Misinformation

Li, Hao
University of Toronto

Wei Li
University of California, Riverside

December, 2008

Abstract

A candidate has private information about her own quality and about her rival's quality. She can run an informative campaign which generates a public signal about either her quality ("positive campaign") or that of her rival ("negative campaign"). Both the type of campaign and its informativeness are signals about the qualities of the candidates. In a separating equilibrium, a candidate runs a positive campaign if she has a good signal about her own quality and a negative campaign if she has a bad signal about her rival's quality. When the candidate has both good news about herself and bad news about her rival, the type of the campaign generally depends both on the accuracies of her private signals and on relative accuracy, but with a small marginal cost of running more informative campaigns, the candidate chooses a positive campaign if her good news is more accurate than her bad news and a negative campaign if the reverse is true. Competing campaigns are less informative in equilibrium than unopposed campaigns, because misinformation is less effective when countered by competing campaigns.

JEL classification: D72, D82, D83

Keywords: positive and negative campaign, direct and indirect campaigning, horizontal and vertical separation

I. Introduction

Much attention has been paid to the issue of positive versus negative advertising in political campaigns, in both academia and popular press. The focus is on which type of campaign — praising one's own qualifications or discrediting that of the rival's — is more effective in influencing the voters. The effectiveness of a campaign, however, also depend on the informativeness of the campaign. Some campaign advertisements are "feel-good" ads that do not reveal much about the candidate's quality, while others give more specific information about a candidate's records and character. The informativeness of a campaign may be thought of as how much a candidate is willing to let the voters learn about either herself or her rival. Therefore both the type and the informativeness of campaign may matter to the voters' decision.

This paper considers a model in which a candidate uses the informativeness as well as the type of the campaign to influence a representative voter's beliefs, which determine the election outcome. In the basic model, a candidate has private information about herself and a rival candidate. She can choose one campaign, modeled here as a costly public information structure, but not its realization, to convey her private information about her quality or her rival's to the voter. She controls both the target of the campaign which can be either her own quality or her rival's, as well as the informativeness of the campaign, which is the accuracy of the public signal the voter observes.

Since the candidate has private information, her private beliefs about the types of both candidates differ from those of the voter after she has deviated from her equilibrium campaign choice. "Misinformation" is an attempt by the candidate to mislead the voter into forming a higher belief about her own quality than her private belief, or a lower belief about her rival's quality. However, since the candidate does not control the realization of the campaign signal, misinformation is necessarily imperfect as the voter's interim beliefs after observing the candidate's campaign choice are partially corrected by the realized campaign signal. We show that if the voter has a lower interim belief about the candidate's quality given her strategy than her private belief, the candidate prefers to "accentuate the positive" by running an informative campaign about her own quality. In this case, since the realized campaign signal agrees with her private information in expectation, ex post the voter forms a higher opinion of her quality. On the other hand, if the voter has a higher interim belief about the candidate's quality given her strategy than her private belief, then she prefers to "obfuscate the negative" by running an uninformative campaign because in expectation, any informative public signal is likely to lower the voter's ex post belief of her quality. Thus, the level of campaign informativeness can be used as a signal of the quality of the candidate or her rival. Furthermore, the more accurate a candidate's signal is, the more her campaign choice affects the voter's belief, and thus the stronger her incentive is to misinform. Consequently, the level of informativeness of a campaign required for the candidate to credibly signal her private information increases as well.

In our model the candidate can only run one campaign. One natural question is therefore whether a candidate should run an informative campaign about herself or an informative campaign about her rival. In each campaign, the candidate presents direct evidence about the target of the campaign, which we label *direct campaigning*. We show that there is *horizontal separation* in direct campaigning: the candidate runs a positive informative campaign about herself if she has good news about her own quality, while she runs a negative informative campaign about her rival if she has bad news about her rival's quality. This result holds regardless of the accuracies of her private signals about herself and her rival. Intuitively, to do the opposite, for example, to run a negative campaign when she has good news about herself but also good news about her rival's quality, means to lie against her own information, which is less effective because the realized campaign signal is likely to reveal that her rival is good. Moreover, she also loses an opportunity to signal her own quality, which in expectation is supported by the public signal.

In addition to direct campaigning, the candidate with the best news from her perspective — her private information suggests that her quality is high while her rival's is low — also engages in *indirect campaigning*. Even though she can run only one informative campaign, by increasing the informativeness of that campaign, she can suggest, without presenting any direct evidence, that her rival's quality is low in a positive informative campaign; or that her own quality is high in a negative campaign. Whether the candidate should run a highly informative positive campaign or a highly informative negative campaign depends on how accurate her private signals about her own quality and about her rival's quality are, and on the relative accuracy of the two signals. A basic comparative statics result is that when the candidate's private information about her own quality (or about her rival's quality) is inaccurate, an improvement in the accuracy increases the gain from misinformation through indirect campaigning relative to direct campaigning, because a greater accuracy in this case makes direct campaigning riskier, as the voter's posterior belief about the candidate's quality (or about the rival's quality) is more responsive when the campaign signal turns out bad for the candidate (or good for the rival). Thus, when the candidate's private information about her own quality is not accurate, and her private information about her rival's quality is even less so, the gain from misinformation through indirect campaigning relative to direct campaigning at the same level of informativeness is smaller when the target is the rival than when the target is the candidate herself. Moreover, when the candidate's private information about her rival's quality is less accurate than her information about her own quality, the level of informativeness is lower in a direct negative campaign targeting the rival than in a direct positive campaign targeting the candidate herself, and this further reduces the relative gain from misinformation through indirect campaigning that targets the rival. Thus, for the candidate to credibly inform the voter of her favorable private signals about her quality and about the rival's quality, a positive campaign is less costly than a negative campaign, as the former involves indirect campaigning that targets the rival while the latter involves indirect campaigning that targets the candidate herself.

We also show that when the marginal cost of running more informative campaigns is small, the equilibrium choice of a candidate with both good news about herself and bad news about the rival depends only on which piece of her private information is more accurate. In particular, a strongly informative positive campaign is used to credibly inform the voter of both private signals when the good news is more accurate, and a negative campaign is run if the bad news is more accurate. This is because when the marginal cost of campaign is small, accurate private information about the candidate's own quality means that misinformation through a direct positive campaign requires a high level, making misinformation targeting herself through indirect campaigning relatively more attractive. As a result, it is costly for the candidate to credibly signal both the good news about her own quality and the bad news about her rival through a strong negative campaign that involves in part indirect campaigning that targets the candidate herself.

We extend our analysis to a model of competing campaigns. In order to make this competing model comparable to the model with single campaign in terms of the amount of information available, we assume that the private signals of the two candidates are perfectly correlated. We show that competition makes misinformation less effective, and thus candidates runs less informative campaigns in comparison with the single campaign case. There are two reasons for this result. First, there is less incentive for a candidate to misinform the voter about the his own quality or about the his rival's quality, because with a positive probability misinformation is countered ex post by a campaign run by the candidate's rival. Second, for the same level of campaign, the voter is less likely to be misinformed even if the candidate's campaign is not countered ex post by the rival's campaign, because the candidate's choice of campaign becomes a less effective way of changing the voter's perception before the voter observes the realized campaign signals.

Moreover, unlike the model of single campaigns, the model of competing campaigns exhibits *vertical separation*, in the sense that a candidate with the most favorable private information strictly prefers to run highly informative campaigns. This is because under competition, the effectiveness of indirect campaigning depends on the private signals of the candidate, in contrast to the case of single campaign. Due to signal correlation, a candidate with the most favorable private information has a lot to lose if the candidate deviates and runs campaign at a lower level, because the rival is likely to run no campaign.

In existing papers, advertising, political or otherwise, is either modeled as directly informative because they contain hard information about the candidate's own (Nelson 1974, Coate 2004), or indirectly informative (Milgrom and Roberts 1986, Prat 2002). Milgrom and Roberts (1986) shows that the amount of money a firm spends on advertising can signal its product quality. Prat (2002) considers an electoral campaign setting where voters do not learn useful information from the campaign advertising itself. Rather, they use the amount of money spent on the campaign, in terms of campaign contributions, as a costly signal of the interest groups who have private information about the candidate's quality. Kotowitz and Mathewson (1979) study a setting where consumers' purchase decision based on their expectations of the quality of an experience good. They show that a monopolist can use advertising to mislead the consumers into having higher expectations than the actual quality of the good, at least in the short run.

Mostly closely related to our paper, Polborn and Yi (2006) considers a model in which each candidate has two characteristics, of which he can only reveal one. He runs an informative and truthful campaign, which can be a positive one about one good characteristic of his own or a negative one about one bad characteristic of his rival. The voters rationally estimate the characteristic not revealed. Thus a candidate is more likely to choose a positive campaign when his own characteristic is good and/or his rival's is good too. Our model differs from Polborn and Yi (2006) in that the candidate chooses the informativeness of the campaign signal the voter receives, which influences the voters' beliefs about his own quality and his rival's quality.

II. Setup

There are two candidates, A and B. Each candidate i, $i = A, B$, is either qualified, referred to as her unobserved "quality" being 1 and denoted as $i = 1$, or unqualified, denoted as $i = 0$. It is common knowledge between the candidates, and a median voter to be described later, that the prior probability that $i = 1$ is $\frac{1}{2}$, and that the candidates' qualities are independent.

Candidate i receives an imperfect binary signal $t_i^i \in \{h, l\}$ about her own quality, and another signal $t_j^i \in \{h, l\}$ about her rival j's quality. We assume that the signal structures are symmetric:

$$\Pr(t_i^i = h | i = 1) = \Pr(t_i^i = l | i = 0) = q,$$

and

$$\Pr(t_j^i = h | j = 1) = \Pr(t_j^i = l | j = 0) = r,$$

with $q, r \in \left(\frac{1}{2}, 1\right)$. We allow q and r to differ, representing the idea that candidate i's information about her own quality and her rival's quality may have different accuracies. The two signals t_i^i and t_j^i are assumed to be independent. We may allow the candidates' signals about i, t_i^i and t_i^j, to be correlated conditional on the quality of candidate i. We refer to the vector (t_i^i, t_j^i) as the "type" of candidate i. Type of each candidate is private.

Each candidate may choose to run a "campaign." Candidate i may choose one of two kinds of campaigns: a "positive" campaign, which is modeled as an information structure of a binary public signal about the candidate's own quality, and a "negative" campaign, which is an information structure of a public signal about the rival's quality. The informativeness of the campaign, is referred to as "level" of the campaign. Specifically, if $s_p \in \{+, -\}$ is the public signal from a positive campaign run by candidate i, the level κ_p^i of the campaign is

$$\kappa_p^i = \Pr(s_p = + | i = 1) = \Pr(s_p = - | i = 0).$$

Similarly, if $s_n \in \{+, -\}$ is the public signal from a negative campaign, the level κ_n^i of the campaign is

$$\kappa_n^i = \Pr(s_n = + | j = 1) = \Pr(s_n = - | j = 0).$$

Candidate i must choose both the kind, positive or negative, and the level, κ_p^i or κ_n^i, of the campaign. Note that the candidate does not control the realization of the public signal. Without loss of generality, we assume that κ_p^i and κ_n^i are at least as great as $\frac{1}{2}$. If candidate i chooses either a positive campaign with level equal to $\frac{1}{2}$, or a negative campaign with level equal to $\frac{1}{2}$, we say that the candidate does not run an (informative) campaign. We assume that the public signal of a campaign is independent of the private signals of either candidate.

There is a median voter that updates his belief about the two candidates' qualities after observing their choices of campaigns, and the campaign signals. The voter's prior belief is the same as the candidates': candidate i is qualified with probability $\frac{1}{2}$, and the candidates' qualities are independent. Let π_i be the voter's posterior belief that candidate i's quality is 1. We assume that candidate i's payoff is given by

$$\pi_i - \pi_j - f(\kappa^i),$$

where κ^i is either κ_p^i if a positive campaign is chosen, or κ_n^i if a negative campaign is chosen, and where $f(\cdot)$ is a common "cost" function for running a campaign. Note that the cost is assumed to be independent of the kind of campaign. We adopt the normalization that $f\left(\frac{1}{2}\right) = 0$, and for simplicity assume that f is differentiable with $f' > 0$.

A few remarks about the setup follow.

First, we have assumed that a candidate cannot run both a positive and a negative campaign. This assumption allows us to provide insights about the choice of the kind of campaign. It is also consistent with the central idea of this paper that campaigns, as information technologies, are to generate public signals.

Second, the voter in our setup updates his belief about the two candidates based on the kind and the level of each campaign as well as the realized public campaign signal. Since a candidate does not control the realization of the public signal generated by her campaign, the level of campaign is an important ingredient in our analysis of the signaling equilibrium. The assumption that the voter directly observes the informativeness of a campaign is of course an abstraction, but in practice the public may be able to indirectly infer it from, for example, campaign spending disclosures. Or how easily the claims in a campaign can be verified or proven false.

Third, we have chosen to model the payoffs of the two candidates in reduced form, instead of modeling

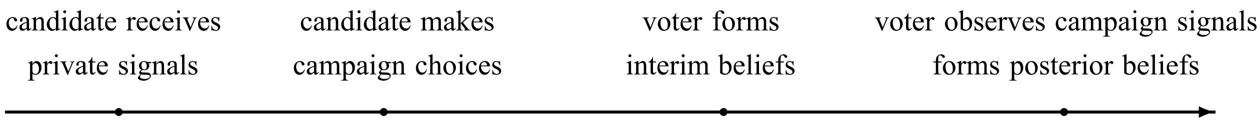

Figure 1. Timeline of the single campaign game

the voting game after the campaigns. The reduced form can be made more general without affecting the results qualitatively. In particular, we have assumed that a candidate's payoff depends linearly on the difference between the voter's posterior belief about her quality and the belief about her rival's quality. While admittedly restrictive, it allows us to highlight the dependence of the choice between positive campaigns and negative campaigns on the relative accuracy of a candidate's signals about her own quality and about her rival's quality.

III. Single Campaign

In this section we consider the case where only candidate A runs an active campaign. The timing of the single campaign game is summarized in Figure 1. Because this is a signaling game with candidate A as the only player, we drop all superscripts from the notation. Let the four private types of candidate A be $t = \{lh, hh, ll, hl\}$, where, for example, type $t = lh$ is when she has a bad signal $t_A = l$ about her own quality and a good signal $t_B = h$ about her rival's quality. A strategy of A specifies, for each of the four private types, the kind of campaign, positive or negative, and the corresponding level, κ_p or κ_n. We adopt perfect Bayesian equilibrium as the solution concept. As is standard, the posterior beliefs in candidate A's payoff are computed by using Bayes' rule and A's equilibrium signaling strategy.

We first derive a few preliminary results, which are more robust than our basic setup suggests, before constructing the equilibrium.

A. *Direct campaigning*

A campaign choice by candidate A affects the median voter's posterior beliefs π_A about her own quality and π_B about her rival in two ways. First, after observing the campaign choices, including both whether it is positive or negative, and the corresponding level of the campaign, κ_p or κ_n, but before the realization of the public signal generated by the campaign, the voter updates his beliefs about the qualities of the two candidates from the prior beliefs, computing interim beliefs according to Bayes' rule and the equilibrium signaling strategy of candidate A. Second, after observing the realized public signal, the voter uses the interim beliefs to arrive at π_A and π_B, according to Bayes' rule and the level of the campaign that generated the signal.

To fix ideas, let us first consider the case of positive campaign, with level κ. Denote as χ the voter's interim belief, conditional on that candidate A runs a positive campaign of level κ, that candidate A's quality is 1. Let $\hat{\chi}$ be the private belief of candidate A that her quality is 1 conditional on her type t. For now, suppose that $\hat{\chi}$ and χ are fixed. Then, candidate A's expectation of the voter's posterior belief π_A about her own quality is given by

$$\pi(\hat{\chi},\chi;\kappa)=(\hat{\chi}\kappa+(1-\hat{\chi})(1-\kappa))\frac{\chi\kappa}{\chi\kappa+(1-\chi)(1-\kappa)}+(\hat{\chi}(1-\kappa)+(1-\hat{\chi})\kappa)\frac{\chi(1-\kappa)}{\chi(1-\kappa)+(1-\chi)\kappa},$$

where the first fraction is the voter's posterior about the candidate's quality after observing a plus public signal ($s_p=+$), and the second fraction is the voter's posterior after observing a minus public signal ($s_p=-$). The above can be rewritten as

$$\pi(\hat{\chi},\chi;\kappa)=\chi+(\hat{\chi}-\chi)(2\kappa-1)\left(\frac{\chi\kappa}{\chi\kappa+(1-\chi)(1-\kappa)}-\frac{\chi(1-\kappa)}{\chi(1-\kappa)+(1-\chi)\kappa}\right).$$

The following result is immediate.

LEMMA 1: *(i)* $\pi(\hat{\chi},\chi;\kappa)=\hat{\chi}$ *if* $\chi=\hat{\chi}$*; (ii)* $\pi(\hat{\chi},\chi;\kappa)$ *decreases in* κ *if* $\hat{\chi}<\chi$*; and (iii)* $\pi(\hat{\chi},\chi;\kappa)$ *increases in* κ *if* $\hat{\chi}>\chi$.

Both a positive campaign to change the voter's perception of one's own quality and a negative campaign to change the voter's perception of her rival's quality are "direct campaigning," because the candidate allows the voter to receive a direct public signal about the target of the campaign. By the above lemma, if the candidate is privately more confident about her quality than the voter is, and if these beliefs are exogenously fixed, then she can increase her average perceived type by choosing a more informative positive campaign. In this case, an informative positive campaign about the candidate "accentuates the positive." On the flip side, a candidate can "obfuscate the negative" by reducing the informativeness of her campaign signal if she is privately less confident about her own quality than the voter. Naturally, the opposite holds for a negative campaign: the candidate lowers the voter's perception about her rival by running an informative campaign if she has worse news about the rival than the voter.

In any separating equilibrium, the private beliefs of each type t about her own quality and the rival's quality are equal to the interim beliefs of the voter on the equilibrium path, that is, after the voter observes the kind and the level of campaign that type t is supposed to run. It follows immediately from part (i) of Lemma 1 that in any separating equilibrium, type lh runs no campaign. This is because this type can always choose to run no campaign and obtain a payoff equal to its complete information payoff, while any informative campaign with level greater than $\frac{1}{2}$, positive or negative, is costly but has no effect on either the candidate's own quality perceived by the voter, or her rival's perceived quality.

In our model, there is *horizontal separation* through direct campaigning in the following sense. In any separating equilibrium, type hh strictly prefers its equilibrium level κ_p of informative positive campaign to the equilibrium level κ_n run by type ll, and vice versa for type ll. That is, in a separating equilibrium, the kind of campaign, positive or negative, is associated with whether the candidate has good news about her own quality or bad news about her rival's quality. In the proposed separating equilibrium, type hh succeeds in communicating to the voter her private belief that she is qualified with probability $\Pr(A = 1|t = hh) = q$ by running a positive campaign of level κ_p. By deviating to the negative campaign of level κ_n run by type ll, the candidate only succeeds in understating her private belief about her own quality, leading the voter to believe that her own quality is 1 with probability $\Pr(A = 1|t = lh) = 1 - q$ instead of q. This loss in payoff is the same as when type hh deviates from running the equilibrium level of positive campaign to running no campaign. In the meanwhile, by part (iii) of Lemma 1, just as type lh, this candidate fails to lower the voter's perception about her rival's quality to the level achieved by type ll in equilibrium, because type hh's private belief about the rival's quality is $\Pr(B = 1|t = hh) = r$, which is higher than the voter's interim belief $\Pr(B = 1|t = ll) = 1 - r$. It follows that in any separating equilibrium type hh prefers to running the positive campaign of level κ_p to running the negative campaign of level κ_n. A symmetric argument establishes that type ll prefers to running the equilibrium level of negative campaign to running a campaign of level κ_p.

It may seem unintuitive that horizontal separation in our model does not depend on the relative accuracy of the candidate's private signals about her own quality and about her rival's quality. After all, if q is just above $\frac{1}{2}$ so that type hh has little positive to say about herself, but $r > q$ so that she has accurate information about her rival, it might seem that discrediting the rival is more important than proving one's own quality. However, since the candidate can only run one campaign, type hh has more to lose from pretending to be type ll than type lh, $2q - 1$ versus 0, in terms of a missed opportunity of informing the voter of her own quality, but the same to gain, $r - \pi(r, 1-r; \kappa_n)$, in terms of an expected decrease in the voter's posterior belief about her rival's quality. If running a negative campaign of level κ_n is sufficient for type ll to deter type lh from imitating, type hh strictly prefers running a positive campaign of level κ_n, even though doing so acknowledges that her rival is qualified as well.

Define $\underline{\kappa}_p$ as the level of informative positive campaign that makes type lh indifferent between running and not running a campaign, assuming that running the positive campaign of level $\underline{\kappa}_p$ leads to the interim belief of q that the candidate is qualified and r that the candidate's rival is qualified. This is given by

$$1 - q - r = \pi(1 - q, q; \underline{\kappa}_p) - r - f(\underline{\kappa}_p). \tag{1}$$

Due to the assumption that the payoff to a candidate depends linearly on the difference between the voter's posterior belief about her quality and the belief about her rival's quality, the above equation determines a level of positive campaign that depends only on the accuracy q of the candidate's private signal about her own quality. Furthermore, for q there is a unique $\underline{\kappa}_p$ that satisfies the above equation: the right-hand-side of equation (1) ranges between $1 - q - r - f(1)$, when $\underline{\kappa}_p = 1$, and $q - r$, when $\underline{\kappa}_p = \frac{1}{2}$, and is decreasing by part (ii) of Lemma 1.

Lemma 1 implies that the least costly level of the informative positive campaign in any separating equilibrium is $\underline{\kappa}_p$. This is because, in any separating equilibrium with level κ_p, the voter's belief after observing the choice of campaign is that candidate A is qualified with probability q. By part (ii) of Lemma 1, the expected posterior belief of the voter about A's quality is decreasing in the level of positive campaign for type lh. Thus, any level $\kappa_p < \underline{\kappa}_p$ is insufficient to deter type lh from deviation.

Since $\pi\left(\hat{\chi}, \chi; \frac{1}{2}\right) = \chi$ regardless of $\hat{\chi}$, part (iii) of Lemma 1 implies that the maximum gain for type lh in deviating to any informative positive campaign, in terms of changing the voter's posterior belief about her quality, is $q - (1 - q)$, which is the loss for type hh in a separating equilibrium from not running a campaign. Since in the separating equilibrium with level $\underline{\kappa}_p$, type lh is indifferent between running the campaign and not running, type hh strictly prefers running the campaign to not running.

Similarly, we can define $\underline{\kappa}_n$ as the level of informative negative campaign that makes type lh indifferent between running and not running a campaign, assuming that running the negative campaign of level $\underline{\kappa}_n$ leads to the interim belief of $1-r$ that the candidate's rival is qualified and $1-q$ that the candidate herself is qualified. This is given by

$$1 - q - r = 1 - q - \pi(r, 1-r; \underline{\kappa}_n) - f(\underline{\kappa}_n). \tag{2}$$

A similar argument as above then implies that the least costly level of the informative negative campaign in any separating equilibrium is $\underline{\kappa}_n$, at which type ll strictly prefers running the campaign to not running. It can easily verified that

$$\pi(\chi, 1-\chi; \kappa) = 1 - \pi(1-\chi, \chi; \kappa),$$

for any $\chi \in [0, 1]$ and any κ, so the above equation takes a symmetric form as equation (1).

B. *Indirect campaigning*

Lemma 1 suggests that the kind of campaign run in equilibrium depends on whether the candidate has good news about herself or bad news about her rival. Within each campaign, however, the candidate uses the level of her campaign as a costly signal for separation: a more informative positive campaign is needed for type hh to separate from type lh when the candidate's private signal about her own quality is more accurate; and similarly, a more informative negative campaign is needed for type ll to separate from type lh if her signal about her rival's quality is more accurate. Intuitively, the more accurate is her private signal, the more she has to gain by misleading the voter into thinking she is qualified (or her rival is unqualified). For notational brevity, define

$$\delta(\chi; \kappa) = \pi(1-\chi, \chi; \kappa),$$

which is a candidate's expectation of the voter's posterior belief about either her own quality or her rival's quality, when her private belief about the quality is $1-\chi$ and the voter's interim belief is χ after observing the candidate runs a campaign of level κ. Then the following lemma holds.

LEMMA 2: *Suppose that* $\chi > \frac{1}{2}$. *(i)* $\delta(\chi; \kappa) \in [1-\chi; \chi]$; *(ii)* $\delta(\chi; \kappa) - (1-\chi)$ *increases in* χ; *and (iii) there exists* $\xi(\kappa) \in \left(\frac{1}{2}, 1\right)$ *such that* $\chi - \delta(\chi; \kappa)$ *increases in* χ *if* $\chi \in \left[\frac{1}{2}, \xi(\kappa)\right)$, *and decreases if* $\chi \in [\xi(\kappa), 1]$.

In any separating equilibrium, $\delta(q;\kappa)-(1-q)$ is type lh's gain from misinformation through deviating to running a positive campaign of level κ, whenever doing so yields an interim belief of q. Similarly, since $\pi(r,1-r;\kappa)=1-\pi(1-r,r;\kappa)$, type lh's gain from misinformation through deviating to a negative campaign of level κ takes the symmetric form of $\delta(r;\kappa)-(1-r)$. Part (ii) then implies that the gain from misinformation is increasing in the accuracy of the candidate's private information. To interpret part (ii), for the case of positive indirect campaign, let us write $\delta(\chi;\kappa)$ as

$$\frac{\chi\kappa}{\chi\kappa+(1-\chi)(1-\kappa)}+\frac{\chi(1-\kappa)}{\chi(1-\kappa)+(1-\chi)\kappa}-\chi,$$

where the first fraction is the voter's posterior about the candidate's quality after getting a plus public signal from the campaign, and the second fraction is the voter's posterior after getting a minus signal. Thus, an increase in the interim belief raises the gain from misinformation $\delta(\chi;\kappa)-(1-\chi)$ by increasing both posterior beliefs.

An immediate implication of part (ii) of Lemma 2 is that the level of direct campaigning $\underline{\kappa}_p$ and $\underline{\kappa}_n$ required to signal one's own quality or the rival's quality increases in the accuracy of the candidate's private signal. To see this, denote as $\underline{\kappa}(\chi)$ the level of positive campaign in the least costly separating equilibrium as a function of the interim belief of the voter χ, as determined by equation (1). By part (ii) of Lemma 1 and part (ii) of Lemma 2, this function is strictly increasing, with

$$\underline{\kappa}'(\chi)=-\frac{\partial\delta(\chi;\underline{\kappa}(\chi))/\partial\chi+1}{\partial\delta(\chi;\underline{\kappa}(\chi))/\partial\kappa-f'(\underline{\kappa}(\chi))}>0.$$

Note that $\underline{\kappa}(\chi)$ is strictly increasing even if the marginal cost of direct campaigning, f', is arbitrarily small. Similarly, the level of negative informative campaign $\underline{\kappa}_n$ required to signal the rival's low quality is given by $\underline{\kappa}(r)$, and increases in r. It follows that $\underline{\kappa}_p\geq\underline{\kappa}_n$ if $q\geq r$. Thus, if the candidate has a more accurate private signal about her own quality than about her rival's quality, type hh must use a higher level of positive campaign to separate from type lh than the level of negative campaign that type ll uses to separate from type lh.

The above result that the more accurate is a candidate's private information about herself or about her rival, the more informative a campaign — and thus a more costly one — she has to run, may seem counterintuitive. One may think that a better candidate or one has more accurate news that her rival is unqualified should spend less on campaign activities, expecting to be supported by public campaign signals that the voter observes later. However, in our model, the voter's evaluation of the campaign signals depends on the candidate's campaign choices. Thus, for example, if the candidate has a more accurate signal that she is qualified, the voter forms a higher interim belief after observing her campaign choices, which increases the incentive for a low type candidate to misinform. As a result, it becomes necessary for a qualified candidate to choose a higher level of campaign to discourage imitation. In this sense, a candidate, who is privately informed about her quality or about her rival's lack of it, uses direct campaigning to discourage deviation by a candidate with worse news instead of convincing the voter by presenting persuasive direct evidence.

Given our result that both type hh and type ll separate themselves from type lh by engaging in informative direct campaigning, we next turn to the question of what campaign choices the candidate must make to credibly signal both good news for her own quality and bad news for her rival's. There is no a priori reason why type hl should choose a positive or a negative campaign. But clearly, if she chooses to run a positive campaign for example, she must run a campaign with a level higher than $\underline{\kappa}_p$ to separate from type hh. In this case, type hl runs a more informative campaign about her own quality and does not provide any direct evidence about the her rival's quality, we refer to this high level of positive campaign as "indirect campaigning". Similarly, type hl can run a negative campaign with a level higher than $\underline{\kappa}_n$ to suggest that her own quality is high without giving the voter direct evidence, a public signal about herself.

To understand indirect campaigning, first observe that in a separating equilibrium, type hl can not run both a positive campaign and a negative campaign if they involve different levels of informativeness: she should choose the one with a lower cost. Define $\overline{\kappa}_p$ as the level of informative positive campaign that makes type hh indifferent between running this campaign and running her equilibrium positive campaign of the level $\underline{\kappa}_p$, assuming that running the positive campaign of level $\overline{\kappa}_p$ leads to the interim belief of q that herself is qualified and $1 - r$ that her rival is qualified. This is given by:

$$q - r - f(\underline{\kappa}_p) = q - (1 - r) - f(\overline{\kappa}_p).$$

For both the campaign at the level $\underline{\kappa}_p$ and the campaign at the level $\overline{\kappa}_p$, the candidate's expectation of the voter's belief about her own quality does not depend on whether her type is hh or hl. By part (i) of Lemma 1, the expectation is q from her direct campaigning. The indirect campaigning about the quality of her rival also does not depend on whether her type is hh or hl, because she does not provide the voter with a public signal. As a result, the higher is r, the stronger is type hh's incentive to use a higher level of positive campaign as an indirect signal of her rival's low quality. Similarly, define $\overline{\kappa}_n$ as the level of informative negative campaign that makes type ll indifferent between running this campaign and running her equilibrium negative campaign of the level $\underline{\kappa}_n$, assuming that running the negative campaign of level $\overline{\kappa}_n$ leads to the interim belief of $1-r$ that her rival is qualified and q that herself is qualified. This is given by

$$1 - q - (1 - r) - f(\underline{\kappa}_n) = q - (1 - r) - f(\overline{\kappa}_n).$$

Thus, if the candidate has more accurate private information about her own quality than about her rival's quality, i.e., if $q > r$, then type ll has a stronger incentive to engage in indirect campaigning relative to type hh, because the former has more to gain than the latter. From the perspective of type hl, we have $f(\overline{\kappa}_p) < f(\overline{\kappa}_n)$ if and only if

$$f(\underline{\kappa}_p) + (2r - 1) < f(\underline{\kappa}_n) + (2q - 1),$$

which is equivalent to

$$r - \delta(r; \underline{\kappa}(r)) < q - \delta(q; \underline{\kappa}(q)),$$

by the definitions of $\underline{\kappa}_p$ and $\underline{\kappa}_n$ in equation (1) and (2). The left-hand-side of the above condition can be interpreted as the gain from misinformation through an indirect positive campaign targeting the rival, $r - (1 - r)$, relative to the gain from misinformation through a direct negative campaign of level $\underline{\kappa}(r)$ targeting the rival, $r - (1 - \delta(r; \underline{\kappa}(r)))$. Similarly, the right-hand-side has the interpretation as the gain from misinformation through an indirect negative campaign targeting the candidate herself, $q - (1 - q)$, relative to the gain from misinformation through a direct positive campaign of level $\underline{\kappa}(q)$ targeting the candidate herself, $\delta(q; \underline{\kappa}(q)) - (1 - q)$. Thus, for the candidate to credibly inform the voter that she has both good news about her own quality and bad news about her rival's quality, a positive campaign of level $\overline{\kappa}(q)$ is less costly than a negative campaign of level $\overline{\kappa}(r)$, if relative to direct campaigning, misinformation through indirect campaigning is less effective when the target is the rival than when the target is the candidate herself.

If $q = r$, clearly $\overline{\kappa}_p = \overline{\kappa}_n$. Instead, if $r < q < \xi(\underline{\kappa}(q))$, then

$$r - \delta(r; \underline{\kappa}(r)) < r - \delta(r; \underline{\kappa}(q)) < q - \delta(q; \underline{\kappa}(q)),$$

where the first inequality follows from part (ii) of Lemma 1 because $\underline{\kappa}(r) < \underline{\kappa}(q)$, and the second inequality follows from part (iii) of Lemma 2. Similarly, if $q < r < \xi(\underline{\kappa}(r))$, then

$$q - \delta(q; \underline{\kappa}(q)) < q - \delta(q; \underline{\kappa}(r)) < r - \delta(r; \underline{\kappa}(r)),$$

so that a negative campaign of level $\overline{\kappa}_n$ is less costly than a positive campaign of level $\overline{\kappa}_p$.

To understand the above result, we note that by part (iii) of Lemma 2, for any fixed level κ of direct campaigning, the gain from misinformation through indirect campaigning relative to direct campaigning, $\chi - \delta(\chi; \kappa)$, is not monotone in the accuracy χ of the candidate's private signal. To see why, for the case when direct campaigning is positive, let us write $\chi - \delta(\chi; \kappa)$, which is always positive by part (i) of Lemma 2, as

$$\left(\chi - \frac{\chi(1-\kappa)}{\chi(1-\kappa) + (1-\chi)\kappa}\right) - \left(\frac{\chi\kappa}{\chi\kappa + (1-\chi)(1-\kappa)} - \chi\right).$$

Observe that the first difference is how much the voter adjusts downwards his posterior belief about the candidate's quality upon receiving a minus campaign signal, while the second difference is how much he adjusts upwards his posterior belief upon receiving a plus signal. That the candidate gains more from misinformation through indirect campaigning than through direct campaigning means that she has more to lose when the campaign signal is minus than there is to gain when the signal is plus since the voter's interim belief is already above $\frac{1}{2}$. Furthermore, the second difference always decreases in χ, that is, the higher is the voter's interim belief, the less responsive his posterior belief is to a plus signal. In contrast, the first difference is not monotone in χ: the decrease in the posterior belief after a minus campaign signal is large when the interim belief χ is close to $\frac{1}{2}$ but becomes smaller when χ increases. By taking derivatives, we can verify that for any $\kappa \in \left(\frac{1}{2}, 1\right)$, the difference of the differences, $\chi - \delta(\chi; \kappa)$, is a concave function of χ, starting at 0 when $\chi = \frac{1}{2}$, increasing and reaching a maximum at $\chi = \xi(\kappa)$, and then decreasing to 0 when $\chi = 1$. Thus, when the candidate's private information about her own quality (or about her rival's quality) is not accurate, an improvement in the accuracy increases the relative gain from misinformation through indirect campaigning, because a greater accuracy in this case means that direct campaigning is riskier, as the voter's posterior belief about the candidate's quality (or about the rival's quality) is more responsive when the campaign signal turns out to be minus for the candidate (or plus for the rival). It follows that when the candidate's private information about her own quality is not so accurate, so that $q < \xi(\kappa(q))$, and her private information about her rival's quality is even less accurate, so that $r < q$, the gain from misinformation through indirect campaigning relative to direct campaigning at the same level of $\underline{\kappa}(q)$ is smaller when the target is the rival than when the target is the candidate herself. Since the least costly separating level of direct campaigning is lower in a negative campaign than a positive campaign when $r < q$, implying the former is more effective than the latter, the gain from misinformation through indirect campaigning that targets the rival relative to direct negative campaigning at level $\underline{\kappa}(r)$ targeting the rival is even smaller, implying that a positive campaign of level $\overline{\kappa}_p$ is less costly than a negative campaign of level $\overline{\kappa}_n$.

To find sufficient conditions on the parameters of the model for $q < \xi(\underline{\kappa}(q))$, we write the condition that determines $\xi(\kappa)$ for any $\kappa \in \left(\frac{1}{2}, 1\right)$ as

$$\frac{2}{\kappa(1-\kappa)} = \frac{1}{(\xi(\kappa)\kappa + (1-\xi(\kappa))(1-\kappa))^2} + \frac{1}{(\xi(\kappa)(1-\kappa) + (1-\xi(\kappa))\kappa)^2}.$$

Using the above condition and using the fact that $\chi - \delta(\chi;\kappa)$ is concave in χ and reaches the maximum at $\chi = \xi(\kappa)$, we can show that $\xi(\kappa) > \kappa$ if and only if $\kappa \in \left(\frac{1}{2}, \hat{\kappa}\right)$, where $\hat{\kappa} = \frac{1}{2} + \frac{1}{2}\sqrt{\sqrt{2}-1}$. Suppose that the cost function f satisfies $\underline{\kappa}(\hat{k}) < \hat{\kappa}$, or equivalently,

$$f(\hat{\kappa}) < \delta(\hat{\kappa};\hat{\kappa}) - (1-\hat{\kappa}).$$

Then, by part (ii) of Lemma 2, there is a unique $\hat{q} \in \left(\frac{1}{2}, \hat{\kappa}\right)$ such that $\underline{\kappa}(\hat{q}) = \hat{\kappa}$. Furthermore, since $\underline{\kappa}(\hat{q}) < \hat{\kappa}$, we have

$$\delta(\hat{q};\hat{q}) - f(\hat{q}) > 1 - \hat{q}.$$

Thus, there is a range of values of q smaller than $\hat{q}$ such that

$$\delta(q;q) - f(q) > 1 - q,$$

or equivalently $q < \underline{\kappa}(q)$. It follows that for any such q we have $q < \underline{\kappa}(q) < \underline{\kappa}(\hat{q}) < \hat{\kappa}$, and thus $q < \xi(\underline{\kappa}(q))$.

If the candidate has accurate private signals about her own quality and about her rival's quality, then whether it is more or less costly for type hl to run a strong positive campaign of level $\overline{\kappa}_p$ or a strong negative campaign of level $\overline{\kappa}_n$ depends on which of the following two opposite effects of relative accuracy of the candidate's two private signals dominates. On one hand, if $r > \xi(\underline{\kappa}(r))$ and if q is even greater than r, then by part (iii) of Lemma 2, we have

$$r - \delta(r; \underline{\kappa}(r)) > q - \delta(q; \underline{\kappa}(r)),$$

so that the gain from misinformation through indirect campaigning relative to direct campaigning at the same level of $\underline{\kappa}(r)$ is greater when the target is the rival than when the target is the candidate herself. This implies that the cost for type hl to credibly inform the voter about the good news for her own quality and the bad news for her rival tends to be higher through a positive campaign than through a negative campaign. On the other hand,

$$q - \delta(q; \underline{\kappa}(r)) < q - \delta(q; \underline{\kappa}(q))$$

because $r < q$ implies $\underline{\kappa}(r) < \underline{\kappa}(q)$, so that the gain from misinformation through indirect campaigning relative to a direct positive campaign is increased by the fact that the level required for the direct positive campaign is higher than the level required for the direct negative campaign. The first effect dominates if the marginal cost of campaign f' is large, so that there is little difference between $\underline{\kappa}(r)$ and $\underline{\kappa}(q)$ even though q is greater than r.

To conclude the analysis in this subsection, we provide a sufficient condition on the campaign cost function f such that the campaign choice of type hl depends only on the relative accuracy of the candidate's private signals about her own quality and about her rival's quality. Using the expression for $\underline{\kappa}'(\chi)$ derived earlier, we can show by taking derivatives that the function $\chi - \delta(\chi; \underline{\kappa}(\chi))$ is increasing for any $\chi \geq \frac{1}{2}$ if and only if

$$\frac{\partial\delta(\chi; \underline{\kappa}(\chi))/\partial\chi - 1}{\partial\delta(\chi; \underline{\kappa}(\chi))/\partial\chi + 1} < \frac{\partial\delta(\chi; \underline{\kappa}(\chi))/\partial\kappa}{\partial\delta(\chi; \underline{\kappa}(\chi))/\partial\kappa - f'(\underline{\kappa}(\chi))}.$$

By part (iii) of Lemma 2, the above is satisfied when $\chi < \xi(\underline{\kappa}(\chi))$ because the left-hand-side is negative and the right-hand-side is positive. If $\chi \geq \xi(\underline{\kappa}(\chi)$, the above remains true if the marginal cost of campaign f' is sufficiently small. Thus, when informative campaigns involve some fixed cost and little marginal cost in the relevant range of levels of informativeness, the candidate credibly signals both her good news about her own quality and bad news about her rival through a strong positive campaign if her good news is more accurate than the bad news, and through a strong negative campaign if the opposite is true. Intuitively, when the marginal cost of campaign is small, the dominant force in determining the gain from misinformation through indirect campaigning relative to direct campaigning, whether the target is the rival or the candidate herself, is that the level of direct campaigning, $\underline{\kappa}(\chi)$, required for successful misinformation is highly responsive to the accuracy of the candidate's private signal about the target of direct campaigning, χ. Thus, if the candidate has more accurate private information about her own quality than about her rival, misinformation through a direct positive campaign requires a high level, increasing the relative gain from misinformation through indirect campaigning that targets the candidate herself. As a result, it is costly for the candidate to credibly signal both the good news about her own quality and the bad news about her rival through a strong negative campaign that involves in part indirect campaigning that targets the candidate herself. The candidate will instead choose a strong positive campaign.

C. *Least costly separating equilibrium*

We now proceed to state the least costly separating equilibrium. A natural set of out-of-equilibrium-path beliefs is: the candidate is of type lh if $\kappa_p \in \left(\frac{1}{2}, \underline{\kappa}_p\right)$ or $\kappa_n \in \left(\frac{1}{2}, \underline{\kappa}_n\right)$; regardless of whether type hl runs a positive or a negative campaign in the equilibrium, the candidate is of type hh if $\kappa_p \in (\underline{\kappa}_p, \overline{\kappa}_p)$ and type hl if $\kappa_p \geq \overline{\kappa}_p$, and is of type ll if $\kappa_n \in (\underline{\kappa}_n, \overline{\kappa}_n)$ and type hl if $\kappa_n \geq \overline{\kappa}_n$. It can be verified straightforwardly that the above set of beliefs supports the least costly separating equilibrium described in the previous two subsections. Type lh has no incentive to deviate to any positive campaign of level $\kappa_p \in \left(\frac{1}{2}, \underline{\kappa}(q)\right)$, or any negative campaign of level $\kappa_n \in \left(\frac{1}{2}, \underline{\kappa}(r)\right)$. For example, the deviation payoff from a positive campaign is $1 - q - r - f(\kappa_p)$ by part (i) of Lemma 1, which is strictly less than its equilibrium payoff of $1 - q - r$. We claim that type hh has no incentive to deviate to a positive campaign of any level $\kappa_p < \underline{\kappa}(q)$, and symmetrically type ll has no incentive to deviate to a negative campaign of any level $\kappa_n < \underline{\kappa}(r)$. To establish this claim, note that given the above out-of-equilibrium-path beliefs, the deviation payoff for type hh is

$$\pi(q, 1-q; \kappa_p) - r - f(\kappa_p) = 1 - \pi(1-q, q; \kappa_p) - r - f(\kappa_p).$$

Since $\kappa_p < \underline{\kappa}(q)$, from part (ii) of Lemma 1 we have that the deviation payoff is strictly less than $1 - \pi(1-q, q; \underline{\kappa}(q)) - r - f(\kappa_p)$, which by the definition of $\underline{\kappa}(q)$ is equal to $q - f(\underline{\kappa}(q)) - r - f(\kappa_p)$. Thus, the deviation payoff for type hh is strictly less than her equilibrium payoff. A symmetric argument establishes that type ll has no incentive to deviate to a negative campaign of any level $\kappa_n < \underline{\kappa}(r)$. We have the following proposition.

PROPOSITION 1: *In the least costly separating equilibrium, type lh candidate runs no campaign. Type hh runs an informative positive campaign of level $\underline{\kappa}(q)$ while type ll runs an informative negative campaign of level $\underline{\kappa}(r)$. If $r < q < \xi(\underline{\kappa}(q))$, type hl runs a positive campaign of level $\overline{\kappa}_p$; if $q < r < \xi(\underline{\kappa}(r))$, type hl runs a negative campaign of level $\overline{\kappa}_n$.*

As is standard in a signaling game, there are other separating equilibria of the single campaign game that involves higher levels of campaigns for type hh and type ll. For example, since type hh strictly prefers her equilibrium campaign choice of running a positive campaign of level $\underline{\kappa}(q)$ to any lower level, there are values of κ_p greater than $\underline{\kappa}_p(q)$ that are equilibrium levels of positive campaign for type hh as part of a separating equilibrium, supported by the out-of-equilibrium belief that the candidate is of type lh if she runs a positive campaign of any level $\tilde{\kappa}_p$ below κ_p. However, such separating equilibria can be ruled out by applying the standard out-of-equilibrium-path belief refinement such as the "Intuitive Criterion" of Cho and Kreps (1987). To see this, suppose that the separating level is $\kappa_p > \underline{\kappa}_p$. Then, by part (ii) of Lemma 1, for any $\tilde{\kappa}_p \in (\underline{\kappa}_p, \kappa_p)$, type lh strictly prefers no campaign to running a positive campaign of level $\tilde{\kappa}_p$, regardless of the out-of-equilibrium-path belief about her own quality associated with $\tilde{\kappa}_p$. However, by part (i) of Lemma 1, type hh benefits from such a deviation if the interim belief is that she is type hh. Thus, there is no out-of-equilibrium-path belief that satisfies the Intuitive Criterion that supports the separating level of $\kappa_p > \underline{\kappa}(q)$.

In contrast to the separation of types hh and ll from type lh, due to the linear payoff structure and the nature of indirect campaigning in our model, $\overline{\kappa}_p$ and $\overline{\kappa}_n$ are respectively the unique level separating type hl from type hh in a positive campaign, and from type ll in a negative campaign. For the same reasons, there is no *vertical separation* of type hl from types hh and ll in our basic model. If in equilibrium, type hl runs an informative positive campaign, both type hl and type hh are indifferent between running a campaign of level $\overline{\kappa}_p$ and $\underline{\kappa}_p$. Similarly, if in equilibrium, type hl runs an informative negative campaign, type ll is also indifferent between a campaign of level $\underline{\kappa}_n$ and $\overline{\kappa}_n$. To see this, take the positive campaign for example. The voter's interim and posterior belief about the candidate given either $\underline{\kappa}_p$ or $\overline{\kappa}_p$ are both q. Thus the gain for type hh to pretend to be type hl, which is $r - (1 - r)$, is precisely the loss for type

hl to pretend to be type hh. Given the common campaign cost function and the linear payoff structure, both types are indifferent between running a positive campaign of level $\underline{\kappa}_p$ and level $\overline{\kappa}_p$.

It is straightforward to modify the basic model to generate vertical separation, for example, by assuming that type hl faces a smaller marginal campaign cost than type hh, or that payoff complementarity exist so that type hl has more to lose from overstating the rival's quality than type hh to gain from understating it. Further, such modifications may be natural in some circumstances. However, since the focus of this paper is on effects of misinformation, we are content with maintaining the assumptions of a common campaign cost function and the linear payoff structure. Instead, we ask if the absence of vertical separation is due to the assumption in our model that the voter has no other independent information that can be used to evaluate the candidates. This turns out not to be the case if the information that the voter receives is independent of the candidate's campaign choices. In the rest of this section we use two extensions to illustrate this point. The extensions also help elaborate on the equilibrium campaign choice of type hl. In the next section we will introduce competing campaigns to deliver the vertical separation of the type with the most favorable private signals from the two middle types.

Without loss of generality, we consider only signals about the candidate that is actively campaigning. Suppose that with probability v the voter receives a perfect signal about candidate A's quality. This reduces type lh's incentive to misinform the voters by running a positive campaign of any level κ_p, as the gain from such misinformation becomes $(1-v)(\delta(q;\underline{\kappa}_p)-(1-q))$. Type hh's incentive to deviate from running an informative positive campaign to not running a campaign also increased, because the loss in terms of a lower expected posterior belief of the voter about her quality becomes $(1-v)(q-(1-q))$ due to the voter's independent opportunity of learning her quality. Thus, the least costly level $\underline{\kappa}_p$ of positive campaign to prevent type lh from deviating is still defined by the indifference condition of type lh:

$$(1-v)(\delta(q;\underline{\kappa}_p)-(1-q)) = f(\underline{\kappa}_p).$$

It follows from part (ii) of Lemma 1 that an increase in v decreases $\underline{\kappa}_p$. On the other hand, since type lh does not misinform the voter about herself in a negative campaign, the effectiveness of such a campaign is unaffected, and so the least costly level $\underline{\kappa}_n$ of a direct negative campaign does not depend on v. For indirect campaigning, the possible public signal makes any such misinformation targeting the candidate herself less effective, reducing the gain to $(1-v)(q-(1-q))$, while leaving any indirect campaigning that targets the rival unaffected. Either way, vertical separation remains absent in this extension, because the loss to type hl from deviating to running a weak negative campaign of level $\underline{\kappa}_n$, or a weak positive level $\underline{\kappa}_p$, is still equal to the gain from misinformation to type ll by running a strong negative campaign, or type hh by running a strong positive campaign. However, there are implications to the equilibrium campaign choice of type hl: the gain from misinformation through indirect campaigning relative to direct campaigning is now given by $(1-v)(q-\delta(q;\underline{\kappa}_p))$ when the target is the candidate herself, and remains $r-\delta(r;\underline{\kappa}_n))$ when the target is the rival. Since an increase in v reduces the least costly separating level of direct positive campaign $\underline{\kappa}_p$, there is now a smaller relative gain from misinformation about the candidate, even though an increase in v decreases the effectiveness of both indirect campaigning and direct campaigning. This makes it less likely that type hl of the candidate uses a strong positive campaign to inform the voter of her good news about her own quality and bad news about the rival's quality.

Another way to allow the voter to receive independent information about candidates' qualities is to assume that with probability 1 the voter receives another imperfect binary signal about the candidate's quality, which is conditionally independent from the campaign signal generated by A's positive campaign. Let the binary signal be $\tilde{s}=\{+,-\}$, and let the informativeness be $\tilde{k}$, given by

$$\tilde{k}=\Pr(\tilde{s}=+|A=1)=\Pr(\tilde{s}=-|A=0).$$

Next, let $\Delta(q;\kappa,\tilde{k})$ be the expectation of type lh of the voter's posterior belief about A's quality, given that the interim belief of the voter is q. Then, $\Delta(q;\kappa,\tilde{k})$ is given by

$$\begin{aligned}
&((1-q)\kappa\tilde{k}+q(1-\kappa)(1-\tilde{k}))\frac{q\kappa\tilde{k}}{q\kappa\tilde{k}+(1-q)(1-\kappa)(1-\tilde{k})}\\
+\quad&((1-q)\kappa(1-\tilde{k})+q(1-\kappa)\tilde{k})\frac{q\kappa(1-\tilde{k})}{q\kappa(1-\tilde{k})+(1-q)(1-\kappa)\tilde{k}}\\
+\quad&((1-q)(1-\kappa)\tilde{k}+q\kappa(1-\tilde{k}))\frac{q(1-\kappa)\tilde{k}}{q(1-\kappa)\tilde{k}+(1-q)\kappa(1-\tilde{k})}\\
+\quad&((1-q)(1-\kappa)(1-\tilde{k})+q\kappa\tilde{k})\frac{q(1-\kappa)(1-\tilde{k})}{q(1-\kappa)(1-\tilde{k})+(1-q)\kappa\tilde{k}},
\end{aligned}$$

where the first fraction is the voter's updated posterior that A is qualified when both the realized campaign signal and the independent public signal are $+$; the second fraction is the posterior when the realized campaign signal the independent public signal are $+$ and $-$ respectively; the third fraction is the posterior when the signals are $-$ and $+$ respectively; and the fourth fraction is when the signals are both $-$.

Redefine the least costly level $\underline{\kappa}_p$ of positive campaign by

$$1 - q - r = \Delta(q; \underline{\kappa}_p, \tilde{k}) - r - f(\underline{\kappa}_p).$$

In writing the left-hand-side of the above equation we have used part (i) of Lemma 1. To see why the campaign level $\underline{\kappa}_p$ defined above remains the least costly level to separate type hh and hl when the informativeness $\tilde{k}$ of the independent public signals are sufficiently close to $\frac{1}{2}$, note that the loss of type hh in terms of the expected decrease in the voter's posterior belief about her quality is now $q - \pi(q, 1-q; \tilde{k})$, which is equal to $\delta(q; \tilde{k}) - (1-q)$. The gain of type lh is the expected increase in the voter's posterior belief about her quality from deviating to running the positive campaign of level $\underline{\kappa}_p$ is $\Delta(q; \underline{\kappa}_p, \tilde{k}) - (1-q)$. Type lh's gain is strictly smaller than type hh's loss from the corresponding deviation when $\tilde{k}$ is sufficiently close to $\frac{1}{2}$, because $\delta\left(q; \frac{1}{2}\right) = q$ while $\Delta\left(q; \underline{\kappa}_p, \frac{1}{2}\right) = \delta(q; \underline{\kappa}_p) < q$. Furthermore, it can be verified that at $\tilde{k} = \frac{1}{2}$, the first derivative of $\Delta(q; \kappa, \tilde{k})$ with respect to $\tilde{k}$ equals 0 and the second derivative is negative. Since $\Delta\left(q; \kappa, \frac{1}{2}\right)$ equals $\delta(q; \kappa)$, and by part (ii) of Lemma 1 is decreasing in κ, by definition $\underline{\kappa}_p$ decreases as $\tilde{k}$ increases slightly from $\frac{1}{2}$, so that the direct positive campaigning also becomes less effective when the voter observes an (almost) uninformative public signal, just like the case of the perfect signal above.

There remains no vertical separation in this extension. The loss to type hl from deviating to running a weak positive campaign of level $\underline{\kappa}_p$ is unchanged from the benchmark model and still equal to the gain from misinformation to type hh by running a strong positive campaign; and the loss to type hl from deviating to running a negative campaign of level $\underline{\kappa}_n$ becomes $q - \pi(q, 1-q; \tilde{k})$, which is equal to $\delta(q; \tilde{k}) - (1-q)$, and is equal to the gain to type ll by running a strong weak campaign. As to the implications to the equilibrium campaign choice of type hl, note that the gain from misinformation through indirect campaigning relative to direct campaigning is now given by $\delta(q; \tilde{k}) - \Delta(q; \underline{\kappa}_p, \tilde{k})$ when the target is the candidate herself, and remains $r - \delta(r; \underline{\kappa}_n)$ when the target is the rival. An increase in $\tilde{k}$ reduces $\delta(q; \tilde{k})$ by part (ii) of Lemma 1; at the same time, when $\tilde{k}$ is sufficiently close to $\frac{1}{2}$, an increase in $\tilde{k}$ has an arbitrarily small direct effect on $\Delta(q; \underline{\kappa}_p, \tilde{k})$, while the indirect effect is positive because $\Delta\left(q; \underline{\kappa}_p, \frac{1}{2}\right)$ equals $\delta(q; \underline{\kappa}_p)$, which is decreasing in $\underline{\kappa}_p$ and we already know that $\underline{\kappa}_p$ decreases as $\tilde{k}$ increases. As a result, there is now a smaller gain from misinformation about the candidate through indirect campaigning relative to direct campaigning when $\tilde{k}$ increases from $\frac{1}{2}$. Thus, as in the case of a perfect additional signal, it is less likely that type hl candidate uses a strong positive campaign to inform the voter of her good news about her own quality and bad news about the rival's quality.

IV. Competing Campaigns

In this section, we add to our basic model the element of competition. Namely, both candidate A and B simultaneously and independently choose a campaign which influence the voters' posterior beliefs about their quality. Recall from Section II that candidate A's signal is $(t_A^A, t_B^A) \in \{h, l\}$ while candidate B's signal is $(t_A^B, t_B^B) \in \{h, l\}$. For simplification, assume throughout this section that $q = r$, that is, each candidate's signal about herself and about her rival has the same accuracy. To separate the role of competition from that of information, we focus on the limiting case when the candidate and her rival's signals are perfectly correlated. This way, the difference in their campaign choices are driven by competition, rather than the presence of additional information available to the voter.

Formally, suppose that candidate A and B's signals about A's quality are distributed such that with probability u, they are perfectly correlated conditional on A's quality, with

$$\Pr(t_A^A = h \text{ and } t_A^B = h | A = 1) = \Pr(t_A^A = l \text{ and } t_A^B = l | A = 0) = q;$$

and

$$\Pr(t_A^A = l \text{ and } t_A^B = l | A = 1) = \Pr(t_A^A = h \text{ and } t_A^B = h | A = 0) = 1 - q.$$

With probability $1 - u$, candidate A and B's signals about A's quality are conditionally independent, with

$$\Pr(t_A^A = h | A = 1) = \Pr(t_A^A = l | A = 0) = \Pr(t_A^B = h | A = 1) = \Pr(t_A^B = l | A = 0) = q.$$

The candidates' signals about B's quality are symmetrically distributed. We focus on the case where u is equal to 1, and thus each candidate knows that her rival's signals are identical to her own. We construct the above model with u possibly less than 1 for technical reasons: in the ensuing equilibrium analysis, it is necessary to define out-of-equilibrium-path beliefs by considering the case of $u < 1$ and then taking the limit as u converges to 1. As in the case of single campaign, there are four private types for each candidate, lh, hh, ll and hl. By construction, regardless of the value of u, we have $\Pr(t_A^A = h | A = 1) = \Pr(A = 1 | t_A^A = h) = q$, as in the case of single campaign. Also, $\Pr(t_B^A = h | B = 1) = \Pr(B = 1 | t_B^A = h) = q$.

Finally, due to perfect correlation, the belief about a candidate's quality conditional on two h signals remains q. These features make the model of competing campaigns comparable to the single campaign model in terms of the information available to the two candidates.

A. *Horizontal separation*

For types lh, hh and ll, we posit equilibrium behavior similar to that in the previous section. Each candidate i runs no campaign if her private type is lh; runs a positive campaign of level $\underline{\tau}_p > \frac{1}{2}$ if the private type is hh; and runs a negative campaign of level $\underline{\tau}_n > \frac{1}{2}$ if the type is ll. Further, as in the case of single campaign, there is horizontal separation of types hh and ll, in the sense that each of the two types strictly prefers running their equilibrium kind of campaign to running the opposite kind of campaign of the other type. Unlike in the single campaign case, however, we will show that type hl randomizes between a strong positive campaign of level $\overline{\tau}_p > \underline{\tau}_p$ and a strong negative campaign of level $\overline{\tau}_n > \underline{\tau}_n$ with equal probabilities.

Consider first type lh of candidate A, who runs no campaign in equilibrium. Due to perfect correlation, her rival, candidate B, has received two identical private signals and is thus randomizing between a strong positive campaign and a strong negative campaign. Given the posited equilibrium strategy, after observing no campaign by candidate A and an informative campaign of level $\overline{\tau}_p$ or $\overline{\tau}_n$ by candidate B, the voter's interim belief about A's quality is equal to $1-q$, and the interim belief about B's quality is q. Since type lh's private belief about her own quality and that about B's quality are the same as the corresponding interim beliefs of the voter, the same logic behind part (i) of Lemma 1 implies that the equilibrium payoff to type lh is $1-q-q$, regardless of the campaign choice B ends up making through randomization.

What if type lh deviates to imitate type hh and runs a weak positive campaign of level $\underline{\tau}_p$? With perfect correlation of private signals across the candidates, the interim belief of the voter about candidate A's quality is undefined. The reason is that a weak positive campaign by candidate A suggests her signal is hh, which differs from candidate B's signal lh, as suggested by B's equilibrium campaign choice. However, as long as $u < 1$, the belief about candidate A's quality conditional on one h signal from candidate A and one l signal from candidate B is given by

$$\Pr(A=1|t_A^A=h,t_A^B=l)=\frac{\frac{1}{2}(1-u)q(1-q)}{\frac{1}{2}(1-u)q(1-q)+\frac{1}{2}(1-u)q(1-q)}=\frac{1}{2}$$

regardless of the value of u. Due to the above observation, we specify that the out-of-equilibrium-path interim belief of the voter in this case is that candidate A's quality is 1 with probability $\frac{1}{2}$. Then, type lh of candidate A is indifferent between running no campaign and running a weak positive campaign of level $\underline{\tau}_p$ if

$$1-q-q \quad = \quad \frac{1}{2}\left(\pi\left(1-q,\frac{1}{2};\underline{\tau}_p\right)+\Pi\left(1-q,\frac{1}{2};\underline{\tau}_p,\overline{\tau}_p\right)\right)-q-f(\underline{\tau}_p), \tag{3}$$

where as defined before, $\pi\left(1-q,\frac{1}{2};\underline{\tau}_p\right)$ is candidate A's expectation of the voter's posterior belief that her quality is 1 when her own private belief is $1-q$, the voter's interim belief is $\frac{1}{2}$ and the level of her campaign is $\underline{\tau}_p$; and $\Pi\left(1-q,\frac{1}{2};\underline{\tau}_p,\overline{\tau}_p\right)$ is candidate A's expectation of the voter's posterior belief that her quality is 1 when her own private belief is $1-q$, the voter's interim belief is $\frac{1}{2}$, and the qualities of two public signals are $\underline{\tau}_p$ and $\overline{\tau}_p$, given by:

$$\begin{aligned}
\Pi\left(1-q,\frac{1}{2};\underline{\tau}_p,\overline{\tau}_p\right) \quad = \quad & ((1-q)\underline{\tau}_p\overline{\tau}_p+q(1-\underline{\tau}_p)(1-\overline{\tau}_p))\frac{\underline{\tau}_p\overline{\tau}_p}{\underline{\tau}_p\overline{\tau}_p+(1-\underline{\tau}_p)(1-\overline{\tau}_p)}\\
+ \quad & ((1-q)\underline{\tau}_p(1-\overline{\tau}_p)+q(1-\underline{\tau}_p)\overline{\tau}_p)\frac{\underline{\tau}_p(1-\overline{\tau}_p)}{\underline{\tau}_p(1-\overline{\tau}_p)+(1-\underline{\tau}_p)\overline{\tau}_p}\\
+ \quad & ((1-q)(1-\underline{\tau}_p)\overline{\tau}_p+q\underline{\tau}_p(1-\overline{\tau}_p))\frac{(1-\underline{\tau}_p)\overline{\tau}_p}{(1-\underline{\tau}_p)\overline{\tau}_p+\underline{\tau}_p(1-\overline{\tau}_p)}\\
+ \quad & ((1-q)(1-\underline{\tau}_p)(1-\overline{\tau}_p)+q\underline{\tau}_p\overline{\tau}_p)\frac{(1-\underline{\tau}_p)(1-\overline{\tau}_p)}{(1-\underline{\tau}_p)(1-\overline{\tau}_p)+\underline{\tau}_p\overline{\tau}_p}.
\end{aligned}$$

Note that $\underline{\tau}_p$ and $\overline{\tau}_p$ enter the function $\Pi\left(1-q,\frac{1}{2};\underline{\tau}_p,\overline{\tau}_p\right)$ symmetrically.

Equation (3) defines $\underline{\tau}_p$, the least costly level of campaign that separates type lh from type hh. A symmetric expression defines $\underline{\tau}_n$, which is equal to $\underline{\tau}_p$ in this model. By symmetry, we drop the subscripts and simply write $\underline{\tau}$. To study the properties of the function $\Pi\left(1-q,\frac{1}{2};\underline{\tau},\overline{\tau}\right)$, we rewrite it as

$$\Pi\left(1-q,\frac{1}{2};\underline{\tau},\overline{\tau}\right)=\frac{1}{2}-\left(\frac{1}{2}-(1-q)\right)\left(1-\frac{4\underline{\tau}(1-\underline{\tau})\overline{\tau}(1-\overline{\tau})}{(\underline{\tau}\overline{\tau}+(1-\underline{\tau})(1-\overline{\tau}))(\underline{\tau}(1-\overline{\tau})+(1-\underline{\tau})\overline{\tau})}\right).$$

The above expression immediately implies the following lemma, which identifies two key properties of the function $\Pi\left(1-q,\frac{1}{2};\underline{\tau},\overline{\tau}\right)$.

LEMMA 3: *(i)* $\Pi\left(1-q,\frac{1}{2};\underline{\tau},\overline{\tau}\right)<\frac{1}{2}$*; and (ii)* $\Pi\left(1-q,\frac{1}{2};\underline{\tau},\overline{\tau}\right)$ *decreases with both* $\underline{\tau}$ *and* $\overline{\tau}$*.*

To see that there is horizontal separation in this model of competing campaigns, we first show that type hh of candidate A strictly prefers running a positive campaign of level $\underline{\tau}$ to running no campaign. Due to perfect correlation, candidate B has identical private signals and in equilibrium runs a positive campaign of level $\underline{\tau}$. By the same logic as in part (i) of Lemma 1, the equilibrium payoff for candidate A from running a positive campaign of level $\underline{\tau}$ is $q-q-f(\underline{\tau})$. If type hh deviates and runs no campaign, imitating the equilibrium campaign choice of type lh, then the candidates send opposing signals about A's quality. Following the same logic as above, we specify this out-of-equilibrium-path belief about A's quality as $\frac{1}{2}$. Then, type hh's deviation payoff is $\frac{1}{2}-q$. From the definition of $\underline{\tau}$, type hh strictly prefers running a positive campaign of level $\underline{\tau}$ to running no campaign if and only if the payoff loss for type hh from switching to no campaign is greater than the payoff gain for type lh from misinformation, or

$$q-\frac{1}{2}>\frac{1}{2}\left(\pi\left(1-q,\frac{1}{2};\underline{\tau}\right)+\Pi\left(1-q,\frac{1}{2};\underline{\tau},\overline{\tau}\right)\right)-(1-q).$$

By part (ii) of Lemma 1 and part (i) of Lemma 3, both $\pi\left(1-q,\frac{1}{2};\underline{\tau}\right)$ and $\Pi\left(1-q,\frac{1}{2};\underline{\tau},\overline{\tau}\right)$ are smaller than $\frac{1}{2}$, the maximum posterior belief they can induce in the voter if they run uninformative campaigns. The above inequality always holds and thus there is strict separation between type lh and type hh. A symmetric argument establishes that there is strict separation between type lh and type ll.

To consider horizontal separation, suppose that type hh switches to running a negative campaign of level $\underline{\tau}$. Due to perfect correlation, her rival's private signals are identical and in equilibrium is running a positive campaign of level $\underline{\tau}$. The interim beliefs of the voter about the qualities of the two candidates are undefined, but using the same logic as before, by allowing u to be less than 1 and then taking the limit as u goes to 1, we can specify both beliefs as $\frac{1}{2}$. The deviation payoff to type hh is then

$$\frac{1}{2}-\Pi\left(q,\frac{1}{2};\underline{\tau},\underline{\tau}\right)-f(\underline{\tau}),$$

as candidate A's private belief of B's quality is q. It is straightforward to verify that

$$\Pi\left(q,\frac{1}{2};\underline{\tau},\underline{\tau}\right)=1-\Pi\left(1-q,\frac{1}{2};\underline{\tau},\underline{\tau}\right),$$

and thus from part (i) of Lemma 3 that type hh's deviation payoff is strictly lower than her equilibrium payoff of $q-q-f(\underline{\tau})$.

B. *Vertical separation*

In contrast with the case of single campaign, in competing campaigns, there is vertical separation between type hl and types hh and ll. That is, type hl strictly prefers either the strong positive campaign of level $\overline{\tau}$ or the strong negative campaign of level $\overline{\tau}$ to either imitating type hh by running a weak positive campaign of level $\underline{\tau}$ or imitating type ll by running a weak negative campaign of level $\underline{\tau}$.

Consider first type hh of candidate A. Given the posited strategies, her equilibrium payoff from running a weak positive campaign of level $\underline{\tau}$ is $q - q - f(\underline{\tau})$. Suppose that type hh deviates to running a strong positive campaign of level $\overline{\tau}$. Due to perfect correlation her rival has identical private signals and runs a weak positive campaign of level $\underline{\tau}$. Similar to what we have argued in the previous subsection, upon observing the deviating campaign choice by candidate A and the equilibrium choice by candidate B, the voter's interim beliefs are such that candidate A's quality is 1 with probability q while candidate B's quality is 1 with probability $\frac{1}{2}$. For type hh to be indifferent between her equilibrium choice of campaign and this deviation, we must have

$$q - q - f(\underline{\tau}) \quad = \quad q - \pi\left(q, \frac{1}{2}; \overline{\tau}\right) - f(\overline{\tau}), \tag{4}$$

where in writing the deviation payoff on the right-hand-side we use the fact that running a strong positive campaign of level $\overline{\tau}$ does not change type hh's expectation of the voter's posterior belief about her own quality, by part (i) of Lemma 1; and the fact that type hh's private belief about her rival's quality is q. Equation (3) and (4) respectively define $\underline{\tau}$ and $\overline{\tau}$.

Now consider type hl of candidate A. Due to perfect correlation, her rival runs no campaign in equilibrium. Type hl's equilibrium payoff from running a strong positive campaign of level $\overline{\tau}$ is then given by $q - (1 - q) - f(\overline{\tau})$. Suppose that type hl deviates and runs a weak positive campaign of level $\underline{\tau}$. Then given the weak positive campaign by candidate A and no campaign by candidate B, the voter's interim beliefs are such that candidate A's quality is 1 with probability q while candidate B's quality is 1 with probability $\frac{1}{2}$. Type hl's deviation payoff is then

$$q - \frac{1}{2} - f(\underline{\tau}).$$

There is strict separation between type hl and type hh if the payoff loss for type hl from switching to a weak positive campaign is greater than the payoff gain for type hh from misinformation, or

$$\frac{1}{2} - (1-q) > q - \pi\left(q, \frac{1}{2}; \overline{\tau}\right).$$

This inequality holds because $\pi\left(q, \frac{1}{2}; \overline{\tau}\right) > \frac{1}{2}$: A's private belief about B's quality is higher than the voter's interim belief, and thus the minimum posterior belief of the voter is $\frac{1}{2}$. A symmetric argument establishes that type hl strictly prefers running a strong negative campaign of level $\overline{\tau}$ to running a weak negative campaign of level $\underline{\tau}$, so that there is vertical separation between type hl and type ll as well.

Intuitively, there is separation between type hl and types hh and ll because competition now affects the effectiveness of indirect campaigning, in contrast to the case of single campaign. In the competing campaigns model, because of signal correlation, type hh has less to gain from the indirect campaigning of misinforming the voter that her rival has low quality, because it is undermined by the positive campaign run by her rival. In comparison, type hl has more to lose from not running the more informative positive campaign because in equilibrium her rival does not run an informative campaign to counter her campaign. We stress that this vertical separating result is not driven by the presence of additional information to the voter compared to the single campaign model. In fact, in the single campaign case, even if the voter has an additional public signal about the qualities of the candidates, the change in the effectiveness of indirect campaigning is the same for type hh and type hl, and thus the two types remain unseparated.

Due to the absence of vertical separation, in the single campaign model with symmetry, i.e., $q = r$, the equilibrium campaign choice of type hl is indeterminate in the sense that type hl may randomize arbitrarily between the strong positive campaign of level $\overline{\kappa}$ and the strong negative campaign of the same level. This is no longer the case in the competing campaigns model. To see this, consider the two deviations that type lh may choose, a weak positive campaign of level $\underline{\tau}$ and the weak negative campaign of the same level. Suppose that candidate B chooses a strong positive campaign with probability $1 - \eta \in [0, 1]$, and choose a strong negative campaign with probability η. Then, by deviating to the weak positive campaign of level $\underline{\tau}$, type lh of candidate A obtains an expected payoff of

$$(1-\eta)\pi\left(1-q, \frac{1}{2}; \underline{\tau}\right) + \eta\Pi\left(1-q, \frac{1}{2}; \underline{\tau}, \overline{\tau}\right) - q - f(\underline{\tau}),$$

while her deviation payoff from running the weak negative campaign of level $\underline{\tau}$ is

$$(1-q) - \left((1-\eta)\Pi\left(q, \frac{1}{2}; \underline{\tau}, \overline{\tau}\right) + \eta\pi\left(q, \frac{1}{2}; \underline{\tau}\right)\right) - f(\underline{\tau}).$$

Thus, type lh prefers the positive campaign of level $\underline{\tau}$ to the negative campaign of the same level if and only if

$$(1-2\eta)\left(\pi\left(1-q,\frac{1}{2};\underline{\tau}\right)-\Pi\left(1-q,\frac{1}{2};\underline{\tau},\overline{\tau}\right)\right)>0.$$

It can be directly verified that

$$\pi\left(1-q,\frac{1}{2};\underline{\tau}\right)=\Pi\left(1-q,\frac{1}{2};\underline{\tau},\frac{1}{2}\right).$$

Since $\Pi\left(1-q,\frac{1}{2};\underline{\tau},\overline{\tau}\right)$ is decreasing in $\overline{\tau}$ by part (ii) of Lemma 3, type lh can be indifferent between the positive campaign of level $\underline{\tau}$ and the negative campaign of the same level only when $\eta=\frac{1}{2}$. It follows that in any separating equilibrium type hl must be run the positive campaign of level $\overline{\tau}$ and the negative campaign of the same level with equal probabilities.

The reason that we are able to pin down the equilibrium mixing of the highest type hl in the present model is that, with competing campaigns, type hl uses indirect campaigning to inform the voter that she has both bad news for her rival's quality in addition to good news for her own quality, influencing the effectiveness of the potential misinformation campaigns that the lowest type lh may run. In particular, suppose that $\eta<\frac{1}{2}$, so that type hl is more like to run the strong positive campaign of level $\overline{\tau}$ than the strong negative campaign of the same level. Then for candidate B, who is the lowest type, the weak positive campaign becomes less effective because it is more likely to be contradicted by the strong positive campaign of A. Instead, candidate B is more attempted to deviate to a weak negative campaign. Such a link is absent in the single campaign model.

C. *Campaign levels in competition*

We now combine the analysis in the previous two subsections and describe the symmetric, least costly separating equilibrium of the competing campaigns model. To specify out-of-equilibrium-path beliefs, note that only the beliefs after unilateral deviations are relevant. One set of such beliefs is: the deviating candidate is type lh if she runs a positive campaign of level $\tau_p\in\left(\frac{1}{2},\underline{\tau}\right)$ or a negative campaign of level $\tau_n\in\left(\frac{1}{2},\underline{\tau}\right)$; the deviating candidate is type hh if she runs a positive campaign of level $\tau_p\in(\underline{\tau},\overline{\tau})$; and is type ll if she runs a negative campaign of level $\tau_n\in(\underline{\tau},\overline{\tau})$; and if the candidate runs a positive campaign of level $\tau_p>\overline{\tau}$ or a negative campaign of level $\tau_n>\overline{\tau}$, she is believed to be of type hl. We are able to specify the above beliefs as in the case of single campaign, because the voter can infer who is the deviating candidate, and his beliefs about the signals of the non-deviating candidate are the equilibrium beliefs. This completes the specification of the out-of-equilibrium beliefs, as we have already described the voter's beliefs when he cannot infer which of the two candidates has deviated.

It can be straightforwardly verified that the above set of beliefs supports the least costly separating equilibrium as described in the previous two subsections. First, type lh has no incentive to deviate to any positive or negative campaign of level $\tau \in \left(\frac{1}{2}, \underline{\tau}\right)$. This is because, due to perfect correlation the rival candidate in equilibrium runs either a strong positive campaign or a strong negative campaign of level $\overline{\tau}$, so for example, the deviation payoff from a positive campaign is

$$\frac{1}{2}\left(1 - q + \Pi(1-q, 1-q; \tau, \overline{\tau})\right) - q - f(\tau),$$

which is equal to $1 - q - q - f(\tau)$ and is strictly less than its equilibrium payoff. Second, type hh has no incentive to deviate to a positive campaign of any level $\tau_p < \underline{\tau}$, and symmetrically type ll has no incentive to deviate to a negative campaign of any level $\tau_n < \underline{\tau}$. Given the above out-of-equilibrium-path beliefs, the argument for this claim is identical to the corresponding case under single campaigns, because due to the perfect correlation the rival candidate runs a weak positive campaign of level $\underline{\tau}$, and so the expected posterior belief of the voter about the rival is unaffected by the deviation. Third, type hh has no incentive to deviate to a positive campaign of any level $\tau_p \in (\underline{\tau}, \overline{\tau})$, and a symmetric claim for type ll holds. This is simply because, given the above out-of-equilibrium-path beliefs, type hh cannot improve the expected posterior belief of the voter about her own quality with such a deviation. Fourth, type hl has no incentive to deviate to any campaign of level $\tau \in (\underline{\tau}, \overline{\tau})$. The reason is that the rival candidate does not run an informative campaign, so for example, the deviation payoff from a positive campaign is simply $q - q - f(\tau)$, which is strictly less than its equilibrium payoff of $q - (1 - q) - f(\overline{\tau})$. Thus we have:

PROPOSITION 2: *Suppose that $r = q$ and $u = 1$. In the symmetric, least costly separating equilibrium of the competing campaigns model, type lh candidate runs no campaign. Type hh runs an informative positive campaign of level $\underline{\tau} > \frac{1}{2}$ while type ll runs an informative negative campaign of the same level $\underline{\tau}$. Type hl randomizes between a positive campaign of level $\overline{\tau} > \underline{\tau}$ and a negative campaign of level $\overline{\tau} > \underline{\tau}$ with equal probabilities.*

We now compare the equilibrium separating levels of campaigns $\underline{\tau}$ and $\overline{\tau}$ to the corresponding levels of campaigns $\underline{\kappa}$ and $\overline{\kappa}$ in the symmetric single campaign model. We want to know if competition raises and lowers the intensity of campaigns. By equation (3), we have

$$f(\underline{\tau}) = \frac{1}{2}\left(\pi\left(1-q,\frac{1}{2};\underline{\tau}\right) + \Pi\left(1-q,\frac{1}{2};\underline{\tau},\overline{\tau}\right)\right) - (1-q)$$

in the competing campaigns case, while

$$f(\underline{\kappa}) = \pi(1-q,q;\underline{\kappa}) - (1-q)$$

in the single campaign case. We already know that

$$\Pi\left(1-q,\frac{1}{2};\underline{\tau},\overline{\tau}\right) < \pi\left(1-q,\frac{1}{2};\underline{\tau}\right).$$

It is straightforward to verify that

$$\pi\left(1-q,\frac{1}{2};\underline{\tau}\right) < \pi(1-q,q;\underline{\tau})$$

for any $\underline{\tau}$ since $q > \frac{1}{2}$, and thus

$$\Pi\left(1-q,\frac{1}{2};\underline{\tau},\overline{\tau}\right) < \pi\left(1-q,\frac{1}{2};\underline{\tau}\right) < \pi(1-q,q;\underline{\tau}).$$

Since the function $\pi(1-q,q;\kappa)$ is decreasing in κ (part (ii) of Lemma 1), we must have $\underline{\tau} < \underline{\kappa}$.

Thus, competition reduces the least costly level of campaigns that separate type hh and type ll from the lowest type lh. There are two separate reasons for this comparison result. First, because of competition, type lh candidate has a smaller incentive to deviate because, regardless of whether she chooses the weak positive campaign or the weak negative campaign, with probability $\frac{1}{2}$, it will be directly countered by her rival's campaign. Second, the voter is less likely to be misinformed even without being countered by the rival, because the gain for type lh, in terms of changing the voter's interim belief, is $q-\frac{1}{2}$ in the competing campaign case, as opposed to $q-(1-q)$ in the single campaign case.

Finally, we compare $\overline{\tau}$ in the competing campaigns case to $\overline{\kappa}$ in the single campaign case. By equation (4), we have

$$f(\overline{\tau}) = f(\underline{\tau}) + q + \pi\left(1-q,\frac{1}{2};\overline{\tau}\right) - 1$$

in the competing campaigns model, while

$$f(\overline{\kappa}) = f(\underline{\kappa}) + 2q - 1$$

in the single campaign model. Since

$$\pi\left(1-q,\frac{1}{2};\overline{\tau}\right) < \pi\left(1-q,q;\overline{\tau}\right) < q,$$

where the second inequality follows from part (i) of Lemma 2, and since we already know that $\underline{\tau} < \underline{\kappa}$, we find that $\overline{\tau} < \overline{\kappa}$. This shows that the level of campaigns that separate the highest type hl from the middle types hh and ll is also reduced by competition. Again there are two separate reasons for this result. First, for any fixed campaign level run by types hh and ll, there is now less incentive for the two types to deviate and engage in indirect campaigning to misinform the voter. For example, for type hh, the voter's interim belief is raised from $1-q$ to just $\frac{1}{2}$, as opposed to q in the single campaign case. Second, competition has already reduced the equilibrium campaign level for the middle types ($\underline{\tau} < \underline{\kappa}$).

We summarize the above comparison results in the following proposition.

PROPOSITION 3: *The levels of campaign informativeness in the competing campaign case are lower than the corresponding levels in the single campaign case:* $\underline{\tau} < \underline{\kappa}$ *and* $\overline{\tau} < \overline{\kappa}$*.*

V. Discussion and conclusion

In this model, both the type and the level of a campaign reveal information about a candidate's private information about herself and her rival. Therefore she may have an incentive to misinform the voter by changing the information structure of the campaign signal the voter receives. A crucial element of the model is that the candidate does not control the realized campaign signal by making the campaign choices. This idea has already appeared in an industrial organization context, for example in monopoly pricing by Ottaviani and Moscarini (2001) and price competition by Damiano and Li (2007). However, in their papers consumers acquire private signals as a result of information choices made by the sellers, while in the present paper, the candidate uses campaign choices to signal her private information. How to generalize this to richer type and signal space is a topic for further research.

References

Cho, In-Koo and David Kreps, "Signaling Games and Stable Equilibria," *Quarterly Journal of Economics*, May 1987, *102* (2), 179–222.

Coate, Stephen, "Pareto-Improving Campaign Finance Policy," *American Economic Review*, June 2004, *94* (3), 628–655.

Damiano, Ettore and Hao Li, "Information Provision and Price Competition," February 2007. University of Toronto, mimeo.

Kotowitz, Yehuda and Frank Mathewson, "Advertising, Consumer Information, and Product Quality," *Bell Journal of Economics*, Autumn 1979, *10* (2), 566–588.

Milgrom, Paul and John Roberts, "Price and Advertising Signals of Product Quality," *Journal of Political Economy*, August 1986, *94* (4), 796–821.

Nelson, Phillip, "Advertising as Information," *Journal of Political Economy*, July-August 1974, *82* (4), 729–754.

Ottaviani, Marco and Giuseppe Moscarini, "Price Competition for an Informed Buyer," *Journal of Economic Theory*, 2001, *101* (2), 457–493.

Polborn, Mattias K. and David T. Yi, "Informative Positive and Negative Campaigning," *Quarterly Journal of Political Science*, 2006, *1* (4), 351–371.

Prat, Andrea, "Campaign Advertising and Voter Welfare," *Review of Economic Studies*, October 2002, *69* (4), 999–1017.