

On Indeterminacy and Growth under Progressive Taxation and Productive Government Spending*

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Abstract

This paper examines the theoretical interrelations between equilibrium (in)determinacy and economic growth in a one-sector representative-agent model of endogenous growth with progressive taxation of income and productive flow of public spending. We analytically show that if the demand-side effect of government purchases is weaker, the economy exhibits an indeterminate balanced-growth equilibrium and belief-driven growth fluctuations when the tax schedule is sufficiently progressive or regressive. If the supply-side effect of public expenditures is weaker, indeterminacy and sunspots arise under progressive income taxation. In sharp contrast to traditional Keynesian-type stabilization policies, our analysis finds that raising the tax progressivity may destabilize an endogenously growing economy with fluctuations driven by agents' self-fulfilling expectations.

Keywords: Equilibrium (In)determinacy, Endogenous Growth, Progressive Income Taxation, Productive Government Spending.

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1 Introduction

Since the early 1990's, there has been an extensive literature that explores the macroeconomic effects of tax policy within various endogenous growth models. As it turns out, the vast majority of previous theoretical studies postulate a constant tax rate of income, which is used to finance government purchases of goods and services that could be “wasteful” in that they do not contribute to production or utility.¹ These assumptions, although commonly adopted for the sake of analytical simplicity, are not necessarily the most realistic *vis-à-vis* those observed in the actual data. Motivated by this gap in the existing literature, we examine a one-sector endogenous growth model with empirically-relevant characteristics in industrialized countries: progressive taxation of income and productive flow of public spending. Specifically, this paper provides a comprehensive analytical investigation of the interrelations between tax progressivity, equilibrium (in)determinacy and economic growth. Our work is valuable not only for its theoretical insights, but also for its important implications about the (de)stabilization role of tax policies in an endogenously growing economy.

In this paper, we systematically study the (local) stability effects of Guo and Lansing's (1998) nonlinear tax structure in a prototypical one-sector representative-agent model of endogenous growth with inelastic labor supply and productive public expenditures *a la* Barro (1990).² The Guo-Lansing taxation scheme possesses a progressive/regressive property, characterized by a single parameter, whereby the household's tax rate is an increasing/decreasing function of its taxable income relative to some baseline level. Our analyses are focused on the economy's unique balanced growth path along which output, consumption, physical capital and government spending all grow at a common positive rate. In particular, we analytically show that the relationship between indeterminacy and growth depends crucially on (i) the relative strength between the demand-side and supply-side effects of government purchases, and (ii) the sign and level of the slope parameter in the tax schedule that governs its progressivity feature.³

¹See, for example, Barro (1990), Jones and Manuelli (1990), King and Rebelo (1990), Rebelo (1991), Barro and Sala-i-Martin (1992), Futagami, Morita and Shibata (1993), Pecorino (1993), Glomm and Ravikumar (1994, 1997), Cazzavillan (1996), Turnovsky (1996, 1997, 1999), Zhang (2000), Baier and Glomm (2001), Yamarik (2001), Palivos, Yip and Zhang (2003), Chen (2006), Greiner (2007), and Hu, Ohdoi and Shimomura (2008), among others.

²Li and Sarte (2004) examine the growth and redistributive effects of the Guo-Lansing progressive policy rule in a one-sector endogenously growing economy with heterogeneous agents and public production services.

³Under a flat-rate income tax, Cazzavillan (1996) examines a one-sector endogenous growth model with inelastic labor supply and government purchases entering both the household's utility and the firm's production functions. In this case, the model economy exhibits multiple balanced growth paths and belief-driven growth fluctuations when the household preferences display increasing returns-to-scale in private consumption and public spending. Chen (2006) shows that Cazzavillan's results are qualitatively robust to incorporating the stock of public capital, rather than the flow of government spending, into the production technology. Moreover,

If the demand-side effect of public spending is weaker than its supply-side counterpart, we find that the economy exhibits equilibrium indeterminacy and belief-driven growth fluctuations under regressive income taxation or when the tax progressivity is positive and higher than a critical value. In either specification, start from a particular balanced growth path, and suppose that agents become optimistic about the future of the economy. Acting upon this belief, the representative household will reduce consumption and increase investment today, which in turn lead to another dynamic trajectory. Due to a dominating supply-side effect of government expenditures, the after-tax return on investment is shown to be monotonically increasing along the positively-sloped transitional path as the ratio of public spending to physical capital rises. As a result, agents' initial optimistic expectations are validated and the alternative path becomes a self-fulfilling equilibrium. By contrast, our model displays local determinacy and equilibrium uniqueness when the tax progressivity is zero, or positive but not sufficiently high. The above results together imply that under progressive income taxation, raising the tax progressivity can destabilize the economy by generating endogenous fluctuations caused by agents' animal spirits, provided the supply-side effect of government purchases is stronger.

If public expenditures exert a relatively weaker impact on the economy's supply side, we find that the economy exhibits an indeterminate balanced-growth equilibrium under progressive income taxation. In this case, when the household deviates from the original balanced growth path because of its optimism, the equilibrium after-tax marginal product of capital is shown to be rising along the downward-sloping transitional path as the ratio of government spending to physical capital falls. On the contrary, indeterminacy and sunspots do not arise within this formulation when the taxation scheme is flat or regressive. These results jointly imply that in sharp contrast to traditional Keynesian-type stabilization policies, changing the tax schedule from being flat or regressive to progressive will magnify the business cycle and thus destabilize the economy, provided the demand-side effect of government purchases is stronger.

This paper is related to recent work of Greiner (2006) who studies the growth and stability effects of a progressive tax policy in an endogenously growing one-sector representative-agent economy. Our analysis differs from his in three aspects. First, Greiner incorporates the stock of public capital into the firm's production function, whereas we consider the flow of productive government spending. Second, we maintain the assumption of balanced budget, whereas Greiner also examines the model with public debt. Third and most importantly, the

Zhang (2000) finds that various forms of local stability properties or Hopf bifurcations may arise in Cazzavillan's model when the social technology exhibits increasing returns in private capital and public expenditures.

baseline level of income in our tax schedule is set equal to output per capita on the balanced-growth equilibrium path, whereas Greiner postulates the economy-wide average income as the benchmark. Consequently, the balanced growth path in Greiner's model is always locally determinate because of constant equilibrium (average and marginal) tax rates.⁴ By contrast, indeterminacy and sunspots may arise within our model under time-varying taxation of income in equilibrium.

The remainder of this paper is organized as follows. Section 2 describes the model and analyzes the equilibrium conditions. Section 3 derives the economy's unique balanced growth path and the associated Jacobian matrix that governs its local stability properties. Section 4 analytically examines the interrelations between tax progressivity, equilibrium (in)determinacy and economic growth. Section 5 concludes.

2 The Economy

We incorporate a progressive/regressive income tax schedule *a la* Guo and Lansing (1998) into Barro's (1990) one-sector model of endogenous growth with productive government spending. The economy is populated by a unit measure of identical infinitely-lived households. Each household provides fixed labor supply and maximizes its discounted lifetime utility

$$U = \int_0^{\infty} \frac{c_t^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt, \quad \sigma \geq 1, \quad (1)$$

where c_t is consumption, $\rho > 0$ denotes the subjective rate of time preference, and σ represents the inverse of the intertemporal elasticity of substitution in consumption. Based on the empirical evidence for this preference parameter in the mainstream macroeconomics literature, our analysis is restricted to the cases with $\sigma \geq 1$. We also assume that there are no fundamental uncertainties present in the economy.

The budget constraint faced by the representative household is

$$c_t + i_t = (1 - \tau_t)y_t, \quad (2)$$

where i_t is gross investment, y_t is output and τ_t represents a proportional income tax rate. Output is produced by the following Cobb-Douglas technology (Barro, 1990):

⁴As in Greiner (2006), Slobodyan (2006) considers the same progressive taxation scheme in a one-sector endogenous growth model with elastic labor supply, variable capital utilization and public production services. However, Slobodyan's model may possess an indeterminate balanced-growth equilibrium because of aggregate increasing returns-to-scale in production and the congestion that government spending is subject to (Barro and Sala-i-Martin, 1992).

$$y_t = Ak_t^\alpha g_t^{1-\alpha}, \quad A > 0, \quad 0 < \alpha < 1, \quad (3)$$

where k_t is the household's capital stock and g_t is the productive service flow of government spending.⁵ Notice that $1 - \alpha$ captures the degree of positive external effect that public expenditures exert on the production process, and that the technology (3) exhibits constant returns-to-scale with respect to k_t and g_t such that sustained economic growth will arise in equilibrium. Investment adds to the stock of physical capital according to the law of motion

$$\dot{k}_t = i_t - \delta k_t, \quad k_0 > 0 \text{ given}, \quad (4)$$

where $\delta \in (0, 1)$ is the capital depreciation rate.

For the income tax rate, we adopt the sustained-growth version of Guo and Lansing's (1998, p.485, footnote 4) nonlinear tax structure and postulate τ_t as

$$\tau_t = 1 - \eta \left(\frac{y_t^*}{y_t} \right)^\phi, \quad \eta \in (0, 1), \quad \phi \in (\underline{\phi}, 1), \quad (5)$$

where y_t^* denotes a benchmark level of income that is taken as given by the representative household. In our model with endogenous growth, y_t^* is set equal to the level of per capita income on the economy's balanced growth path whereby $\frac{\dot{y}_t^*}{y_t^*} = \theta > 0$ for all t . The parameters η and ϕ govern the level and slope of the tax schedule, respectively. When $\phi > (<)0$, the tax rate τ_t is monotonically increasing (decreasing) with the household's income y_t , *i.e.* agents with income above y_t^* face a higher (lower) tax rate than those with income below y_t^* . When $\phi = 0$, we recover Barro's (1990) model in which all households face the constant tax rate $1 - \eta$ regardless of the level of their taxable income.

With regard to the progressivity features of the above taxation scheme, we first note that the marginal tax rate τ_{mt} , defined as the change in taxes paid by the household divided by the change in its taxable income, is given by

$$\tau_{mt} = \frac{\partial(\tau_t y_t)}{\partial y_t} = \tau_t + \eta \phi \left(\frac{y_t^*}{y_t} \right)^\phi. \quad (6)$$

Next, we restrict the analyses to an environment with $0 < \tau_t, \tau_{mt} < 1$ such that (i) the government does not have access to lump-sum taxes or transfers, (ii) the government cannot confiscate all productive resources, and (iii) households have incentive to provide factor services to the production process. Along the economy's balanced-growth equilibrium path where

⁵Alternatively, g_t can be interpreted as the stock of public capital with a depreciation rate of 100%. Allowing for not-fully-depreciated public capital will introduce another state variable to our model's dynamical system. This is an extension that is worth pursuing in future research.

$y_t = y_t^*$, these considerations imply that $0 < \eta < 1$ and $\frac{\eta-1}{\eta} < \phi < 1$. Moreover, to guarantee the convexity of the household's budget set, the after-tax marginal product of capital $(1 - \tau_{mt})MPK_t$ must be a strictly decreasing function of k_t , which in turn requires that $\phi > \frac{\alpha-1}{\alpha}$ on the balanced growth path. It follows that the lower bound on the slope parameter of the tax schedule (5) is determined by

$$\underline{\phi} = \max \left\{ \frac{\eta-1}{\eta}, \frac{\alpha-1}{\alpha} \right\}. \quad (7)$$

Given the postulated restrictions on η and ϕ , equation (6) shows that the marginal tax rate τ_{mt} is higher than the average tax rate τ_t when $\phi > 0$. In this case, the tax schedule is said to be “progressive”. When $\phi = 0$, the average and marginal tax rates coincide at the value $1 - \eta$ and the tax schedule is said to be “flat”. When $\phi < 0$, the tax schedule is “regressive”.

In making decisions about how much to consume and invest over their lifetimes, agents take into account the effect in which the tax schedule influences their net earnings. As a result, it is the marginal tax rate of income that will govern the household's economic decisions. The first-order conditions for the representative agent with respect to the indicated variables and the associated transversality conditions (TVC) are

$$c_t : \quad c_t^{-\sigma} = \lambda_t, \quad (8)$$

$$k_t : \quad \dot{\lambda}_t \left[\underbrace{\eta(1-\phi) \left(\frac{y_t^*}{y_t} \right)^\phi}_{(1-\tau_{mt})} \underbrace{\alpha \frac{y_t}{k_t}}_{MPK_t} - \delta \right] = \rho \lambda_t - \dot{\lambda}_t, \quad (9)$$

$$\text{TVC} : \quad \lim_{t \rightarrow \infty} e^{-\rho t} \lambda_t k_t = 0, \quad (10)$$

where (8) equates the marginal utility of consumption to its marginal cost λ_t , which is the Lagrange multiplier on the household's budget constraint (2) that also captures the shadow price of capital. Equation (9) is the Keynes-Ramsey condition that characterizes how the stock of physical capital evolves over time, and (10) is the transversality condition.

The government sets the income tax rate τ_t according to (5), and balances its budget at each point in time. Hence, the instantaneous government budget constraint is given by

$$g_t = \tau_t y_t. \quad (11)$$

Substituting (11) into the household's budget constraint (2), together with the law of motion for capital (4), yields the following aggregate resource constraint for the economy:

$$c_t + \dot{k}_t + \delta k_t + g_t = y_t. \quad (12)$$

3 Balanced Growth Path

We focus on the economy's balanced growth path (BGP) along which output, consumption, physical capital and government spending exhibit a common, positive constant growth rate θ . To facilitate the subsequent dynamic analyses, we adopt the following variable transformations: $z_t \equiv \frac{c_t}{k_t}$ and $x_t \equiv \frac{g_t}{k_t}$. Using (3), (5), and (11), the transformed variable x_t can be expressed as

$$x_t = A^{\frac{1}{\alpha}} \left[1 - \eta \left(\frac{y_t^*}{y_t} \right)^\phi \right]^{\frac{1}{\alpha}}. \quad (13)$$

Using these variable transformations, the model's equilibrium conditions (with $\frac{\dot{y}_t^*}{y_t^*} = \theta$ imposed) can be collapsed into the following autonomous dynamical system:

$$\frac{\dot{z}_t}{z_t} = \frac{1}{\sigma} [\alpha(1 - \phi)(A - x_t^\alpha)x_t^{1-\alpha} - \delta - \rho] - Ax_t^{1-\alpha} + x_t + z_t + \delta, \quad (14)$$

$$\frac{\dot{x}_t}{x_t} = \frac{\phi(x_t^\alpha - A)(\theta - Ax_t^{1-\alpha} + x_t + z_t + \delta)}{\alpha x_t^\alpha - \phi(1 - \alpha)(A - x_t^\alpha)}. \quad (15)$$

A balanced-growth equilibrium is characterized by a pair of positive real numbers (z^*, x^*) that satisfy $\dot{z}_t = \dot{x}_t = 0$. Since $y_t = y_t^*$ along the economy's balanced growth path, it is immediate from equation (13) that

$$x^* = [A(1 - \eta)]^{\frac{1}{\alpha}}. \quad (16)$$

From (14) and (16), it is then straightforward to show that

$$z^* = \frac{1}{\sigma} \left\{ A^{\frac{1}{\alpha}}(1 - \eta)^{\frac{1-\alpha}{\alpha}} [\sigma - \alpha\eta(1 - \phi)] + (1 - \sigma)\delta + \rho \right\} - [A(1 - \eta)]^{\frac{1}{\alpha}}, \quad (17)$$

and that the common (positive) rate of economic growth on the unique BGP is given by

$$\theta = \frac{1}{\sigma} \left[\alpha\eta(1 - \phi)A^{\frac{1}{\alpha}}(1 - \eta)^{\frac{1-\alpha}{\alpha}} - \delta - \rho \right]. \quad (18)$$

It follows that the growth effect of tax progression is negative, *i.e.* $\frac{\partial \theta}{\partial \phi} < 0$. When income taxation becomes more progressive as ϕ rises, the resulting after-tax rate of return on capital investment will fall, which in turn leads to a lower growth rate of output.

In terms of the BGP's local dynamics, we analytically compute the Jacobian matrix J of the dynamical system (14)-(15) evaluated at (z^*, x^*) . The determinant and trace of the Jacobian are

$$Det = \underbrace{\frac{z^* \eta (1 - \phi) A_{\alpha}^{\frac{1}{\alpha}} (1 - \eta)^{\frac{1-\alpha}{\alpha}}}{\sigma (1 - \eta)}}_{(+)} \frac{\phi (\eta - \alpha)}{\Omega}, \quad (19)$$

$$Tr = z^* + \frac{\phi \eta (\eta - \alpha) A_{\alpha}^{\frac{1}{\alpha}} (1 - \eta)^{\frac{1-\alpha}{\alpha}}}{\alpha (1 - \eta) \Omega}, \quad (20)$$

where

$$\Omega = 1 - \frac{\phi \eta (1 - \alpha)}{\alpha (1 - \eta)} \geq 0 \quad \text{when} \quad \phi \leq \hat{\phi} = \frac{\alpha (1 - \eta)}{(1 - \alpha) \eta} > 0, \quad (21)$$

and

$$\hat{\phi} = \frac{\alpha (1 - \eta)}{(1 - \alpha) \eta} \begin{matrix} \geq 1 \\ < 1 \end{matrix} \quad \text{when} \quad \alpha \begin{matrix} \geq \\ < \end{matrix} \eta. \quad (22)$$

The local stability property of our balanced-growth equilibrium path is determined by comparing the eigenvalues of J that have negative real parts with the number of initial conditions in the dynamical system (14)-(15), which is zero in our model economy because x_t and z_t are both non-predetermined jump variables. As a result, the BGP displays local determinacy and equilibrium uniqueness when both eigenvalues have positive real parts. If one or two eigenvalues have negative real parts, then the BGP is locally indeterminate and can be exploited to generate endogenous growth fluctuations driven by agents' self-fulfilling expectations or sunspots.

4 Analysis of Dynamics

Since $\sigma \geq 1$ and $A, z^* > 0$, the preceding analysis shows that the interrelations between the government's tax policy rule and macroeconomic (in)stability depend on (i) the demand-side effect of public expenditures represented by the BGP ratio of government purchases to output, *i.e.* $\left(\frac{g_t}{y_t}\right)^* = 1 - \eta$, (ii) the supply-side effect of government spending represented by the degree of positive external effect that g_t exerts on the production process $(1 - \alpha)$, and (iii) the sign and level of the slope parameter in the tax schedule ϕ that governs its progressivity feature. In this section, we systematically and comprehensively analyze the local dynamics around the model's balanced growth path in three parametric specifications.⁶

⁶When $\alpha = \eta$, the supply-side and demand-side effects of government spending are equal. Substituting $\alpha = \eta$ into (19) and (20) shows that the two eigenvalues of the Jacobian matrix are zero and $z^* > 0$. In this

4.1 When $\phi \neq 0$ and $1 - \eta < 1 - \alpha$

In this case, the demand-side effect of government spending is weaker than its supply-side counterpart, and the most-binding constraint on ϕ turns out to be a positive BGP marginal tax rate of income ($\tau_m^* > 0$), thus $\underline{\phi} = \frac{\eta-1}{\eta}$. Since $\alpha < \eta$, condition (22) states that $0 < \hat{\phi} = \frac{\alpha(1-\eta)}{(1-\alpha)\eta} < 1$. Using (19), (20) and (21), Figure 1 summarizes the model's local stability properties as the tax progressivity parameter ϕ changes over its feasible range. In the “*indet*” region, the Jacobian's determinant is negative ($Det < 0$) under regressive income taxation with $\frac{\eta-1}{\eta} < \phi < 0$ and $\Omega > 0$; or when the tax schedule is “sufficiently” progressive with $\hat{\phi} < \phi < 1$ and $\Omega < 0$. Hence, the economy's balanced-growth equilibrium exhibits local indeterminacy and belief-driven growth fluctuations.

The intuition for the above result can be understood with Figure 2 which depicts the phase diagram for these indeterminate configurations. Using (14) and (15), it is straightforward to show that the equilibrium loci $\dot{z}_t = 0$ and $\dot{x}_t = 0$ are upward sloping, and that the associated positively-sloped stable arm (denoted as *SS*) is steeper than the $\dot{z}_t = 0$ locus, followed by $\dot{x}_t = 0$. Next, start from a particular balanced growth path characterized by (z^*, x^*) , and suppose that agents become optimistic about the future of the economy. Acting upon this belief, households will invest more and consume less today, which in turn lead to another dynamic trajectory $\{z'_t, x'_t\}$ that begins at (z'_0, x'_0) with $z'_0 < z^*$ and $x'_0 < x^*$. Figure 2 shows that for this alternative path to become a self-fulfilling equilibrium, the after-tax return on investment $(1 - \tau_{mt})MPK_t$ must be monotonically increasing along the transitional path *SS* as the ratio of government spending to physical capital $x_t \equiv \frac{g_t}{k_t}$ rises. From (3), (5), (6) and (11), it can be shown that

$$\frac{d[(1 - \tau_{mt})MPK_t]}{dx_t} = \alpha(1 - \phi) \left[\frac{1 - \alpha}{\frac{g_t}{y_t}} - 1 \right]. \quad (23)$$

Since $\frac{g_t}{y_t}$ increases with respect to x_t (see equation 3), and $x'_t < x^*$ before the economy converges back to the original BGP, $\frac{g_t}{y_t} < \left(\frac{g_t}{y_t}\right)^* = 1 - \eta$ during the transition. It follows that $\frac{d[(1 - \tau_{mt})MPK_t]}{dx_t} > 0$ because of a dominating supply-side effect of public expenditures $\left(\frac{1 - \alpha}{1 - \eta} > 1\right)$, hence validating agents' initial optimistic expectations.

When the tax schedule is “relatively” less progressive with $0 < \phi < \hat{\phi}$ and thus $\Omega > 0$ (see equation 21), we find that both eigenvalues of the Jacobian matrix J have positive real parts ($Det > 0$ and $Tr > 0$), indicating the presence of local determinacy and equilibrium uniqueness, as in the “*deter*” region of Figure 1. In this specification, when households become

case, the economy will undergo a saddle-node bifurcation that may cause the hard loss of stability, *i.e.* the disappearance of the balanced growth path.

optimistic and decide to raise their investment expenditures today, the mechanism described in the previous paragraph that makes for multiple equilibria, *i.e.* an increase in the equilibrium after-tax marginal product of capital, will generate divergent trajectories away from the original balanced growth path. This implies that given the initial capital stock k_0 , the period-0 levels of the household's consumption c_0 as well as the government's productive spending g_0 are uniquely determined such that the economy immediately jumps onto its balanced-growth equilibrium (z^*, x^*) , and always stays there without any possibility of deviating transitional dynamics. As a consequence, equilibrium indeterminacy and endogenous growth fluctuations can never occur in this setting.

4.2 When $\phi \neq 0$ and $1 - \eta > 1 - \alpha$

In this case, government spending exerts a relatively stronger impact on the economy's demand side, and the most-binding constraint on $\underline{\phi}$ is that the equilibrium after-tax return on investment is monotonically decreasing in physical capital, hence $\underline{\phi} = \frac{\alpha-1}{\alpha}$. Next, substituting $\alpha > \eta$ into (22) yields $\hat{\phi} > 1$, which exceeds the feasible upper bound of the tax progressivity (see equation 5). Using (19) and (21), it is then straightforward to show that $Det < 0$ and $\Omega > 0$ under progressive income taxation with $0 < \phi < 1$, hence the economy exhibits equilibrium indeterminacy and sunspot fluctuations.

Figure 3 presents the phase diagram for this indeterminate formulation. As in Figure 2, the $\dot{x}_t = 0$ locus is flatter than $\dot{z}_t = 0$, and the associated stable arm SS is the steepest; however, all of them are now downward sloping, indicating that z_t and x_t move in the opposite direction. When the household deviates from the original BGP (z^*, x^*) and decreases today's consumption due to its optimism about the economy's future, the resulting dynamic trajectory $\{z'_t, x'_t\}$ will begin at (z'_0, x'_0) with $z'_0 < z^*$ and $x'_0 > x^*$. Figure 3 shows that when $x_t \equiv \frac{g_t}{k_t}$ falls along the convergent transitional path SS , the equilibrium after-tax marginal product of capital $(1 - \tau_{mt})MPK_t$ must be rising in order to justify $\{z'_t, x'_t\}$ as a self-fulfilling equilibrium path. Using (23) and the same subsequent arguments, we find that this requisite condition is satisfied, *i.e.* $\frac{d[(1 - \tau_{mt})MPK_t]}{dx_t} < 0$, because the demand-side effect of public expenditures is stronger $\left(\frac{1 - \alpha}{1 - \eta} < 1\right)$.

When the tax schedule is regressive with $\frac{\alpha-1}{\alpha} < \phi < 0$, it can easily be shown that the Jacobian's determinant and trace are both positive ($Det > 0$ and $Tr > 0$). This implies that the economy displays local determinacy, and thus any belief-driven deviation from the initial balanced-growth equilibrium (z^*, x^*) results in divergent trajectories that will violate the transversality condition (10). It follows that indeterminacy and sunspots do not arise in our model under regressive income taxation provided $1 - \eta > 1 - \alpha$.

4.3 When $\phi = 0$ and $\alpha \neq \eta$

In this case, we recover Barro's (1990) model with a flat tax schedule whereby $\tau_t = \tau_{mt} = 1 - \eta$. Substituting $\phi = 0$ into (15) shows that the ratio of government purchases to physical capital x_t remains unchanged over time, which in turn implies that the dynamical system (14)-(15) becomes degenerate. Resolving our model with $\phi = 0$ and $\alpha \neq \eta$ leads to the following single differential equation in $z_t \equiv \frac{c_t}{k_t}$ that describes the equilibrium dynamics:

$$\frac{\dot{z}_t}{z_t} = z_t + \delta + [A(1 - \eta)]^{\frac{1}{\alpha}} + \frac{1}{\sigma} \left[\alpha \eta A^{\frac{1}{\alpha}} (1 - \eta)^{\frac{1-\alpha}{\alpha}} - \delta - \rho \right] - A^{\frac{1}{\alpha}} (1 - \eta)^{\frac{1-\alpha}{\alpha}}, \quad (24)$$

which has a unique interior solution z^* that satisfies $\dot{z}_t = 0$ along the balanced-growth equilibrium path. We then linearize (24) around the BGP and find that its local stability property is governed by the explosive eigenvalue $z^* > 0$. Consequently, our endogenously growing economy exhibits local determinacy and equilibrium uniqueness under flat income taxation since there is no initial condition associated with (24).

4.4 Discussion

To offer further insights from the preceding analyses, Figure 4 plots the combinations of ϕ (the tax progressivity) and $\left(\frac{g_t}{y_t}\right)^* = 1 - \eta$ (the BGP ratio of government spending to output) that summarize all the possibilities of our model's local dynamics. Using (5) and (6), the balanced-growth equilibrium marginal tax rate can be written as

$$\tau_m^* = \phi + (1 - \phi) \left(\frac{g_t}{y_t}\right)^*. \quad (25)$$

The lower half of Figure 4 depicts that when $1 - \eta < 1 - \alpha$ (as in section 4.1), the locus $\tau_m^* = 0$ is a downward-sloping and convex curve, and the area below is not feasible because it exhibits $\tau_m^* < 0$. Under regressive income taxation ($\phi < 0$), the zone above the locus $\tau_m^* = 0$ in the southwest quadrant displays equilibrium indeterminacy and belief-driven fluctuations. This finding turns out to be qualitatively consistent with that in Schmitt-Grohé and Uribe's (1997) no-growth one-sector real business cycle model in which government purchases are postulated to be wasteful or useless.

Next, along the economy's balanced growth path, equation (21) can be re-written as

$$\Omega = 1 - \frac{\phi(1 - \alpha) \left[1 - \left(\frac{g_t}{y_t}\right)^* \right]}{\alpha \left(\frac{g_t}{y_t}\right)^*}. \quad (26)$$

We then find that when the tax schedule is progressive ($0 < \phi < 1$), the locus $\Omega = 0$ is an upward-sloping and concave curve in the southeast quadrant, and that it divides the regions

labeled as “*deter*” ($\Omega > 0$) and “*indet*” ($\Omega < 0$). This, together with the previous paragraph, implies that a shift of income taxation away from being regressive toward progressive will stabilize the economy against aggregate fluctuations driven by animal spirits. However, in sharp contrast to traditional Keynesian-type stabilization policies, raising the tax progressivity within the southeast quadrant transforms the BGP from being locally determinate to locally indeterminate *ceteris paribus*. It follows that a more progressive tax schedule may destabilize our model economy by generating endogenous growth fluctuations, provided the supply-side effect of public expenditures is stronger.

The upper half of Figure 4 illustrates that when $1 - \eta > 1 - \alpha$ (as in section 4.2), regressive taxation with $\phi < 0$ works like a conventional automatic stabilizer which leads to local determinacy and mitigates the magnitude of business cycles. On the contrary, fluctuations in output growth caused by agents’ self-fulfilling expectations or sunspots will occur under progressive taxation with $\phi > 0$. These results jointly imply that if the demand-side effect of government spending is stronger, changing the tax policy from being regressive to progressive will magnify the business cycle and thus destabilize the economy. This turns out to be exactly the opposite to what emerges in the lower half of Figure 4, thereby highlighting the important role that the relative strength between the demand-side and supply-side impact of public expenditures plays in determining the BGP’s macroeconomic (in)stability. Finally, when the taxation scheme is flat with $\phi = 0$ (as in section 4.3), the vertical axis of Figure 4 shows that our model always exhibits local determinacy and equilibrium uniqueness.

5 Conclusion

This paper has systematically explored the theoretical interrelations between progressive taxation of income, equilibrium (in)determinacy and economic growth in a one-sector representative-agent model of endogenous growth with productive government spending. When the supply-side effect of public expenditures is stronger, we analytically show that the economy exhibits an indeterminate balanced-growth equilibrium and belief-driven growth fluctuations under regressive or sufficiently progressive income taxation. When the demand-side effect of government purchases is stronger, indeterminacy and sunspots arise in our model with a progressive tax schedule. These results together imply that in sharp contrast to a conventional automatic stabilizer, raising the tax progressivity may destabilize an endogenously growing economy by generating fluctuations driven by agents’ changing non-fundamental expectations.

This paper can be extended in several directions. For example, it would be worthwhile to incorporate variable labor supply (Palivos, Yip and Zhang, 2003), national debt (Greiner,

2007), or multiple production sectors (Hu, Ohdoi and Shimomura, 2008) into our analytical framework. In addition, we can follow the primal approach of Chari, Christiano and Kehoe (1994) to derive the Ramsey second-best optimal fiscal policy in our endogenously growing economy with progressive income taxation and productive government spending (Park and Philippopoulos, 2002; and Economides, Park and Philippopoulos, 2011). These possible extensions will allow us to examine the robustness of this paper's results, as well as further enhance our understanding of the relationship between a progressive tax schedule and macroeconomic (in)stability in an endogenous growth model with public production services. We plan to pursue these research projects in the near future.

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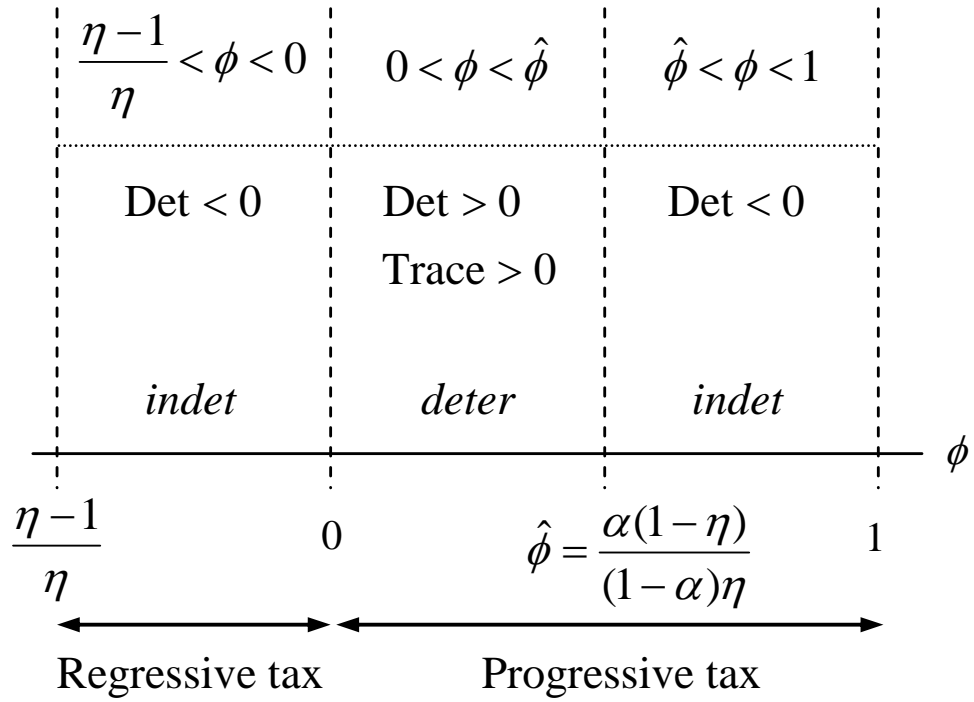


Figure 1. When $\phi \neq 0$ and $(1 - \eta) < (1 - \alpha)$

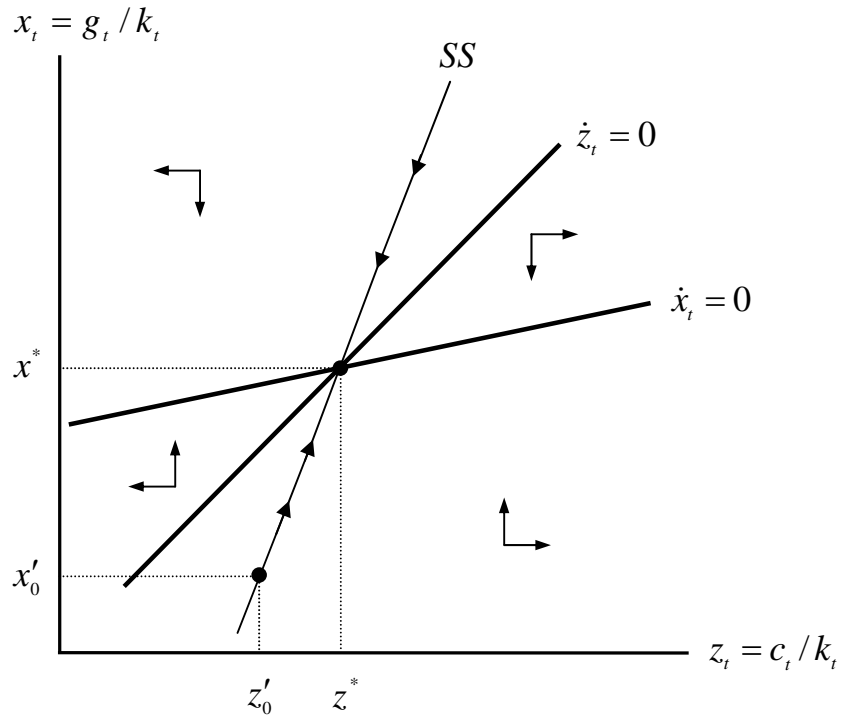


Figure 2. When $\phi \neq 0$ and $(1 - \eta) < (1 - \alpha)$: Indeterminacy

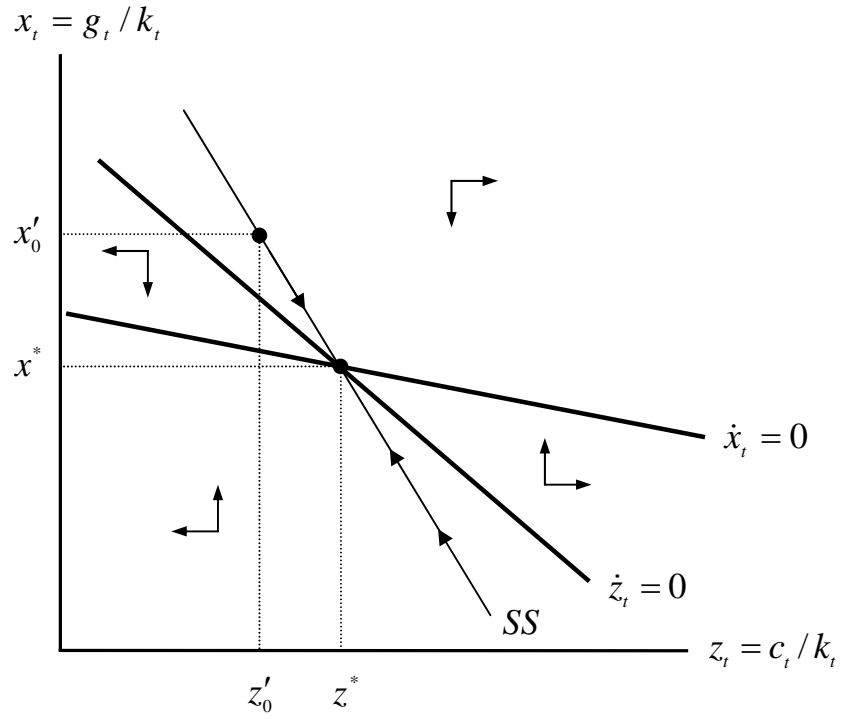


Figure 3. When $\phi \neq 0$ and $(1 - \alpha) < (1 - \eta)$: Indeterminacy

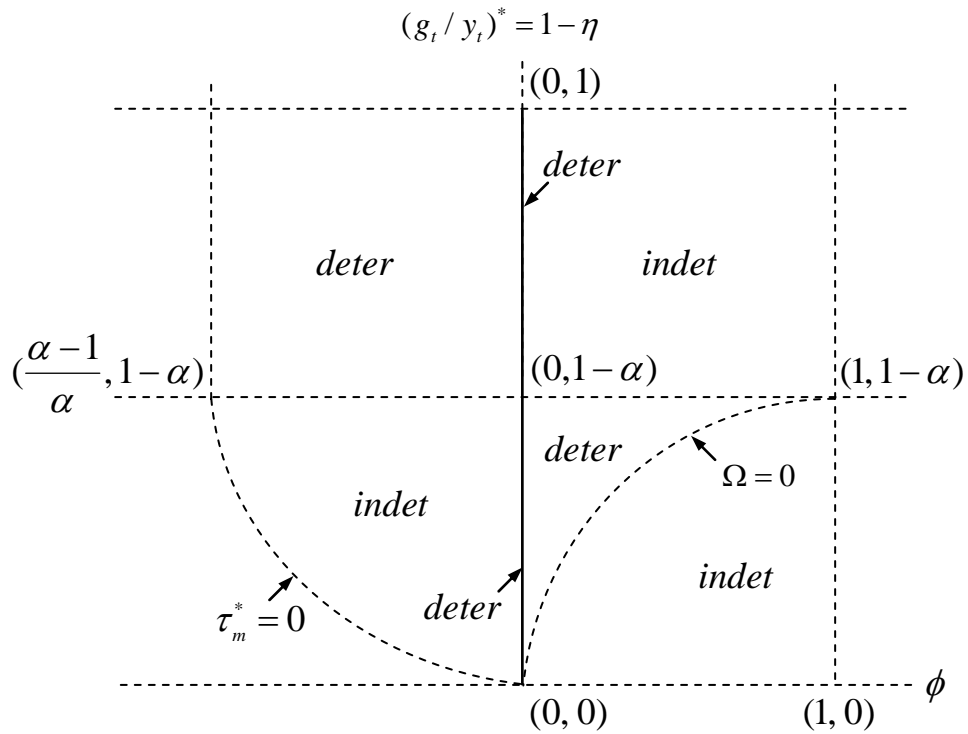


Figure 4. Tax Policy and Equilibrium (In)determinacy