OPTIMAL ROSCAS

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1. Abstract

We are able to show that in an environment without an intertemporal technology but with an indivisible good, a ROSCA-like form which gives members fixed "turns" at receiving a pot is in fact an optimal mechanism. We are also able to show that in the face of enforcement problems a well-designed ROSCA can in fact overcome these enforcement problems, even without any outside enforcement or social sanctions beyond exclusion from the ROSCA.

An interesting feature of the operation of an optimal ROSCA is that the ROSCA can be regarded as substituting for credit markets, with implied interest rates that will vary depending on how far the constrained optimal allocations are from a first-best allocation. This allows us to understand when the introduction of an intertemporal technology or credit markets would displace (as in Anderson, Baland, and Moene 2009) or improve upon (as in Besley, Coate, and Loury 1993) the ROSCA as an allocation mechanism.

2. Introduction

Consider a set of people who are all members of a Rotating Savings and Credit Association (ROSCA). The defining characteristic of this ROSCA is that every period every member contributes some amount of money to the ROSCA, and, taking turns, every period a single member "takes the pot", receiving the entire sum of members' contributions to the ROSCA that period.

ROSCAs of this sort are very common in low income countries. But they seem to be designed to make total income *less* smooth for any given member, which seems odd, given the importance that smooth consumption is given in other parts of the literature on financial arrangments in low income settings. It's often argued that ROSCAs are important because they make it possible for members to make occasional purchases of indivisible durable goods, but it's been understood since Besley, Coate, and Loury (1993) that the ordinary use of credit

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markets appears to make similar indivisible purchases feasible at a lower cost. On the other hand, the ROSCA seems to be admirably well-designed if its function is to permit the occasional purchase of indivisible goods in a setting in which borrowing and saving is infeasible (or perhaps just very expensive).

Problems with credit markets in low-income countries are widely documented. Why savings should be infeasible is less clear, though apparent constraints on savings have been carefully documented in a variety of settings (e.g., Dupas and Robinson 2013; Adams and Fitchett 1992). However, Anderson and Baland (2002) argue that differences in desired savings within households may provide an explanation. They collect data from households in a large slum in Kenya, and from the ROSCAs to which people in these households belong. Some stylized facts about the ROSCAs they observe: married women are the most likely participants; reports from some of these women indicate that they're motivated to participate in ROSCAs because they need to save for indivisible expenditures; and, in the words of one of their respondents, "joining a [ROSCA] is the only way to save some money. If I leave it at home, it will disappear."[1] Another way the value of money "left at home" can disappear is in environments with high inflation; ROSCAs in Indonesia (Lindauer 1971) and Northern Cyprus (Khatib-Chahidi 1995) have avoided this problem by pegging contributions to the price of some commodity such as rice or gold.

Motivated by this kind of evidence, we explore a simple model in which people receive utility from both a consumption good and from the flow of services derived from possession of some indivisible consumer durables. The environment we consider is close to that found in Anderson, Baland, and Moene (2009), and as in that paper, our principal concern is with the issue of commitment or enforcement; but unlike that paper we assume that saving is infeasible.[2] Anderson, Baland, and Moene (2009) provide stylized descriptions of two different ROSCA forms, differing as to how priority in receipt of the indivisible durables is received. One of the main contributions of that paper is to

1. This quote is taken to allude to the possibility that money can't be committed to savings because another household member (perhaps with different savings preferences) may reallocate it. In a formally related model, Tanaka and Nguyen (2009) locate the commitment problem not within the household, but within the individual, where the latter is assumed to be a sophisticated hyperbolic discounter.

2. Klonner (2003) also assumes an environment without any intertemporal technology, and considers whether bidding ROSCAs might serve as optimal risk-sharing mechanisms in settings with risk and private information.

show that neither of those two stylized ROSCA forms can survive without some exogenous enforcement technology—in the absence of such a technology, any member who has most recently received the "pot" will always prefer to leave the ROSCA rather than to continue.

While Anderson, Baland, and Moene (2009) consider stylized versions of particular institutions, they explicitly do not consider the mechanism design question of whether these particular forms are optimal given the assumed environment. That is the task we undertake here. We are able to show that in an environment without an intertemporal technology but with an indivisible good, a ROSCA-like form which gives members fixed "turns" at receiving a pot is in fact optimal. We are also able to show that in the face of enforcement problems a well-designed ROSCA can in fact overcome these enforcement problems, even without any outside enforcement or social sanctions beyond exclusion from the ROSCA.

ROSCAs are an optimal mechanism in the environment we consider. An interesting feature of the operation of an optimal ROSCA is that the ROSCA can be regarded as substituting for credit markets, with implied interest rates that will vary depending on how far the constrained optimal allocations are from a first-best allocation. This allows us to understand when the introduction of an intertemporal technology or credit markets would displace (as in) or improve upon (as in Besley, Coate, and Loury 1993) the ROSCA as an allocation mechanism.

An interesting feature of the optimal ROSCAs we describe is that contributions do not vary over time in a first-best allocation. This matches the contribution schedules assumed in recent treatments of ROSCAs in the economics literature. However, when ROSCA members are less patient (have lower discount factors) the optimal contribution schedule for a given member may vary across time, depending how many turns remain until that member receives the pot. Though such variable contribution schedules are not described in the economics literature, they are described by other social scientists in a variety of different settings. For example, ROSCAs with variable contributions are mentioned by Sethi (1995) (p. 174) in India, in Cameroon by Niger-Thomas (1995) (p. 106), and described in careful detail in pre-revolutionary rural China by Fei (1939) (pp. 267–274).

3. Model

3.1. **Preferences, Commodities, and Prices.** People derive utility from a consumption good $c \in \mathbb{R}_+$ and from a flow of services from consumer durables. The consumer durables are indivisible, and must be purchased in integer quantities, but each durable owned by the consumer yields a flow of services over many periods. For simplicity, there is no depreciation of these durables, so that a consumer who owns $d \in \mathbb{N} = \{0, 1, 2, \dots\}$ durables will receive d units of services in every period hence.

We follow Besley, Coate, and Loury (1993) in taking the utility function to be quasi-linear in services. There is time, indexed by t. Thus, let momentary utility at date t be given by $u(c_t) + d_{t-1}$, so that one derives services from durables brought into the period. Unlike Besley, Coate, and Loury (1993) we allow future utility to be discounted, using a common factor β. Thus, lifetime utility for a person with a sequence of consumptions and durables $\{(c_t, d_t)\}$ is

$$\sum_{t=0}^{\infty} \beta^t [u(c_t) + d_{t-1}] = \sum_{t=0}^{\infty} \beta^t u(c_t) + (1 - \beta)^{-1} \sum_{t=0}^{\infty} \beta^t \Delta d_{t-1},$$

where $\Delta d_t = d_t - d_{t-1}$ for $t \geq 0$, and where $d_{-1} = 0$.

The consumption good c is taken as numéraire; a durable costs p. Each period a consumer receives an endowment y of the numéraire good. Our timing convention is that the purchase of a durable good at a particular period begins to yield services only in the subsequent period; thus, a single durable received at time t yields one additional util in every period after t, or total discounted lifetime utility of $\beta/(1-\beta)$ from the consumer's point of view at time t.

Throughout, we will often find it instructive to consider a special case, in which the utility function $u(c) = \log(c)$. This choice of utility allows us to abstract from issues of scale, allowing a representation of the problem which depends only on the ratio of individual consumption to per capita income. Similarly, taking durable services to be perfectly substitutable across periods (or, from the social planner's point of view, across people) allows us to focus on incentive-related reasons for scheduling the assignment of durables to different ROSCA members, avoiding any 'service-smoothing' motive which might otherwise confuse the issue.

3.2. **Autarkic Investment.** Now, consider the problem facing an autarkist in this environment who already possesses d durables. Each period he receives an endowment y, and must allocate it between consumption and the purchase of additional durables.

His decision to purchase additional durables or not is necessarily a discrete decision. The problem is stationary, so if it doesn't make sense for him to purchase any additional durables this period, he will consume y, and behave in exactly the same fashion in all future periods.

Then the discounted utility of consuming income without making any further purchases of durables is given by $(u(y)+d)/(1-\beta)$. If $p \geq y$ then it will never be feasible for the autarkist to purchase the durable. Otherwise, assuming $p < y$ and comparing this with the discounted utility of purchasing an additional unit of d, we see that an autarkist will choose to purchase another durable if and only if

$$u(y-p)+d+\frac{\beta}{1-\beta}u(y)+\frac{\beta}{1-\beta}(d+1) \geq \frac{u(y)}{1-\beta}+\frac{d}{1-\beta}.$$

This simplifies to

$$u(y)-u(y-p) \leq \frac{\beta}{1-\beta},$$

or $\log(y/(y-p)) \leq \beta/(1-\beta)$ for the case of logarithmic utility. This requirement is simply a statement that the utility cost of purchasing a durable (the left-hand side) is less than the benefit, equal to the discounted value of the flow of services from purchasing the durable.

3.3. **Optimal Group Investment.** Now consider a group of N people, all with identical preferences and endowments. Though they are assumed not to be able to share durables or the flow of services from those durables, the optimal rate of investment in durable goods is nevertheless generally greater than it would be for a set of N autarkists, in the sense that for some prices, incomes, and discount factors a ROSCA of size N would make durable purchases where an autarkist would not. To see this, consider the problem facing a planner maximizing the sum of utilities of people in the group. The planner takes as given some number of durables d at the beginning of the period, and solves the dynamic program

$$V(d)=\max_{c_i,d_i,d'}\sum_{i=1}^{N}u(c_i)+d_i+\beta V(d') \tag{1}$$

subject to a collective budget constraint

$$\sum_i c_i+p(d'-d) \leq Ny \tag{2}$$

and subject to the requirement that total durables allocated within the period not exceed the total available, or $\sum_i d_i = d$.

Proposition 1. *The optimal allocation of consumption* $\{c_i^*\}$ *and investment in durables* Δd^* *are determined by*

$$c_i^* = y-\frac{p}{N}(\Delta d) \qquad \text{for } i=1,\ldots,N;$$

and

$$\Delta d^* = \arg\max_{\Delta d \in \mathbb{N}} Nu\left(y - \frac{p}{N}\Delta d\right) + \frac{\beta}{1-\beta}\Delta d. \tag{3}$$

Proof. The optimal assignment of consumption must satisfy a set of N first order conditions. In the general case these are

$$u'(c_i) = \mu,$$

with μ the value of the multiplier on the budget constraint. Since μ doesn't depend on i it's (unsurprisingly) the case that every member of the group will be assigned identical consumption, so that (using the budget constraint) $c_i = y - \frac{p}{N}(d' - d)$ for $i = 1, \dots, N$.

The decision to invest in additional durables is discrete, just as it was for the autarkist, so the solution to the investment decision is not characterized by first order conditions, but by a set of inequalities. Let $V_m(d)$ be the value function for a planner who brings d durables into the period with a policy of purchasing an additional m durables in every period. Since we know the optimal consumption conditional on these investments, (1) implies

$$V_m(d) = Nu(y - mp/N) + d + \beta V_m(d + m).$$

The linearity of the utility function in d implies that $V_m(d + m) = V_m(d) + m/(1 - \beta)$, so it follows that $V_m(d) = Nu(y - mp/N) + d + \beta V_m(d) + m\beta/(1 - \beta)$, and $V_m(d) = (1 - \beta)^{-1}[Nu(y - mp/N) + d + m\beta/(1-\beta)$. Then the optimal investment policy simply involves choosing the maximal element of the set $\{V_m(d)\}$. Modifying each of these functions by multiplying by a constant $(1 - \beta)$ and subtracting the common d doesn't affect the choice problem, which appears in (3). $\square$

Reasoning as in the single agent case, the planner prefers to invest one unit of the durable good each period rather than none when

$$Nu(y - p/N) + \tfrac{\beta}{(1-\beta)} \geq Nu(y). \tag{4}$$

The left hand side is the change in utility from one extra durable. The cost p is divided between the N members of the group. The benefit to the group is the flow benefit of one extra durable that occurs ad infinitum from next period on.

The marginal cost in the group case is

$$U_0^N := N\left(u(y) - u(y - p/N)\right).$$

$U_0^1 = u(y) - u(y - p)$ as above. By the concavity of the utility function this marginal cost is decreasing in N. To see this, differentiate the marginal cost to get

$$\frac{\mathrm{d}U_0^N}{\mathrm{d}N} = (u(y) - u(y - p/N)) - u'(y - p/N)(p/N),$$

which is negative by the concavity of u. The reason is that with concave utility cost sharing improves aggregate utility because it reduces the variability in consumption over the N agents.

The marginal cost of a second extra unit is

$$U_1^N := N\left(u(y - p/N) - u(y - 2p/N)\right).$$

Note that

$$\begin{aligned}\frac{\mathrm{d}U_1^N}{\mathrm{d}N} = ((u(y - p/N) - u(y - 2p/N)) - u'(y - 2p/N)(p/N))\\ + \left(u'(y - p/N) - u'(y - 2p/N)\right) p/N.\end{aligned}$$

This is again negative. Let $m = \max\{i \in \mathbb{N} : y - (m+1)p/N > 0\}$. Let $U_i^N := u(y - ip/N) - u(y - (i+1)p/N)$, for $i = \{0, 1, \ldots, m\}$. The value of m is increasing in N and y, and decreasing in p. The marginal utility U_i^N is increasing in i because utility u is strictly concave. The solution is $\Delta d = i$ when $U_i^N \leq \beta/(1-\beta) < U_{i+1}^N$. The solution for Δd is a constant because if $\Delta d < i$ it always is beneficial to increase Δd to i, because the marginal utility cost is lower than the marginal benefit of an extra durable and never optimal to set $\Delta d > i$ because in that case the marginal cost exceeds the marginal benefit.

Notice that, because the dynamic program is linear in the d_i, the planner is entirely indifferent to which particular people are given durables—all she cares about is the total durables within the group. Notice also that the corresponding condition for the autarkist is a special case of this, with $N = 1$. An easy corollary for the case of logarithmic utility gives conditions under which the planner will purchase exactly one durable every period.

Corollary 1. *The optimal rate of investment in durables for a group of size N will be one durable per period if and only if*

(5)
$$\frac{N[u(y) - u(y - p/N)]}{1 + N[u(y) - u(y - p/N)]} \leq \beta < \frac{N[u(y - p/N) - u(y - 2p/N)]}{1 + N[u(y - p/N) - u(y - 2p/N)]}.$$

Proof. From (3), for $\Delta d^* = 1$ we must have

$$Nu\left(y - \frac{p}{N}\right) + \frac{\beta}{1-\beta} \geq Nu(y)$$

and

$$Nu\left(y-\frac{p}{N}\right)+\frac{\beta}{1-\beta}>Nu\left(y-2\frac{p}{N}\right)+\frac{\beta}{1-\beta}2.$$

We can use the first of these to find a lower bound on β such the planner will choose $\Delta d \geq 1$, and use the second to find an upper bound on β such that the planner will choose $\Delta d < 2$. □

When considering ROSCAs of optimal size, so far as allocations go there's no loss of generality in considering only ROSCAs that satisfy the condition (**??**), so for practical purposes we can regard this condition as a restriction on N for ROSCas of optimal size.

In the logarithmic case (5) becomes

$$\frac{-N[\log(1-\sigma/N)}{1-N\log(1-\sigma/N)}\leq\beta<\frac{N[\log(1-\sigma/N)-\log(1-2\sigma/N)]}{1+N[\log(1-\sigma/N)-\log(1-2\sigma/N)]},$$

where $\sigma = p/y$ is the share of the cost of the durable relative to per person income.

As before, the case of autarky is a special case with $N = 1$. Also note that if $\sigma \geq N$ this expression is not well-defined, but of course in this case purchasing the durable would not be feasible in any case.

Notice that in the general case of (5) these inequalities depend on four quantities: β, y, p, and N, while in the logarithmic case the scale-free nature of the preferences reduces the number of quantities to three: β, σ and N.

3.4. **When ROSCAs have value.** Now consider interpreting the group of the previous section as a ROSCA. It will be optimal for the ROSCA to purchase exactly one unit of the durable in every period so long as income y, the cost share of the durable σ/N, and the common discount factor β satisfy (5). This is the case of interest: if β and σ were such that it was never optimal to buy any durables, there would be no point to having a ROSCA; while if β and σ were such that it was optimal to buy *more* than one durable each period it turns out that we can achieve identical outcomes by dividing up the group into *two* ROSCAs, each buying a single durable.

There would also be no point to the ROSCA if β and σ were such that an autarkist would always buy a durable. Thus, we say that a ROSCA of size N has value whenever it would be optimal for such a ROSCA to purchase at least one durable every period while an autarkist would not. Adopting (5), the condition for the ROSCA to have value is

$$(6) \qquad \frac{N[u(y) - u(y - p/N)]}{1 + N[u(y) - u(y - p/N)]} \leq \beta < \frac{[u(y) - u(y - p)]}{1 + u(y) - u(y - p)]}.$$

For the case of logarithmic utility, by rearranging the first inequality in (5) we see that this will be the case whenever

$$1 - \exp\left(\frac{\beta}{\beta - 1}\right) < \sigma \leq N\left(1 - \exp\left(\frac{\beta}{N(\beta - 1)}\right)\right).$$

The right-most expression is positive and strictly increasing in both N and β. It converges to N as $\beta \to 1$, and converges to $\beta/(1-\beta)$ as $N \to \infty$. Since it's increasing in N it's immediately clear that in the logarithmic case larger ROSCAs may have value where smaller ones do not. This depends only on the discount factor β and the share parameter σ. More particularly, provided only that $\sigma/N < 1$, there's a value of β large enough such that the ROSCA (or even the autarkist, with $N = 1$) will purchase the durable. Conversely, for any $\sigma < \beta/(1-\beta)$ and $\beta \in (0, 1)$ such that the autarkist doesn't purchase the durable, there's some ROSCA large enough for it to be optimal for that ROSCA to make such a purchase, and hence have value.

However, the indivisibility of durables means that a somewhat larger N won't always help. For example, if $\sigma = 1$ and $\beta = 1/2$, then no ROSCA of any size would ever purchase the durable. Even when $\sigma < \beta/(1-\beta)$ rather large increases in size may be necessary for a ROSCA to have value. For example, if we take $\beta = 0.8$ and $\sigma = 2$, then the smallest ROSCA which will have value would be a ROSCA of three people—just moving from one to two would have no benefit over autarky.

Though a ROSCA of size N may have value, it does not follow that N is necessarily the optimal size.[3] Consider a countably infinite population, from which individuals are selected to form a ROSCA of size N. The value of the ROSCA can be thought of as the increase in the sum of individual utilities which arise from participation in the ROSCA, but while a ROSCA of size N may have positive value, an alternate ROSCA of size $N' \neq N$ might well have a greater value. A ROSCA of size N and value V is said to be of optimal size there does not exist a ROSCA of size N' with value V' such that $V' > V$.

3.5. **When ROSCAs are constrained.** Suppose that a ROSCA of size N has value, but that any member of the group can leave whenever

3. Or necessarily the optimal composition, but in the environment of identical people we consider the issue of composition does not arise.

she chooses. Then keeping the ROSCA together requires that every member be at worst indifferent between staying and leaving.

If a ROSCA member leaves, they are assumed to take their assigned durables and become (and remain) autarkists. Since the ROSCA has value, this implies that the autarkist will never choose to buy any additional durables, and the discounted utility of an autarkist assigned d_i durables before defection will be $(u(y) + d_i)/(1 - \beta)$.

3.5.1. *Claim:* $|d_i - d_j| \leq 1$. In the case with full commitment any allocation of durables across agents is consistent with optimality. But with limited commitment, it will be optimal to assign durables in such a way that there is never one agent with two more durables than any other agent. This follows from the fact that taking one of the durables from the agent with more and giving it to the agent without will relax the former's sustainability constraint...

3.5.2. *Claim: It's optimal to give new durables in turn.* Given the previous claim, this just follows from discounting, by a similar argument: the agent who has to wait longest to receive a new durable is the one with the binding sustainability constraint.

3.6. **TODO Stationary optimal ROSCA design.** In any period, different ROSCA members will find it more or less costly to remain in the ROSCA depending on how soon they expect to next receive the durable. Just from the assumption of exponential discounting, then, we can see that the member who's just received the durable will face the highest cost of remaining in the ROSCA; the member who received in the previous period faces the second highest costs, and so on.

An optimal ROSCA design will take these differing constraints into account, and, if need be, compensate each member for the time they must wait by offering them different levels of consumption. In an optimal stationary arrangement, the consumption of player i may not be the same every period, but instead will repeat every N periods. The surplus derived from participating in the ROSCA over autarky can then be expressed recursively. For a member with i periods before receiving the durable, we have

$$V_i = u(c_i) - u(y) + \beta V_{i-1} \qquad \text{for } i = 1, \ldots, N \text{ and,} \tag{7}$$
$$V_0 = V_N + 1/(1 - \beta).$$

The requirement that no member ever prefers autarky to participation is then

$$V_i \geq 0 \qquad \text{for } i = 1, \ldots, N. \tag{8}$$

The discounted values for each member $i = 1, \dots, N$ can also be written as

$$
\begin{aligned}
(9) \quad V_i = {} & (1-\beta^N)^{-1} \sum_{j=0}^{i-1} \beta^j \left(u(c_{i-j}) - u(y)\right) \\
& + (1-\beta^N)^{-1} \sum_{j=0}^{N-i-1} \beta^{i+j} \left(u(c_{N-j}) - u(y)\right) + \frac{\beta^i}{(1-\beta)(1-\beta^N)}
\end{aligned}
$$

In particular, the value for the person with longest to wait before receiving the pot will be

$$
\begin{aligned}
(10) \quad V_N = \Bigg[& (u(c_N) - u(y)) + \beta \left(u(c_{N-1}) - u(y)\right) + \dots \\
& + \beta^{N-2} \left(u(c_2) - u(y)\right) + \beta^{N-1} \left(u(c_1) - u(y)\right) \\
& + \frac{\beta^N}{(1-\beta)} \Bigg] (1-\beta^N)^{-1}.
\end{aligned}
$$

The following lemma gives us conditions for an optimal ROSCA which help to rule out both negative transfers to the ROSCA, and which restricts the size of the ROSCA, and which allows us to restrict our attention to the case in which at most one of the constraints (9) is binding.

Lemma 1. *If the ROSCA has value, then*

(1) *Consumption* $c_i \leq y$ *for all* $i = 1, \dots, N$*;*
(2) $V_i \leq V_{i-1}$ *for* $i = 2, \dots, N$*;*
(3) *If any constraint (8) is binding, then among the binding constraints will be* $V_N = 0$*;*
(4) *For all* $i \neq N$ *consumption if* $c_i < y$ *then* $V_i > 0$.

Proof. First, suppose that the ROSCA has no members constrained. Then consumption will be equal across members in every period, and $c_i \leq y$ from the budget constraint (2). Alternatively, suppose that member i is constrained. Then i is indifferent between being in ROSCA and not. But if her consumption $c_i > y$ then the ROSCA size is not optimal; the sum of surpluses from the ROSCA will be greater if i leaves, as this will both relax the ROSCA's budget constraint and also shorten the time required to win the pot.

Second, since $c_i \leq y$, we have $u(c_i) - u(y) \leq 0$. Substituting this into (7) yields the monotonicity result that $V_i \leq V_{i-1}$..

Third, from this monotonicity of $\{V_i\}$ it follows that if any member is constrained then member N must be constrained.

Fourth, consider the contrary; we have $c_i < y$ but $V_i = 0$. two members are constrained then these must be N and $N-1$. But if $N-1$ is constrained then $V_{N-1} = 0$, and $V_N = u(c_N) - u(y) + \beta V_{N-1} = 0$ implies that $c_N = y$. But then using the argument above N can't the the optimal size of the ROSCA; member N is indifferent between being in the ROSCA and not, while the remaining members of the ROSCA would benefit from more frequent receipt of the durable if N were to leave. □

An ordering of all the surpluses V_i turns out to be more than we need; all that is really required is the weaker condition that $V_N \leq V_i$ for $i = 1, \ldots, N-1$. Even with this weaker condition, we know we'll never need to be concerned with any sustainability constraint save $V_N \geq 0$.

The structure of the sharing rule for the ROSCA turns out take one of three forms. In the first form, no limited commitment constraints bind, and allocations are first best. In the second form, the limited commitment constraint for N binds, and N has autarkic levels of welfare and $c_N = y$. The third form is intermediate, with the member N periods from receiving the pot indifferent between remaining in the ROSCA or not, but having $c_N < y$. Further, these different forms can all be indexed by the value of the discount factor β. We consider each in turn below.

3.6.1. *First best levels of welfare.* As we've seen above, the first-best arrangement is characterized by c_i equal to a constant c. In this case, the constraint $V_N \geq 0$ can be written

$$u(c) - u(y) + \frac{\beta^N}{1 - \beta^N} \geq 0.$$

There exists a critical $\bar{\beta}$ which satisfies this with equality; for values of $\beta > \bar{\beta}$ allocations will be first best. Note that the critical $\bar{\beta}$ will vary with y, p, and N. Alternatively, for fixed β we can regard this as a restriction on the size of the ROSCA. The term $\beta^N/(1-\beta^N)$ is decreasing (without bound) in N. Since $u(c) - u(y) = u(y - p/N) - u(y) < 0$, for any fixed β there's a maximal N which satisfies the inequality.

Note that this first best allocation is exactly what we'd expect to observe if each member took turns receiving the entire aggregate endowment of Ny every N periods, and engaged in borrowing and lending at an interest rate $1/\beta - 1$.

3.6.2. **TODO** *Autarkic levels of welfare.* Now suppose that the constraint $V_N = 0$ is binding, but the V_i are such that this is the only sustainability constraint that binds. Given complete freedom to schedule consumption over time, we can think of the member N periods from receiving the durable as solving the problem

$$V_N = 0 = \max_{c_i}(1-\beta^N)^{-1}\sum_{i=1}^{N}\beta^{N-i}[u(c_i)-u(y)] + \frac{\beta^N}{(1-\beta)(1-\beta^N)}$$

subject to the resource constraint $\sum_{i=1}^{N} c_i \leq Ny-p$. Notice an interesting feature of this stationary problem: the act of choosing consumption for other members of the ROSCA is the same as the act of choosing the path of *future* consumption for a given member. In fact, this formulation of the problem is exactly what it would be for an *autarkist* who received an endowment of Ny every N periods and purchased a durable in that same period, and who could save (or borrow) at an interest rate of zero.

In line with this interpretation, the solution to this problem must satisfy a sort of Euler equation

$$u'(c_i) = \beta u'(c_{i-1}), \tag{11}$$

where we bear in mind that c_{i-1} is *next* period's consumption for a member who presently has i turns to wait to receive the durable. Also, by the inverse function theorem, we have $c_{i-1} = \beta/u'(c_i)$.

If we fix consumption c_N, then the Euler equation determines the consumption profile $\{c_i(c_N,\beta)\}_{i=1}^{N}$ over N periods as a function of this initial condition and the discount factor. This consumption profile must satisfy the resource constraint $\sum_{i=1}^{N} c_i(c_N,\beta) = Ny-p$. Since the sustainability constraint for N must be satisfied with equality

$$\sum_{i=1}^{N}\beta^{N-i}[u(c_i(c_N,\beta)) - u(y)] + \frac{\beta^N}{1-\beta} = 0.$$

This equation and the Euler equation (12) determine a number $\underline{\beta}$, an initial consumption c_N, and the rest of the consumption profile. The value $\underline{\beta}$ is the smallest value of the discount factor consistent with the functioning of the ROSCA. When $\beta = \underline{\beta}$, then the person N periods away from receiving the durable will be indifferent between staying in the ROSCA or becoming an autarkist. What about the other people in the ROSCA? For values of $\beta < \underline{\beta}$ the the social surplus function $\sum_{i=1}^{N} V_i$ will be zero. But at $\beta = \underline{\beta}$ this social surplus will have a discontinuous jump from zero to some positive number, as the ROSCA becomes viable and the ROSCA collectively purchases the indivisible durable.

3.6.3. *Intermediate levels of welfare.* For values of $\beta \in (\underline{\beta}, \bar{\beta})$ the ROSCA can function, but will deliver levels of welfare less than the first best because the provision of incentives requires variation in consumption across periods. We can describe this variation and the optimal operation of the ROSCA in this case by formulating a planner's problem, maximizing an equally-weighted sum of utilities. Since from the planner's point of view the sum of utilities in a stationary allocation will be constant, we can think of her as solving the static problem

$$\max_{\{c_i\}} \sum_{i=1}^{N} u(c_i)$$

subject only to the budget constraint

$$\sum_{i=1}^{N} c_i \leq Ny - p$$

and the sustainability constraint for N, simplified to become

$$\sum_{i=1}^{N} \beta^{N-i}(u(c_i) - u(y)) + \frac{1}{\beta^N - \beta} \geq 0.$$

If we associate with the budget constraint a multiplier η and with the sustainability constraint a multiplier μ, we obtain first order conditions

$$u'(c_j)\left[1 + \beta^{-j}\mu\right] = \eta \qquad \text{for } j = 1, \ldots, N.$$

Combining conditions for c_j and c_{j-1} we obtain an expression for the growth rate of marginal utility

$$\frac{u'(c_{j-1})}{u'(c_j)} = \frac{\mu + \beta^j}{\beta\mu + \beta^j};$$

alternatively, we can express this as an Euler equation

$$u'(c_j) = \beta R_j u'(c_{j-1}), \tag{12}$$

where 'returns' R_j vary across periods (or people), with

$$R_j = \frac{\mu + \beta^{j-1}}{\mu + \beta^j} \qquad \text{for } j = 2, \ldots, N,$$

and

$$R_1 = \frac{\mu + \beta^{-N}}{\mu + \beta}$$

the returns facing a member of the ROSCA depending on the number of periods remaining until they receive the durable.

The parameter μ measures the marginal cost of the sustainability constraint to the planner, so at the first best when this constraint isn't binding we have $\mu = 0$, and we recover $R_i = 1/\beta$, consistent with our discussion above. Similarly, when the survival of the ROSCA hinges on whether the person N periods away from receiving the durable can just be indifferent between autarky and remaining in the ROSCA (i.e., at $\beta = \underline{\beta}$) we've observed that the social welfare function is discontinuous; then μ is unbounded as $\beta \to \underline{\beta}$, and $R_i \to 1$.

It's straight-forward to check that R_i is decreasing in i, so that returns are increasing across time within a ROSCA cycle for an individual. One interpretation of this is that ROSCA members save an increasing amount (their consumption falls) as the period in which they receive the durable approaches. A second interpretation is that the ROSCA treats receipt of the pot as a sort of loan, the principal of which must be repaid with interest over the course of N periods according to a particular amortization schedule. Just such an interpretation can be found in the detailed description of a ROSCA in pre-revolutionary China (Fei 1939, pp. 267–274).

At any of these intermediate values of β the Euler equation and expressions for R_i determine the consumption profile across the cycle as a function of μ, β, and some initial c_N. Holding fixed $\beta \in (\underline{\beta}, \bar{\beta})$, this consumption profile must be consistent with the resource constraint and with the requirement that $V_N = 0$; thus, for any discount factor in this range we can completely determine the path of consumption.

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