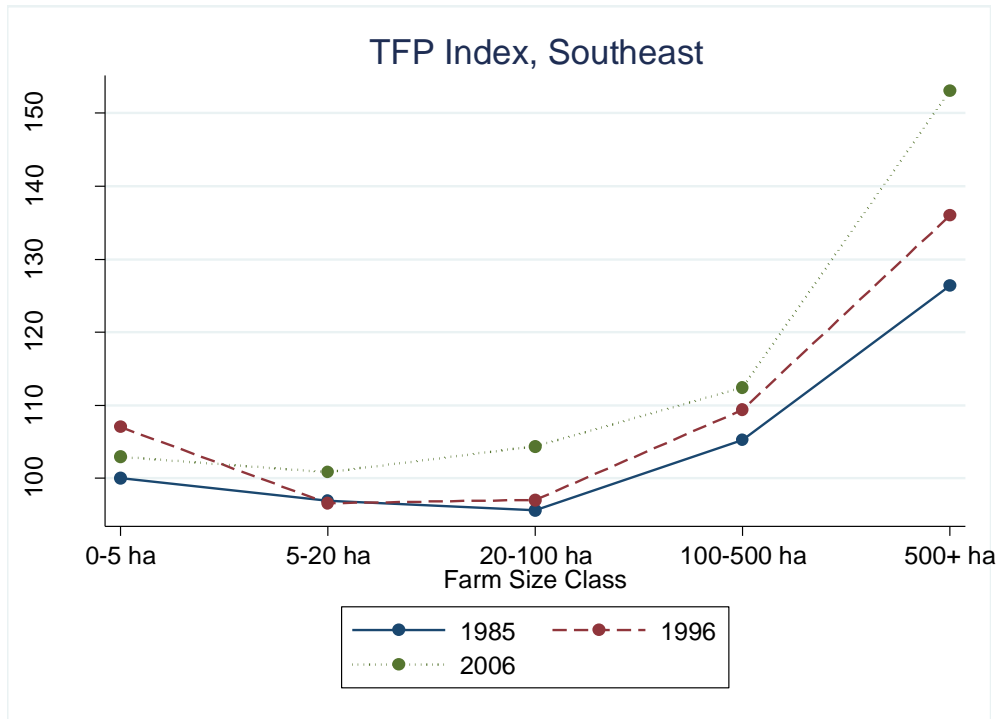
**Figure 10**



**Figure 11**



**Figure 12**



**Figure 13**

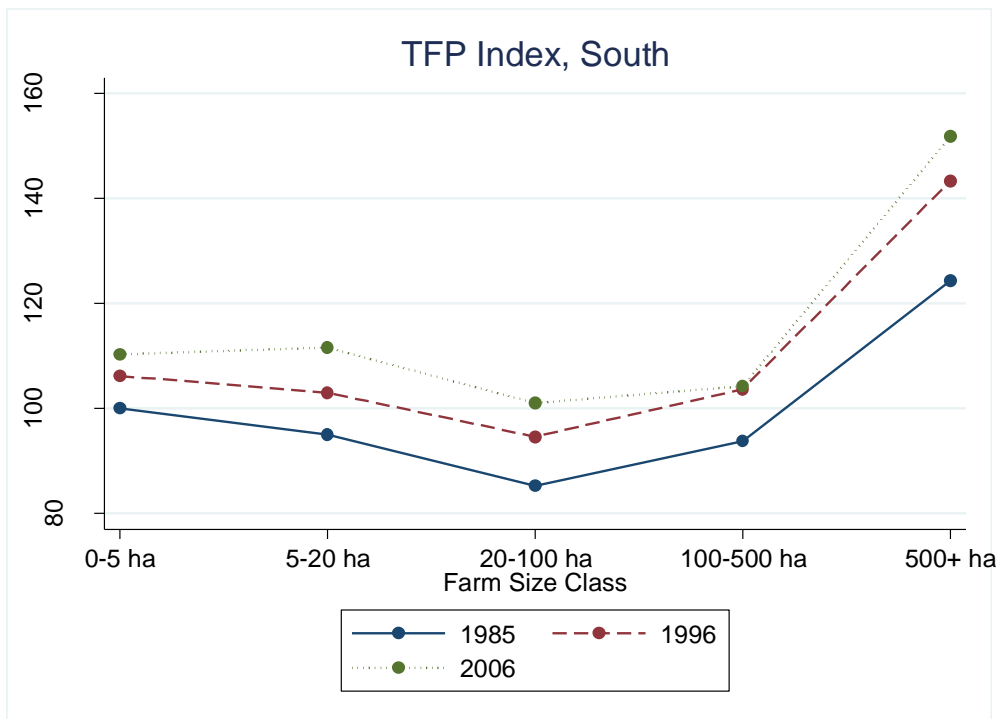
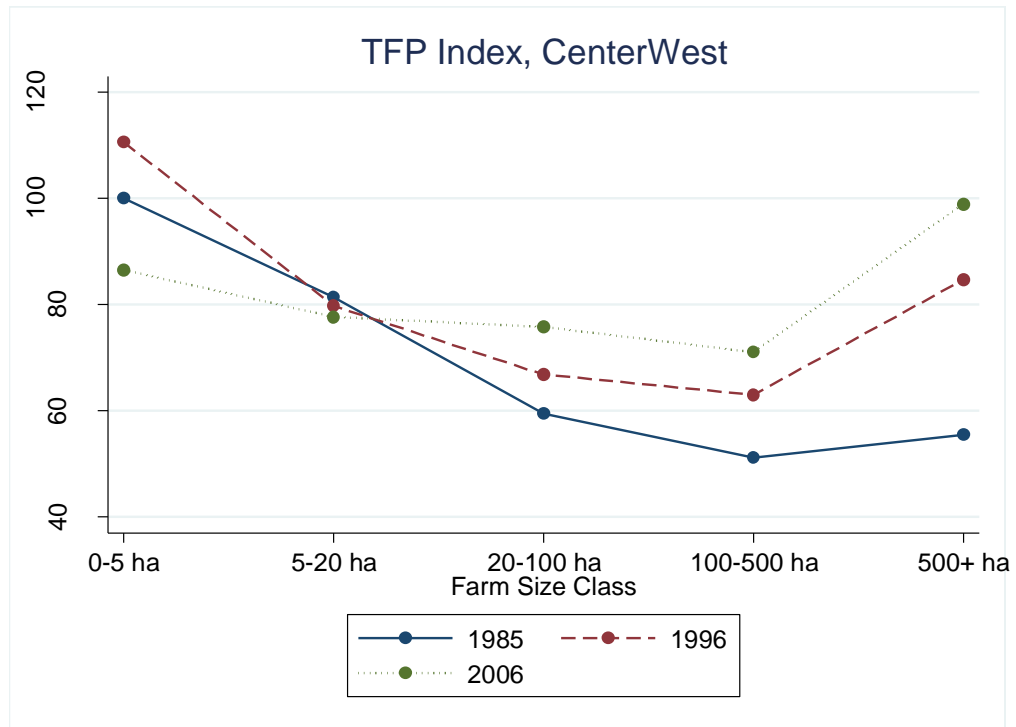


Figure 14



**Table 5. Percentage Difference in TFP Relative to 0-5 ha Farms, 1985**

	North	Northeast	Southeast	South	Center-West
5-20 ha	-15.39* (0.086)	2.26 (0.642)	-3.09 (0.383)	-5.02 (0.226)	-18.63*** (0.004)
20-100 ha	-29.77** (0.035)	-8.68 (0.327)	-4.41 (0.536)	-14.75** (0.019)	-40.54*** (0.001)
100-500 ha	-35.87* (0.053)	-14.30 (0.276)	5.26 (0.641)	-6.19 (0.592)	-48.88*** (0.005)
500 + ha	-28.61 (0.382)	-14.79 (0.439)	26.36 (0.150)	24.38 (0.209)	-44.55* (0.094)
N	1888	16465	17742	7382	3038

Base farm size bin, 0-5 ha P-values from significance tests are in parentheses: \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ .

**Table 6. Percentage Difference in TFP Relative to 0-5 ha Farms, 1996**

	North	Northeast	Southeast	South	Center-West
5-20 ha	-13.22 (0.131)	-5.46 (0.248)	-9.76*** (0.009)	-3.01 (0.427)	-27.90*** (0.003)
20-100 ha	-38.80*** (0.008)	-15.24* (0.076)	-9.32 (0.184)	-10.90* (0.092)	-39.58*** (0.007)
100-500 ha	-47.27** (0.013)	-18.05 (0.163)	2.19 (0.850)	-2.38 (0.842)	-43.02** (0.034)
500 + ha	-30.06 (0.383)	0.78 (0.970)	27.05 (0.147)	34.93* (0.085)	-23.42 (0.471)
N	1888	16465	17742	7382	3038

Base farm size bin, 0-5 ha. P-values from significance tests are in parentheses: \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$



**Table 7. Percentage Difference in TFP Relative to 0-5 ha Farms, 2006**

	North	Northeast	Southeast	South	Center-West
5-20 ha	-14.03 (0.293)	-0.31 (0.952)	-2.06 (0.623)	1.20 (0.813)	-10.20 (0.440)
20-100 ha	-34.27** (0.048)	-12.92 (0.164)	1.39 (0.846)	-8.40 (0.306)	-12.37 (0.323)
100-500 ha	-47.70** (0.038)	-23.02* (0.070)	9.21 (0.442)	-5.48 (0.670)	-17.86 (0.405)
500 + ha	-25.13 (0.547)	-7.20 (0.707)	48.68** (0.018)	37.62* (0.074)	14.38 (0.695)
N	1888	16465	17742	7382	3038

Base farm size bin, 0-5 ha. P-values from significance tests are in parentheses: \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

**Table 8. Joint Tests of Equality in Farm Size Class Indicator Variables Relative to 0-5 ha Farms**

	Brazil	North	Northeast	Southeast	South	Center-West
1985 All Farms	0.000	0.002	0.000	0.000	0.000	0.000
1985 5-500 ha Farms	0.000	0.097	0.000	0.000	0.000	0.001
1996 All Farms	0.000	0.000	0.000	0.000	0.000	0.000
1996 5-500 ha Farms	0.000	0.004	0.000	0.000	0.000	0.002
2006 All Farms	0.000	0.000	0.000	0.000	0.000	0.000
2006 5-500 ha Farms	0.219	0.225	0.000	0.093	0.001	0.802
N	46515	1888	16465	17742	7382	3038

P-values are presented here.

## V. Conclusion

We have sought to address an important weakness of the development economics literature on the inverse relationship between farm size and productivity. We argued that the most commonly used measure—land productivity—is problematic and potentially misleading. While this point is commonly noted in the literature, the use of land productivity or other partial measures of productivity remains widespread. Furthermore, where movements are made towards use of a more comprehensive and appropriate measure, such as TFP, the distinction is seldom made and is left undeveloped. The conceptual discussion provides a framework for assessing the implications and appropriateness of the choice of productivity measure used. Theoretically it is clear that an inverse relationship between yield and farm size is neither necessary nor sufficient for an inverse relationship to exist between farm size and TFP. This is an important point, as empirical evidence on the yield and farm size relationship has driven the modern debate on the existence and determinants of the IR, and is often referenced in policy making, particularly in issues of land reform.

How much does this critique matter? We conduct an empirical analysis of the Brazilian experience, using a pseudo-panel from 1985 to 2006 to contrast the yield – farm size relationship with the TFP – farm size relationship and how they themselves and their correlation have changed over time. In short, the choice of productivity measure matters. As in many developing country contexts, there exists an inverse relationship between yield and productivity in each period for Brazil, and within each of its five distinct macro regions in each period. In contrast, the TFP and farm size relationship varies across time and space.

In Brazil's poorer regions, the North and Northeast, yield serves as a decent yet crude proxy for TFP, as both reveal an overall inverse relationship. In contrast, in the South and Southeast, and increasingly in the Center-West as well, the use of yield is clearly inappropriate – the TFP and farm size relationship is characterized by an overall U-shape that has changed over time. The regional analysis of the TFP and farm size relationship shows that (1) yield is often inappropriate as a proxy for TFP; (2) the relationship between TFP and farm size is non-linear, best characterized by a U-shape in most cases; and (3) the very largest farms, those with more than 500 ha, are important to consider when assessing any relationship between farm size and productivity. In the case of Brazil, the large farms have become more productive than the medium sized farms, and have been the principal force transforming the inverse relationship into a U-shaped relationship.

Moving forward, the development economics literature on the IR would benefit from more careful treatment of the productivity measures being used in empirical assessments. While yield may be an appropriate proxy for more preferred measures in setting of low productivity and slow productivity growth, such an assumption clearly requires justification, as exemplified by the Brazilian experience.

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## Appendix A

Proof of expression (22).

Allowing  $x \in \{\pi_A, q\}$  to generalize equations (20) and (21), respectively, we have:

$$\frac{\partial TFP}{\partial A} = \left(\frac{\partial x}{\partial A}\right) \left(\frac{1}{c(q)}\right) - \frac{\left(\frac{\partial c(q)}{\partial A}\right) x}{c(q)^2}$$

$$\frac{\partial TFP}{\partial A} = \left(\frac{\partial x}{\partial A}\right) \left(\frac{1}{c(q)}\right) \left[1 - \frac{\left(\frac{\partial c(q)}{\partial A}\right) x}{c(q) \left(\frac{\partial x}{\partial A}\right)}\right]$$

$$\frac{\partial TFP}{\partial A} = \left(\frac{\partial x}{\partial A}\right) \left(\frac{1}{c(q)}\right) \left[1 - \frac{\left(\frac{\partial c(q)}{\partial A}\right) \left(\frac{1}{c(q)}\right)}{\left(\frac{\partial x}{\partial A}\right) \left(\frac{1}{x}\right)}\right] = \left(\frac{\partial x}{\partial A}\right) \left(\frac{1}{c(q)}\right) \left[1 - \frac{\left(\frac{\partial c(q)}{\partial A}\right) \left(\frac{A}{c(q)}\right)}{\left(\frac{\partial x}{\partial A}\right) \left(\frac{A}{x}\right)}\right]$$

$$\frac{\partial TFP}{\partial A} = \left(\frac{\partial x}{\partial A}\right) \left(\frac{1}{c(q)}\right) \left[\frac{\varepsilon_{x,A}}{\varepsilon_{x,A}} - \frac{\varepsilon_{c(q),A}}{\varepsilon_{x,A}}\right]$$

$$\frac{\partial x}{\partial A} = \left(\frac{\partial TFP}{\partial A}\right) c(q) \left[\frac{\varepsilon_{x,A}}{\varepsilon_{x,A} - \varepsilon_{c(q),A}}\right]$$

## Appendix B

**Table A1: Data Cleaning and Sample Size, by Region and Farm Size Class**

Region	Farm Size Class (ha)	N	Less Farm Size Outliers	Less Tails of Yield Distribution	Less Yield Outliers	Percent Dropped from Cleaning
North	0-5	315	315	307	307	1.6%
	5-20	420	420	410	403	4.0%
	20-100	459	459	449	437	4.8%
	100-500	443	443	433	433	2.3%
	500-	323	315	307	305	3.2%
Northeast	0-5	3674	3674	3600	3600	1.4%
	5-20	3712	3712	3636	3636	0.9%
	20-100	3705	3705	3629	3625	2.2%
	100-500	3514	3514	3442	3441	2.1%
	500-	2159	2127	2083	2083	1.2%
Southeast	0-5	3850	3850	3772	3772	1.2%
	5-20	3991	3991	3911	3911	1.6%
	20-100	4024	4024	3942	3942	2.0%
	100-500	3896	3896	3818	3818	0.6%
	500-	2235	2215	2169	2169	0.9%
South	0-5	1726	1726	1690	1690	0.1%
	5-20	1754	1754	1718	1712	2.4%
	20-100	1769	1769	1733	1730	2.2%
	100-500	1523	1523	1491	1490	2.2%
	500-	751	745	729	726	2.6%
Center-West	0-5	537	537	525	520	3.2%
	5-20	619	619	605	605	2.3%
	20-100	681	681	667	667	1.2%
	100-500	672	672	658	658	1.9%
	500-	613	595	583	581	2.4%
<i>Total</i>		47365	47281	46307	46261	2.3%

